

CONSERVATION LAWS AND PREFERRED FRAMES IN RELATIVISTIC GRAVITY. I. PREFERRED-FRAME THEORIES AND AN EXTENDED PPN FORMALISM*

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ABSTRACT

We present and discuss a revised version of the Parametrized Post-Newtonian (PPN) Formalism. By regrouping the old PPN parameters, we obtain a new set of parameters which are used to classify theories of gravity according to four attributes: curvature of space-geometry, nonlinearity of gravity, existence of a preferred Universal rest-frame, and validity of conservation laws for momentum. We study a variety of preferred-frame theories of gravity, and show that new terms must be added to the PPN metric in order to analyze such preferred-frame theories from any chosen frame of reference. These new terms depend on the velocity of the observer's chosen frame relative to the preferred Universal rest-frame singled out by such theories.

I. INTRODUCTION AND SUMMARY

Because of the recent interest in high-precision experiments to test gravitational theories, a *theoretical framework* encompassing a very broad and populous class of metric gravitational theories has been developed to systematize the comparison between theory and experiment. This framework has taken the form of a *Parametrized Post-Newtonian (PPN) formalism* (Eddington 1922; Robertson 1962; Schiff 1967; Nordtvedt 1968*b*; Will 1971*a*), which treats the post-Newtonian limits of metric theories of gravity in terms of a series of metric parameters whose values vary from theory to theory. Hand in hand with the PPN metric go equations of motion describing the response of matter to the metric (the usual "divergence of the stress-energy tensor vanishes" for stressed matter, or geodesic motion for test bodies). The PPN formalism has been used to study the "classical tests" of general relativity, to search for new tests (Nordtvedt 1968*a, c*, 1970*a*, 1971*a, b*; Will 1971*b, d*; Thorne, Will, and Ni 1971), and to analyze a wide class of contemporary metric theories of gravity (Nordtvedt 1970*b*; Ni 1972).

In this paper we introduce a revised and, in certain respects, extended version of the PPN formalism. We view the original PPN formalism as having an empirical emphasis or flavor; each parameter was simply a coefficient of a different term in the post-Newtonian metric. In the extended PPN we have found it desirable to regroup the metric parameters. The new parameters (algebraic combinations of old parameters) are now individually related to different physical or conceptual aspects of metric theories. In terms of these new parameters, classification of metric theories of gravity

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becomes simpler and more concise. Every metric theory of gravity¹ can be categorized by four attributes:

i) *Curvature of space-geometry*.—Metric theories of gravity predict that the spatial metrical properties of objects are describable by a non-Euclidean, or “curved” geometry, but differ in their predictions for the strength of this curvature. The amount of *space-curvature* which a standard mass produces in a given theory is measured by a parameter γ .

ii) *Nonlinearity of gravity*.—The superposition law for gravity in most theories is nonlinear (most theories predict that gravity itself also produces gravity), and the parameter β measures the extent of nonlinearity of post-Newtonian gravity. Note that γ and β are the usual Eddington-Robertson-Schiff parameters used to describe the “classical tests” of general relativity.

iii) *Preferred Universal rest-frame*.—Some theories of gravity single out or produce a “preferred” frame, related to the mean rest-frame of the Universe. These theories have recently been shown to predict observable solar-system effects related quantitatively to the solar system’s velocity through this frame (§§ II, III; see also Will 1971*d*, and Paper II in this series). Three parameters α_1 , α_2 , and α_3 suffice to measure the strength and specific manner in which motion in the preferred frame affects the post-Newtonian metric. All α ’s are zero in theories which have no such “preferred-frame” effects (such as general relativity).

iv) *Conservation laws for energy-momentum*.—Some metric gravitational theories do not possess a complete set of conserved energy-momentum quantities for isolated gravitating systems. A set of parameters ζ_i measures the effects resulting from a breakdown of conservation laws. For point-mass gravitational systems, two parameters ζ_1 and ζ_2 suffice in the post-Newtonian metric; for perfect-fluid systems, four are needed,

TABLE 1
THE NEW PPN PARAMETERS AND THEIR INTERPRETATION*

PARAMETER	WHAT IT MEASURES, RELATIVE TO GENERAL RELATIVITY	VALUE IN GENERAL RELATIVITY	EQUIVALENT PARAMETER EXPRESSIONS IN THE OLD FORMALISM†	
			a) Nordtvedt	b) Will
γ	How much space-curvature is produced by unit rest mass ($g_{\alpha\beta}$)?	1	γ	γ
β	How much nonlinearity is there in the superposition law for gravity (g_{00})?	1	β	β
α_1	To what extent and in what manner does the theory	0	$8\Delta - 4\gamma - 4$	$7\Delta_1 + \Delta_2 - 4\gamma - 4$
α_2	single out a preferred	0	$\alpha'' - 1$	$\Delta_2 + \zeta - 1$
α_3	Universal rest-frame?	0	$4\alpha'' - \alpha''' - 2\gamma - 1$	$4\beta_1 - 2\gamma - 2 - \zeta$
ζ_1	How much and what kind of	0	$\alpha''' - \chi$	ζ
ζ_2	violation of conservation of	0	$2\beta - \alpha' - 1$	$2\beta + 2\beta_2 - 3\gamma - 1$
ζ_3	total momentum does the	0	Absent	$\beta_3 - 1$
ζ_4	theory predict?	0	Absent	$\beta_4 - \gamma$

* Adapted from Thorne, Will, and Ni (1971).

† The metric in the old formalism was valid only in a coordinate frame at rest in the Universe rest-frame (see § II).

¹ Exception: Whitehead’s (1922) theory is too complex to fit into the PPN formalism. However, it has been shown to disagree violently with gravimeter measurements of the tides of the solid Earth (Will 1971*d*), and hence is no longer a viable theory.

TABLE 2
METRIC THEORIES OF GRAVITY AND THEIR PPN PARAMETER VALUES*

Theory and Its Parameters†	γ	β	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
General relativity.....	1	1	0	0	0	0	0	0	0
Scalar-tensor theories‡ (ω, Λ).....	$(1 + \omega)/(2 + \omega)$	$1 + \Lambda$	0	0	0	0	0	0	0
Vector-metric theory (K)	1	1	0	$K^2/(1 + \frac{1}{2}K^2)$	0	0	0	0	0
(§ IIIb; Appendix).....									
Stratified theories:									
a) Page and Tupper (1968) (a, c)	a	$1 + c$	$-4(1 + a)$	0	$-2(1 + a)$	0	$1 + a + 2c$	0	$-a$ or $-(1 + a)$
b) Yilmaz (1958, 1962).....	1	1	-8	0	-4	0	-2	0	-1 or -2§
c) Papapetrou (1954a, b, c).....	1	1	-8	-4	0	0	0	0	0
d) Lagrangian stratified theory (Ni 1972).....	1	1	-8	0	0	0	0	0	0
e) General stratified theory (Ni 1972) (p, q).....	1	$1 - q$	-8	0	-4	0	$p - 2q - 2$	0	-1 or -2§
f) Coleman (1971) (p).....	1	1	-8	0	-4	0	$p - 2$	0	-1 or -2§
g) Rosen (1971a, b) (λ).....	λ	$\frac{1}{2}(3 + \lambda)$	$-4(1 + \lambda)$	0	0	0	0	0	0

* See Ni (1972) for detailed discussion of all the theories in this table except the vector-metric theory (see § IIIb, Appendix), and Coleman's (1971) theory.

† Several of these theories contain adjustable parameters whose values may be fixed either by fitting to experimental results or by appealing to cosmological or philosophical arguments.

‡ Brans-Dicke Theory is the special case $\Lambda = 0$.

§ The value of ζ_4 for many theories depends on what matter density one chooses as source for the gravitational fields: $\rho = T_{ii}u^i u^i$ = component of stress-energy tensor along four-velocity of matter [$\zeta_4 = -\gamma$]; or $\rho = \text{trace}(T_{ii})$ [$\zeta_4 = -(1 + \gamma)$]. See Ni (1972) for discussion of this point.

ζ_1 , ζ_2 , ζ_3 , and ζ_4 . More complicated models for matter may require additional ζ 's to specify the complete post-Newtonian metric. Theories which do possess conservation laws (such as general relativity) predict that all ζ 's should be zero (see Will 1971c for a discussion of conservation laws using the original PPN formalism).

Table 1 presents a summary of this new version of the PPN formalism. Each of the new parameters has an equivalent expression in the old formalisms of Nordtvedt (1968b) and Will (1971a), and these expressions are given in this table.

Table 2 lists the values of these parameters for all viable metric theories which have been analyzed to date (Ni 1972) and for one member of a new class of metric theories which are presented in this paper for the first time (§ III).

In table 3 the parameter dependences of a variety of experimental tests are presented using the new PPN formalism. Some of the effects listed in table 3 are new and are discussed in Paper II of this series.

This then, is the new PPN formalism in summary, and the remainder of this paper is devoted to details. In § II we present and discuss the metric for the extended PPN formalism. This metric contains additional metric terms which are functions of the velocity of the observer's chosen coordinate system relative to some "preferred Universal rest-frame," and we show that these additional terms are necessary in order to handle "preferred-frame" theories of gravity from any frame of reference more easily and consistently. Section III contains detailed discussions of two classes of "preferred-frame" theories: (a) theories which introduce the preferred frame as a basic postulate, called "stratified" theories (see Ni 1972), and (b) theories in which the preferred frame is established by cosmological fields (vector fields or second tensor fields) which are determined by the Universe as a whole. Concluding remarks are presented in § IV. An Appendix gives detailed calculation of the PPN metric of a vector-metric theory as an example of (b) above.

TABLE 3
PPN PARAMETER DEPENDENCE OF VARIOUS EXPERIMENTAL TESTS

Experimental Test or Observable Effect	PPN Parameter Dependence
Light deflection, radar time delay.....	$\frac{1}{2}(1 + \gamma)$
Perihelion shift:	
a) Classical.....	$\frac{1}{3}(2 + 2\gamma - \beta)$
b) Due to motion through Universal rest-frame*	$\alpha_1, \alpha_2, \alpha_3$
Geodetic precession of a gyroscope.....	$\frac{1}{3}(1 + 2\gamma)$
Dragging of inertial frames.....	$\frac{1}{2}(1 + \gamma + \frac{1}{4}\alpha_1)$
Equivalence-principle breakdown ($m_{\text{passive}} \neq m_{\text{inertial}}$):	
a) Isotropic.....	$-(4\beta - \gamma - 3 - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2)$
b) Anisotropic.....	$-(\alpha_2 + \zeta_2 - \zeta_1)$
Perturbations in Earthbound gravimeter measurements:	
a) Variation in G	
i) Due to field of Sun and planets.....	$4\beta - \gamma - 3 - \zeta_2$
ii) Due to motion through Universal rest-frame†.....	$\alpha_2 + \alpha_3 - \alpha_1$
iii) Anisotropy due to motion through rest-frame†.....	α_2
b) Other effects:	
i) Due to external field gradients.....	α_1
ii) Due to internal structure of the Earth.....	$4\beta - \gamma - 3 - \alpha_2 + \zeta_1 - 2\zeta_2$
Periodic perturbations in orbits due to motion through Universe rest-frame*.....	$\alpha_1, \alpha_2, \alpha_3$

* See Nordtvedt and Will (1972) (Paper II).

† See Will (1971d), Nordtvedt and Will (1972) (Paper II).

II. THE EXTENDED PPN FORMALISM

a) *Key Features of the Old Formalism*

We first outline PPN formalism as described in Nordtvedt (1968b) and Will (1971a). The PPN framework has the following key features.

i) *Coordinate System*

The formalism uses a coordinate system in which, at large distances from the matter, the metric becomes nearly Minkowskian.

ii) *Matter Variables*

Gravity is produced by matter and energy, and the various matter variables which enter into the metric are: masses of particles (m_i), rest-mass densities of fluid elements (ρ), specific internal energies (Π), and pressures (p), as measured in local inertial frames comoving with the matter. The metric also contains positions and velocities (v) of elements of matter as measured *relative to the coordinate system*.

iii) *PPN Parameters*²

The PPN parameters are γ , β , α_1 , α_2 , α_3 , ζ_1 , ζ_2 , ζ_3 , ζ_4 (see table 1).

iv) *Metric of the Old PPN Formalism*³A. *Perfect-fluid metric:*

$$\begin{aligned} g_{00} &= 1 - 2U + 2\beta U^2 - (2\gamma + 2 + \alpha_3 + \zeta_1)\Phi_1 \\ &\quad - 2[(3\gamma + 1 - 2\beta + \zeta_2)\Phi_2 + (1 + \zeta_3)\Phi_3 + 3(\gamma + \zeta_4)\Phi_4] + \zeta_1 \mathcal{A}, \\ g_{0\alpha} &= \frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1)V_\alpha + \frac{1}{2}(1 + \alpha_2 - \zeta_1)W_\alpha, \\ g_{\alpha\beta} &= -(1 + 2\gamma U)\delta_{\alpha\beta}. \end{aligned} \quad (1)$$

Here U is the Newtonian gravitational potential given by

$$U(\mathbf{x}, t) = \int \frac{\rho(\mathbf{x}', t)d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|}; \quad (2)$$

the other potentials appearing in equations (1) are

$$\begin{aligned} \Phi_1 &= \int \frac{\rho' v'^2 d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|}, & \Phi_2 &= \int \frac{\rho' U' d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|}, \\ \Phi_3 &= \int \frac{\rho' \Pi' d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|}, & \Phi_4 &= \int \frac{p' d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|}, \\ \mathcal{A} &= \int \frac{\rho' [\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')]^2 d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3}, \\ V_\alpha &= \int \frac{\rho' v'_\alpha d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|}, & W_\alpha &= \int \frac{\rho' \mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}') (x - x')_\alpha d\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3}. \end{aligned} \quad (3)$$

² Each of the authors of this paper originally used his own set of PPN parameters. By using the parameters given in table 1, we have united our separate notations, in addition to using parameters with more conceptual significance. Therefore, here we use the "new" parameters to review the old formalism.

³ In this series of papers, Greek indices will take the values 1, 2, and 3; Roman indices will take the values 0, 1, 2, 3; and summation over repeated indices will be employed. Commas will denote partial differentiation, and semicolons will denote covariant differentiation. We will use units for which the velocity of light is unity and the Newtonian gravitational constant in the outer regions of the solar system, as measured today in the rest frame of the Universe, is unity.

The amount of rest-mass contained in a proper column $d\mathcal{V}$ of fluid is given by (see Will 1971a for discussion)

$$\rho d\mathcal{V} = \rho(1 + \frac{1}{2}v^2 + 3\gamma U)dx. \quad (4)$$

B. Point-mass metric:

$$\begin{aligned} g_{00} = & 1 - 2 \sum_k \frac{m_k}{r_k} + 2\beta \left(\sum_k \frac{m_k}{r_k} \right)^2 - (2\gamma + 1 + \alpha_3 + \zeta_1) \sum_k \frac{m_k v_k^2}{r_k} \\ & - 2(1 - 2\beta + \zeta_2) \sum_k \frac{m_k}{r_k} \sum_{j \neq k} \frac{m_j}{r_{jk}} + \zeta_1 \sum_k \frac{m_k}{r_k^3} (\mathbf{v}_k \cdot \mathbf{r}_k)^2, \\ g_{0\alpha} = & \frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1) \sum_k \frac{m_k}{r_k} v_k^\alpha + \frac{1}{2}(1 + \alpha_2 - \zeta_1) \sum_k \frac{m_k}{r_k^3} (\mathbf{v}_k \cdot \mathbf{r}_k) r_k^\alpha, \\ g_{\alpha\beta} = & - \left(1 + 2\gamma \sum_k \frac{m_k}{r_k} \right) \delta_{\alpha\beta}. \end{aligned} \quad (5)$$

Here $\mathbf{r}_k = \mathbf{x} - \mathbf{x}_k$, $\mathbf{r}_{ik} = \mathbf{x}_i - \mathbf{x}_k$, $r_k = |\mathbf{r}_k|$, and

$$m_k = \int_{k\text{th body}} \rho d\mathcal{V}. \quad (6)$$

v) *Choice of Gauge*

An infinitesimal coordinate transformation

$$(x^i)^\dagger = x^i + \delta x^i, \quad (7)$$

changes the metric to

$$(g_{ij})^\dagger = g_{ij} - \delta x_{i;j} - \delta x_{j;i}. \quad (8)$$

We have chosen $\delta x^1, \delta x^2, \delta x^3$ in order to make the spatial part of the metric ($g_{\alpha\beta}$) diagonal and isotropic. We have chosen δx^0 of the form

$$\delta x^0 = \lambda \chi_{,0}, \quad (9)$$

where λ is an adjustable parameter, and where

$$\begin{aligned} \chi(\mathbf{x}, t) = & - \int \rho(\mathbf{x}', t) |\mathbf{x} - \mathbf{x}'| d\mathbf{x}' \quad [\text{fluid}], \\ = & - \sum_k m_k |\mathbf{x} - \mathbf{x}_k| \quad [\text{point mass}], \end{aligned} \quad (10)$$

in order that the metric component g_{00} contain no term of the form

$$\begin{aligned} \mathcal{B}(\mathbf{x}, t) = & \int \int \frac{\rho' \rho'' (\mathbf{x} - \mathbf{x}') \cdot (\mathbf{x}' - \mathbf{x}'')}{|\mathbf{x} - \mathbf{x}'| |\mathbf{x}' - \mathbf{x}''|^3} d\mathbf{x}' d\mathbf{x}'' \quad [\text{fluid}], \\ = & - \sum_k \frac{m_k \mathbf{r}_k \cdot d\mathbf{v}_k / dt}{r_k} \quad [\text{point mass}]. \end{aligned} \quad (11)$$

b) Lorentz Transformations and Preferred Frames

Recent theoretical evidence has led us to conclude that *the PPN metric in the form of equations (1) and (5) is valid for any metric theory of gravity only in a coordinate system which is at rest relative to a "preferred Universal rest-frame."* In fact, we will

show (§ IIc) that *there is a more general PPN metric which is valid in any chosen coordinate frame which moves at velocity $\mathbf{w}$ relative to the "Universal rest-frame,"* and that the old PPN metric is a special case of this general metric corresponding to $\mathbf{w} = [\text{speed of chosen frame relative to Universe}] = 0$. Three pieces of evidence support this conclusion:

i) A Lorentz transformation of the old PPN metric from one coordinate frame to another coordinate frame whose outer region moves with velocity $\mathbf{w}$ ($|\mathbf{w}|$ assumed small) relative to the first does not always yield the PPN metric in its initial form but introduces new metric functions which depend on $\mathbf{w}$. Only when the PPN parameters satisfy the conditions

$$\alpha_1 = 0, \quad \alpha_2 = 0, \quad \alpha_3 = 0, \quad (12)$$

do *no* such $\mathbf{w}$ -terms arise (Will 1971c; Nordtvedt 1969). (More precisely, when $\alpha_1 = \alpha_2 = \alpha_3 = 0$, any $\mathbf{w}$ -terms which do arise have no physical meaning; they can always be removed by an infinitesimal gauge transformation. See § IIc for details.)

ii) A calculation by one of us (Will 1971d) showed that the Newtonian gravitational constant G as measured by a Cavendish experiment on Earth should depend on the Earth's velocity relative to some "preferred frame," for any theory whose PPN parameters violated equations (12).

iii) A compilation and study by Ni (1972) of contemporary metric theories of gravity gives several specific examples of theories which single out such a "preferred Universal rest-frame." These theories all predict that at least one of the α -parameters should be nonzero. We will discuss these theories in more detail in § IIIa.

There is additional motivation for using a more general PPN metric which is valid in any coordinate frame. Many computations of solar-system effects are simplified by working in the rest frame of the system being considered. Also, comparison of experimental results with the predictions of theory becomes more straightforward when the experimental data and the theoretical predictions are analyzed in the same coordinate system. *Therefore, for use in any frame moving at speed $\mathbf{w}$ relative to the Universal rest-frame, we will adopt a more general PPN metric which contains additional metric terms depending on $\mathbf{w}$.* Just what these additional terms are is spelled out in the next subsection.

c) The General PPN Metric

We take as our starting point the PPN metric valid in the mean rest-frame of the Universe (eq. [1] or [5]). To obtain the form of the metric valid in a frame moving with velocity $\mathbf{w}$ relative to this frame, we apply a Lorentz transformation to the metric, assuming throughout that $\mathbf{w}$ is small, so that our post-Newtonian expansion is still valid.⁴ The transformation from the Universal rest-frame coordinates $(\mathbf{x}, t)$ to the moving coordinates $(\boldsymbol{\xi}, \tau)$ has the form (to post-Newtonian order):

$$\begin{aligned} \tau &= t(1 + \tfrac{1}{2}w^2 + \tfrac{3}{8}w^4) - \mathbf{x} \cdot \mathbf{w}(1 + \tfrac{1}{2}w^2) + O(5) \times t, \\ \boldsymbol{\xi} &= \mathbf{x} - (1 + \tfrac{1}{2}w^2)\mathbf{w}t + \tfrac{1}{2}(\mathbf{x} \cdot \mathbf{w})\mathbf{w} + O(4) \times \mathbf{x}. \end{aligned} \quad (13)$$

The matter velocity in the two frames, $\mathbf{v} = d\mathbf{x}/dt$ and $\boldsymbol{\nu} = d\boldsymbol{\xi}/d\tau$, are related by

$$\boldsymbol{\nu} = \mathbf{v} - \mathbf{w} + O(3). \quad (14)$$

⁴ The post-Newtonian metric is an "expansion" in terms of small dimensionless quantities which characterize the gravitating system: Newtonian gravitational potential $U \sim m_k/r_k$, velocity v^2 , specific internal energy Π , pressure relative to density p/ρ , all of which are less than 10^{-5} in the solar system. In order to simplify the "bookkeeping" of the terms in the metric, one assigns them "orders of smallness" (Chandrasekhar 1965). In our notation $U \sim m_k/r_k$ is $O(2)$, v is $O(1)$, v^2 is $O(2)$, and so on. We will also assume that $\mathbf{w}$ is $O(1)$.

In order to express the metric functions in terms of (ξ, τ) and $\mathbf{v}$ we make use of the expression given by Chandrasekhar and Contopoulos (1967):

$$\frac{1}{|\xi - \xi'|} = \frac{1}{|x - x'|} [1 + \frac{1}{2}(\mathbf{n}' \cdot \mathbf{w})^2 - (\mathbf{n}' \cdot \mathbf{w})(\mathbf{n}' \cdot \mathbf{v}) + O(4)], \quad (15)$$

where

$$\mathbf{n}' = (\mathbf{x} - \mathbf{x}')/|\mathbf{x} - \mathbf{x}'|.$$

We use the standard transformation law ($x^0 = t$, $\xi^0 = \tau$),

$$g_{ij}(\xi, \tau) = \frac{\partial x^k}{\partial \xi^i} \frac{\partial x^l}{\partial \xi^j} g_{kl}(x, t), \quad (16)$$

along with equations (1), (4), (13), (14), and (15) to obtain a metric which has the same functional form (in terms of ξ , τ , $\mathbf{v}$, etc.) as the old metric (in terms of x , t , $\mathbf{v}$, etc.) except for additional terms which depend on $\mathbf{w}$ (we quote only the perfect-fluid case; an equivalent result is obtained using the point-mass metric eq. [5]):

$$\begin{aligned} \delta g_{00} &= (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - (2\alpha_3 - \alpha_1 + \alpha_2 + \zeta_1 - 1)w^\alpha V_\alpha \\ &\quad + (\alpha_2 + \zeta_1 - 1)w^\alpha W_\alpha + \alpha_2 w^\alpha w^\beta U_{\alpha\beta}, \\ \delta g_{0\alpha} &= \frac{1}{2}(\alpha_1 - \alpha_2 + \zeta_1 - 1)w^\alpha U + \frac{1}{2}(\alpha_2 - \zeta_1 + 1)w^\beta U_{\alpha\beta}, \\ \delta g_{\alpha\beta} &= 0, \end{aligned} \quad (17)$$

where the potential $U_{\alpha\beta}$ is given by (cf. eq. [10])

$$\begin{aligned} U_{\alpha\beta} &= \chi_{,\alpha\beta} + \delta_{\alpha\beta} U, \\ &= \int \frac{\rho(\xi', \tau)(\xi - \xi')_\alpha (\xi - \xi')_\beta}{|\xi - \xi'|^3} d\xi', \quad [\text{fluid}] \\ &= \sum_k \frac{m_k r_k^\alpha r_k^\beta}{r_k^3}. \quad [\text{point mass}] \end{aligned} \quad (18)$$

The introduction of the new variable $\mathbf{w}$ in the PPN metric also introduces an additional gauge freedom. By making a coordinate transformation of the form

$$\begin{aligned} \tau^\dagger &= \tau - \frac{1}{2}(\alpha_2 + \zeta_1 - 1)w^\alpha \chi_{,\alpha}, \\ \xi^\dagger &= \xi, \end{aligned} \quad (19)$$

and using equation (8), we can transform the metric into a gauge in which no term of the form $w^\alpha W_\alpha$ appears. Note that this gauge transformation is completely independent of the gauge change (eq. [9]) which led to the form of the old PPN metric in equation (1). The final result for the new PPN metric for perfect fluids and for point masses, is given in table 4.

Table 4 shows clearly the usefulness of the new parameters α_1 , α_2 , and α_3 . These parameters label the terms in the metric which depend on $\mathbf{w}$ and indicate, for a given theory, *how large and what kind of solar-system effects will be produced by motion relative to the Universal rest-frame*. Paper II in this series (Nordtvedt and Will 1972) will catalog such solar-system effects and present evidence to put experimental limits on the α 's, thereby ruling out many theories of gravity which were thought at one time to be viable.

We close this section with a recipe for computing the PPN parameter values for any metric theory of gravity:

- i) Pick a coordinate system whose metric becomes Minkowskian far from the local

TABLE 4
THE PPN METRIC VALID IN ANY FRAME

Perfect-Fluid Metric
$g_{00} = 1 - 2U + 2\beta U^2 - (2\gamma + 2 + \alpha_3 + \zeta_1)\Phi_1 + \zeta_1 \mathcal{A}$ $- 2[(3\gamma + 1 - 2\beta + \zeta_2)\Phi_2 + (1 + \zeta_3)\Phi_3 + 3(\gamma + \zeta_4)\Phi_4]$ $+ (\alpha_1 - \alpha_2 - \alpha_3)w^2 U + \alpha_2 w^\alpha w^\beta U_{\alpha\beta} - (2\alpha_3 - \alpha_1)w^\alpha V_\alpha,$ $g_{0\alpha} = \frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1)V_\alpha + \frac{1}{2}(1 + \alpha_2 - \zeta_1)W_\alpha$ $+ \frac{1}{2}(\alpha_1 - 2\alpha_2)w^\alpha U + \alpha_2 w^\beta U_{\alpha\beta},$ $g_{\alpha\beta} = -(1 + 2\gamma U)\delta_{\alpha\beta}.$
Point-Mass Metric
$g_{00} = 1 - 2 \sum_k \frac{m_k}{r_k} + 2\beta \left(\sum_k \frac{m_k}{r_k} \right)^2 - 2(1 - 2\beta + \zeta_2) \sum_k \frac{m_k}{r_k} \sum_{j \neq k} \frac{m_j}{r_{jk}}$ $- (2\gamma + 1 + \alpha_3 + \zeta_1) \sum_k \frac{m_k v_k^2}{r_k} + \zeta_1 \sum_k \frac{m_k}{r_k^3} (v_k \cdot r_k)^2$ $+ (\alpha_1 - \alpha_2 - \alpha_3)w^2 \sum_k \frac{m_k}{r_k^3} + \alpha_2 \sum_k \frac{m_k}{r_k} (w \cdot r_k)^2 - (2\alpha_3 - \alpha_1) \sum_k \frac{m_k w \cdot v_k}{r_k},$ $g_{0\alpha} = \frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1) \sum_k \frac{m_k v_k^\alpha}{r_k} + \frac{1}{2}(1 + \alpha_2 - \zeta_1) \sum_k \frac{m_k}{r_k^3} (v_k \cdot r_k) r_k^\alpha$ $+ \frac{1}{2}(\alpha_1 - 2\alpha_2)w^\alpha \sum_k \frac{m_k}{r_k} + \alpha_2 \sum_k \frac{m_k}{r_k^3} (w \cdot r_k) r_k^\alpha,$ $g_{\alpha\beta} = - \left(1 + 2\gamma \sum_k \frac{m_k}{r_k} \right) \delta_{\alpha\beta}.$

distribution of matter. It does not matter whether the coordinate system is moving or at rest relative to the Universe, although for many theories (cf. § III, and Appendix) *calculations are simplified* if the “rest” coordinate frame of the Universe is used.

ii) Use the standard techniques of Chandrasekhar (1965) or Einstein, Infeld, and Hoffman (1938) to compute the post-Newtonian metric of the matter in this coordinate system.

iii) Compare this metric term by term with the equations in table 4 and read off the values of the PPN parameters. Even if you used the “rest” coordinate frame, in which $w = 0$, you will still be able to obtain values for all the PPN parameters.

iv) To obtain the metric valid in any chosen frame, use the equations in table 4, along with the PPN parameter values just computed, the velocity w of the chosen frame relative to the Universe, and the velocities v of matter relative to the chosen frame (the other variables—mass, density, etc.—are all determined in frames comoving with each element of matter and are unaffected by this change in coordinate frame).

III. THEORIES OF GRAVITY WITH A PREFERRED UNIVERSAL REST-FRAME

There are a variety of gravitational theories which single out some “preferred frame” in the Universe, and which have been shown to predict observable effects due to motion through this frame. These theories fall into two classes. (a) Theories which postulate the existence of a preferred frame directly and incorporate it into their calculational rules. These theories endow the Universe with “prior geometry,” and use

this prior-geometric structure as a foundation for the theory.⁵ In § IIIa we discuss a particularly simple subclass of these theories, known as “stratified theories with time-orthogonal conformally flat space slices” (Ni 1972). (b) Theories which assume *no* “prior geometry,” but which possess, in addition to the metric, vector or tensor cosmological fields whose source is the matter in the Universe. In these theories, the particular structure of the vector and tensor fields determines (in a Machian sense) the “preferred frame.” These theories are examined in § IIIb and in an Appendix.

a) *Stratified Theories With Time-Orthogonal Conformally Flat Space Slices*

This is a particularly simple subclass of theories of gravity which postulate a “prior geometry.” These theories are devised using the following prescription:⁶ The Universe’s large-scale distribution of matter determines a preferred reference frame whose space slices (“strata”) are conformally flat, although the full spacetime is not. In this preferred frame, the metric has the form

$$ds^2 = e^{2f(\varphi)} dt^2 - e^{2g(\varphi)} (dx^2 + dy^2 + dz^2), \quad (20)$$

where φ is a scalar field. In geometric, coordinate-free language, such theories have (i) a background, flat metric η ; (ii) a Universal time coordinate t (a scalar field) which is covariantly constant and has timelike gradients with respect to η ; (iii) a scalar gravitational field φ ; and (iv) a metric g constructed from η , t , and φ by

$$g = e^{2g(\varphi)} \eta + [e^{2f(\varphi)} - e^{2g(\varphi)}] dt \otimes dt. \quad (21)$$

These theories differ from one another by their choice of the functions $f(\varphi)$ and $g(\varphi)$ and by their field equations for φ .

Ni (1972) has analyzed theories of this type, whose authors include Papapetrou (1954a, b, c), Yilmaz (1958, 1962), Page and Tupper (1968), and Rosen (1971a, b). Two additional theories were devised by Ni himself: a Lagrangian stratified theory and a general stratified theory (Ni 1972). A special case of Ni’s “general stratified theory” is Coleman’s (1971) theory. Einstein (1912) (*not* general relativity!) and Whitrow and Morduch (1960, 1965) devised stratified theories of gravity, but their theories disagree violently with light-deflection and time-delay experiments, and so will not be considered further.

We will not discuss any of these theories individually; the reader is referred to Ni (1972) for details. We will only present Ni’s results, in the form of PPN parameter values for each theory (table 2). For completeness, table 2 also gives the PPN parameter values for general relativity and for scalar-tensor theories (generalizations of Brans-Dicke-Jordan theory; see Ni 1972 for details and references). We conclude this subsection with a theorem concerning the PPN parameter α_1 for stratified theories:

In every stratified theory of gravity with time-orthogonal, conformally flat space slices, the PPN parameter α_1 has the value

$$\alpha_1 = -4(1 + \gamma). \quad (22)$$

In order to agree with light-deflection and radar time-delay experiments ($\gamma \sim 1$; see Thorne *et al.* 1971 for a review), these theories must therefore have

$$\alpha_1 \sim -8. \quad (23)$$

Note that all the stratified theories in table 2 obey equation (22). In Paper II of this series, we will make use of this result to show that *all the stratified theories in table 2 disagree violently with experiment.*

⁵ For a detailed discussion of “prior-geometry” see Misner, Thorne, and Wheeler (1972).

⁶ This discussion is adapted from Ni (1972).

The proof of this theorem goes as follows. Pick any stratified theory with time-orthogonal conformally flat space slices. Using a coordinate system at rest with respect to the Universe, compute the post-Newtonian metric due to an arbitrary configuration of matter. To put this metric into the "standard" PPN gauge, it may be necessary to apply an infinitesimal gauge transformation (eq. [9])

$$t^{\dagger} = t + \lambda \chi_{,0}, \quad x^{\dagger} = x, \quad (24)$$

for some value of λ . Since $g_{0\alpha}$ was initially identically zero (by assumption; cf. eq. [20]) in the new gauge it becomes

$$(g_{0\alpha})^{\dagger} = -\lambda \chi_{,0\alpha} = -\lambda V_{\alpha} + \lambda W_{\alpha}. \quad (25)$$

By comparing this with the PPN metric in table 4 we obtain

$$\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1) = -\lambda, \quad (26)$$

$$\frac{1}{2}(1 + \alpha_2 - \zeta_1) = \lambda. \quad (27)$$

Adding equations (26) and (27) we obtain, finally,

$$\alpha_1 = -(4\gamma + 4), \quad (28)$$

independent of gauge λ . Q.E.D.

b) Lagrangian-based Metric Theories with Additional Vector and Tensor Fields

Here we take an approach which is different from that of § IIIa, and analyze preferred-frame theories of gravity based on a Lagrangian formulation which initially assumes *no* preferred frames (*no* "prior geometry").

Since gravitation appears to be a long-range, purely attractive force, one might expect the Universe's global matter distribution to affect local gravitational physics, and to play a dominant role in establishing any "preferred" reference frame. In fact, from this "Machian" point of view, the mystery seems to be: *How can a metric theory of gravity avoid at all having a preferred frame related to the Universe rest-frame?* General relativity and the Brans-Dicke theory obviously avoid preferred-frame effects (cf. their parameter values $\alpha_1 = \alpha_2 = \alpha_3 = 0$ in table 2). Before we create theories which *do* have preferred frames, we must first shed some light on how these exceptional theories avoid them.

Consider a local gravitational system, such as the solar system, which is embedded in the Universe. We separate the computation of the metric into two parts: a Universal or cosmological solution, and a "local" solution. From this viewpoint, the Universe affects the local gravitational physics of the system by establishing the boundary conditions (at a boundary "far" from the matter) for the various fields generated by the local system. The local system "feels" its relationship to the Universe via the asymptotic field values of the fields present; metric g_{ij} , scalar field φ , vector field K_i , second-rank tensor field C_{ij} , and so on. Several conclusions follow:

1) *A theory which contains solely a metric field yields local gravitational physics which is identical in all frames which are asymptotically Minkowskian (called "asymptotic Lorentz frames"), and which does not change with Universe evolution. In addition, the Newtonian gravitational constant G is unaffected by the proximity of matter.* All this follows from the invariance properties of η_{ij} (the asymptotic form of g_{ij}), the only field coupling the local system asymptotically to the Universe, and from general covariance, which allows us to find a coordinate system in which the metric field takes this Minkowski form at the boundary between the Universe and the local system.

2) *A theory which contains a metric field and a scalar field φ yields physics which is identical in all asymptotic Lorentz frames, but which may vary with Universe evolution;*

G may be affected by proximity of matter. These conclusions follow from invariance of both η_{ij} and φ under Lorentz transformations, but now φ may vary with Universe evolution and may depend on the proximity of matter.

3) *A theory which contains a vector field K_i and/or an additional second-rank tensor field C_{ij} yields local physics which may depend on motion relative to a preferred Universe rest-frame, and which may vary with evolution of the Universe; G may be affected by proximity of matter.* This follows because the asymptotic values of K_i and C_{ij} are not invariant under Lorentz transformations (an exception would be $C_{ij[\text{asymptotic}]}$ proportional to η_{ij}).

In summary, it is the Lorentz invariance of the asymptotic forms of the fields φ and g_{ij} which makes it impossible to have preferred-frame effects in theories containing solely those fields. Thus we must appeal to vector- or tensor-metric theories for preferred-frame effects.

It is commonly believed that cosmological vector fields and additional second-rank tensor fields are absent from physics. The Hughes-Drever experiment (measurement of the isotropy of inertial mass) and a variety of laboratory "ether-drift" experiments have been used by Dicke (1964; see also Peebles and Dicke 1962, Peebles 1962) to rule out these fields. Closer examination of these arguments shows that *the experimental evidence rules out only those vector and "second tensor" cosmological fields which couple directly to matter*. Since these experiments were performed under conditions where the effects of gravity were negligible, *they do not rule out vector and second-tensor fields which couple only to gravity*.

We now proceed to devise such theories. We assume these theories are derivable from a coordinate-invariant Lagrangian, with no *a priori* assumption of a special coordinate frame or of any prescribed form for the tensors in the theory. All physical quantities (metric, vector fields, tensor fields, matter variables) must be calculated in terms of other physical quantities. The Lagrangian for such theories has the general form

$$L = L_m(q, p, g_{ij}) + L_g(g_{ij}, g_{ij,k}, K_i, K_{i,j}, C_{ij}, C_{ij,k}, \dots), \quad (29)$$

where q and p symbolically represent the dynamical variables of matter, g_{ij} is the metric, and $K_i, C_{ij}, \dots$ are possible cosmological vector and tensor fields. The matter Lagrangian is related to the stress-energy tensor for matter and nongravitational fields by the functional derivative

$$\frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{ij}} (\sqrt{-g} L_m) \equiv T_{ij}. \quad (30)$$

The additional fields in the Lagrangian do not couple directly to matter; this leads to the equation of motion for matter

$$T^{ij}_{;j} = 0, \quad (31)$$

which yields geodesic motion for test bodies. This form for the Lagrangian thus guarantees that the theory satisfy the postulates of a "metric" theory (for a discussion of these postulates see Thorne and Will 1971).

For simplicity, we restrict ourselves to theories which have linear field equations for all the supplementary fields; nevertheless, a wide variety of theories is possible. For vector-metric theories, the general Lagrangian has the form

$$\begin{aligned} L_{\text{VM}} = & L_m + k_1 R + k_2 K_i K^i R + k_3 K_i K_j R^{ij} \\ & + k_4 K_{i;j} K^{i;j} + k_5 K_{i;j} K^{j;i} + k_6 K_i{}^{;i} K_j{}^{;j}, \end{aligned} \quad (32)$$

where R_{ij} and R are the Ricci tensor and scalar formed from the metric; $k_1, \dots, k_6$ are dimensionless coupling constants. Tensor-metric theories have even more structure (we assume C_{ij} is symmetric):

$$\begin{aligned} L_{\text{TM}} = & L_m + k_1' R + k_2' C_{ij} C^{ij} R + k_3' (C_{ij} g^{ij})^2 R \\ & + k_4' C_{ij} g^{ij} C_{kl} R^{kl} + k_5' C_{ik} C^k_j R^{ij} \\ & + k_6' C_{ij;k} C^{ij;k} + k_7' C_{ij;k} C^{ik;j} + k_8' C_{ik;k} C^{ij;j} \\ & + k_9' C_{ij;j} g^{kl} C_{kl;i} + k_{10}' C_{ij;k} C_{lm;k} g^{ij} g^{lm}. \end{aligned} \quad (33)$$

We point out here that the above equation (33) alone does not reveal the full richness of possible Lagrangian terms for tensor-metric theories. If C_{ij} is a nonsingular field, one can define Christoffel symbols in terms of C_{ij} . Also, third-rank tensors S^i_{jk} can be produced by taking the difference of the Christoffel symbols

$$S^i_{jk} = \Gamma^i_{jk}(\mathbf{g}) - \Gamma^i_{jk}(\mathbf{C}).$$

Covariant derivatives can be defined with respect to each tensor, g_{ij} or C_{ij} , and tensor densities can be formed using either

$$\sqrt{-g}$$

or

$$\sqrt{|\det C|}.$$

All these quantities can be used to construct Lagrangian terms.

A future paper in this series will give a complete study of these theories. Such a study will discuss consistency of the field equations, the post-Newtonian limit and the PPN parameter values, cosmological solutions, and positive-definiteness of free-field energies.

To show that theories of this type can produce apparent agreement with the "three classical tests" of general relativity, yet also possess preferred-frame effects, we now quote the PPN parameter values for a simple illustrative case of a vector-metric theory. Detailed calculations have been relegated to an Appendix. This theory has as its action integral

$$A = \int d^4x \sqrt{-g} [G_0 L_m + R + K_{i;j} K_{k;l} g^{ik} g^{jl}], \quad (34)$$

where G_0 is the *unrenormalized* gravitational constant. This theory corresponds to the case (eq. [32]) $k_1 = k_4 = G_0^{-1}$, $k_2 = k_3 = k_5 = k_6 = 0$.

The calculations in the Appendix yield a PPN metric with parameter values (see also table 2)

$$\begin{aligned} \gamma &= \beta = 1, \\ \alpha_1 &= \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0, \quad \alpha_2 = K^2 / (1 + \frac{1}{2} K^2). \end{aligned} \quad (35)$$

Here K^2 is the squared magnitude of the vector field, evaluated far from the matter, i.e.,

$$K^2 = [g^{ij} K_i K_j]_{\text{far from matter}}. \quad (36)$$

The gravitational constant is renormalized in this theory, and so changes in time as the Universe evolves:

$$G(t) = \frac{G_0}{1 + \frac{1}{2} [K(t)]^2}. \quad (37)$$

IV. CONCLUSIONS

In this paper, we have presented a new version of the Parametrized Post-Newtonian formalism, and have discussed preferred-frame theories of gravity. In Paper II of this series we will use this new formalism to compute observable solar-system effects predicted by preferred-frame theories and will present experimental evidence to rule out many of these theories.

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APPENDIX

A VECTOR-METRIC THEORY OF GRAVITY

Here we discuss in detail the vector-metric theory described in § IIIb. This theory has as its action integral (cf. eq. [34])

$$A = \int d^4x \sqrt{-g} [G_0 L_m + R + K_{i;j} K_{k;l} g^{ik} g^{jl}], \quad (\text{A1})$$

where L_m is the matter Lagrangian, R is the scalar curvature formed from the metric g_{ij} , K_i is the cosmological vector field, and G_0 is the (unrenormalized) gravitational constant. By varying the action with respect to g_{ij} and K_i in the usual way, we obtain the field equations

$$K_{i;j}{}^{;j} = 0, \quad (\text{A2})$$

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi G_0 T_{ij} + \Theta_{ij}, \quad (\text{A3})$$

where R_{ij} is the Ricci tensor, T_{ij} is the stress-energy tensor for matter and nongravitational fields, and Θ_{ij} is the vector-field stress-energy tensor given by

$$\begin{aligned} \Theta_{ij} = & K_{i;m} K_j{}^{;m} + K_{m;i} K_m{}^{;j} - \frac{1}{2} K^m{}_{;n} K_m{}^{;n} g_{ij} \\ & + \frac{1}{2} (K^m S_{ij} - K_i S^m{}_j - K_j S^m{}_i)_{;m}, \end{aligned} \quad (\text{A4})$$

where

$$S_{ij} = K_{i;j} + K_{j;i}. \quad (\text{A5})$$

It is straightforward, though tedious, to verify by using equations (A2), (A4), and (A5) that

$$\Theta_{ij}{}^{;j} = 0, \quad (\text{A6})$$

and thus that

$$T_{ij}{}^{;j} = 0. \quad (\text{A7})$$

We now use these equations to compute the metric produced by a configuration of perfect fluid, in the post-Newtonian approximation, using the standard techniques of Chandrasekhar (1965). (Equivalent results can be obtained for a system of point masses using the standard EIH [Einstein *et al.* 1938] method.) The metric is expanded about the flat-space Minkowski metric according to

$$g_{ij}(\mathbf{x}, t) = \eta_{ij} + h_{ij}(\mathbf{x}, t) \quad (\text{A8})$$

where, far from the matter, h_{ij} tends asymptotically toward zero. We also expand the vector field K_i about its asymptotic value. However, in order to simplify the calculation, we will work in a coordinate system in which the asymptotic vector field has only a time component; i.e., asymptotically

$$K_\alpha = 0. \quad (\text{A9})$$

Then, in this coordinate system, we have

$$|K|^2 \equiv \eta^{ij} K_i K_j = K_0^2, \quad (\text{A10})$$

far from the configuration of fluid. The frame in which $K_0 = K$ and $K_\alpha = 0$ is presumably the rest-frame of the Universe's smoothed-out distribution of matter. The expansion of the vector field can thus be written

$$K_0 = K + \varphi(x, t), \quad (\text{A11})$$

$$K_\alpha = k_\alpha(x, t), \quad (\text{A12})$$

where φ is $O(2)$ and k_α is $O(3)$ (cf. n. 4), and both go to zero far from the fluid.

The perfect-fluid stress-energy tensor is given by (cf. Will 1971a)

$$T_{ij} = (\rho + \rho\Pi + p)u_i u_j - p g_{ij}, \quad (\text{A13})$$

where u_i is the four-velocity of the fluid. The field equation (A3) can be written in the form

$$R_{ij} = 8\pi G_0(T_{ij} - \frac{1}{2}Tg_{ij}) + (\Theta_{ij} - \frac{1}{2}\Theta g_{ij}), \quad (\text{A14})$$

where T and Θ are the traces of T_{ij} and Θ_{ij} , respectively.

We wish to obtain h_{00} to $O(4)$, $h_{0\alpha}$ to $O(3)$, and $h_{\alpha\beta}$ to $O(2)$. The calculation proceeds along the following lines.

i) Calculate h_{00} to $O(2)$. This corresponds to the theory's Newtonian limit. To the required order, we have

$$\begin{aligned} R_{00} &= \frac{1}{2}\nabla^2 h_{00}, & T_{00} &= T = \rho, \\ \Theta_{00} &= -\frac{1}{2}K^2\nabla^2 h_{00}, & \Theta &= -K^2 R_{00}. \end{aligned} \quad (\text{A15})$$

Equation (A14) then becomes

$$\nabla^2 h_{00} = 8\pi\rho[G_0/(1 + \frac{1}{2}K^2)], \quad (\text{A16})$$

whose solution is

$$h_{00} = -2U[G_0/(1 + \frac{1}{2}K^2)]. \quad (\text{A17})$$

Because we have chosen units so that the gravitational constant as measured far from the solar system, today is unity, we have

$$\frac{G_0}{1 + \frac{1}{2}K^2} = G \equiv 1 \quad [\text{today}]. \quad (\text{A18})$$

Note that if K varies slowly as the Universe evolves, then the gravitational constant G may vary from its present value with time at a rate

$$\frac{1}{G} \left(\frac{dG}{dt} \right) = -\frac{K}{1 + \frac{1}{2}K^2} \left(\frac{dK}{dt} \right). \quad (\text{A19})$$

ii) Calculate $h_{\alpha\beta}$ to $O(2)$. By making a particular choice of gauge:

$$h^i_{i,\alpha\beta} - h^\gamma_{\beta,\alpha\gamma} - h^\gamma_{\alpha,\beta\gamma} = 0, \quad (\text{A20})$$

we give the field equation for $R_{\alpha\beta}$ (eq. [A14]) the simple form (to the required order)

$$\nabla^2 h_{\alpha\beta} = 8\pi\rho\delta_{\alpha\beta}, \quad (\text{A21})$$

whose solution is

$$h_{\alpha\beta} = -2U\delta_{\alpha\beta}. \quad (\text{A22})$$

iii) Calculate φ and k_α to $O(2)$ and $O(3)$. The field equation (A2) for $i = 0$ becomes, to the necessary order,

$$\nabla^2(\varphi + KU) = 0. \quad (\text{A23})$$

Since φ must vanish far from the fluid, we have

$$\varphi = -KU. \quad (\text{A24})$$

If we choose the gauge in which

$$\frac{1}{2}h^\beta_{\beta,0} - h^\beta_{0,\beta} = U_{,0}[K^2/(1 + \frac{1}{2}K^2)], \quad (\text{A25})$$

then equation (A2) for $i = \alpha$ becomes

$$\nabla^2 \left[k_\alpha - \frac{1}{2}Kh_{0\alpha} + \frac{1}{4}K \left(1 + \frac{K^2}{1 + \frac{1}{2}K^2} \right) \chi_{,0\alpha} \right] = 0, \quad (\text{A26})$$

where χ is given by equation (10). Equation (A26) has the solution

$$k_\alpha = \frac{1}{2}Kh_{0\alpha} - \frac{1}{4}K \left(1 + \frac{K^2}{1 + \frac{1}{2}K^2} \right) \chi_{,0\alpha}. \quad (\text{A27})$$

iv) Calculate $h_{0\alpha}$ to $O(3)$. With the choice of gauge (eq. [A25]), the field equation (A14) simplifies to

$$\nabla^2 h_{0\alpha} = -16\pi\rho v_\alpha - \frac{1}{2} \left(1 + \frac{K^2}{1 + \frac{1}{2}K^2} \right) \chi_{,0\alpha}, \quad (\text{A28})$$

or

$$h_{0\alpha} = 4V_\alpha - \frac{1}{2} \left(1 + \frac{K^2}{1 + \frac{1}{2}K^2} \right) \chi_{,0\alpha}. \quad (\text{A29})$$

Using the fact that

$$\chi_{,0\alpha} = V_\alpha - W_\alpha, \quad (\text{A30})$$

we get

$$h_{0\alpha} = \frac{1}{2} \left(7 - \frac{K^2}{1 + \frac{1}{2}K^2} \right) V_\alpha + \frac{1}{2} \left(1 + \frac{K^2}{1 + \frac{1}{2}K^2} \right) W_\alpha. \quad (\text{A31})$$

v) Calculate h_{00} to $O(4)$. To the appropriate order, equation (A14) along with equations (A13), (A17), (A24), (A25), and (A27) gives

$$\nabla^2 h_{00} = 8\pi\rho + 2\nabla^2(U^2) + 16\pi\rho v^2 + 8\pi\rho\Pi + 24\pi p + 16\pi\rho U, \quad (\text{A32})$$

whose solution is

$$h_{00} = -2U + 2U^2 - 4\Phi_1 - 4\Phi_2 - 2\Phi_3 - 6\Phi_4. \quad (\text{A33})$$

The final form of the metric is

$$\begin{aligned} g_{00} &= 1 - 2U + 2U^2 - 4\Phi_1 - 4\Phi_2 - 2\Phi_3 - 6\Phi_4, \\ g_{0\alpha} &= \frac{1}{2} \left(7 - \frac{K^2}{1 + \frac{1}{2}K^2} \right) V_\alpha + \frac{1}{2} \left(1 + \frac{K^2}{1 + \frac{1}{2}K^2} \right) W_\alpha, \\ g_{\alpha\beta} &= -(1 + 2U)\delta_{\alpha\beta}. \end{aligned} \quad (\text{A34})$$

By comparing equation (A34) with the metric in table 4, we can read off the values of the PPN parameters, as listed in table 2.

Thus this theory does predict that local gravitational physics should depend on motion relative to the preferred Universal rest-frame (since $\alpha_2 \neq 0$), and should vary with evolution of the Universe (since K may vary with time; cf. eq. [A19]). Notice that, in a Robertson-Walker universe with expansion parameter $a(t)$, K_i evolves according to (the dot denotes d/dt).

$$\begin{aligned} \dot{K}_0 + 3\dot{K}_0(\dot{a}/a) - 3(\dot{a}/a)^2 K_0 &= 0, \\ K_\alpha &= 0. \end{aligned} \quad (\text{A35})$$

However, this theory predicts that the local gravitational constant G should be *unaffected* by nearby matter. In the presence of matter (and in the Universal rest-frame), this locally measured G is given by (see Nordvedt 1970b; Will 1971d)

$$G_{\text{local}} = 1 - (4\beta - \gamma - 3 - \zeta_2)U_{\text{ext}}. \quad (\text{A36})$$

Since $\beta = \gamma = 1$, and $\zeta_2 = 0$ for this theory, we find G_{local} is unaffected by nearby matter. This can be seen in another way, as follows: Consider a gravitating system in the presence of an external mass, whose Newtonian gravitational potential is U_{ext} . When the local system and the external mass are analyzed together in an asymptotically flat coordinate system, the metric and the vector field are given by, to first order (cf. eqs. [A24], [A34]),

$$g_{00} = 1 - 2U - 2U_{\text{ext}}, \quad K_0 = K(1 - U - U_{\text{ext}}). \quad (\text{A37})$$

The gravitational constant G is determined by the value of K_0 far from the system; i.e. (cf. eq. [A17]),

$$G = \frac{G_0}{1 + \frac{1}{2}K^2}. \quad (\text{A38})$$

We now look at the local gravitational system itself using a coordinate system which becomes asymptotically flat (to some desired precision) far from the local system but between the local system and the external mass. The transformation from the first coordinate system to this one has the form

$$t_{\text{new}} = t_{\text{old}}(1 - U_{\text{ext}}), \quad (\text{A39})$$

which gives

$$(g_{00})_{\text{new}} = 1 - 2U, \quad (K_0)_{\text{new}} = K(1 - U). \quad (\text{A40})$$

Thus the transformation which removes the external field from the metric also removes the external field from the vector field. Therefore, the gravitational constant as measured in the local system is unaffected by the external mass.

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