

## Accurate Positions for Radio Sources

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Fringe-rate residuals from observations with the Goldstack interferometer ( $d/\lambda = 10^8$  at 3.8 cm) have been analyzed. The positions for eleven sources, and the baseline, are obtained to an accuracy of about 0.2 arc sec. Five sources are coincident with their associated optical objects to about 0.2 arc sec.

### INTRODUCTION

The technique of very-long-baseline interferometry (VLB) provides a powerful way to measure the positions of radio sources to high accuracy. In an earlier paper (Cohen and Shaffer 1971) we reported results from observations at 13 cm between California and Australia. The analysis was carried out with the fringe rate and the time delay, which together give intersecting arcs on the sky. The fringe rate arc was the more accurate but we could not use fringe rates alone, because we made only brief observations of each source. The accuracy was typically  $1 \times 3$  arc sec.

The new data derive from observations at 3 cm between California and Massachusetts. A number of sources have been observed over a wide hour-angle range and the fringe rates can be used by themselves. The accuracy is improved by an order of magnitude over that of the earlier method.

### OBSERVATIONS AND ANALYSIS

The observations were made with the Goldstack interferometer, which consists of the 64-m telescope at the NASA Deep Space Network station at Goldstone, California, and the 37-m telescope at Haystack Observatory in Massachusetts. There were three observing sessions, as shown in Table 1. Observations were made with the Mark I VLB

TABLE 1  
Goldstack observing sessions

Day	Date (1971)	UT	$\nu$ (MHz)
1	17/18 Feb.	1600-1600	7840
2	27 Feb.	0000-2400	7840
3	17 Nov.	0100-1145	7850

digital recording system (Clark *et al.* 1968), with a bandwidth of 330 kHz and integration time of 160 sec. Hydrogen-maser frequency standards were used at both stations. Further information on the observations may be found in two papers that contain the fringe amplitude data (Cohen *et al.* 1971, Shaffer *et al.* 1972).

The primary reduction to find correlation coefficients was done with programs prepared by B. G. Clark (day 1) and by A. R. Whitney (days 2 and 3). We found no inconsistencies attributable to the use of two different programs. In most cases the observations spanned 6 to 10 hr and the fringe-rate residuals alone were used to find the baseline and source positions. Relativistic effects were ignored, but the minor corrections for frequency offset and retarded baseline were necessary (Cohen and Shaffer 1971). Each day's set of fringe-rate residuals was fitted with a model containing several adjustable parameters: namely, two atmospheric coefficients, one constant-frequency offset, and 24-hr sine and cosine components for each source. The fitting was done graphically by trial-and-error.

(a) *Atmospheric model.* The ionosphere was ignored: at  $\lambda = 3.5$  cm it has a small effect. The troposphere over each station was assumed spherical with a 7-km scale-height. The equivalent thicknesses at zenith were left as free parameters and found together with the sine and cosine components. The values found are 235, 243, and 263 cm at Haystack and 212, 203, and 218 cm at Goldstone for days 1, 2, and 3 respectively.

(b) *Cosine components (RA).* In the Appendix it is shown that various small terms generate fringe-rate residuals with a 24-hr period. Clock errors, increments in right ascension (RA), and increments in the hour angle of the interferometer ( $h$ ) are

inextricably mixed together. In an attempt to sort them out, it was assumed that  $h$  was constant and that UT1-UTC was exactly given by the *BIH* Circular *D*. Internal consistency was obtained by the requirement that the three sources which were observed twice (see Table 4 and note that OJ 287 does not count because it has larger and uncertain errors) have the same RA, and simultaneously that the three values of  $h$  (Table 2) agree with each other. Very good consistency was obtained in this way. The mean discrepancy in the common RAs is  $0.005^s$  and the three values of  $h$  agree to  $0.001^s$ .

External consistency was then obtained by rotating the coordinate system until the radio RAs agreed, in the mean, with the optical RAs for a selected set of sources. This comparison is discussed later.

(c) *Sine components (Dec)*. Errors in the observing frequency and in the rotation rate of the earth are mixed with increments in declination ( $\delta$ ) and in the equatorial projection of the baseline ( $b_2/c$ ). To separate them, it was assumed that the observing frequency and the rotation rate are known exactly. Internal consistency was then obtained in a manner analogous to that used for the RAs. The declinations of the three sources observed twice were forced to agree, and simultaneously the three values of  $b_2/c$  were forced to agree. In this case, however, offsets in declination are weighted by  $\tan \delta$  (see Appendix), which varies strongly among the sources. The three common sources in Table 4 have values of  $\Delta\delta$  which vary by a factor of 10, but they represent equivalent errors, roughly  $0.05 \cot \delta$  arc sec. In Table 2, the values of  $b_2/c$  have a standard error which is equivalent to  $0.04 \cot \delta$  arc sec. The internal consistency is on the order of that obtained with the RAs, except that at low declinations the error can be very large.

External consistency was again obtained by comparing the radio declinations with *a priori* values. The procedure is to adjust  $b_2/c$  to make  $\Sigma \Delta\delta \tan \delta = 0$ . However, only two sources near the equator, PKS 2134+004 and 3C 273, are useful in this sum because the *a priori* positions of the rest are not accurate enough. This point is discussed further in the Appendix. The reference declination for 3C 273 is the optical value given in Table 5. For PKS 2134+004 results of Adgie, Crowther, and Gent (1972) and of Pauliny-

TABLE 2

Baseline parameters derived from the observations

Day	$b_2/c$ ( $\mu\text{sec}$ )	Hour angle of interferometer ( $^\circ$ )
1	12843.645	177.25987
2	.638	.25985
3	.647	.25986

$b_2$  is the equatorial projection of the baseline and  $c$  the velocity of light.

Toth *et al.* (1972) were used. These positions all come from conventional radio interferometers and agree closely at  $\delta = 00^\circ 28' 26''$  (1950.0).

## RESULTS

(a) *Baseline*. The baseline is the vector between the intersections of the axes of the two telescopes. In Table 2 the second column gives  $b_2/c$ , the light-travel time across the equatorial projection of the baseline of length  $b$ , and the third column gives the Greenwich hour angle of the baseline. If we adopt the standard value for the velocity of light,  $c = 299792.5 \text{ km sec}^{-1}$ , then

$$\langle b_2 \rangle = 3,850,428.0 \text{ m.}$$

The standard error is 1.1 m, but systematic errors may be larger. The final adjustment was made by fitting the measured values of  $\delta$  to *a priori* values, and systematic errors would tend to be thrown into  $b_2$ . This remark is stronger for the hour angle of the interferometer. The three values of  $h$  (Table 2) have a standard error which must be less than systematic errors.

The polar component of the baseline is not determined from a fringe-rate analysis, so that the telescope coordinates cannot be completely determined. However, if we assume the coordinates of one telescope, then  $b_2$  and  $h$  determine the longitude of the other. Table 3 shows a possible solution. The LS 35 values for the coordinates of the 64-m telescope at Goldstone are given; they come from space-craft tracking data. The value derived for the longitude at Haystack (Table 3) is  $0.00015^\circ$  (0.5 arc sec) east of the conventional value, as given in the American Ephemeris and Nautical Almanac (AENA). The disagreement

TABLE 3  
Geocentric coordinates of the two antennae

Station	Radius (km)	Latitude (°)	Longitude (°)
Goldstone	6371.9937	35.24435	116.88954
Haystack	(6368.4997)	(42.43166)	71.48818

is several times larger than typical uncertainties associated with the right ascensions, and is marginally significant. It is of the same order and sign as the discrepancy already noted in JPL *vs.* Smithsonian longitudes (Gaposchkin and Lambeck 1971). The radius and latitude of Haystack are not determined uniquely, but a possible combination is shown in Table 3 in parentheses. These values are in agreement with values in AENA, although the latter are not given to high precision.

The polar position given in Circular *D* of the *BIH* was included in the baseline analysis, although it had only a marginal effect. All values are with respect to the Conventional International Origin for the pole.

(b) *Sources.* The source positions are listed in Table 4. Right ascensions are with respect to the mean of optical positions. Declinations are fitted by *a priori* values for PKS 2134+004 and 3C 273. The errors in Table 4 are presumed to arise in three ways.

1. Noise is measured by the final residuals,

which range from 0.3 to 0.7 mHz (standard error). They represent positional errors smaller than 0.1 arc sec except in the declination of PKS 2134+004, where they are 1.0 arc sec.

2. Insensitivity in the parameter fitting seems to be about 1–2 mHz. This should be independent on the different days and is tested with the internal consistency of the baselines and source positions, as discussed in the preceding section. This consistency is equivalent to about 2 mHz also. The accuracy of a typical measurement is better than this; however, because the consistency discussion is mainly based on total differences and not on rms deviations or some equivalent, we adopt  $\pm 1.5$  mHz as the estimate of the error introduced by ambiguities in the fitting.
3. Errors of measurement or procedure which introduce a consistent bias are unknown. They will mainly affect the baseline, however, because the positions were rotated to agree with optical positions. The largest error of this type might come from the calculation for precession, which was made to the midpoint of the observing span for each source, and not separately for each observation.

The errors shown in Table 4 are given by the

TABLE 4  
Source coordinates derived from observations with the Goldstack interferometer

Source	Day	RA (1950.0)	Error	Dec (1950.0)	Error
3C 84	1	03 <sup>h</sup> 16 <sup>m</sup> 29.571 <sup>s</sup>	$\pm 0.010^s$	41° 19' 51.82"	$\pm 0.15''$
	3	29.565	0.010	51.87	0.15
NRAO 150	3	03 55 45.255	0.010	50 49 20.30	0.15
3C 120	2	04 30 31.604	0.010	05 14 60.49	0.45
	3	31.606	0.010	59.89	0.45
OJ 287	2	08 51 57.267	0.015*	20 17 58.53	0.4*
	3	57.249	0.012*	58.82	0.3*
3C 273	1	12 26 33.242	0.010	02 19 42.6	1.1
3C 274	1	12 28 17.565	0.010	12 40 01.66	0.2
3C 279	1	12 53 35.825	0.010	−05 31 07.65	0.45
3C 345	2	16 41 17.613	0.010	39 54 11.15	0.15
2134+004	2	21 34 05.212	0.010	00 28 28.6	4.4
VRO 42.22.01	1	22 00 39.362	0.010	42 02 08.55	0.15
	3	39.369	0.010*	08.49	0.15*
3C 454.3	2	22 51 29.524	0.010	15 52 54.36	0.2

\* Errors possibly larger.

quadratic combination of 1.5 mHz and the formal standard error of the residuals, or by  $0.010^s$  or  $0.15$  arc sec, whichever is larger. Starred cases in Table 4 have larger, and somewhat unreliable, errors because they were observed over a short hour-angle range.

(c) *Comparison with optical positions.* All the sources in this study are identified with optical objects, but for only a few is the optical position known to a few tenths of a second of arc. Table 5 shows the comparison for these sources; it gives seconds of time for the right ascensions, and seconds of arc for the declinations. The position for 3C 345 is determined from FK4 stars, through the intermediary of several series of fainter stars all measured at one epoch. The position for 3C 273 is the mean of the two independent optical investigations reported by Hazard *et al.* (1971); these were both made from AGK3 stars, which are on the FK4 system. The other determinations in Table 5 are also from AGK3 stars, except 3C 279, which is measured from stars in the Smithsonian catalog. The starred cases were measured by Couper (1972) and  $0.3$  arc sec is 'probably' the 'realistic figure for the overall error'; this includes possible discrepancies between the AGK3 and the FK4.

The radio right ascensions have been rotated to make them agree in the mean with the optical values; i.e.  $\langle \Delta\alpha \rangle = 0$ . The mean absolute discrepancy, however, seems small,  $\langle |\Delta\alpha| \rangle = 0.005^s$ , and is on the order of the discrepancies within the radio data alone. The agreement may well be fortuitous, but it also suggests that the errors in RA are smaller than the values shown in Table 5.

This remark applies mainly to the tabulated optical errors, since they are larger than the radio errors.

The declination fit in Table 5 is worse than the RA fit but is still reasonable, in view of the tabulated errors. The Kristian and Sandage (1970) value for 3C 120 is possibly discordant with Couper's value and with the radio value. In every other case, the positions agree to within the sum of the errors.

Two additional sources, 3C 274 and 3C 454.3, have optical positions known to better than  $0.5$  arc sec (Griffin 1963, Kristian and Sandage 1970). In both cases the radio and optical positions agree within the combined errors.

The radio position for 3C 273 given by Hazard *et al.* (1971) is derived from lunar occultations, and the connection to the FK4 system is less direct than with the optical measurements. For this reason, we prefer to use only their mean optical position in Table 5. However, they did show that the radio and optical objects are coincident to about  $0.2$  arc sec. This is true for the other four objects in Table 5 also. These five objects are widely spread across the sky, and the optical positions presumably have independent errors with respect to the FK4 system. The radio positions therefore are based on the FK4 system also. Since  $\alpha_{\text{radio}} - \alpha_{\text{optical}}$  (Table 5) is less than the tabulated error in RA, we may assume that the errors in RA (Table 4) refer to the FK4 system. This remark may not be true for the declinations, where the scatter is bigger.

## CONCLUSIONS

Fringe-rate analysis of VLB observations gives positions and baselines accurate to  $0.2$  arc sec

TABLE 5  
Comparison of the radio positions with previously published optical positions

Source	$\alpha_{\text{radio}}$	$\alpha_{\text{optical}}$	Errors radio $\times$ optical (sec)	$\delta_{\text{radio}}$	$\delta_{\text{optical}}$	Errors radio $\times$ optical (arc sec)	Reference
3C 120	31.605 <sup>s</sup>	31.605 <sup>s</sup>	$0.01 \times 0.02^*$	60.19"	60.10"	$0.3 \times 0.3^*$	C
		31.60	$0.01 \times 0.015$		58.9	$0.3 \times 0.3$	K
OJ 287	57.258	57.257	$0.012 \times 0.02^*$	58.68	58.22	$0.3 \times 0.3^*$	C
3C 273	33.242	33.239	$0.01 \times 0.01$	42.6	43.28	$1.1 \times 0.1$	H
3C 279	35.825	35.82	$0.01 \times 0.02$	-07.65	-07.65	$0.45 \times 0.25$	K
3C 345	17.613	17.625	$0.01 \times 0.016$	11.15	10.99	$0.15 \times 0.13$	M

References for optical positions: C = Couper 1972; K = Kristian and Sandage 1970, H = Hazard *et al.* 1971, M = Murray *et al.* 1971.

\* Optical error assumed to be  $0.02^s$  or  $0.3$  arc sec, as in Couper (1972).



except in the declination of low-declination objects. The relative positions are as accurate as optical positions measured from AGK3 stars.

The positions for the compact components of 11 radio sources are tabulated. Five of these agree with their associated optical objects to about 0.2 arc sec.

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## APPENDIX

The zero-order expression for the fringe rate is (Cohen and Shaffer 1971)

$$f_0 = -(\Omega v b_2/c) \cos \delta \sin(H-h)$$

where  $\Omega$  = rotation rate of the earth,  $b_2$  = equatorial projection of the baseline,  $v$  = rf frequency,  $c$  = velocity of light,  $\delta$  = declination,  $H$  = hour angle of the source, and  $h$  = hour angle of the interferometer. To first order

$$\begin{aligned} \Delta f = & \{(\Omega v b_2/c) \tan \delta \Delta \delta - \Omega v \Delta(b_2/c) \\ & - (b_2/c) \Delta(\Omega v)\} \cos \delta \sin(H-h) \\ & + (\Omega v b_2/c) \{\Delta(RA) - \Delta(h) \\ & - \Delta(GST)\} \cos \delta \cos(H-h) \end{aligned}$$

where GST = Greenwich sidereal time. In making the analysis, it is assumed that  $\Delta(\Omega v) = \Delta(GST) = 0$ . Corrections to  $\delta$  and  $b_2/c$  are found from the observed sine component, and corrections to RA

and  $h$  are found from the observed cosine component.

In matching the declinations to *a priori* positions it is assumed that the errors for the various sources are independent, and the fit is made with the condition  $\Sigma(\delta_{\text{radio}} - \delta_{\text{a priori}}) \tan \delta = 0$ . This is equivalent to saying that the correction to  $b_2/c$  is calculated from all the final declination offsets, each being weighted by  $\tan \delta$ . In practice, however, only the two sources at low declination have  $\delta_{\text{a priori}}$  insensitive enough to be useful in the sum, and the others are not used. The declinations are to some extent on an independent scale therefore. If the reference position for PKS 2134+004 were to change by 1 arc sec the change in  $b_2$  would be 10 cm, and the change in  $\delta$  for 3C 345 would be only 0.006 arc sec.

## REFERENCES

- Adgie, R. L., Crowther, J. H., and Gent, H., 1972, *Monthly Not. Roy. Astron. Soc.*, in press.
- Clark, B. G., Kellermann, K. I., Bare, C. C., Cohen, M. H., and Jauncey, D. L., 1968, *Astrophys. J.*, **153**, 705.
- Cohen, M. H., Cannon, W., Purcell, G. H., Shaffer, D. B., Broderick, J. J., Kellermann, K. I., and Jauncey, D. I., 1971, *Astrophys. J.*, **170**, 207.
- Cohen, M. H., and Shaffer, D. B., 1971, *Astron. J.*, **76**, 91.
- Couper, H. A., 1972, *Astrophys. Letters*, **10**, 121.
- Gaposchkin, E. M., and Lambeck, K., 1971, *J. Geophys. Res.*, **76**, 4855.
- Griffin, R. F., 1963, *Astron. J.*, **68**, 421.
- Hazard, C., Sutton, J., Argue, A. N., Kenworthy, C. M., Morrison, L. V., and Murray, C. A., 1971, *Nature*, **233**, 89.
- Kristian, J., and Sandage, A., 1970, *Astrophys. J.*, **162**, 391.
- Murray, C. A., Tucker, R. H., and Clements, E. D., 1971, *Roy. Observ. Bull.* No. 162, 215.
- Pauliny-Toth, I. I. K., Kellermann, K. I., Davis, M. M., Fomalont, E. B., and Shaffer, D. B., 1972, *Astron. J.*, **77**, 265.
- Shaffer, D. B., Cohen, M. H., Jauncey, D. L., and Kellermann, K. I., 1972, *Astrophys. J.*, **173**, L147.

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