

Photoelectric Observations of UU Piscium

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UU Piscium was monitored in the V color. The amplitude of the light variation was 0.03 mag. This is caused by the elliptical shapes of the component stars of UU Piscium. The phase of secondary minimum occurred at 0.55. The cause of this may be that the period of rotation of the component stars of UU Piscium is not synchronized with their orbital revolutions, or their orbit is elliptical. Previous observations of UU Piscium by N. L. Magalashvili and J. I. Kumsishvili seem to indicate that the orbit is elliptical.

I. INTRODUCTION

THE variable star UU Piscium is the brighter component of the visual double star ADS 191. The distance between the components of this double is 11.6 sec of arc (Jeffers, Van den Bos, and Greeby 1963). The difference in visual magnitude of the two components is 1.61 (Eggen 1963).

MacCormack (1934) found UU Piscium to be a double-lined spectroscopic binary. From an analysis of 55 spectrographic plates she found the period of variation in radial velocity was 0^d84166. She also commented that no significant difference could be noted in the strength of the lines of the two members of the system whose spectral type is A9s.

The light variation of UU Piscium was investigated by Magalashvili and Kumsishvili (1957). The light variation was found to have an amplitude of 0.09 mag. Using MacCormack's period for the radial-velocity variations, they found that their photometric data, using normal points, could represent the light curve of an eclipsing binary with a depth of primary eclipse of 0.09 mag and of secondary eclipse of 0.07. Thus they concluded that the stars did indeed eclipse one another. They also noted that the secondary eclipse occurred at phase 0.45. This resulted in the brighter components of ADS 191 being listed in *The General Catalogue of Variable Stars* (Kukarkin *et al.* 1958) as the variable UU Piscium. It is listed as an Algol-type eclipsing binary with an amplitude of 0.10 mag.

During the fall of 1957 and 1958, Cester (1959) obtained 53 spectrograms of UU Piscium. From his analysis of the radial-velocity curve, he found that the orbit is circular. He also deduced that the inclination of the orbit is about 19°.

This would rule out a light variation of the order of 0.10 mag. Thus, another photometric study of this star was made.

II. OBSERVATIONS

The observations of UU Piscium were obtained during the fall and winter of 1967. The observations were made with the 12-inch refractor of Georgetown College Observatory. The photometer employed a V filter (Corning No. 3384) and a 1P21 photomultiplier.

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Table I gives the particulars of the variable, comparison, and check stars used in this study as listed by Hoffleit (1964).

Since the focal-plane diaphragm of the photometer used in this study was one minute of arc, the light of both components of ADS 191 was measured.

Observations were made on a total of 11 nights for which the extinction coefficients could be determined. The comparison and variable stars were observed consecutively with occasional observations of the check star. Each observation of a star was followed by an observation of the sky brightness near the star being observed.

The observations of the check star were reduced relative to the comparison star and the standard deviation of the measurements is 0.010 mag. Thus the comparison star is not variable.

The extinction east and west of the meridian found from the comparison star observations was used to correct the delta magnitude observations for differential extinction. A total of 474 observations of UU Piscium were obtained. These are listed in Table II.

III. PERIOD ANALYSIS AND LIGHT ELEMENTS

The amplitude of the light variations of UU Piscium found in this study is 0.03 mag. Due to the fact that times of minimum light could not be determined accurately, the conventional method of finding the times of minimum light and fitting a simple linear relation to determine the period and epoch could not be applied.

The phase of the light variations was determined using MacCormack's period and epoch (MacCormack 1934). The light curve resembled a cosine wave whose frequency was twice that of the period. Therefore a

TABLE I. Coordinates, magnitudes, and spectral types of the observed stars.

Star	HD	α (1900)	δ (1900)	m_V	Sp.
Variable (UU Psc)	1061	0 ^h 09 ^m 50 ^s	+8° 16'	5.87	sgA9
Comparison (32 Psc)	225003	23 57 23	+7 56	5.59	dF0
Check (31 Psc)	224995	23 57 17	+8 24	6.29	A5

TABLE II. Observations of UU Piscium.

J.D. $\odot$ 2439000+	Δm	J.D. $\odot$ 2439000+	Δm	J.D. $\odot$ 2439000+	Δm	J.D. $\odot$ 2439000+	Δm	J.D. $\odot$ 2439000+	Δm	J.D. $\odot$ 2439000+	Δm
765.552	0.115	766.778	0.111	810.608	0.106	813.638	0.101	841.628	0.098	848.498	0.126
.555	0.130	.780	0.104	.611	0.104	.641	0.101	.633	0.088	.501	0.125
.558	0.090	.783	0.116	.613	0.115	.643	0.107	.636	0.116	.503	0.126
.560	0.071	.785	0.107	.616	0.111	.645	0.126	.639	0.108	.507	0.128
.564	0.063	.787	0.115	.621	0.110	.657	0.106	.643	0.112	.510	0.126
.566	0.115	.790	0.109	.623	0.105	.659	0.118	844.470	0.089	.514	0.123
.570	0.096	.792	0.116	.630	0.107	.663	0.114	.472	0.096	.516	0.138
.572	0.100	.795	0.128	.633	0.100	.669	0.115	.475	0.092	.518	0.122
.575	0.098	.797	0.130	.635	0.120	.672	0.106	.478	0.087	.521	0.124
.578	0.114	.800	0.134	.638	0.112	.675	0.115	.480	0.089	.523	0.129
.580	0.111	.803	0.111	.641	0.126	.677	0.108	.484	0.094	.526	0.130
.583	0.100	.806	0.116	.650	0.097	.680	0.114	.486	0.093	.529	0.132
.586	0.083	776.550	0.118	.655	0.093	.683	0.117	.489	0.085	.532	0.138
.589	0.099	.553	0.113	.659	0.106	.686	0.119	.492	0.091	.536	0.117
.591	0.116	.555	0.122	.663	0.106	837.491	0.111	.494	0.085	.541	0.113
.595	0.110	.558	0.121	.673	0.118	.506	0.112	.497	0.087	.543	0.122
.597	0.102	.560	0.119	.678	0.115	.520	0.110	.499	0.089	.546	0.127
.600	0.098	.563	0.098	.680	0.113	.527	0.123	.502	0.089	.549	0.120
.605	0.058	.565	0.115	.683	0.119	.533	0.114	.506	0.092	.552	0.120
.607	0.110	.568	0.108	.686	0.118	.540	0.117	.508	0.092	.555	0.108
.610	0.142	.571	0.117	.689	0.116	.548	0.113	.511	0.091	.558	0.108
.613	0.101	.573	0.118	.702	0.101	.554	0.117	.513	0.095	.561	0.111
.615	0.110	.578	0.106	.706	0.137	.560	0.120	.515	0.086	.563	0.101
.619	0.119	.580	0.099	.709	0.121	.567	0.139	.518	0.092	.567	0.109
.622	0.122	.583	0.117	.712	0.106	.579	0.133	.520	0.083	.570	0.094
.625	0.115	.585	0.103	.719	0.101	.586	0.132	.523	0.083	.573	0.098
.630	0.115	.588	0.118	813.504	0.098	.593	0.129	.527	0.087	.575	0.104
.633	0.089	.590	0.088	.509	0.095	.598	0.123	.530	0.082	.579	0.110
.635	0.105	.594	0.124	.512	0.093	.604	0.123	.533	0.085	.581	0.111
.638	0.116	.596	0.116	.514	0.095	.609	0.107	.535	0.085	.585	0.100
.640	0.112	.599	0.118	.516	0.094	.616	0.141	.538	0.085	.587	0.091
.643	0.118	.601	0.107	.521	0.098	841.492	0.084	.541	0.089	.590	0.088
.645	0.115	.604	0.101	.523	0.096	.495	0.096	.543	0.075	.593	0.116
.648	0.104	.607	0.099	.525	0.087	.499	0.100	.546	0.084	.598	0.103
.651	0.116	.612	0.105	.528	0.093	.503	0.086	.550	0.088	.602	0.118
.654	0.125	.615	0.114	.531	0.090	.506	0.089	.553	0.086	.606	0.099
.657	0.109	.617	0.102	.536	0.090	.509	0.090	.554	0.091	.611	0.091
.659	0.104	.620	0.101	.538	0.102	.512	0.097	.557	0.087	.616	0.092
.662	0.107	.623	0.106	.541	0.097	.515	0.075	.559	0.085	.620	0.096
.665	0.110	.626	0.109	.543	0.104	.517	0.092	.562	0.088	.624	0.092
.668	0.111	.628	0.111	.545	0.105	.520	0.088	.564	0.083	857.466	0.099
.671	0.114	.630	0.110	.549	0.095	.526	0.088	.569	0.085	.469	0.116
.673	0.106	.633	0.111	.552	0.103	.529	0.096	.571	0.088	.473	0.111
.676	0.108	.635	0.103	.554	0.109	.532	0.078	.573	0.089	.474	0.108
.679	0.105	.637	0.106	.556	0.098	.534	0.060	.576	0.086	.476	0.096
.681	0.097	.640	0.108	.559	0.108	.537	0.091	.578	0.090	.478	0.103
.684	0.095	.642	0.102	.561	0.103	.541	0.070	.582	0.087	.479	0.095
.687	0.105	.645	0.088	.565	0.102	.544	0.079	.584	0.099	.481	0.093
.690	0.114	.648	0.099	.572	0.101	.547	0.071	.586	0.098	.482	0.091
.692	0.116	.651	0.114	.575	0.117	.550	0.104	.589	0.104	.483	0.095
.695	0.111	.654	0.076	.580	0.105	.554	0.088	.594	0.088	.485	0.095
766.713	0.092	.659	0.092	.582	0.098	.557	0.074	.597	0.091	.487	0.095
.717	0.095	810.479	0.107	.585	0.097	.560	0.083	.600	0.084	.488	0.100
.721	0.104	.501	0.087	.588	0.094	.564	0.092	.603	0.080	.490	0.095
.725	0.091	.505	0.088	.590	0.108	.566	0.088	.608	0.087	.491	0.095
.728	0.103	.508	0.088	.593	0.110	.569	0.087	.614	0.091	.493	0.093
.731	0.087	.514	0.083	.596	0.102	.573	0.086	.618	0.098	.494	0.105
.733	0.090	.520	0.088	.598	0.109	.576	0.090	.621	0.091	.496	0.095
.736	0.097	.541	0.091	.603	0.107	.578	0.087	.623	0.091	.497	0.106
.739	0.108	.547	0.108	.605	0.099	.581	0.087	.627	0.095	.499	0.105
.741	0.098	.549	0.096	.607	0.090	.583	0.087	.631	0.101	.500	0.093
.744	0.104	.552	0.102	.609	0.090	.589	0.093	.636	0.098	.502	0.100
.746	0.093	.557	0.103	.611	0.100	.592	0.096	.639	0.113	.503	0.098
.750	0.089	.562	0.095	.614	0.102	.595	0.083	848.458	0.119	.504	0.095
.752	0.097	.567	0.101	.616	0.103	.597	0.097	.461	0.111	.506	0.096
.756	0.118	.577	0.110	.618	0.109	.600	0.091	.467	0.111	.507	0.106
.758	0.108	.580	0.098	.621	0.115	.604	0.085	.471	0.103	.508	0.088
.762	0.108	.589	0.126	.623	0.118	.607	0.084	.476	0.120	.510	0.091
.765	0.110	.593	0.093	.625	0.116	.610	0.096	.481	0.128	.512	0.089
.767	0.114	.595	0.094	.629	0.101	.613	0.102	.484	0.122	.513	0.093
.770	0.105	.599	0.104	.631	0.102	.619	0.106	.487	0.124	.515	0.102
.772	0.107	.602	0.104	.633	0.108	.622	0.096	.489	0.123	.517	0.091
.775	0.109	.605	0.126	.635	0.105	.625	0.098	.493	0.129	.518	0.089

TABLE II (continued)

J.D.⊙ 2439000+	Δm	J.D.⊙ 2439000+	Δm	J.D.⊙ 2439000+	Δm	J.D.⊙ 2439000+	Δm	J.D.⊙ 2439000+	Δm	J.D.⊙ 2439000+	Δm
857.520	0.085	857.531	0.096	857.544	0.095	857.556	0.084	857.571	0.095	857.589	0.097
.523	0.086	.533	0.094	.545	0.092	.559	0.082	.574	0.094	.591	0.092
.524	0.089	.535	0.098	.547	0.087	.561	0.075	.578	0.084	.594	0.097
.526	0.096	.537	0.089	.549	0.104	.563	0.094	.581	0.084	.596	0.095
.527	0.089	.539	0.086	.552	0.093	.566	0.099	.583	0.092	.599	0.106
.529	0.086	.541	0.096	.554	0.094	.569	0.098	.586	0.092	.601	0.087

Fourier series of the form

$$\Delta m_{\text{obs}} = A_0 + A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t \\ + B_1 \sin \omega t + B_2 \sin 2\omega t + B_3 \sin 3\omega t \\ = f(A_0, A_1, A_2, A_3, B_1, B_2, B_3, P), \quad (1)$$

where $\omega = 2\pi/\text{period}$, was fitted by the method of least squares to the observations for different values of the period from 0.75 to 0.90 days. The best fit was achieved for the period of 0.841678 d. The rms error of this fit was 0.0096 mag.

The method of differential corrections was applied to Eq. (1) in order to evaluate the accuracy of the period. Corrections to the calculated Fourier coefficients and period may be found from

$$\Delta m_{\text{obs}} - \Delta m_{\text{calc}} = \frac{\partial f}{\partial A_0} \Delta A_0 + \frac{\partial f}{\partial A_1} \Delta A_1 + \dots + \frac{\partial f}{\partial P} \Delta P \quad (2)$$

if it is assumed that the changes in the Fourier coefficients and period are sufficiently small that the higher-order terms may be ignored. Now Eq. (2) is rewritten upon the substitution of (1) as

$$\Delta m_{\text{obs}} - \Delta m_{\text{calc}} = \Delta A_0 + \Delta A_1 \cos \omega t + \Delta A_2 \cos 2\omega t \\ + \Delta A_3 \cos 3\omega t + \Delta B_1 \sin \omega t + \Delta B_2 \sin 2\omega t + \Delta B_3 \sin 3\omega t \\ + \Delta \omega t (-A_1 \sin \omega t - 2A_2 \sin 2\omega t + 3A_3 \sin 3\omega t + B_1 \cos \omega t \\ + 2B_2 \cos 2\omega t + 3B_3 \cos 3\omega t), \quad (3)$$

where Δm_{obs} = the observed magnitude, Δm_{calc} = the calculated magnitude which is found by

$$\Delta m_{\text{calc}} = A_0 + A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t \\ + B_1 \sin \omega t + B_2 \sin 2\omega t + B_3 \sin 3\omega t.$$

This equation was solved by the method of least squares. The values of the coefficients and period were adjusted and the solution of the equation repeated until the

TABLE III. Fourier coefficients of the observed variation of UU Piscium.

A_0	A_1	A_2	A_3
0.10016	0.00524	0.01171	0.00154
B_1	B_2	B_3	
-0.00244	0.00415	-0.00195	

value of the period solved for could no longer be adjusted. This took three iterations for the data of UU Piscium. The period found was $0^{\text{d}}841678 \pm 0^{\text{d}}000008$.

In order to find the epoch which would give minimum light at phase zero, Eq. (1) was differentiated. This was used to adjust the epoch to Julian Date 2439765.175. The Fourier coefficients fitting the data for this epoch are listed in Table III. The rms error of the curve is 0.0096 mag. This curve is shown in Fig. 1. Thus the resulting light elements from this study of UU Piscium are

$$t_{\text{min}} = 2439765^{\text{d}}175 + 0^{\text{d}}841678E. \quad (4) \\ \pm 8$$

IV. DISCUSSION OF THE LIGHT CURVE

Almost all the light variation is caused by the distortion into ellipsoids of the two stars making up this system. This is easily seen from the coefficients of Table III. The A_2 term, due mainly to ellipticity, is the only significant term. The contribution of the ellipticity effect may be calculated by the relationship given by Merrill (1970). This expresses the coefficients as a function of the limb darkening, gravity darkening, inclination of the orbit, and the masses and radii of the component stars. From the elements of the spectrographic orbit (Cester 1959) listed in Table IV, estimates may be made of these quantities using the present knowledge of the masses and radii of stars.

LIGHT VARIATION OF UU PISCUM

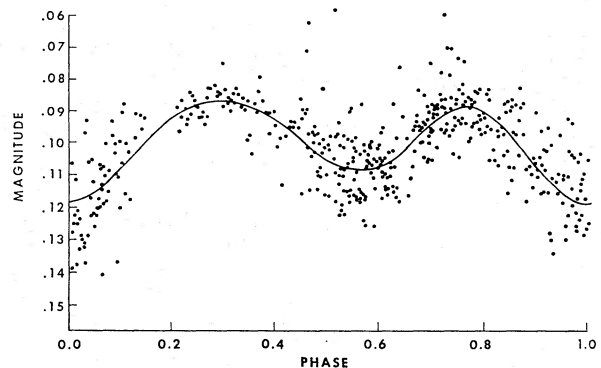


FIG. 1. Light variation of UU Piscium.

TABLE IV. Spectrographic elements of UU Piscium.

$V_0 = 2.6 \pm 1.2$ km/sec	$a_1 \sin i = 1\,040\,000$ km
$K_1 = 89.9 \pm 2.1$ km/sec	$a_2 \sin i = 1\,120\,000$ km
$K_2 = 96.8 \pm 2.1$ km/sec	$m_1 \sin^3 i = 0.29$
$e = 0$	$m_2 \sin^3 i = 0.27$
$P = 0.841658$	$m_1/m_2 = 0.93 \pm 0.03$
$T_0 = 243\,6094.121 \pm 0.005$	

The spectral type of UU Piscium was found by Berger (1962) to be F0 IV. The mass and radius of stars near this spectral type [i.e., τ Cyg, ψ Vel, Z Her, RS CVn, and α Aql from Harris, Strand, and Worley (1963); Popper (1967); Brown, Davis, and Allen (1967)] have an average value of $1.37 M_\odot$ and $1.65 R_\odot$. Photometric radii of stars of luminosity classes III, IV, and V have been calculated by Gray (1968). The radii for giants and main-sequence stars near spectral class F0 are almost equal and have a value of $1.65 R_\odot$. Combining these values with the spectroscopic data, the results are

$$\begin{aligned} i &= 38^\circ, \\ a_1 + a_2 &= 2.45 \times 10^6 \text{ km}, \\ r_1 = r_2 &= 0.33. \end{aligned}$$

Using a limb-darkening coefficient of 0.6 and a gravity darkening of 1.0, together with an inclination of 38° and assuming that the radii of both stars are equal, the Fourier coefficient of the A_2 term was found to be -0.013 .

Upon converting the observed magnitudes into intensities, and subtracting the light of ADS 191 B, the A_2 term was found to be -0.013 . Thus the ellipticity effect accounts for the majority of the light variations. The reflection effect contributes negligibly to the light variation since both stars comprising UU Piscium are equal in radii and spectral type.

However, inspection of the light curve shown in Fig. 1 indicates that the minimum of the secondary falls at phase 0.55. In his analysis of the spectroscopic data, Cester (1959) found that the eccentricity of the orbit was zero. Thus these two facts seem to contradict one another.

One explanation for the phase of secondary not falling at phase 0.50 may be due to nonsynchronism in the rotational and orbital periods of the components of UU Piscium. However, since the stars are so close together, their rotation should be synchronized with their revolution. Or the orbit may be eccentric. A lower limit of the eccentricity may be obtained from (Binnendijk 1960):

$$e \cos \omega = \frac{\pm \pi (D - \frac{1}{2})}{1 + \operatorname{cosec}^2 i}, \quad (5)$$

where e =eccentricity, ω =longitude of periastron, D =phase of secondary minimum, and i =inclination of

the orbit. The maximum shift of the phase of secondary minimum occurs when the longitude of periastron is 0 or 180 deg. Substituting an inclination of 38° and phase of secondary of 0.55 yields an eccentricity of 0.04. This is larger than the eccentricity found previously by MacCormack (1934) of 0.027 ± 0.016 (p.e.) and by Cester (1959) of 0.16 ± 0.015 (p.e.). It was using this eccentricity (0.016) that Cester found the inclination to be 19° .

The period of rotation of the line of apsides of a binary star with the approximate characteristics of UU Piscium would be shorter than any yet observed. The ratio of the orbital period of revolution to the period of rotation of the line of apsides may be calculated by (Schwarzschild 1958)

$$\begin{aligned} \frac{\text{orbital period}}{\text{apsidal period}} &= 15 \left[k_1 \left(\frac{R_1}{D} \right)^5 \frac{M_2}{M_1} + k_2 \left(\frac{R_2}{D} \right)^5 \frac{M_1}{M_2} \right] \\ &\quad \times \frac{1 + \frac{3}{2}e^2 + \frac{1}{8}e^4}{(1-e^2)^5}, \quad (6) \end{aligned}$$

where $k_{1,2}$ =numerical constants which depend upon the density distributions of the stars, $M_{1,2}$ =masses of the stars, $R_{1,2}$ =radii of the stars, D =distance between the center of the stars, and e =eccentricity of the orbit.

By extrapolating the calculated values (Schwarzschild 1958) of k for stars of spectral class B0-A0, to that for an F0 star, k is 0.005. If the observed values (Schwarzschild 1958) of k for stars in spectral class O9 through A5 are extrapolated to F0, k is 0.0018. The period of rotation of the line of apsides for the k obtained for the calculated and observed values is 1400 and 3500 days, respectively.

The photoelectric observations of Megalashvili and Kumishvili (1957) show the phase of secondary minima to be 0.45. The time interval between the observations reported by Megalashvili and Kumishvili (1957) and those of this paper is approximately 4800 Julian days. Thus apsidal motion may account for the differences in the observed light curves.

The observations of the radial-velocity curves were taken over a period of three years by MacCormack and of two years by Cester. Thus apsidal motion may have distorted the radial-velocity curves.

Thus from the data presented in this paper, and that of earlier observations of UU Piscium, the best model for this star system is one consisting of two nearly identical F0-type stars, with radii of 0.33, whose orbit is inclined at an angle of 38° and is undergoing apsidal rotation.

As we pointed out earlier, if apsidal motion is the cause of the discrepancies in the observed data, UU Piscium will have the smallest known period of rotation of the line of apsides. Future observations of UU Piscium are needed to confirm this.

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