

OPTICAL POSITIONS FOR 21 3C OBJECTS

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SUMMARY

Optical positions relative to AGK have been obtained for 21 3C objects using (i) direct photographs taken with the Cambridge 17/24-inch Schmidt telescope; (ii) positions measured by Murray, Tucker and Clements and modified by our recalibration of their secondary reference stars and (iii) positions measured from 48-inch Mt Palomar Schmidt Telescope plates. The external standard errors have been estimated by comparison with the most accurate radio positions resulting in: for R.A. ± 0.15 arc sec, and for Dec a value varying between ± 0.12 arc sec at $\delta = +40^\circ$ and ± 0.20 arc sec at $\delta = +77^\circ$ and $+4^\circ$.

1. INTRODUCTION

We present in Table I photographic positions for 21 objects identified with 3C radio sources of small angular diameter. The objects were chosen for us by Mr B. Elsmore of the Mullard Radio Astronomy Observatory, Cambridge. They are: the 16 measured by Murray, Tucker & Clements (1971), plus 3C 84, 216, 286, 380 and 454.3. Originally the purpose was to check 'absolute' radio positions measured by Elsmore & Mackay (1970). Subsequently some of these objects have been used as calibrators for radio positional surveys.

An ideal calibrator has a small angular diameter and gives a strong radio flux. A source that is acceptable at one stage of instrumental development may not always remain so. Further developments may render it unacceptable. This has happened here. Of the 16 sources measured by Murray *et al.* only five are now regarded as suitable, and of the additional sources added to the programme, only three (Smith 1971).

Preliminary optical positions for 13 of the objects have already been published by us (1970). Results given in the present paper may differ slightly because of additional data. In particular, the data for faint QSO's have been strengthened by plates taken with the Mt Palomar 48-inch Schmidt Telescope by Dr J. V. Peach. The results derived from these plates are given in Cols 5 and 10 of Table I.

2. OBSERVATIONAL

The photographs were taken with the 17/24-inch classical Schmidt Telescope at the Cambridge Observatories. This telescope has been described by Redman (1953). Before commencing the present observations, an improvement was made to the drive control unit (Argue 1969).

The plate scale, $8 \mu\text{m}$ per arc sec, is small by the usual astrometric standards, making our results more susceptible to measuring errors and various effects that can lead to a displacement of the image across the photographic plate. These will be reviewed below in Section 8, but it might be helpful to state now that the measur-

TABLE

Measured optical positions for 3C objects. The final values are in Columns 6 and 11 with estimates of 2, 4 and 5 and 7, 9 and 10 respectively, with the corresponding internal standard error and number of Columns 3 and 8 the mean difference between Cambridge and Murray et al. for the secondary reference et al. corrected by Columns 3 and 8 respectively; Columns 5 and 10 the positions measured from

(1)	(2)	(3)	(4)	(5)	(6)
	(time sec)	(time sec)	R.A. (time sec)	(time sec)	(h m s)
3C 9	—	-0.045 ±0.003; 4	49.899 ±0.005	49.887 ±0.004; 3	0 17 49.890 ± 0.011 ± 0.004
48	49.819 ±0.008; 3	—	—	—	1 34 49.819 ± 0.012 ± 0.008
84	29.548 ±0.009; 5	—	—	—	3 16 29.548 ± 0.014 ± 0.009
93	—	-0.016 ±0.003; 4	51.546 ±0.007	51.520 ±0.013; 3	3 40 51.541 ± 0.010 ± 0.007
138	—	0.024 ±0.006; 4	16.532 ±0.009	16.508 ±0.009; 4	5 18 16.521 ± 0.011 ± 0.008
147	—	0.036 ±0.005; 3	43.500 ±0.011	43.490 ±0.007; 3	5 38 43.492 ± 0.016 ± 0.007
153	—	0.054 ±0.007; 4	44.438 ±0.012	—	6 05 44.438 ± 0.015 ± 0.012
186	—	0.006 ±0.008; 3	56.748 ±0.010	56.755 ±0.009; 3	7 40 56.752 ± 0.013 ± 0.009
196	—	0.003 ±0.003; 3	59.389 ±0.010	59.402 ±0.005; 5	8 09 59.400 ± 0.015 ± 0.005
204	—	0.123 ±0.007; 2	18.148 ±0.016	18.143 ±0.017; 4	8 33 18.146 ± 0.025 ± 0.013
205	—	0.078 ±0.012; 3	10.020 ±0.017	—	8 35 10.020 ± 0.019 ± 0.017
216	—	— ±0.008; 3	—	17.265 ±0.008; 2	9 06 17.265 ± 0.014 ± 0.008
249.1	27.420 ±0.063; 3	0.032 ±0.017; 3	27.348 ±0.026	27.298 ±0.019; 3	11 00 27.316 ± 0.046 ± 0.018
263	09.367 ±0.027; 4	0.045 ±0.011; 4	09.341 ±0.015	09.348 ±0.012; 2	11 37 09.349 ± 0.025 ± 0.010
277.1	—	-0.148 ±0.011; 3	15.130 ±0.016	15.093 ±0.020; 1	12 50 15.118 ± 0.019 ± 0.014
286	49.653 ±0.019; 1	— ±0.004; 4	—	49.660 ±0.016; 1	13 28 49.657 ± 0.012 ± 0.012
288.1	—	0.015 ±0.005; 4	29.926 ±0.014	29.958 ±0.015; 3	13 40 29.940 ± 0.021 ± 0.011
309.1	56.679 ±0.033; 2	0.166 ±0.004; 3	56.642 ±0.014	56.647 ±0.017; 1	14 58 56.648 ± 0.033 ± 0.011
345	17.605 ±0.021; 1	-0.019 ±0.002; 4	17.606 ±0.005	—	16 41 17.606 ± 0.013 ± 0.005
380	13.513 ±0.009; 4	—	—	—	18 28 13.513 ± 0.016 ± 0.009
454.3	29.533 ±0.009; 4	—	—	—	22 51 29.533 ± 0.011 ± 0.009

ing error is ± 0.09 arc sec (internal standard error), and that the combined effect of elastic inhomogeneities in the glass plate, acting when the plate is bent to the classical curvature during exposure, and of emulsion shifts during the darkroom processing, has an upper limit of ± 0.08 arc sec r.m.s. The dominant error is that

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the external standard error given alongside. These final positions are weighted means from Columns plates in the line directly beneath. Columns 2 and 7 give the directly determined Cambridge position; stars (in the sense Cambridge minus Murray et al.); Columns 4 and 9 the position determined by Murray Mt Palomar 48-inch Schmidt Telescope plates.

(7)	(8)	(9)	(10)	(11)
(arc sec)	(arc sec)	Dec (arc sec)	(arc sec)	(° ' ")
—	0.26 ±0.08; 4	16.47 ±0.10	16.36 ±0.13; 4	15 24 16.44 ±0.15 ±0.09
20.40 ±0.21; 3	—	—	—	32 54 20.40 ±0.12 ±0.21
52.19 ±0.05; 5	—	—	—	41 19 52.19 ±0.12 ±0.05
—	0.07 ±0.03; 4	21.83 ±0.10	21.65 ±0.06; 3	4 48 21.70 ±0.20 ±0.06
—	0.13 ±0.07; 4	27.16 ±0.12	26.90 ±0.13; 4	16 35 27.06 ±0.15 ±0.10
—	-0.13 ±0.13; 3	43.09 ±0.16	43.12 ±0.18; 3	49 49 43.10 ±0.12 ±0.15
—	-0.43 ±0.06; 4	48.80 ±0.11	—	48 04 48.80 ±0.12 ±0.11
—	0.01 ±0.06; 3	31.09 ±0.08	31.02 ±0.11; 4	38 00 31.07 ±0.12 ±0.08
—	-0.15 ±0.10; 3	07.56 ±0.14	07.57 ±0.11; 4	48 22 07.57 ±0.12 ±0.11
—	-0.62 ±0.14; 2	03.82 ±0.17	03.88 ±0.16; 4	65 24 03.86 ±0.13 ±0.15
—	-0.32 ±0.01; 3	51.44 ±0.09	—	58 04 51.44 ±0.13 ±0.09
—	—	—	58.56 ±0.10; 2	43 05 58.56 ±0.12 ±0.10
08.84 ±0.18; 3	-0.13 ±0.03; 3	08.49 ±0.07	08.38 ±0.03; 3	77 15 08.41 ±0.17 ±0.03
26.83 ±0.10; 4	-0.13 ±0.04; 4	26.91 ±0.08	26.90 ±0.07; 2	66 04 26.88 ±0.15 ±0.05
—	-0.28 ±0.07; 3	36.44 ±0.11	36.26 ±0.17; 1	56 50 36.39 ±0.13 ±0.10
58.36 ±0.25; 1	—	—	58.50 ±0.16; 1	30 45 58.46 ±0.12 ±0.13
—	-0.06 ±0.04; 4	48.45 ±0.10	48.27 ±0.10; 3	60 36 48.36 ±0.15 ±0.08
11.24 ±0.37; 2	-0.10 ±0.01; 3	11.32 ±0.07	11.43 ±0.16; 1	71 52 11.33 ±0.16 ±0.06
10.81 ±0.25; 1	-0.28 ±0.04; 4	10.71 ±0.07	—	39 54 10.72 ±0.12 ±0.07
40.45 ±0.01; 3	—	—	—	48 42 40.45 ±0.12 ±0.01
54.98 ±0.16; 4	—	—	—	15 52 54.98 ±0.15 ±0.16

due to photographic granularity. For a single measurement, positioning the image within the measuring graticule by eye, the standard error is ± 0.15 arc sec at brightness 14^m . A comparison with automatic positioning by the GALAXY machine has been published (Argue & Kenworthy 1972).

Some of the telescope adjustments were offset to optimize for this programme. The pole of the corrector was displaced vertically with respect to the centre of curvature of the mirror for the reason given in Section 7. The tilt of the polar axis was set to dip 25 arc sec below the photographic refracted pole to compromise between field rotation and drift of the guide star in declination, two effects caused by refraction in the Earth's atmosphere.

The photographic emulsion was Ilford SRO blue sensitive, used without filter. Exposure times varied between 10 and 15 min. Development was in Kodak D19 developer in an open dish with vigorous rocking followed, without intermediate water rinse, by acid fixer and hardener. A water rinse would have swollen the gelatin, increasing the likelihood of displacement of the images. Only after the emulsion has been hardened should it be brought in contact with plain water. After hardening, the plates were washed for 1 hr, drained vertically on to blotting paper for about 1 min and then dried horizontally at room temperature. There were generally three or four plates per field.

The plates were measured by hand using a Zeiss Komess x, y machine. Readings were written down, punched on to paper tape and reduced by computer (TITAN).

During exposure the photographic plate was bent in its holder in the usual way for a classical Schmidt telescope so that the celestial sphere was projected on to the concentric spherical emulsion surface. For measurement the plate was held flat by clamping between two glass flats of thickness 6 mm. Distances on the celestial sphere are related to those on the flattened plate by azimuthal equidistant projection about the centre of the plate, apart from elasticity effects occurring in the bent state. In the reduction program the equatorial coordinates (α, δ), of the reference (AGK 3) stars were transformed by this projection, giving coordinates x', y' analogous to but not identical with the 'standard coordinates' of classical astrometry. x' and y' were then related to the measured x, y by quadratic regression. This can be shown to take adequate account of aberration, differential refraction, maladjustment of the plateholder with regard to focus and tilt, and elastic deformation of the bent photographic plate according to Shepherd's treatment (1953). The plate is circular, diameter 150 mm and thickness about 1 mm.

The unvignetted field of our telescope is $3^\circ.3$. In all cases we imaged the QSO at the centre of the field and measured AGK stars within a radius of 1° . There were on average 23.8 AGK stars per QSO. Nine of the QSO's registered sufficiently strongly on our plates for direct measurement, the results of which are in Cols 2 and 7 of Table I. For the fields measured by Murray *et al.* we remeasured these authors' 'secondary reference stars'. These numbered usually 11 per QSO, were of brightness $m_{pg} \sim 13$ and were situated within a few arc min of the QSO. These stars were used by Murray *et al.* to link the narrow field of the Isaac Newton Telescope or the 26-inch refractor to the wider field covered by the AGK stars, and from our recalibration of them we have revised these authors' results. These revisions are given in Cols 4 and 9 of Table I. Secondary reference stars were also used by us to link measurements on Dr Peach's plates to AGK. Dr Peach's plates were taken with a field flattener and we did not attempt to measure beyond the narrow field covered by the secondary reference stars. We also used for these plates a special reduction program involving linear plate constants only. Strictly speaking, the classical 'standard coordinates' (gnomic projection) reduction would have been more appropriate to the flat field, but over such a small radius the difference is negligible.

3. RESULTS

(i) Nine sources were bright enough for direct measurement on our plates. The results are given in Table I, Cols 2 and 7 with the internal standard error (calculated from the scatter among the plates of that field) in the line directly beneath in the same column. In cases where only one plate had been measured we took a mean from the remaining fields.

(ii) Cols 3 and 8 give the difference between the mean R.A. and Dec for the secondary reference stars for that field as obtained by us and by Murray *et al.* This is the correction we have applied to the QSO positions published by these authors. Its internal standard error and the number of our plates are in the line directly beneath. This standard error was derived from the plate-to-plate scatter of individual secondary reference stars as described in Section 8. The corrected position of Murray *et al.* is given in Cols 4 and 9 with its internal standard error directly beneath. This error is made up of the error in the correction, as just described, plus the measuring error of Murray *et al.* taken as ± 0.13 arc sec for one plate as quoted by these authors. Their number of plates are given in Table VII of their paper.

(iii) The positions measured from Dr Peach's plates are in Cols 5 and 10 with the internal standard error and the number of Dr Peach's plates beneath. This error is again compounded of the error of the secondary reference zero-point in Cols 3 and 8, plus the error in the source position relative to this point, derived from the scatter between individual plates of that field. In cases where only one plate was measured we took a value of ± 0.16 arc sec in each coordinate, which was a mean determined from the remaining fields.

(iv) Finally we formed weighted means from the results in Cols 2, 4 and 5 and 7, 9 and 10. These are given in Cols 6 and 11 with their internal standard errors directly beneath. The weight was taken as inversely proportional to the square of the standard error. Beside each weighted mean we have given our estimate of its external standard error. In R.A. this has the value ± 0.15 arc sec. This is derived in Section 4 by comparison with mean radio positions. This estimated value is large enough to cover the internal standard error, with average value ± 0.082 arc sec, and the uncertainty of fit to AGK having the value $\pm 0.22/\sqrt{n}$ arc sec where n is the number of AGK stars (Section 8; $\langle n \rangle = 23.8$), leaving an excess error which we attribute to outstanding instrumental effects and local systematic errors in AGK. How this outstanding error is divided between our instrument and AGK we cannot say. AGK errors show up in this comparison because the radio R.A.'s are independent of AGK apart from a zero point adjustment.

In Dec the external standard error is made up of three components: (a) an internal error averaging ± 0.097 arc sec over all sources; (b) $\pm 0.19/\sqrt{n}$ for uncertainty of fit to AGK; (c) an uncertainty over declination dependent effects, derived by comparison with radio declinations and having the value given by equation (7.2). It includes systematic errors in AGK because the radio declinations are independent of AGK.

4. COMPARISON WITH RADIO POSITIONS

Fig. 1 shows a comparison of our positions with three radio surveys: Smith (1971), Adgie, Crowther & Gent (1972,) and Wade (1970). Only those sources which are included in at least two of the radio catalogues have been

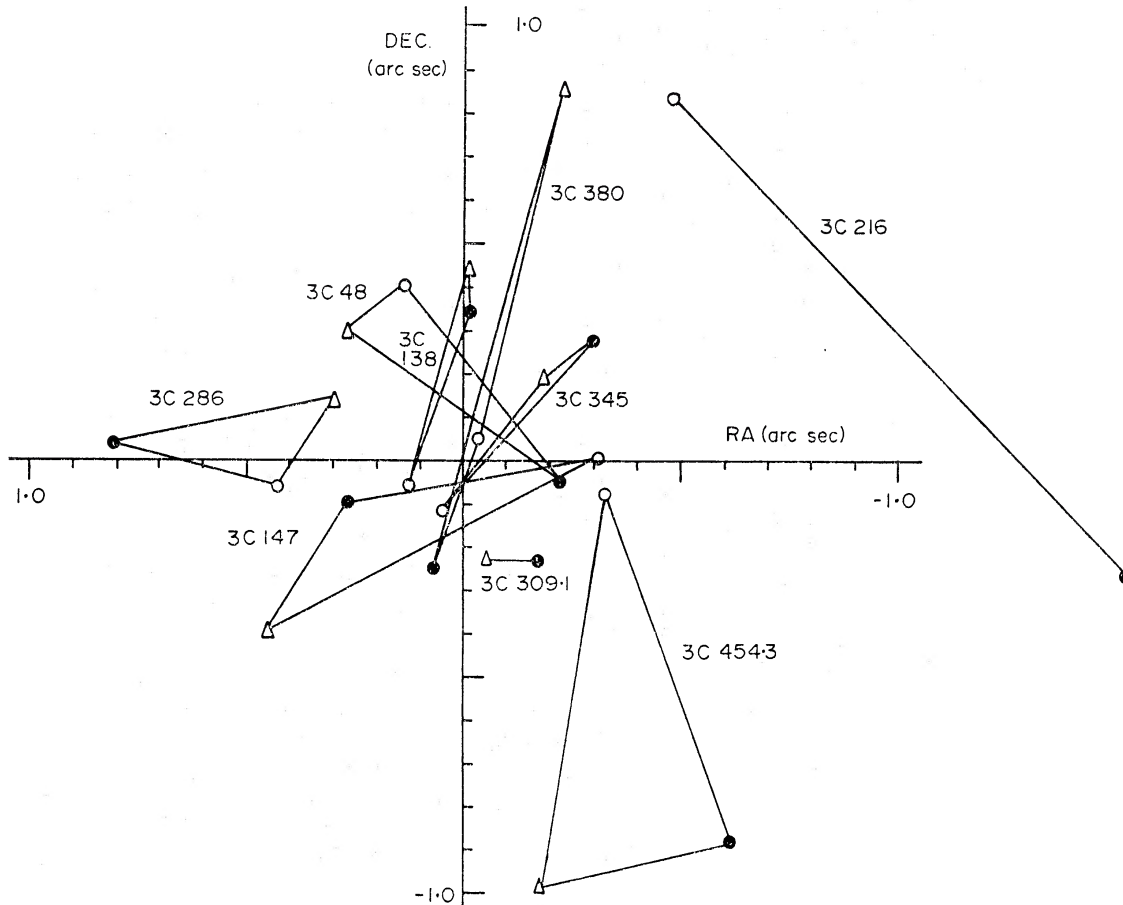


FIG. 1. Comparison of our optical position in Table I with three radio catalogues: Smith (1971, filled circles), Adgie et al. (1972, open circles) and Wade (1970, triangles). Ordinates: radio minus optical, in R.A. Abscissae: the same, in Dec. The R.A.'s of Adgie et al. have been reduced by 0.03 time sec (Argue & Kenworthy 1970). Wade's R.A.'s have been increased by 0.02 time sec (Argue & Kenworthy 1970) and his Dec's by 0.3 arc sec (Kristian & Sandage 1970).

plotted. The diagram shows the difference between radio and optical positions. The optical may be visualized as situated at the origin.

There is seen to be considerable scatter for 3C 216. This source was not one of Smith's calibrators and consequently was not measured by him with his highest possible accuracy. On the other hand 3C 380 also was not one of his calibrators, yet here the interagreement is seen to be fairly good. 3C 454.3 is a calibrator and would have been expected to give better interagreement than is shown on the plot. The optical position derived for this source however turned out to be unsatisfactory as explained in the next section. As to the radio position, the declination of this source is low and this might be considered a reason for the large scatter shown in the diagram. But 3C 138 is equally low, yet is seen to give much better interagreement. The question of a declination dependent effect for the optical positions will be discussed in Section 7.

The radio positions provided the basis for estimating the external standard error of our optical positions. Omitting 3C 216 and 454.3, the remaining eight plotted on Fig. 1 give an r.m.s. difference in R.A. between our optical and the mean radio (as derived from two or all three catalogues) of 0.15 arc sec. We have taken this as our estimate of the external standard error of our measurements in R.A.,

using it in Section 3 and Table I. It is based on an assumed coincidence of radio and optical objects for the eight sources from which it has been derived. This is justified in the next section. As explained in the previous section, this estimate includes effects of local systematic errors in AGK.

The external standard error in declination is derived in Section 7.

5. COMPARISON WITH OPTICAL POSITIONS

Fig. 2 gives a similar comparison of our own with other optical positions. We have taken those published by Murray *et al.* (1971), Barbieri, Capaccioli & Pinto (1971) and Kristian & Sandage (1970).

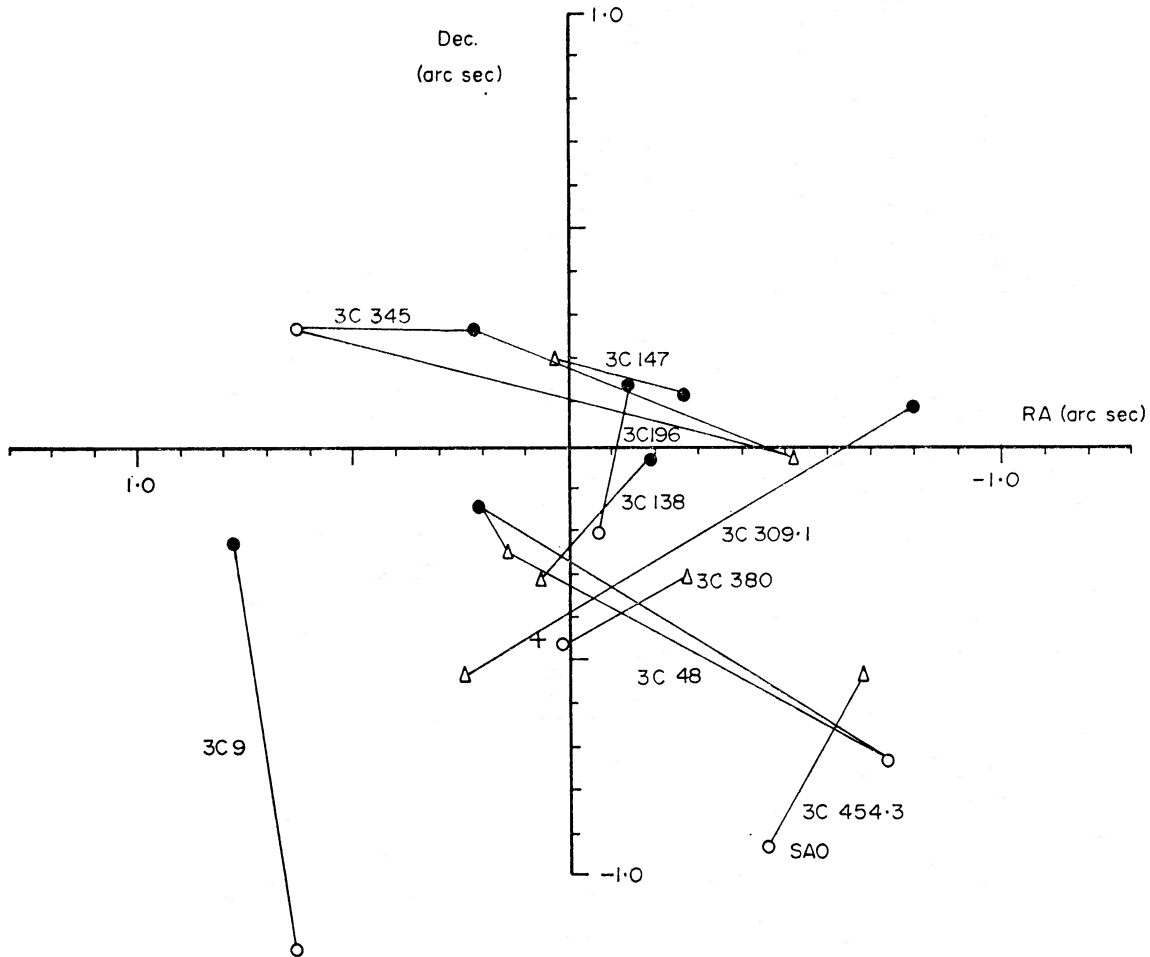


FIG. 2. Comparison of our optical position in Table I with three optical catalogues: Murray *et al.* (1971, filled circles), Barbieri *et al.* (1971, open circles) and Kristian & Sandage (1970, triangles). Ordinates: Catalogue minus Table I, in R.A. Abscissae: the same, in Dec. Upright cross: position for 3C 454.3 measured from Palomar Sky Survey prints (see text).

3C 454.3 again appears low and to the right in the figure, in about the same mean position as the radio object. The displacement on our plates would seem to be connected with the photographic process. 3C 454.3 is not a very suitable object for photographic astrometry. There is an object of comparable brightness very close and to the north. On the POSS prints the images almost merge, that of the QSO having a slightly elongated appearance towards the south. The images

on our plates were quite separate from the object to the north, and less densely exposed than those on the POSS prints. Even so it would seem likely that all photographically determined positions for 3C 454.3 are affected by adjacency effects. To test for this, we measured positions for this object from the POSS prints (O and E), using as comparison frame 12 secondary reference stars with positions determined from our plates. The result is indicated in Fig. 2 by the upright cross. It indicates a southward displacement on the POSS prints relative to our plates. This would explain at least qualitatively the scatter among the optical but not, of course, the radio positions.

6. COMPARISON OF MEAN RADIO AND OPTICAL POSITIONS

We must now consider whether there is any significant difference between the radio and optical positions. If it turns out that there is not, it will be correct to combine the two to give a more accurate mean position.

Differences may be real, for physical reasons, or apparent because of for instance local errors in AGK (in general those optical positions plotted in Fig. 2 have been tied to AGK field by field, whereas the radio have been tied only through a zero-point in R.A.). We have tested for differences by combining the radio positions into means (unweighted) and similarly the optical other than our own. The difference between these means and our position is plotted in Fig. 3(a). Filled circles denote mean radio, open circles mean optical. Our positions are again situated at the origin.

Apart from 3C 454.3, our position in general lies not far from the mean of the radio and optical means ($\lesssim 0.2$ arc sec). This indicates that any real or apparent separation between radio and optical for these particular sources is below our detection limit and hence it is legitimate to combine the two into a single mean which we shall denote by $\langle P \rangle$.

It can be seen from Figs 1 and 2 that the individual radio and optical positions are of approximately equal accuracy, and hence it is adequate to combine them into unweighted means to give $\langle P \rangle$. Each $\langle P \rangle$ is made up of at least two radio and two optical positions but does not include ours. In Fig. 3(b) we compare $\langle P \rangle$ with our position. The error bars signify the internal standard error of $\langle P \rangle$ computed separately for each object from the scatter between the catalogues. Except for 3C 454.3 there are no significant differences from our positions. Omitting 3C 454.3, the r.m.s. difference from our positions is already less than the standard error in $\langle P \rangle$ (~ 0.14 arc sec in each coordinate). To show a significant scatter between our positions and $\langle P \rangle$, the external standard error of the former would need to be significantly larger than that of $\langle P \rangle$, i.e. larger than 0.14 arc sec in each coordinate. Hence the external standard errors given in Table I have not been underestimated.

7. DEPENDENCE ON DECLINATION

A test is shown in Fig. 4 where we compare our declinations in Table I, Col. 11 with Smith's radio declinations (1971). Ordinates are the differences between Smith and us, and abscissae $\tan(\phi - \delta)$ where ϕ is our observatory latitude ($52^\circ 13' N$). A simple $\tan z$ refraction law affecting either set of measurements would show in the diagram as a linear regression.

We have restricted this comparison to Smith's eight calibrators only. He measured these with greater accuracy than his other sources. His results are inde-

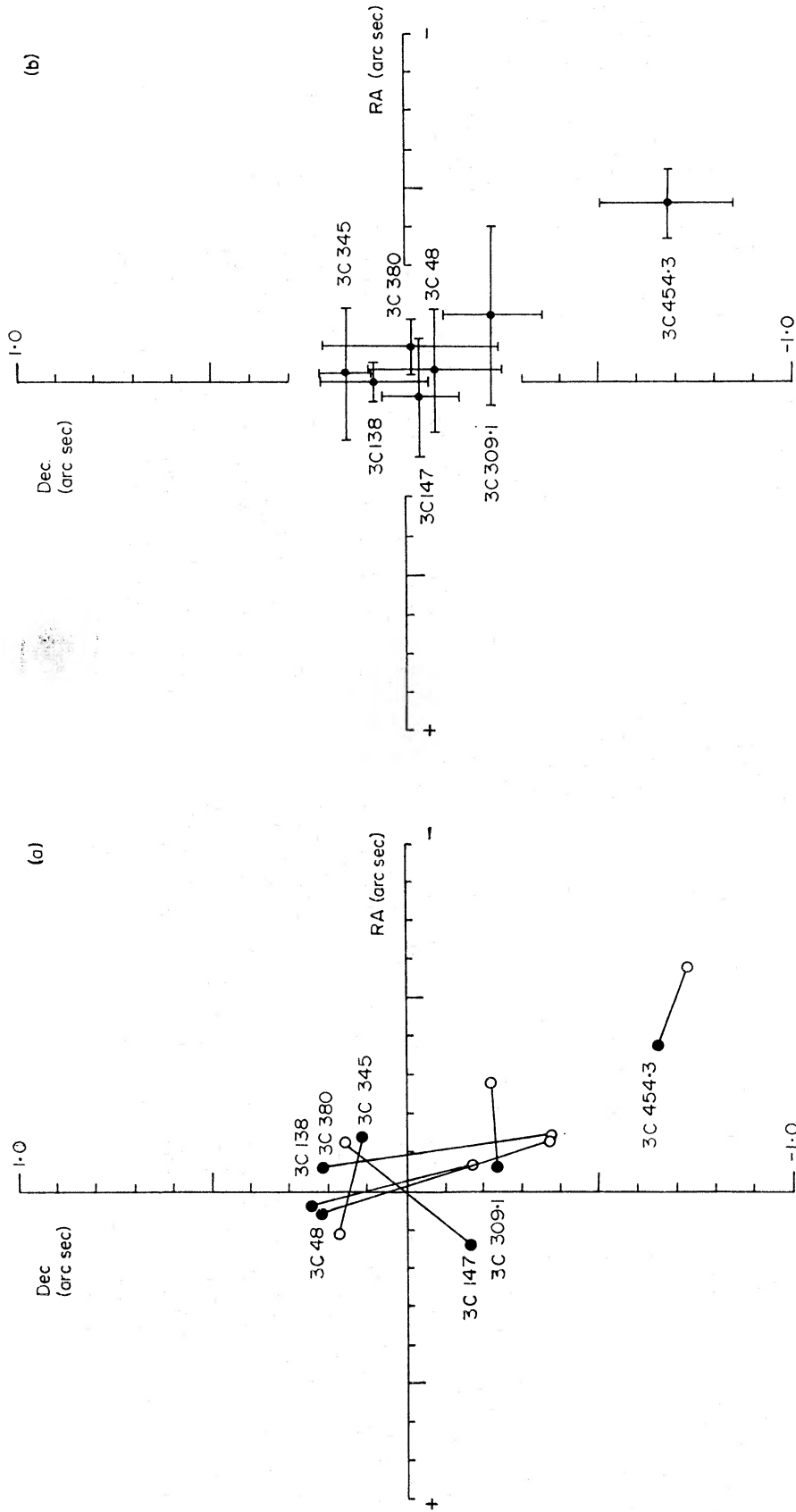


FIG. 3. (a) Comparison of our optical position in Table 1 with the mean of the radio positions plotted in Fig. 1 (filled circles) and with the mean of the optical positions plotted in Fig. 2 (open circles); (b) comparison of our optical position in Table 1 with the mean ($\langle P \rangle$) of the radio and optical positions plotted in Figs 1 and 2. Ordinates: mean minus Table 1, in R.A. Abscissae: the same, in Dec. The scale is the same in both coordinates.

pendent of optically determined declinations, and also of refraction in the Earth's environment (Elsmore & Mackay 1970). They therefore provide a sensitive test for declination dependent effects in the optical results.

We have excluded the catalogue of Adgie *et al.* since it is based on optical calibrators. Wade's catalogue has also been excluded since it contains a declination error of ~ 0.3 arc sec (Kristian & Sandage 1970).

The error bars in Fig. 4 are for ± 1 internal standard error in the mean radio

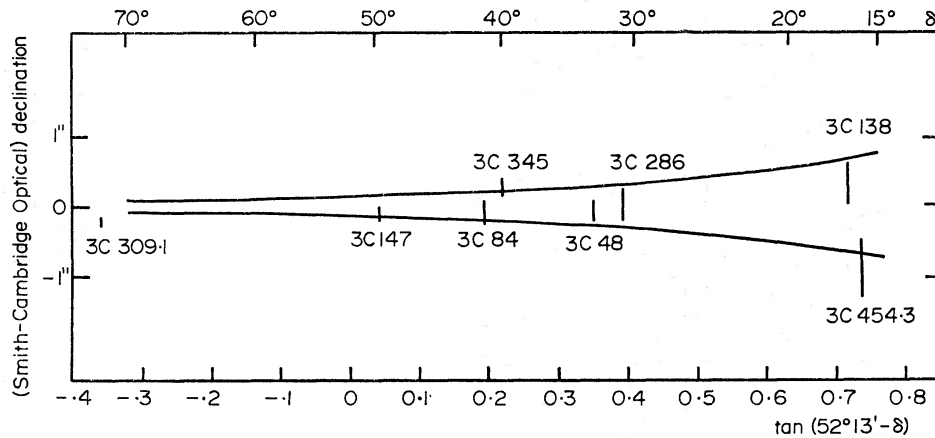


FIG. 4. Comparison of our optical declination in Table I with Smith's radio declination (1971). Ordinates: Difference between radio and optical declination. Abscissae: $\tan(52^\circ 13' - \delta)$. Curves: error limits $\pm 0.2 \cot \delta$ arc sec for Smith's declinations, as set by ± 1.5 mm baseline error. Error bars: ± 1 internal standard error in radio declination (Smith, private communication).

declination, communicated privately by Smith. Our errors are not included. The curves give the limits of error, $\pm 0.2 \cot \delta$, set by an error of ± 1.5 mm in Smith's longest baseline of 1500 m.

In the following discussion we will omit 3C 454.3 for the reason given in Section 5. The remaining points in Fig. 4 give the appearance of a slight trend, upwards towards the right, but statistically this is not significant.

The ordinate y may be regarded as a correction to be added to our measured declination to reproduce Smith's absolute system. A linear regression would be denoted by:

$$y = a + b(x - \langle x \rangle) \quad (7.1)$$

where $x = \tan(\phi - \delta)$. We have seen that a and b are not significantly different from zero. The standard error in the correction y is:

$$\sigma_y = \sqrt{(\sigma_a^2 + (x - \langle x \rangle)^2 \sigma_b^2)} \quad (7.2)$$

where σ_a and σ_b are the standard errors in a and b . We find $\sigma_a = 0.06$, $\sigma_b = 0.18$ and $\langle x \rangle = 0.222$. Hence σ_y has values 0.06 arc sec for 3C 345 at $x \sim \langle x \rangle$, and 0.11 arc sec for 3C 138 at $x = 0.72$. σ_y has been included in the external error in Dec in the last column of Table I. It is the quantity 'Error (c)' in Section 3.

We might have expected optical declination dependent effects to have been caused by atmospheric refraction, telescope flexure and local errors in AGK. With blue sensitive, unfiltered photographic emulsions, QSO's would be expected to

give apparently high measured declinations to the south of the zenith and low to the north, i.e. a trend corresponding to $b < 0$ in (7.1). Telescope flexure would give an effect in the same sense: when the instrument is adjusted so that the pole of the corrector coincides with the centre of curvature of the mirror with the tube pointing vertically upwards, on slewing to the horizontal the pole flexes transversely to a point 0.4 mm below the centre of curvature, leading to a comatic displacement of the photographic image. The maximum displacement in measured position would be 0.2 arc sec for a faint relative to a bright star, in the sense of zenithward displacement of the faint star, the same as for differential refraction of the blue QSO, and increasing with zenith distance. In the present series of observations this effect was reduced by off-setting the centre of curvature of the mirror to give coincidence with the pole of the corrector, not at the zenith but at a point 20° to the south.

Since there is no clear evidence of any such downward trend towards the right in Fig. 4, the effects of refraction and flexure must be small compared with others influencing this plot, that is, refraction and flexure together give an effect less than σ_y in (7.2).

Finally, uncertainties over AGK are included in σ_y on the assumption that Smith's declinations are free from systematic errors.

8. ERRORS

In this section we give an analysis of errors arising at various stages of our measurements.

(a) AGK stars

The r.m.s. difference between measured and catalogue positions for a single image came to 0.31 arc sec in R.A. and 0.26 arc sec in Dec. Although nearly equal, in fact these values are significantly different on account of the large number of measurements from which they have been derived (there were 1284 degrees of freedom). This indicates that there is some additional source of variance present in R.A., either in the catalogue or in our measurements, of amount ± 0.17 arc sec.

In an attempt to examine this further, we have analysed these residuals by the method of variance. Intercomparing different plates of each field, we have found (i) a systematic difference from AGK, peculiar to a given star on all plates of that field, but varying between one star and another, and superimposed on this (ii) for each star a random plate-to-plate variation.

Error (i) might have been due partly to errors in AGK and partly to instrumental effects (e.g. atmospheric dispersion). Its value came to ± 0.22 arc sec in R.A. and ± 0.19 arc sec in Dec. Statistically these two values are not significantly different at the 1 per cent level, nor do they differ significantly from the standard error of AGK 3 which has been given as ± 0.20 arc sec in each coordinate by Dieckvoss (1972, private communication; cf. Dieckvoss 1967). At this point our variance estimates become less reliable because of the reduction in the number of degrees of freedom that results from grouping the plates into fields (giving 226 d.f.). In consequence we cannot use Dieckvoss' estimate of the errors in AGK to divide our Error (i) between AGK and our own instrumental effects. We can however say that our instrumental variance cannot exceed 0.1 arc sec because anything larger would have shown up in the comparison of Error (i) with Dieckvoss'

value at the 1 per cent significance level. This makes it likely that much of the unaccountable ± 0.17 arc sec variance in R.A. is due to AGK.

These conclusions are tentative and can only be strengthened by further data. It would not be justified to delay publication for the sake of elucidating this point.

The random plate-to-plate error (ii) came to ± 0.17 arc sec with no significant difference between R.A. and Dec. This is a precise statement based on 438 degrees of freedom and reinforces our conclusion in the previous paragraph that there is unlikely to have been a significant difference in the behaviour of our instrument between R.A. and Dec.

In passing, we may note that the effect of this random plate-to-plate variance will be reduced by the factor $1/n$ when the number of plates of a field is increased from 1 to n , while the effect of Error (i), the systematic differences from AGK 3, will remain unchanged. Hence the fit to AGK will not be improved for n greater than ~ 4 . Beyond this limit the random errors in AGK become dominant.

Error (ii) contains a component ± 0.09 arc sec due to measuring error, a value determined by remeasuring a plate. As we shall describe below, there are also components ± 0.15 arc sec for photographic noise, ± 0.04 arc sec for fit to AGK ($0.17/\sqrt{23.8}$), and an upper limit of ± 0.08 arc sec for emulsion and elastic shifts. These four components combined would give ± 0.20 arc sec which is not significantly different from the value already given for (ii), viz. 0.17 arc sec. Our figures can only be regarded as giving a rough account of the component variances because the value for photographic noise refers to the secondary reference stars and is probably an overestimate for the brighter AGK stars, while the error of fit refers to the centroid of the distribution of AGK stars measured on the plate and will increase with distance from this centroid. Emulsion and elastic shifts are discussed below.

(b) *Secondary reference stars*

Their positions were determined relative to AGK 3. Intercomparing results for the same group of secondary reference stars in a field, from one plate to another, we derived (iii) a 'between plate' and (iv) a residual variance.

(iii) the 'between plate' variance describes how the secondary reference group as a whole was displaced, relative to the AGK stars, from one plate to another. Its value came to ± 0.10 arc sec, of which ± 0.04 arc sec was attributed to fitting to AGK as before, leaving ± 0.09 arc sec for all other shifts common to the group as a whole. We attribute these to emulsion and elastic shifts and to guiding. Bad guiding may cause an asymmetry in the photographic image, leading to a brightness dependent shift in the measured position. This would cause an apparent displacement in the secondary reference stars as a group, since their range in brightness is small. In a paper on trigonometric parallax (1972) we placed an upper limit of 0.016 arc sec mag^{-1} r.m.s. in R.A. for one plate due to this effect at brightness $m_{\text{pg}} \sim 14^{\text{m}}$. Over the 5^{m} brightness interval between AGK and the secondary reference stars we would therefore expect an upper limit of 0.08 arc sec. This is not significantly different from the quantity we are trying to account for. Hence some or all of the shift of the group may have been due to guiding. The indications in (v) below are that some had been due also to emulsion and elastic effects. In any case, averaged over four plates of a field the effect becomes very small.

The residual variance (iv) came to 0.19 arc sec for a single image. Of this, 0.09 arc sec was attributable to measuring error as before, leaving 0.17 arc sec

which we attribute to photographic granularity. This is the effect of grain on eye estimates of the position of the image. In the trigonometric parallax programme referred to above, we obtained a very similar value (0.15 arc sec) for about the same brightness level. It was also interesting to find in the same programme that the GALAXY automatic measuring machine gave 0.10 arc sec, and reduced the internal measuring error from 0.09 to 0.06 arc sec. Hence the use of GALAXY would lead to some, but not a very large, improvement in accuracy at the secondary reference star level. Possibly the most significant improvement would be at QSO brightness, but we have obtained no figures to confirm this.

(v) Finally, for those QSO's bright enough for measurement on our own plates we obtained by intercomparison an internal standard error of ± 0.25 arc sec in each coordinate for a single image. Allowing 0.09 arc sec for measuring and 0.04 arc sec for fit to AGK as before, there remains 0.23 arc sec for the combined effects of photographic noise, guiding and emulsion and elastic shifts. A guiding shift having an upper limit of ± 0.016 arc sec mag⁻¹, as quoted above, would account for ≤ 0.11 arc sec, leaving ≥ 0.20 arc sec for the remaining effects. Of these, photographic noise at the QSO brightness level would be expected to exceed that found for the secondary reference stars (± 0.17 arc sec) by a substantial margin. It is probably well above the lower limit given, 0.20 arc sec. This implies that our upper limit of 0.11 arc sec for guiding is too high, which in turn implies that the shift of ± 0.09 arc sec derived in (b) above for the secondary reference stars may contain an appreciable component due to emulsion and elastic shifts.

(c) Summary

The variances contributing to a single measured position are set out in Table II. Thus for a single direct measurement of a QSO (not using secondary reference stars) the errors, in arc sec, were: measuring 0.09; fit to AGK 0.04; emulsion and elastic shifts and guiding 0.11; photographic noise 0.20, giving a total of 0.25 arc sec for the internal standard error.

TABLE II
Variances in single hand measured position

Source of variance	Standard error (arc sec)	Remarks
Reference frame:		
Difference from AGK 3	0.22	Value for R.A. for single AGK star
Secondary reference stars:		
Measuring error	0.09	
Fit to AGK	0.04	~ 24 AGK stars
Emulsion and elastic shifts and guiding	0.09	
Photographic noise	0.17	
QSO's:		
Measuring error	0.09	As above
Fit to AGK	0.04	As above
Emulsion and elastic shifts and guiding	0.11	? overestimate
Photographic noise	0.20	? underestimate

9. OUTLOOK

How might the accuracy be improved? Clearly, the initial attack would be on photographic noise. We have seen that GALAXY measurement reduces this effect from 0.17 to 0.10 arc sec for the secondary reference stars, and a similar reduction might be expected at the QSO brightness level, say from 0.20 to 0.12 arc sec. Finer grained emulsions would give an improvement, but only provided that there is an increase in quantum efficiency since it is doubtful whether exposure times could be prolonged to any considerable extent. Another possibility is increased telescope size. Scaling up our instrument by a factor of 2 (maintaining the same f-ratio of 3.8) would reduce the figures given at the end of the previous section for hand measurement to: measuring 0.05; AGK fit 0.04; emulsion and elastic shifts and guiding doubtful, but possibly not much reduced below the previous figure of 0.11; photographic noise 0.10, giving a total of 0.16 arc sec for the internal standard error of a single measurement, compared with 0.25 arc sec for the smaller telescope. Little would be gained by taking a large number of plates of one field because there would still remain a component of external error in linking to AGK. However many plates are measured, there would still remain a difference of 0.22 arc sec r.m.s. from one AGK star (Error (i) in Section 8). For n stars the difference is $0.22/\sqrt{n}$. This is Component (b) of the external error in R.A. given in Section 3 (iv) above. It could be reduced only by increasing n , that is, by widening the field, but there would be limitations set by differential refraction and instrumental effects. Hence an upper limit of ~ 0.04 arc sec might be set by AGK. This is apart from any local systematic effects in this catalogue, about which we are not able to make precise statements.

The standard error for Smith's eight calibrators, to which he devoted most of his effort, was ~ 0.2 arc sec (Smith 1972, private communications). This is of the same order as the external standard errors in our final positions in Table I. The conclusion in the last paragraph of Smith's paper (1971), that 'it is clear that more consistent optical positions will be required before conclusions can be drawn about the physical relationship between optical and radio sources' should not therefore be taken as implying that radio astrometry had outstripped optical. Carefully done, ordinary photographic work could approach quite closely the limit ~ 0.04 arc sec derived in the previous paragraph.

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REFERENCES

- Adgie, R. L., Crowther, J. H. & Gent, H., 1972. *Mon. Not. R. astr. Soc.*, **159**, 233.
 Argue, A. N., 1969. *Observatory*, **89**, 115.
 Argue, A. N. & Kenworthy, C. M., 1970. *Nature*, **228**, 1076.
 Argue, A. N. & Kenworthy, C. M., 1972. *Mon. Not. R. astr. Soc.*, **159**, 31.

- Barbieri, C., Capaccioli, M. & Pinto, G., 1971. *Contr. Obs. Astrofis. Asiago No. 245*.
- Dieckvoss, W., 1967. *Trans. Int. astr. Union, XIII B*, 61.
- Elsmore, B. & Mackay, C. D., 1970. *Mon. Not. R. astr. Soc.*, **146**, 361.
- Kristian, J. & Sandage, A., 1970. *Astrophys. J.*, **162**, 391.
- Murray, C. A., Tucker, R. H. & Clements, E. D., 1971. *R. Obs. Bull. No. 162*.
- Shepherd, W. M., 1953. *Mon. Not. R. astr. Soc.*, **113**, 450.
- Smith, J. W., 1971. *Nature, Phys. Sci.*, **232**, 150.
- Redman, R. O., 1953. *Observatory*, **73**, 180.
- Wade, C. M., 1970. *Astrophys. J.*, **162**, 381.