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Optical Identifications of Radiosources from the B 2 Catalogue. Quasi Stellar Sources

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Summary. This paper presents a homogeneous sample of 70 blue stellar objects, which exhibit the typical ultraviolet excess of Quasars.
These have been found during the programme of optical identification of radiosources from the B 2 catalogue, using *UBV* plates of the 48″ Schmidt telescope of Mount Palomar. Some of these objects have been observed with the Westerbork instrument, to improve the radiopositions and determine the distribution of the spectral indices between 408 and 1415 MHz. Comparing this distribution with the corresponding ones of a sample of galaxies and "empty fields", we estimate that this last category of sources does not contain more than 20% of quasars. Finally we examine the counts as function of the flux of our quasars, on the basis of the M. Schmidt model on the existence of an universal function giving the distribution of the ratios between the radio and the optical flux.

Key words: optical identifications of radiosources – quasar counts

Introduction

In this paper we present data concerning a homogeneous group of 70 radiosources which are identified with quasi stellar objects and have the ultraviolet excess typical of Quasars.
These data are part of a general programme of optical identifications of radiosources in the Bologna B 2 catalogue (Colla *et al.*, 1969), which is essentially complete for flux densities greater than 0.2 f.u. at 408 MHz. Fields for all 1200 radiosources in the region $7^h \leqq$ R.A. $\leqq 18^h30^m$ and $29°20' \leqq \delta \leqq 33°00'$ (0.162 steradians) have now been examined for optical identifications. Although this paper is concerned primarily with the quasi stellar objects, some preliminary results for galaxies and empty fields will also be presented, as these results have some bearing on our conclusions concerning quasi stellar objects.

Summary of the B 2 Identification Programme

The optical identifications have been made using the Palomar Sky Survey prints. The detailed procedure which was used to locate the positions of the radiosources accurately on the prints (to about 2″) has been described by Grueff and Vigotti (1971). For each radiosource photographic reproductions were made of both the red and the blue prints, with an enlargement of a factor 10. Great care was taken to ensure uniformity for all the photographic reproductions. Around the position of every radiosource a rectangular area, whose sides are ± twice the r.m.s. errors in the source coordinate, have been examined[1]. The dimensions of such areas, as a function of the flux density, are reported in the B 2 catalogue. For example, for a radiosource of 0.4 f.u. this area is 36″ × 136″. The adopted search area should have about 91% probability of containing the "true" position of the radiosource. A larger area, with sides equal to ± 3 standard errors, even though it has a probability almost equal to 1.00 of containing the source, has the disadvantage of increasing by a factor of 25 the number of spurious identifications.
The following classification is used:

- G the search area contains one or more galaxies.
- B the search area contains a blue stellar object.
- NI the search area contains optical objects which cannot be classified as G or B.
- E nothing is visible in the search area.

The distribution of the 1200 radiosources into these classes, according to objects in the search area, is presented in Table 1. There are 27 radiosources in whose search area there are both galaxies and blue objects and these have been included in both classes.

[1] The original radiopositions of the B 2 catalogue have been revised. Therefore differences in R.A. and declination, up to 4″ and 12″ respectively, from the published positions are possible.

Table 1. The distribution in optical classes of the 1200 B2 radiosources

Optical class	Number of radiosources
B	307
G	299
E	177
NI	444

We point out that the numbers in Table 1 for G and B are overestimates of the real numbers of identifications with galaxies and blue objects, due to the occasional presence in the search area of objects which are not associated with the radiosource. On the contrary, the number of E sources will certainly be an underestimate of the true number of sources which do not have an optical counterpart on the Palomar Sky Survey prints. We now estimate the true number of radiosources in our sample whose optical counterparts are invisible on the Palomar Sky Survey. This is done by comparing the counts of stars and galaxies in randomly selected fields with those in the search areas for the radiosources. It is difficult to distinguish between the different types of optical objects near the plate limit and we will restrict our attention to the absence or presence of optical objects on the Sky Survey prints, regardless of whether they are galaxies or stars, red or blue.

We introduce the term "invisible source" or "I source" for those radiosources whose optical counterpart, if any, is beyond the limit of the Palomar Sky Survey. It is emphasized that this classification is on the basis of a physical association between radio and optical objects, rather than simply a positional agreement as used for the G, B, NI and E classification. We expect that the "I sources" will comprise all the E sources, most of the NI sources and some of the G and B sources. The number of "I objects" was derived from the number of the E sources in the following way. 62 areas, each of 24 square minutes of arc, were selected at random positions on the Sky Survey prints at declination 30° and between right ascensions 7^h and 18^h30^m. The total number of optical objects, regardless of type and magnitude, were counted in each (see Fig. 1). If ϱ is the density of optical objects derived from such counts, the correction factor C, by which we have to multiply the apparent number of E sources with a certain flux density to obtain the true number of I sources, is

$C = e^{\varrho A}$,

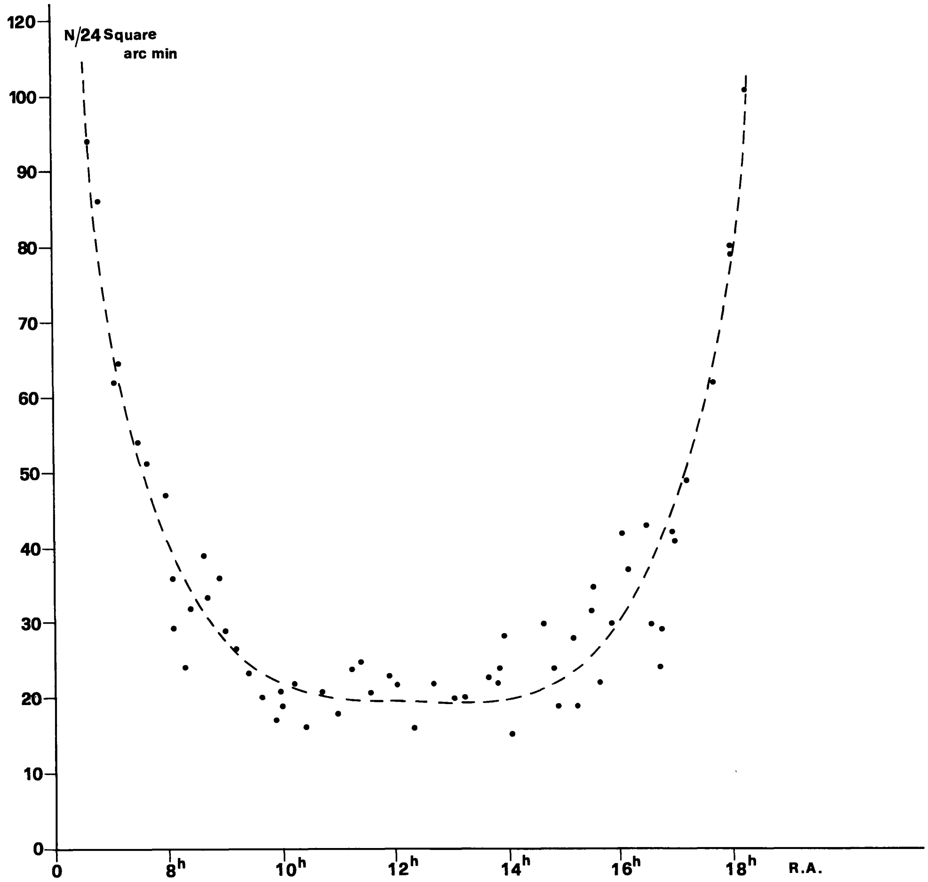

Fig. 1. Density of stars of any magnitude, in area of 24 square arc min, as function of R.A., at 30°

Table 2. Distribution of E and I sources as a function of the flux density

Flux at 408 MHz (f.u.)	Number of E sources $10^h < \text{R.A.} < 15^h$	Correcting factor C	Number of I sources in 0.162 steradian
0.20–0.25	6	19.2	267 ± 110
0.25–0.40	26	3.9	234 ± 46
0.40–0.70	26	3.04	182 ± 36
0.70–1.00	18	1.57	65 ± 15
$\geqq 1.00$	20	1.40	64 ± 14

A being the search area for sources with this given flux density. In Table 2 are reported the numbers of E sources, in the region of $10^h \leqq \text{R.A.} \leqq 15^h$, for various ranges of flux density, the correction factor C and the estimated number of I sources in all the 1200 radiosources. In deriving this result we used only the E sources between 10^h and 15^h because in this interval the star density is minimum and almost constant, giving a reasonably reliable value of C. The estimated total number of I sources for the area of 0.162 steradian is

$$812 \pm 120\,.$$

Most of the uncertainty arises from the weakest radiosources, because of the high value of C and the small number of E sources at such flux densities.
In one of the following paragraphs we will present an independent estimate of the number of I sources, which is in reasonably good agreement with the present estimate.

Blue Stellar Objects

0.076 steradians of the region of sky covered by the present optical identification programme are also included on plates in u, b, v colours[2]), 6.5° by 6.5°, taken with the 48″ Schmidt telescope at Mount Palomar. The plate centers for every field are given in Table 3.
We examined all B fields on these plates to see which ones possess the ultraviolet excess characteristic of quasars. We classified every blue object as "UV" or "not UV" according to whether the dimensions of the image on the ultraviolet plate were respectively bigger or smaller than on the blue plate.
The exposures of the plates were such that the objects classified "UV" should have $U-B \leqq -0.4$ and therefore lie in the area of the $U-B$, $B-V$ diagram which is mainly populated by quasars. In the total of 154 B radiosources in the plates areas, 68 were classified "UV" and 75 "not UV". The remaining 11 were not visible on the plates, mainly because our recent plates were not as sensitive as the original Sky Survey, but also because of optical variations in some objects.

[2]) For the photographic system see Braccesi *et al.* (1969).

Table 3. Coordinates of the centers of the fields covered by plates in u, b, v of the 48″ Schmidt telescope of Mount Palomar

R.A. (1950)	δ(1950)
08 21 05	29 40 20
08 47 20	29 34 20
09 12 50	29 28 30
09 38 30	29 27 00
10 04 40	29 26 44
12 13 33	29 16 00
12 39 53	29 21 10
12 58 48	30 57 06
12 59 37	35 07 49
13 31 12	29 26 35
13 57 01	29 29 15
14 22 52	29 36 40
14 48 43	29 32 00
16 09 42	32 05 06

Table 4. Distribution of the B sources "UV" and "not UV", as a function of flux density

Flux at 408 MHz f.u.	Number of "not UV" sources	Number of "UV" sources
0.20–0.25	22	14
0.25–0.40	28	17
0.40–0.70	24	24
0.70–1.00	1	4
$\geqq 1.00$	0	9

We also examined the U and B plates in the positions of all the other sources which have not been classified as B. With this further search we have found two other stellar objects which were undoubtly "UV" but not blue. These objects have been added to the other 68 "UV", producing a sample of 70 objects with $U-B \leqq -0.4$, down to $m_b = 21.0$[3]).
The 75 blue "not UV" objects are presumably hot stars, present, by chance, in the search area of the radiosources. In Table 4 are reported the distributions of the B sources, "UV" and "not UV", as a function of flux density. We can determine from these data the density of blue "not UV" objects, which is

$$0.071 \pm 0.010 \text{ per square arcmin. for } m_b < 210$$

and also how they are distributed in magnitude.
A differential number count, $N(m)$, given by

$$\log N(m) = 0.3\,\text{m} + \text{const.}$$

[3]) The scale used in our estimate of magnitudes has been derived from SA 57 (Stebbins *et al.*, 1950; Baum, 1963). In order to ensure uniformity with the B photoelectric magnitudes we have made an estimate of magnitudes for the 3 CR quasars.
The r.m.s. differences between our estimates and the photoelectric magnitudes is about 0.4 magnitudes, the zero point shift being negligible.
This value includes the errors in the estimates, those due to photographic copying, the intrinsic inhomogeneity of the P.S.S. plates and some variability of the objects.

fits the data fairly well. These results may be useful for estimating the contamination due to blue "not UV" objects at other magnitudes.

The list of the 70 "UV" objects is given in Table 5.

Radiosources Measured at Westerbork

About 200 B 2 radiosources have been measured at Westerbork to obtain more accurate positions, and also the flux densities at 1415 MHz. These radiosources are not a random sample, but have been selected for various purposes from preliminary identifications obtained using the B 2 catalogue positions. They are all in the declination interval from 31°00′ to 32°40′. Included are 55 B objects, both "UV" and "not UV", which is approximately 80% of the B sources in this region. Although we can expect most of the B "not UV" identifications to be spurious because the colours are not the characteristic of quasars, we decided to measure the B "not UV" sources to check this assumption, including the possibility of quasars with unusual colours.

The observations at Westerbork consisted of observations of 10 m each, taken at 4 (sometime 3 or 5) well separated hour angles. From each group of measurements a field of 42.5′ by 42.5′ has been synthesized. The resolution is 20″ by 40″. The accuracy of the measures, both in position and flux density, is limited essentially by the confusion caused by grating responses from other radiosources. Due to our observing pro-

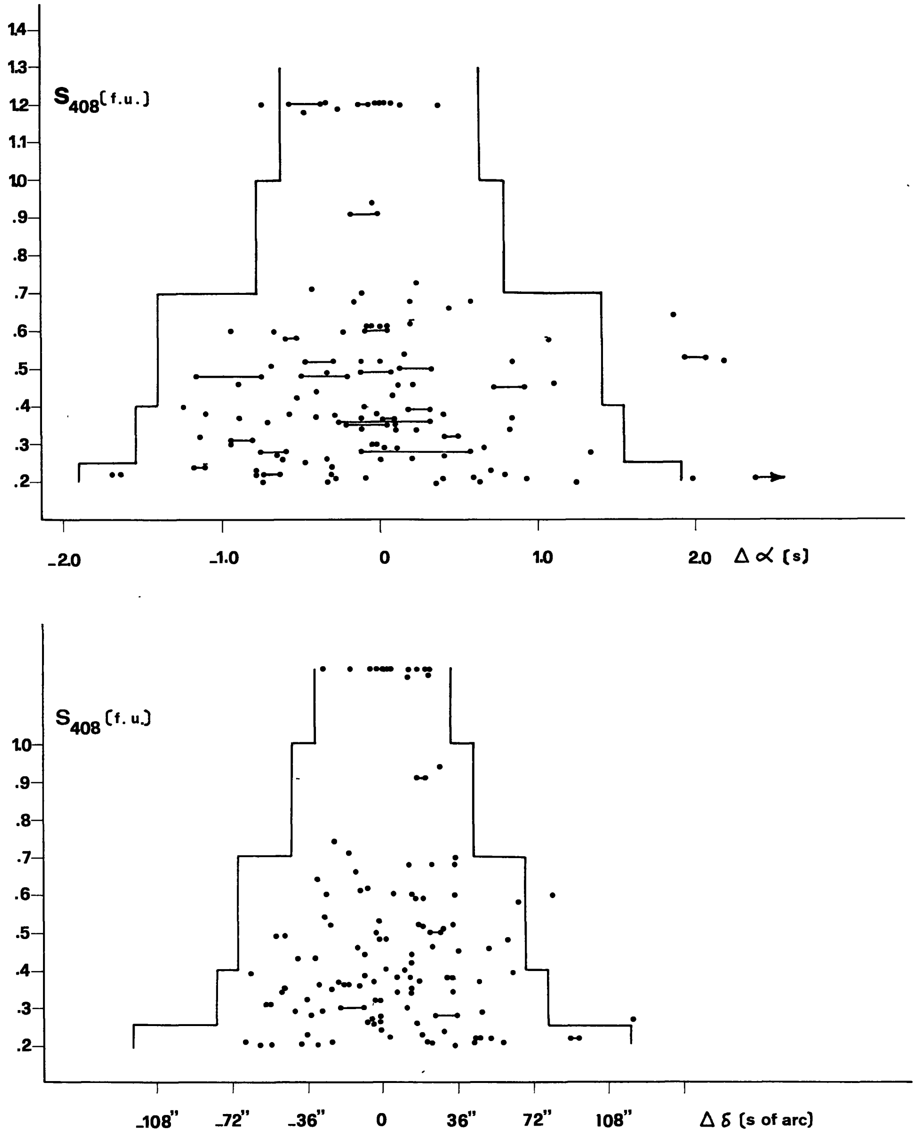

Fig. 2. The distribution of the differences in right ascension and declination between the B 2 positions and the Westerbork positions, as a function of the B 2 flux. Points connected with a bar refer to sources measured twice at Westerbork

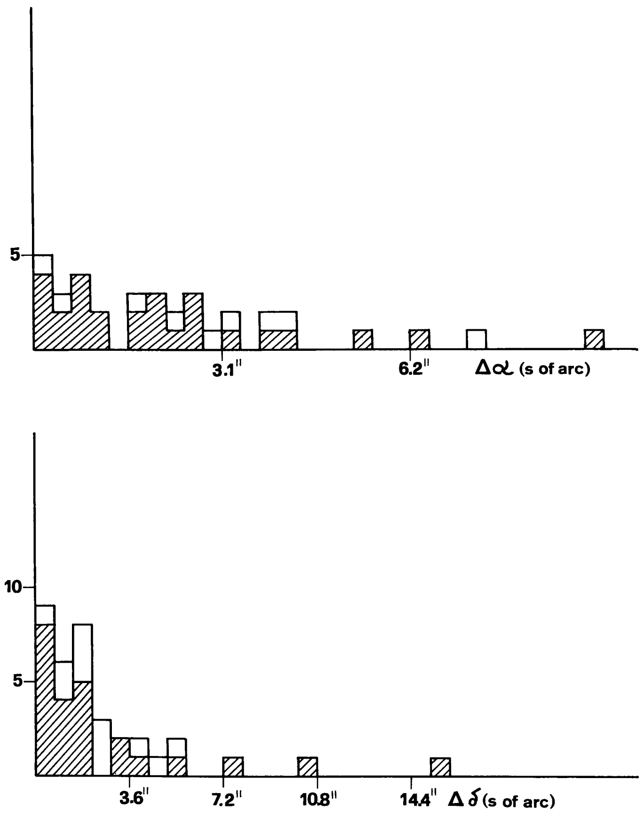

Fig. 3. The distribution of the absolute values of differences in R.A. and declination for the sources measured twice at Westerbork. Dashed squares are radiosources stronger than 0.1 f.u. at 1415 MHz

cedure, which is far from producing a full synthesis, these are particularly strong. Noise and phase stability do not affect the measurements appreciably. The flux densities are obtained by a best-fit of the source profile with the antenna pattern and are therefore not corrected for partial resolution effects. We now have results for 132 of the radiosources observed at Westerbork. They comprise all 55 B sources in programme, 39 NI, 7 E sources and 31 G.

Figure 2 shows the distribution, as a function of the flux density at 408 MHz, of the differences between the right ascensions and the declinations of the B 2 catalogue and those obtained at Westerbork. Since the latter have much smaller errors than the former, these distributions represent, in fact, the error distributions of the Bologna positions. The histograms drawn on the figures represent the assumed sides of the search area, i.e. twice the r.m.s. errors quoted in the B 2 catalogue.

Figure 3 shows the distributions of the absolute values of the differences in right ascension and declination, for 36 radiosources which have two independent sets of Westerbork measures. Generally these are sources belonging to two different fields or observed with different sets of hour angles. Most of the differences (about 70%) are less than 3″, but there are some as large as 10″, in R.A. and 15″ in declination. These large differences are generally produced by confusion arising from grating responses of other sources and give an indication of the maximum errors we can have.

The previous identifications for all the 55 B sources have been checked by comparing the optical positions with the Westerbork positions. A search area of 10″ by 10″, much larger than the error area, was used to allow for possible small discrepances between the radio and optical position. We find that 22 of the 28 identifications suggested for "UV" sources are confirmed and 4 are rejected, while 2 sources could not be measured, being strongly confused.

On the contrary, 15 of the 20 identifications suggested for "not UV" sources are rejected, 3 sources could not be measured, being strongly confused, and only two are confirmed. In addition identifications are confirmed for 3 of the 7 B sources not visible on the *U* and *B* plates. Finally, from examination of all the 132 radiosources measured at Westerbork, two more "UV" sources are recovered, which were lost because the optical object was outside the original Bologna search area.

Figures 4 and 5 show for the "UV" objects the distributions of the differences in R.A. and declination between the optical and the radio positions, for Westerbork and Bologna respectively. The Westerbork data for the 22 "UV" objects are reported in Table 5. The two new "UV" sources, the 2 "not UV" confirmed and the 3 not visible on the plates but also confirmed by the Westerbork positions, are reported in Table 6. The explanation of the columns is the same as for Table 5.

Summarising, the Westerbork measurements show that the sample selected initially with the Bologna positions and the *u* and *b* plates is quite complete. The few rejected "UV" objects are, more or less, what we should expect due to spurious identifications with radioquiet Quasars (Braccesi and Formiggini, 1969). Here after we will use the name of Quasar for our "UV" blue stellar objects.

Using the criterion of agreement in position to 10″ in each coordinate between the Westerbork and the optical positions, we find that all the E sources, 80% of the NI sources, about 50% of the G sources associated with galaxies fainter than 18.0 magnitudes and 70% of the B "not UV" sources consist of I sources. Combining these Westerbork results with the numbers of sources in the various optical classes for the entire area of 0.162 steradian, we estimate that 705 ± 75 of the 1200 radiosources have no optical counterpart on the Palomar plates. Averaging this result with the earlier figure of 812 "I objects" we compute that 760 of the 1200 sources in the full search area (and 360 in the plate area) have optical counterparts beyond the limit of the Palomar Sky Survey.

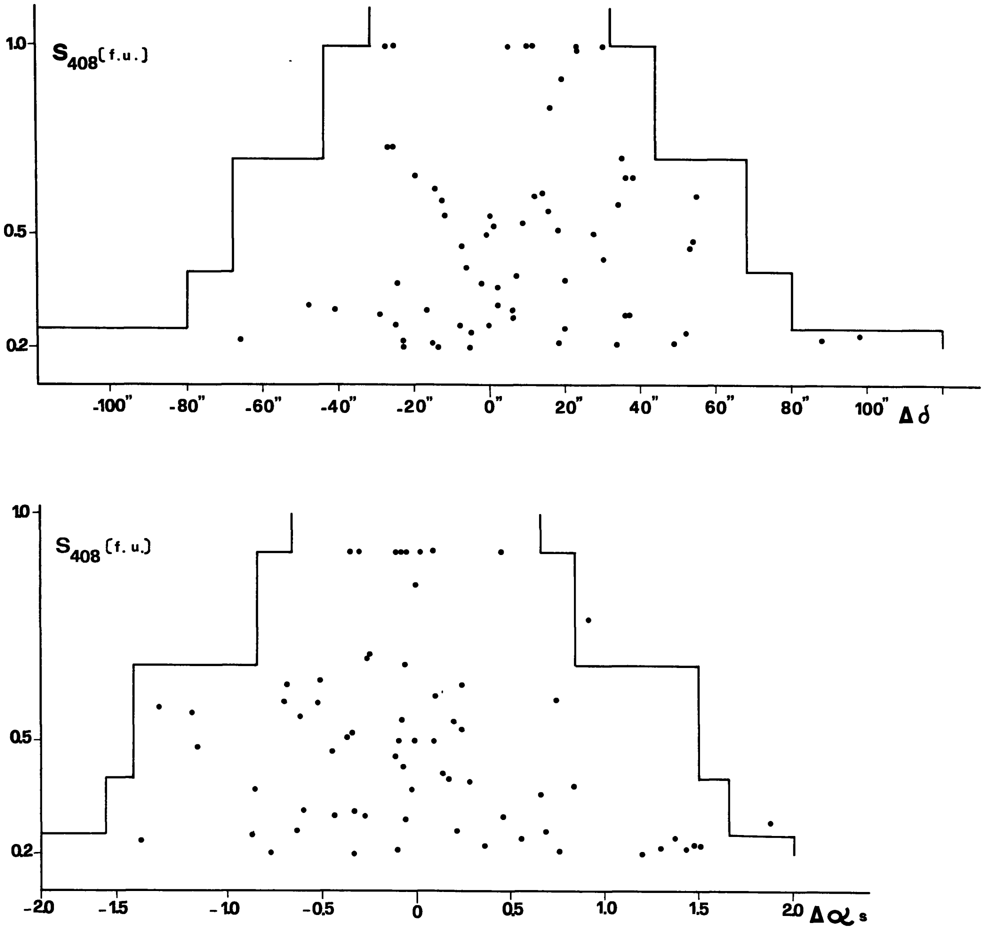

Fig. 4. The distribution of the differences in R.A. and declination between the B 2 positions and the optical positions

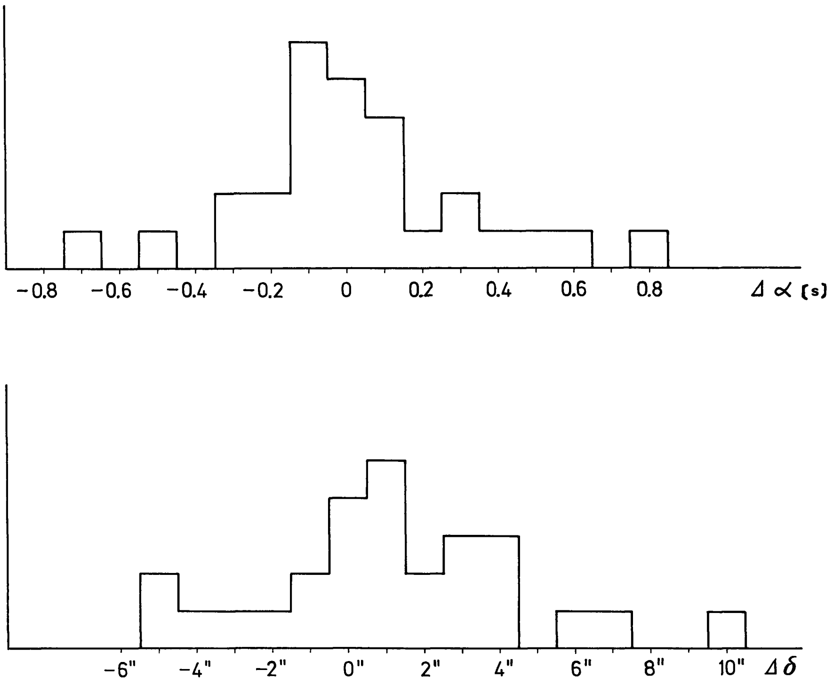

Fig. 5. The distribution of the differences in R.A. and declination, between the optical positions and the Westerbork positions

Table 5. The catalogue of the blue stellar objects having ultraviolet excess

Name	R.A. h m s	Dec ° ′ ″		Δ(R.A.) s	Δ(Dec) ″	S(408) f.u.	S(1415) f.u.	mag	Notes
0810 32	081059.59	324639.8	W	+0.09	+ 27.8	0.50	0.16	18.0	
				−0.03	+ 0.9				
0814 32B	081434.54	324237.6	W	+0.76	+ 61.6	0.43	0.21	19.7	1
				−0.34	102.9				
0836 29	083648.78	293857.6		−0.62	+ 15.6	0.56		20.8	2
0838 32B	083849.14	320426.0	W	−1.46	98.0	0.23		19.4	10
0900 29	090030.12	292624.1		−0.07	+ 30.1	0.43		19.6	
0906 31	090651.99	311817.2	W	−0.11	+ 53.2	0.46	0.16	17.8	3
				−0.22	+ 2.4				
0906 30	090646.73	303617.8		+1.43	+ 17.8	0.21		18.3	
0909 30A	090902.07	303006.8		1.37	− 5.2	0.24		20.0	
0909 30B	090918.51	301751.8		−1.19	+ 33.8	0.57		20.3	
0911 30A	091111.67	302346.7		+0.17	− 67.3	0.40		19.5	
0912 29	091252.83	294630.0		−0.37	+ 18.0	0.51		16.3	
0917 29	091744.53	292904.5		+0.03	− 25.5	0.37		19.7	
0920 31A	092008.09	311219.1	W	−0.01	19.1	0.91	0.28	19.5	
				−0.17	− 0.9				
0920 31B	092048.45	312049.2	W	0.45	− 16.8	0.30	0.17	18.2	
				0.27	+ 4.3				
0922 31B	092244.16	314010.5		−0.44	− 7.5	0.47		20.0	
0922 30	092226.71	303929.8		−0.09	− 12.2	0.55		20.9	
0923 29	092340.90	293419.7		−0.60	+ 1.7	0.31		20.6	
0928 31	092803.25	311549.0	W	−0.35	+ 1.0	0.52	0.12	18.6	
				−0.05	4.5				
0937 30	093722.46	302848.4		0.36	− 23.6	0.22		18.2	
0938 31B	093818.47	311151.9	W	−0.33	− 14.1	0.20	0.09	18.4	5, 7
				+0.60	− 15.0				
0953 30	095343.20	304657.9		0.10	− 14.1	0.62		20.2	8
0955 32B	095525.41	323823.3	W	−0.09	23.3	3.33	1.43	15.9	12
				−0.01	0.4				
1003 29	100319.49	295504.7		−0.51	10.7	0.33		19.2	
1013 30	101305.94	302653.6		1.44	− 66.4	0.22		20.6	
1015 32	101513.03	322842.5		−0.77	− 5.5	0.20		20.0	
1016 30	101646.09	301517.0		−0.01	− 1.0	0.50		21.0	
1017 31	101747.86	315311.1		0.46	− 24.9	2.89	0.95	20.3	14
1208 32A	120805.38	321348.8	W	−0.52	54.8	0.60	0.14	16.2	
				−0.26	3.1				
1213 32A	121317.85	320815.5	W	−0.85	− 2.5	0.37	0.25	19.5	
				0.06	2.3				
1213 30B	121314.74	304524.1		0.14	− 5.9	0.41		19.2	11
1215 30	121521.08	302340.0		0.92	16.1	0.84		15.1	3
1222 32B	122247.57	320212.3	W	−0.33	− 47.7	0.31	0.08	18.1	
				0.51	6.3				
1225 31	122555.94	314512.6	W	−0.06	− 29.4	0.29	0.29	15.6	
				−0.02	− 0.3				
1230 29	123045.60	293042.0		0.20	− 00.0	0.55		20.2	
1234 30	123409.04	303236.2		−1.16	54.2	0.48		21.0	
1236 31	123605.00	310841.1	W	−0.10	48.9	0.21		19.3	10
1238 30A	123855.75	300334.0		0.55	52.0	0.24		19.3	7
1244 32B	124455.40	322522.8	W	−0.10	4.8	1.67	0.43	18.2	12, 13
				−0.09	1.2				
1247 31	124724.23	310448.2		−0.27	6.2	0.30		17.9	
1248 30	124800.19	303259.7		0.09	29.7	1.25	0.50	18.0	12, 14
1255 32	125533.94	324541.3	W	−0.06	35.3	0.70	0.62	18.9	2, 13
				0.07	0.7				
1255 31B	125557.64	314744.1	W	0.84	20.1	0.38	0.06	20.2	1
				1.94	29.1				
1256 31	125607.01	311246.3	W	−1.49	− 97.7	0.20	0.04	18.2	1
				−2.13	−131.9				
1305 32	130511.71	324908.6	W	0.71	32.6	0.21	0.06	19.2	
				0.31	− 4.6				
1308 32	130807.59	323639.8	W	−0.41	9.8	1.18	1.08	19.5	
				0.06	− 1.6				
1327 31	132702.30	312041.8	W	−0.30	23.8	1.19	0.40	19.0	
				−0.04	1.4				

Table 5 (continued)

Name	R.A.	Dec		Δ(R.A.)	Δ(Dec)	S(408)	S(1415)	mag.	Notes
	h m s	° ′ ″		s	″	f.u.	f.u.		
1328 30	132849.63	304559.2		−0.07	11.2	24.59		16.7	9
1340 31	134046.78	315849.5	W	0.08	− 28.5	1.74	0.63	20.5	2
				−0.29	− 3.9				
1348 30B	134837.19	304943.3		0.29	7.3	0.39		19.3	2
1351 31	135151.16	315344.6	W	−0.24	− 27.4	0.73	0.17	17.4	4
				−0.46	− 4.8				
1353 30	135326.14	303851.2		1.34	− 14.8	0.21		18.2	
1355 30	135507.84	305633.3		0.24	9.3	0.53		20.0	
1359 29	135926.08	295624.4		1.88	6.4	0.28		19.3	
1402 29	140246.54	293932.5		0.74	14.5	0.61		20.1	
1410 31	141037.55	314327.0	W	0.55	− 33.0	0.23	0.05	20.5	
				−0.16	3.1				
1411 29	141147.13	295856.3		−0.87	20.3	0.25		18.7	
1419 31	141919.39	313243.7		−0.21	13.7	1.07		20.0	14
1421 30	142121.42	302540.0		1.51	88.0	0.22		19.4	
1422 30	142209.96	304254.2		−0.74	12.2	0.60		20.6	
1427 30	142728.42	303416.8		−1.38	− 13.2	0.59		18.0	
1428 29	142854.46	292632.4		0.66	2.4	0.36		19.6	
1430 31	143055.97	313312.5		−0.43	− 41.5	0.30		19.8	
1431 32	143126.59	322218.4	W	−0.69	35.6	0.65	0.36	20.6	6, 2
				−0.08	4.2				
1435 31	143531.41	313158.3	W	0.21	− 7.7	0.26	0.13	18.0	2
				0.01	0.0				
1443 32	144351.99	322733.5	W	−0.51	− 20.5	0.66	0.19	18.9	
				−0.69	7.5				
1448 30	144843.66	300752.0		−0.64	− 26.0	0.26		19.5	
1451 31	145128.10	312612.8	W	1.20	− 23.2	0.20	0.07	17.5	1
				1.53	− 36.2				
1454 30	145421.34	302256.0		0.24	38.0	0.65		18.9	
1555 30	155524.19	302229.8		0.69	− 0.2	0.26		19.7	2
1620 30	162017.34	300845.0		−0.26	− 27.0	0.73		19.4	

1) Not confirmed by the Westerbork position.
2) Optically variable. This results from a comparison of the image of the object in the PSS prints and in the 48″ Schmidt plates we have, which were taken in the epoch 1968/1970.
3) The object is red but ultraviolet.
4) The radiosource is slightly resolved by the Westerbork beam.
5) There is a main component of 0.06 f.u. displaced by 34″ SE from the optical object. A possible second component of 0.03 f.u. is 43″ NW of the object and can be extended. The radio centroid is displaced from the optical object by about 17″, but in view of the uncertainty on the radiocentroid position we assume the identification as correct. The optical object is fuzzy.
6) Double source with 28″ of separation in EW. The fluxes ratio is 1.2.
7) Very faint cluster of galaxies.
8) Cluster of galaxies. The red magnitude of the brightest member is about 18.0.
9) 3 C 286.
10) Observed at Westerbork, but very confused by grating responses.
11) The optical object is fuzzy.
12) The identification has been already suggested by Olsen (1970).
13) This object belong to the catalogue of QSO's of Braccesi *et al.* (1970).
14) The flux at 21 cm is taken from Fanti *et al.* (1969).

Column 1 gives the source name from the B 2 catalogue.
Column 2 gives the optical right ascension.
Column 3 gives the optical declination. The accuracy in each coordinate is estimated to be better than 1″.
Column 4 contains aW, when the source has been observed at Westerbork (see next paragraph).
Column 5 and 6 give the differences in right ascension and declination between the optical and the radio positions. In first line the radioposition from the B 2 catalogue (corrected) is used; in second line the Westerbork position, when available. For double sources the position of the radiocentroid has been adopted.
Column 7 gives the flux density at 408 MHz.
Column 8 gives the flux density at 1415 MHz, measured at Westerbork. For a few sources this flux has a different origin, referred to in the notes.
Column 9 gives the blue magnitude estimated on the prints.
Column 10 gives notes.

Table 6. Blue stellar objects not belonging to the homogeneous sample

Name	R.A. h m s	Dec ° ′ ″		(R.A.) s	(Dec) ″	S(408) f.u.	S(1415) f.u.	mag	Notes
0858 32	085804.53	323357.3	W	−0.07	− 15.5	0.36	0.12	20.0	1
				−0.09	3.1				
0931 31	093149.62	310420.0	W	+0.12	+ 30.0	0.94	0.33	20.0	1
				0.17	− 3.2				
0934 32	093408.58	322026.5	W	+1.38	+ 20.5	0.21	0.13	20.6	1
				0.80	− 0.5				
1228 31	122826.94	311811.9	W	−1.26	− 54.1	0.32	0.11	19.8	2
				−0.88	− 0.2				
1310 31	131027.54	312853.8	W	0.76	−119.0	0.27	0.16	18.3	3
				−0.06	− 0.2				
1339 31	133951.86	313444.7	W	0.26	− 15.3	0.21	0.07	19.5	2
				0.36	9.9				
1420 32	142021.09	323647.3	W	1.99	− 0.7	0.53	0.39	18.4	3
				0.06	1.1				

1) Not visible on the *u* and *b* plate.
2) Not UV.
3) This source was lost in the original selection, because it is out of the B 2 search area.

Radio Spectra

The radiosource measurements at Westerbork allow us to determine the distribution of spectral indices, between 408 and 1415 MHz. This has been performed separately for radiosources associated with quasars, with galaxies, and for I sources. These sources have been reidentified on the basis of the Westerbork positions. In view of the high accuracy of the Westerbork measurements we believe that the Westerbork E fields constitute practically all the I sources. These distributions are shown in Fig. 6.

We observe two important things.

1) The distribution of spectral indices of "UV" sources, selected at 408 MHz, is the same as that of quasars of higher radiofluxes (Bolton, 1966; Braccesi *et al.*, 1969). The distribution has a long tail of flat spectrum objects ($\alpha < 0.6$) which contains about 30% of all the "UV" sources.

2) The distribution of the spectral indices for galaxies and I sources are similar and clearly different from that of the quasars in not having the flat spectrum tail. The value of the mean spectral index and the widths of the distributions are unusually high, compared with commonly accepted values (Kellermann *et al.*, 1969). We believe that these two features are due to strong resolution effects at 1415 MHz. On the contrary the great similarity of the distribution of the spectral indices of the quasars, reported here, with that obtained by Braccesi *et al.* (1969), shows that for this class of sources the effects of resolution are significantly smaller.

From these spectral data, we can also say that at these faint radiofluxes the bulk of the "I sources" are radiogalaxies beyond the limit of the 48″ Schmidt telescope of Mount Palomar. We can estimate the number of quasars present in the "I sources" by assuming that the two "I sources" with $\alpha < 0.6$ are quasars and that the quasars fainter than the optical limit of the Palomar Sky Survey have the same distribution of spectral indices as the visible quasars. In this way we obtain a rough estimate of 6 quasars included in the 62 "I sources", or about 10%, although a proportion as high as 20% could not be excluded on basis of the present data.

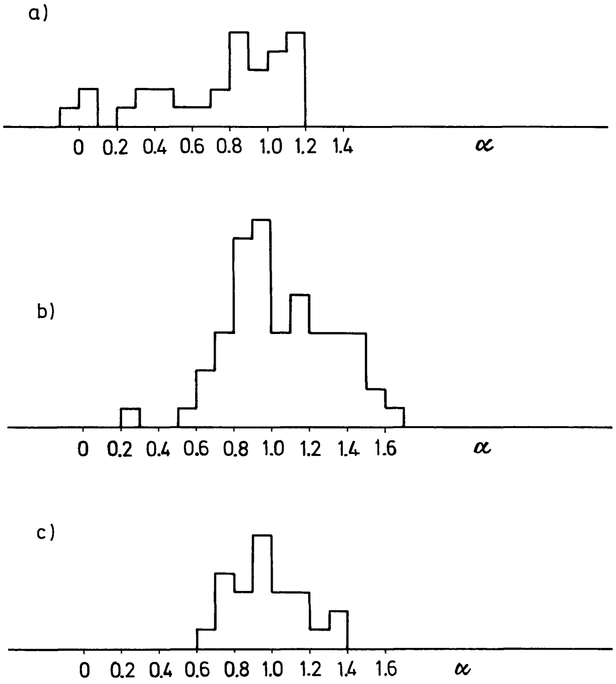

Fig. 6. The distributions of spectral indices for a) "UV" B sources. b) I sources. c) Galaxies

Quasar Counts

Our sample of QSS's[4]) is submitted to a double selection (radio and optical) that makes any count, based on one of the two fluxes, subject to differential losses that change the count itself. A way to overcome this problem is to divide the QSS's into class of constant ratio R between the optical and the radio flux. This parameter is essentially redshift independent, as the K corrections for the radio and optical flux are similar, except for the effects of the emission lines on the optical magnitudes. Therefore R is equal to the ratio of the radio to optical absolute luminosity.

We will now show that in models with strong density evolution, the counts of QSS's of constant R are simply related to the counts of QSO's. Let us consider, for this purposes, two different models.

1) The model of Schmidt (1970) in which is supposed that there is a correlation between the radio and optical luminosity functions.

In this case we have:

$$N_{\mathrm{QSS}}(m, R) = N_{\mathrm{QSO}}(m)\, \psi(R)$$

where $N_{\mathrm{QSO}}(m)$ and $N_{\mathrm{QSS}}(m, R)$ represent respectively the integral counts, up to the magnitude m of the QSO's and of the QSS's with a radio-to-optical ratio R and $\psi(R)$ is the distribution of R.

2) The model in which the radio and optical luminosity functions are independent. The counts of QSS, of constant R, up to the magnitude m is

$$N_{\mathrm{QSS}}(m, R) = \int \varphi(RP_o)\, dP_o n(m, P_o)$$

where the integral count $n(m, P_o)$ of QSO having optical luminosity P_o is weighted by the normalized radio-luminosity function φ computed at $P_{\mathrm{rad}} = RP_o$.

If the strong evolution required by the quasars does not depend on the optical luminosity, so that the counts at various P_o are represented by the same law (of the type $\log n(m) = K \cdot m + \mathrm{const.}$), the total count will also have the same law.

Concluding, in both the hypothesis the count of QSS, at constant R, has to show the same law of the QSO's counts.

The equivalence of the two hypothesis fails if we introduce a maximum red-shift $Z_{\max}$. In the hypothesis of the independency of the two luminosity functions, the $N_{\mathrm{QSO}}(m)$ will show, beyond a certain magnitude, a bending more pronounced than the $N_{\mathrm{QSS}}(m, R)$ because in the last one the radioluminosity function (increasing at the decreasing of P_o) weights more the low optical luminosities, which reach $Z_{\max}$ at fainter magnitudes. However this difference between the two counts should be relevant out of the observed apparent magnitude interval, as will be shown later, in the discussion.

Following the previous discussion, we divide our sample of QSS in two classes according to an index r defined by

$$r = 2.5 \log_{10} S_{408} + m_b$$

such that r is proportional to $\log R$[5]).

The QSS's of our sample with $r > 19.25$, constitute an optical sample complete to $m_b = 21.0$. From this sample, we can determine the distribution $N(m)$, up to that magnitude. Supposing that the relationship between $N(m)$ and m is of the form $\log N(m) = Km + \mathrm{constant}$, we find:

$$K = 0.54 \pm 0.25\,.$$

On the other hand all the objects with $r < 19.25$ form a sample complete down to 0.2 f.u. at 408 MHz. Such objects can be counted as a function of the radio flux density S. Assuming for them the equivalent relationship

$$\log N(S) = -K\, 2.5 \log S + \mathrm{constant}$$

we find:

$$K = 0.64 \pm 0.15\,.$$

As we can see, the values obtained for the slopes have large errors, so that little, if anything, can be concluded from them alone. A better determination of the slope of the QSS counts can be obtained by combining the present sample of QSS's with the 3 CR sample studied by Schmidt (1968) and the 4 C sample studied by Lynds and Wills (1972). For this purpose, we computed the 408 MHz flux densities of the 3 CR radiosources from the spectral data published by Kellermann *et al.* (1969), and by Lynds and Wills (1972), and calculated the corresponding r. Assuming relationships of the forms:

$$\log N(m) = Km + \mathrm{constant}$$

and

$$\log N(S) = -K\, 2.5 \log S + \mathrm{constant}$$

we have for the various classes of r the following results:

$17.25 \leqq r < 18.25$	$K = 0.71 \pm 0.06$
$18.25 \leqq r < 19.25$	$K = 0.69 \pm 0.04$
$19.25 \leqq r < 20.25$	$K = 0.68 \pm 0.05$
$r \geqq 20.25$	$K = 0.64 \pm 0.09$.

[4]) In the following pages we will use the Schmidt's nomenclature, calling "QSS" the quasars derived from radio selection and "QSO" those derived from optical selection. We suppose that both are part of a unique population of objects.

[5]) Ideally $r = 2.5 \log R + 9.0$, where R is the ratio of the radioflux at 500 MHz to the optical flux at 2500 Å (corrected for the emission lines) as used by Schmidt (1968, 1970).

Table 7. The $G(R)$ function deduced from the B 2 identifications

$G(R)$	R
$3.8 \cdot 10^{-1}$	2.1
$1.4 \cdot 10^{-1}$	2.5
$5.0 \cdot 10^{-2}$	2.9
$1.8 \cdot 10^{-2}$	3.3
$6.6 \cdot 10^{-3}$	3.7
$1.9 \cdot 10^{-3}$	4.1
$6.1 \cdot 10^{-4}$	4.4

The weighted average value of K obtained from the various categories of r is

$K = 0.69 \pm 0.04$ [6])

which is equal, within the errors, to the slope of 0.72 ± 0.06 obtained for the counts of QSO's (Braccesi *et al.*, 1969).

In our sample there are 17 objects, with $r < 17.25$, with radio-to-optical ratios which are not represented in the 3 CR sample. Using the slope of 0.70 (average of the values found by us and Braccesi and Formiggini, respectively), we can predict that there should be 3 such objects in 3 CR sample, each with a magnitude brighter than the 15.0$^{\text{th}}$. However we do not consider significant the absence of these 3 objects from the 3 CR sample.

[6]) One can question the reliability of this result on the ground that our magnitudes, estimated from enlargements of the PSS are not only rather inaccurate but affected by systematic errors arising from the intrinsic inhomogeneity of the PSS plates. It can be shown that the two main sources of error, namely:

1) The uncertainty in the estimates of the individual magnitudes,
2) The inhomogeneity of the photographic material used do not affect our results under the conditions,

a) The distribution of the errors in the estimates of the individual magnitudes, expressed on a logarithmic scale i.e. in magnitudes should be independent of the true magnitudes.

b) The magnitude scale is correct, i.e. the differential error $\Delta M/M$ is small, even if the zero point differs from plate to plate.

Under condition a) the errors in the measurements do not influence the slope of a relation of the kind:

$$\log N = Km + \text{constant}$$

(Bracesi *et al.*, 1966).

Condition b) consists of two points

1) $\Delta M/M$ small
2) Zero point shifts.

The first point is easily dismissed: in order to change the slope by 10% one would have to make an error of 0.1 magnitude per magnitude, which is very large indeed.

The second reduces to the following: if the true distribution is

$$\log N = Km + \text{constant} \quad \text{sources per steradian}$$

the distribution observed is:

$$N = \Sigma_i C \Omega_i 10^{K(m+\Delta mi)}$$

or

$$\log N = Km + \log(\Sigma_i C \Omega_i \, 10^{K \Delta m_i})$$

where Ω_i is the solid angle subtended by the i^{th} sample and Δm_i the error in the scale for that sample. This has the same slope as the original distribution.

Counts Beyond 21$^{\text{st}}$ Magnitude

We have already seen from the distribution of spectral indices that approximately 10% of the I sources could be QSS's. From the estimated number of I sources in the plate area (360 ± 50), we can thus deduce an approximate number of 40 QSS's with $m_b > 21.0$ and $S \geqq 0.2$ f.u. at 408 MHz.

If we extrapolate the combined counts of QSS's (Schmidt and ours) with the slope of 0.70, we can predict that there should be 274 ± 130 QSS's in the plate area, fainter than $m_b = 21.0$ and stronger than 0.2 f.u. at 408 MHz. The great uncertainty in the predicted number is due to the presence of only one quasar (3 C 147) in the range $22.25 \leqq r \leqq 23.25$. Even if this object is excluded, the remaining ones lead to a lower limit of 144 ± 20, for the number of QSS's beyond $m_b = 21.0$. This limit seems too high in comparison with our deduced number. Therefore we suspect that the counts for $r < 19.25$ must show a change of slope at magnitudes greater than 21.0.

Discussion

We have seen that the counts of QSS's, at constant R, have the same distribution with magnitude as QSO's, which is consistent with the models of strong density evolution we have considered. In the following discussion we will use Schmidt's model which assumes a correlation of the two luminosity functions. In this case the integral counts of the QSS's are proportional to those of QSO's, where the constant of proportionality $\psi(R)$ represents the distribution of the ratio between the radio and optical emission. The function $\psi(R)$ can be computed by means of the above relation; in Table 7 is given its integral

$$G(R) = \int_R^\infty \psi(R)\, dR$$

$G(R)$ is also drawn in Fig. 7 [7]).

In such a model we have the same distribution, as a function of the magnitude, for every value of R. The QSO data of Braccesi and Formiggini show that the slope is constant up to $m_b = 19.4$ and our counts show that it does not change significantly up to 21$^{\text{st}}$ magnitude. For fainter magnitudes the only data are those of Sandage and Luyten (1969). They show that the slope of $N(m)$ remains steep up to magnitude 21.4. Our pre-

[7]) The $\psi(R)$ function, computed in the same way, by using the 3 CR QSS's shows a good agreement with ours. We point out that the function $\psi(R)$ computed by us differs by about a factor 2 from the one given by Schmidt (1970). This is due to a difference of a factor 2 in the $N(m)$ distribution for QSO's. In fact Schmidt assumes 20000 QSO's, in all the sky, around 18 magnitude, while the counts of Braccesi and Formiggini gives 42000.

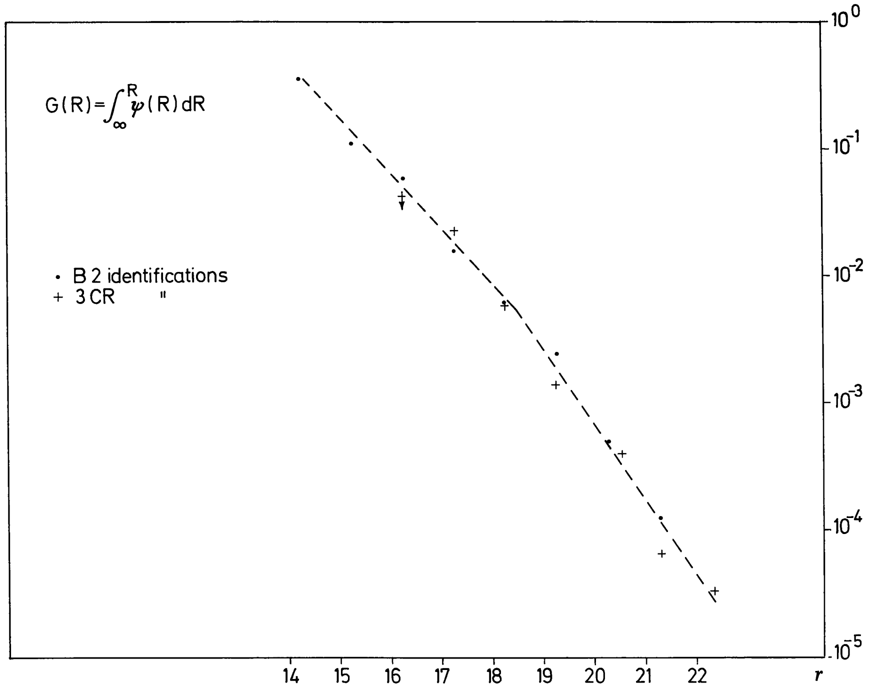

Fig. 7. The function $G(R)=\int_R^\infty \Psi(R)\,dR$. Dots are from the present sample. Crosses from the 3*CR* quasars

dictions of the number of QSS beyond 21st magnitude imply however a change in the slope of $N_{QSO}(m)$. This bending could be due to the presence of a maximum value for redshift, around 2.5, which has been discussed, for instance, by Schmidt (1970). The magnitude at which this bending occurs and the amount of the bending depend on the optical luminosity function. We have computed the $N(m)$ relationship for QSO's using the two samples of QSO's studied by Schmidt (1970) and Braccesi, Grueff and Lynds, (in preparation). We assumed a cosmological model with $\sigma_0 = q_0 = 1$, density evolution $\varrho(z)=\varrho_0(1+z)^6$, and a maximum value for the redshift equal to 2.5. The curve obtained has a slope of about 0.75 down to 20.0 magnitude; then it bends having a slope of about 0.5 between the 21st and 23rd magnitude and it reaches its final slope of 0.0 at about 24th magnitude [8]).

By using this $N(m)$ relationship we can predict that in our plate area the I source class will contain about 44 QSS. This figure is in good agreement with the number of QSS beyond 21st magnitude, estimated from the Westerbork data.

Conclusions

1) Our sample of QSS's is selected on the basis of the positions of the B 2 catalogue and on the basis of a rough estimate of the $U-B$ colour. The measurements made at Westerbork, for about 1/3 of the objects, show that our sample contains 80–90% of correct identifications. The few losses compensate reasonably well the few contaminations.

2) The percentage of QSS's in a sample of radiosources down to 0.2 f.u. at 408 MHz is $12 \pm 2\%$. Comparing this percentage with those found at different radio flux limits (see, for instance Munro, 1971), we can see that the content of QSS in a radio catalogue is slowly decreasing with the radio flux.

3) $64 \pm 9\%$ of the radiosources down to 0.2 f.u. are I sources.

4) The distributions of spectral indices for galaxies and I sources are very similar one to the other and different from the one for QSS's. From the comparison of these distributions we deduce that the majority of the I sources are likely to be galaxies. Only about 10% could be quasars.

5) The counts, at constant *R*, of quasars show a steep slope, similar, within the errors, to the one already found for QSO's. This is in agreement with what is expected on the basis of strong density evolution models. In particular it is in agreement with the model of Schmidt (1970) which suggests a correlation between the radio and the optical luminosity.

6) The small number of QSS's here estimated to be present beyond 21st magnitude, implies a strong bending in the $N(m)$ counts of QSO's beyond the same magnitude.

[8]) Our computed $N(m)$ is different, beyond 21st magnitude from the one computed by Schmidt. This discrepance reflects the different content in absolute magnitude of the two samples of QSO's.

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