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CURRENT STATUS OF DETERMINATIONS OF THE GRAVITATIONAL CONSTANT AND THE MASS OF THE EARTH

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A review is given of determinations of various gravitational constants. The Cavendish gravitational constant, which governs the accuracy of our knowledge of the earth's mass and mean density, is subject to particularly high uncertainty. An improvement in the precision of new determinations of the Cavendish constant should become possible by refining the theory and technique of measurement.

1. Constant quantities of various kinds play an important part in specifying the laws of nature. One quantity of fundamental significance in astronomy, physics, and geophysics is the gravitational constant: the coefficient of proportionality in Newton's law of attraction. This law serves to control the motion of celestial bodies, the shape of their external surfaces, the density distribution in their interiors, and in a certain sense their size as well. The center of gravity of the earth–moon system moves about the sun and the moon moves about the earth in accordance with the Newtonian attraction law. The shape of the earth, its size, and its internal structure in turn determine the character of its precessional, nutational, and other motions. Thus most of the constants in the system of astronomical constants are in the final analysis dependent on the properties of the gravitational field. The numerical value of the gravitational constant is required to calculate the motion taking place under the attraction of natural and artificial celestial bodies, and to study density irregularities within the mass distribution in the earth's interior from anomalies in the gravity field. The gravitational constant is one of the most important constants in cosmology and the theory of gravitation.

In order to solve various problems the gravitational constant is customarily expressed in different systems of units for measurement of mass, length, and time. Depending on the choice for the system of units one obtains different numerical values for the constant, each having its own name.

In celestial mechanics the so-called Gaussian constant k is employed; it is obtained under the convention that the masses of the celestial bodies studied are expressed in solar masses, and the distances in units of length close to the size of the major semiaxis of the earth's orbit. The Gaussian constant is found by using Kepler's third law to establish a relation between the period of revolution about the sun for the center of gravity of the earth–moon system, the major semiaxis of the orbit, and the masses of these celestial bodies. Kepler's law is known to be a consequence of Newton's law of attraction, so that the law of attraction is fundamental for derivation of the Gaussian constant. Its numerical value was originally obtained by setting the mass of the sun and the major semiaxis of the earth's orbit equal to unity, and determining by measurement the revolution period for the center of gravity of the earth–moon system and the ratio of the masses of the earth and moon to the mass of the sun. But since the value of the period and the ratio of the masses are continually being refined from the observations, changes have resulted in the Gaussian constant and thereby in several other astronomical constants related to it. For this reason it is presently regarded as equal to the perfectly exact quantity

$$k = 0.01720209895,$$

and in order for the equation describing the refined third Kepler law to be satisfied, the length of the major semiaxis is changed. The semiaxis thereby becomes slightly different from unity. Thus the unit of distance becomes a quantity determined in accordance with a fixed value for the Gaussian constant; it is called the astronomical unit (a.u.).

The numerical value of the Gaussian constant was determined by Newton himself 120 years prior to Gauss. It agrees with the modern value to six significant figures. Hence the name "Gaussian constant" should be regarded as a tribute to Gauss' services to celestial mechanics as a whole, instead of indicating priority in determining the numerical value of the gravitational constant used in celestial mechanics, as is sometimes considered in referring to his work.

The equations of motion assume a more convenient form for solving various problems in celestial mechanics if one adopts 58.13244 mean solar days as the unit of time, taking the solar mass and the major semiaxis of the earth's orbit equal to unity. In these units the gravitational constant is also equal to unity.

Calculations of the motion of celestial bodies around the earth require a knowledge of the so-called geocentric gravitational constant $fM_\oplus$, which is adopted as one of the fundamental astronomical constants. It represents the product of the gravitational constant f and the mass $M_\oplus$ of the earth, expressed in the metric system of units. There are many methods of determining $fM_\oplus$: from the motion of the moon about the earth, from the motion of artificial earth satellites, and from combined gravimetric and geodetic measurements over the whole earth. The geocentric gravitational constant has assumed an important practical significance with the launching of artificial satellites; it has therefore repeatedly been determined by different methods and is presently known to quite high accuracy.

Table 1 presents some determinations of the geocentric gravitational constant by various methods. We see that the relative accuracy of the current value for the geocentric constant is of the order of 10^{-6}.

At its 12th General Assembly in 1964 the International Astronomical Union adopted a value of $3.98603 \cdot 10^{20}\ cm^3 \cdot sec^2$ for the geocentric gravitational constant, as one of the fundamental astronomical constants. The true value of $M_\oplus$ is considered to fall in the range $3.98600 \cdot 10^{20}\ cm^3 \cdot sec^2 \leq fM_\oplus \leq 3.98606 \cdot 10^{20}\ cm^3 \cdot sec^2$.

2. In order to study the gravitational interaction between terrestrial bodies whose mass and size are expressed in metric units, we must know the gravitational constant f in this system of units. H. Cavendish was the first to make such a determination of the gravitational constant, expressed in the units of measurement adopted at that time. It is worth emphasizing that Cavendish's service in confirming the law of gravitation was greater than simply measuring the numerical value of f. He was the first to demonstrate experimentally that the mutual attraction between bodies on the earth can be detected. A contrary impression had prevailed since Newton's time. In estimating the time required for the mutual approach of two spheres, each 30 cm in diameter and with a density equal to the earth's mean density, Newton had made an error in calculation. He concluded that if the spheres were freed of all forces other than gravitation, and if they were separated by a gap of 0.6 cm, then they would require one month to come together through the influence of their mutual attraction. From this result he drew the far-reaching conclusion that it would be virtually impossible to observe the mutual attraction of terrestrial objects. Actually, however, the time in question is equal to only

TABLE 1

Method of determining $fM_\oplus$	Author, date of publication, reference	Value of $fM_\oplus$, $10^{20}\ cm^3 \cdot sec^2$
Geodesy	Kaula, 1961 [1]	3.986020 ± 0.000028
	Kaula and Uotila, 1962 [2]	3.986043
Motion of moon and radar distances	Yaplee et al., 1963 [3]	3.986057
Distant space vehicles	Sjogren et al., 1964 [4]	3.986009
	Wollenhaupt et al., 1964 [5]	
	Gaposchkin, 1966 [6]	3.986013
	Melbourne, 1969	3.986012 ± 0.000004
Close artificial earth satellites,	Kaula, 1963 [7]	3.986037 ± 0.000012
photographic observations	Kaula, 1963 [8]	3.985993 ± 0.000011

5 min! The prestige of the original discoverer of the law of attraction could not help but delay the first attempt to measure the mutual attraction between terrestrial bodies.

To distinguish the gravitational constant expressed in the metric system of units for measuring mass, length, and time from other forms, we shall call it the Cavendish gravitational constant, or the Cavendish constant.

All experiments for determining the Cavendish constant may be divided into two basic groups. The first of these includes experiments with a torsion balance (the Michell-Cavendish balance), in which the mutual attractive force between test masses is measured by comparing it with the elastic force of the twisted filament. For this purpose one measures the angle by which the filament has twisted due to the moment of the mutual attractive force between the test masses on the balance beam and other test masses brought up from the side (the static method). In an alternative procedure the gravitational constant is established by determining the derivative of the moment of the mutual attractive force between the test masses through measurement of the period of free torsional vibration (the dynamical method).

The second basic group comprises experiments with a vertical balance, in which the mutual attractive force of test masses is determined by comparison with the force of gravity. In these experiments one measures the variation in the weight of a test mass placed on a pan of the balance due to the attraction of a large test mass brought up from below. In a modification of the experiment one observes the variation in the deflection angle of a pendulum from the vertical due to the mutual attraction between a test mass attached to the end of the pendulum and another test mass brought up from the side. The deflection from the vertical reaches an angle such that the moment of gravitational force applied to the center of gravity of the pendulum is equal to the moment of the mutual attractive force between the test masses.

Some authors have set up their experiments in an effort to determine the mean density $\sigma_{\oplus}$ of the earth, which is related to the Cavendish gravitational constant by a simple expression given below. In these experiments the gravitational constant f has merely appeared as an intermediate quantity, so that it sometimes has not been quoted at all.

Table 2 presents a summary of the results of determinations of the Cavendish gravitational constant and the mean density of the earth. In compiling this listing we have in certain cases determined mean values for f or $\sigma_{\oplus}$, and to achieve uniformity we have recomputed the rms errors for several values.

Most of the experiments that have been performed to determine the Cavendish gravitational constant are now only of historical interest. We merely wish to point out that in carrying them out the authors have had to overcome a number of technical difficulties, for example, the experiments of Boys with a miniature quartz apparatus, where the length of the beam in the torsion-balance system was only 2.3 cm and the quartz torsion filament was only a few microns thick, or the experiments of Richarz and Krigar-Menzel, in which 100,000 kg of lead was used for the attracting mass.

The most accurate value for the Cavendish gravitational constant is that derived by Heyl and Chrzanowski in 1942 at the U.S. National Bureau of Standards in Washington [9].

These measurements were performed with a torsion balance having a light beam 20.6 cm long, at whose ends were attached 87-g spherical platinum test masses. The beam was suspended at its center by a tungsten filament 30-35 μ in diameter. The torsion system was placed in a vacuum. Two large test masses made of steel in the form of circular cylinders weighed about 66 kg. The gravitational constant was determined by the dynamical method from measurements of the period for the torsional vibration of the beam with the test masses in the gravitational field of the circular cylinders. A quartz clock was used to measure the torsional vibration period.

To a large extent the success of torsion-balance experiments depends on the stability of the elastic properties of the torsion filament. The elastic force represents the standard force with which the measured force of gravitational interaction between the test masses is compared. Although Heyl and Chrzanowski's experiments were performed with careful attention to the elastic properties of the torsion filament, a systematic error may nevertheless be detected in the values obtained from the interchange of torsion filaments (Table 3).

The difference between the mean values of f corresponding to annealed and nonannealed filaments is equal to $0.0070 \cdot 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot sec^{-2}}$, a value several times as large as the rms error in the determination of f from each of these types of filament.

A difference has also been found in the values of f obtained from a torsion system with different masses. As an illustration we give in Table 4 the

TABLE 2

No.	Author and site of experiments	Date of publ.	Value of gravitational constant f and rms error, 10^{11} $m^3 \cdot kg^{-1} \cdot sec^{-2}$	Mean density $\sigma_{\oplus}$ of earth and rms error, g/cm^3	Remarks and method: (T) torsion balance; (V) vertical balance; (LPVP) long-period vertical pendulum
1	H. Cavendish (Clapham, England)	1798	6.75 * ±0.05	5.45±0.04	Corrected by Baily for Cavendish's error in determining mean value (T)
2	F. Reich (Freiburg, Germany)	1838 1852	6.64 * ±0.06	5.54±0.05	Mean of results of 1852 experiment and 1838 experiment corrected by Reich himself (T)
3	F. Baily (Tavistock Place, England)	1843	6.63 * ±0.07	5.55±0.05	Corrected by Cornu and Baille (T)
4	A. Cornu and J. B. Baille (Paris)	1873 1878	6.64 * ±0.017	5.54±0.014	Mean of 1873 and 1878 results (T)
5	Ph. von Jolly (Munich)	1878	6.47 * ±0.11	5.69±0.10	(V)
6	J. Wilsing (Potsdam, Germany)	1889	6.594 * ±0.015	5.579±0.012	(LPVP)
7	J. H. Poynting (Birmingham, England)	1891	6.70±0.04	5.49±0.03	(V)
8	C. V. Boys (Oxford, England)	1895	6.658±0.007	5.527±0.006	(T)
9	R. von Eötvös (Budapest)	1896	6.657±0.013	5.5S3 * ±0.010	(T)
10	C. Braun (Mariaschein, Austria)	1897	6.649 ±0.002	5.529±0.002	(T)
11	F. Richarz and O. Krigar-Menzel (Spandau citadel, Germany)	1898	6.683 ±0.011	5.505±0.009	(V)
12	G. K. Burgess (Paris)	1902	6.64	5.55	Final result only quoted, without estimate of accuracy
13	P. R. Heyl (Washington	1930	6.670±0.005	5.510 * ±0.004	(T)
14	J. Zahradníček (Brno, Czechoslovakia)	1933	6.66±0.04	5.52 * ±0.04	Mean of resonance and dynamical methods (T)
15	P. R. Heyl and P. Chrzanowski (Washington)	1942	6.673±0.003	5.513 * ±0.003	(T)

*Values of $\sigma_{\oplus}$ and f designated with an asterisk have here been computed from the individual values found experimentally by the authors indicated.

TABLE 3

Tungsten torsion filament	Value of Cavendish gravitational constant f, 10^{-11} $m^3 \cdot kg^{-1} \cdot sec^{-2}$					Mean value of f and rms error, 10^{-11} $m^3 \cdot kg^{-1} \cdot sec^{-2}$
Nonannealed	6.6739	6.6756	6.6769	6.6762	6.6751	6.6755 ± 0.0008
Annealed	6.6670	6.6667	6.6703	6.6707	6.6680	6.6685 ± 0.0016

TABLE 4

Material of test masses at ends of balance beam	Values of Cavendish gravitational constant f and rms error, 10^{-11} $m^3 \cdot kg^{-1} \cdot sec^{-2}$
Gold	6.678 ± 0.003
Platinum	6.664 ± 0.002
Glass	6.674 ± 0.002

value of the gravitational constant determined by Heyl in 1930 for three different test masses at the ends of the torsion-balance beam [10].

With an rms error of $\pm(0.002\text{–}0.003) \cdot 10^{-11}$ $m^3 \cdot kg^{-1} \cdot sec^{-2}$ in the gravitational constant as determined in each series involving the same test-mass material, the difference in f corresponding to these series reaches $0.014 \cdot 10^{-11}$ $m^3 \cdot kg^{-1} \cdot sec^{-2}$. This discrepancy does not result from the difference in the material of the attracting masses, because it has been established from differential experiments that no such dependence exists to within a relative accuracy of at least $3 \cdot 10^{-10}$. It is hard to reach any definite conclusions at the present time regarding the true cause of the discrepancies in f in Table 4. But in any event the empirical data indicate that the final results of the determinations of the gravitational constant do contain some systematic errors associated with the interchange of test masses.

Apart from the 1930 results of Heyl and the 1942 results of Heyl and Chrzanowski, the values of f obtained by Boys and by Braun merit confidence. As is evident from an analysis of the experiments themselves as well as from the formal indicators, the rms errors in the mean result, these experiments were conducted on a relatively high level. A comparison of all the values of f (Table 2) shows that the difference between the results of these experiments is several times the rms error in the determination of f in each individual experiment. This circumstance suggests that uneliminated systematic errors remain in the experiments of different authors.

Thus the results of experiments for determining the gravitational constant contain substantial systematic errors which can hardly be entirely eliminated by taking the mean result of the experiments. For this reason a greater diversity of techniques should be used to establish the Cavendish gravitational constant; as the reference force one may employ the force of gravity, the elastic force of a torsion filament, or other small stable forces. It is desirable to determine the gravitational constant by both dynamical and static methods, to interchange the test masses, and so on. It is very important to set up experiments in different places with a variety of equipment and procedures developed by different authors.

Recently interest has again been stimulated toward determining the Cavendish gravitational constant. At the instigation of L. Éd'ed research was begun in 1966 at the University of Budapest under the direction of the Eötvös scholar Ya. Renner in an effort to make a new determination of the gravitational constant.

A new experiment for determining the gravitational constant by means of a torsion balance has been prepared at the Institute of Geodesy and Geophysics in Trieste in collaboration with the National Physical Laboratory at Teddington.

About 30 years have passed since the experiments of Heyl and Chrzanowski. During this period the techniques for measuring distances, masses, and time have been perfected. Progress has been particularly strong in methods of exact time measurement. The limited accuracy of time measurements has long been one of the obstacles confronting an application of the dynamical method for determining the Cavendish gravitational constant by means of torsion balances. Now, thanks to the availability of photoelectric techniques and quartz frequency standards, time-dependent sources of error have ceased to be the fundamental limitation. Techniques for measuring masses have shown hardly any appreciable change during the last 30 years. Relatively large progress has been made in perfecting the methods and techniques of distance measurement. This circumstance has enabled a new standard of length to be established, with lasers being applied as a procedure for measuring lengths.

One very important factor in the measurement of length is that it is measured between objects (marks, plane surfaces, the generatrices of a cylinder, and so on); thus it is important to prepare the test masses in a form convenient for linear measurements. In this connection it is evidently best to fabricate the test masses not in the form of spheres but in the form of circular cylinders, which furthermore are technologically easier to produce accurately. There is one further difficulty encountered in linear measurements: in the course of an experiment the beam with its test masses will not only execute torsional vibrations about the axis of the torsion filament, but also pendulum vibrations in other degrees of freedom. One must allow for the variation in the distances between the test masses due to these vibrations.

In addition to a knowledge of the value of the mass, density inhomogeneities within the attracting masses assume major importance. Several pioneers as well as some modern investigators therefore considered it desirable to use mercury, filling up a suitable vessel. But this practice does not resolve the problem, because it is difficult to establish accurately the dimensions and shape of the inner and outer surfaces of the vessel filled with mercury, so that the distance between the attracting masses becomes less accurately known.

An important contribution in refining the gravitational constant should come from a new, more exact theory for the instrumentation. One should first of all recognize the nonlinearity of the torsional-vibration problem. Thirty years ago the theory of nonlinear oscillations was only beginning to be developed. Today it offers far wider possibilities for application. Nonlinear vibrations will arise primarily because of the nonlinear dependence of the moment of forces for the mutual attraction of the test masses upon the angle by which the torsion system is deflected from its equilibrium position. The nonlinear dependence of the moment of the angle φ already appears when point masses are considered, but it enters to an even greater extent if test masses of more complicated shape are used. An application of nonspherical attracting masses complicates the theory in connection with an analytic description of the moment of mutual attraction forces. In a refined theory one should examine the question of the influence of nonlinearity due to the resistive forces of residual air, viscosity, and other forces. A more careful theoretical analysis of certain effects neglected in previous experiments is needed. We have in mind here a consideration of the vault where the experiment is conducted: irregularities in the gravity field arising from the attraction of columns, the walls of the building, and the like. These influences appear only in a treatment by a nonlinear theory. There is the problem of allowing for the influence of vibrations of the torsion system in other degrees of freedom, different from the basic torsional vibrations. Experiments under terrestrial conditions are unavoidably subject to the influence of microseisms and vibrations that distort the result. These interference effects, if not completely eliminated, should be taken into account by means of some definite theory for the influence of microseisms.

If one poses the problem of determining the gravitational constant to an accuracy of 10^{-5}–10^{-6}, the number of effects that should be considered becomes incomparably larger.

Since comparatively random quantities are adopted for the units of measurement of mass, length, and time, the Cavendish gravitational constant, like certain other fundamental constants (the velocity of light, the Planck constant, and others), is expressed in terms of irrational numbers. Actually, for the unit of mass one adopts the mass of the prototype preserved at the International Bureau of Weights and Measures in Paris; for the unit of length, 1,650,763.73 wavelengths in vacuo of the radiation corresponding to the transition between the $2p_{10}$ and $5d_5$ levels of the krypton 86 atoms; and for the unit of time, the second is defined as 1/31,556,925.9747 of the tropical year for 1900 January 0^d12^h Ephemeris Time.

One other conceivable approach to the establishment of units of measurement, suggested by Planck, is based on laws reflecting the fundamental properties of an objectively existing world. In the equations describing these laws the constants are set equal to unity. One strives to do this by varying one or several units of measurement (length, mass, time). Lippmann [11] had once suggested using Newton's law of attraction for introducing a unit of time not associated with the diurnal rotation of the earth. If we retain the same units of length (centimeters) and mass (grams) as before, but take the unit of time to be such that the gravitational constant is equal to unity, the new unit of time will be 3870 times greater than the mean second. The proposal has been made that it be called the "natural clock." The tempting idea of introducing natural units of measurement demands a high-precision determination of the fundamental constants, including the Cavendish gravitational constant.

The requirement for an accurate knowledge of the Cavendish gravitational constant begins to be keenly felt in connection with certain theoretical investigations in physics. In particular, there are several theoretical papers [12] devoted to the derivation of formal analytic relations between the fundamental constants, such as the velocity of light, the fine-structure constant, the Planck constant, the Cavendish gravitational constant, and others. A partial verification of these theoretical results can be attempted by substituting the values of the constants obtained experimentally into the derived connecting equations. The fine-structure constant has recently been determined to remarkably high accuracy [13]. The velocity of light and the Planck constant have been determined to a relative accuracy of roughly 10^{-6}. In view of this situation the low accuracy of our knowledge of the Cavendish gravitational constant seems particularly striking. It is very likely that eventually, if analytic relations between the gravitational constant and the other constants are established, the gravitational constant might be specified indirectly in terms of the others rather than by direct measurement of the gravitational effect between test masses.

In order to determine the mass $M_\oplus$ of the earth in metric units, one requires not only the geocentric gravitational constant $fM_\oplus$ but also the Cavendish gravitational constant f, while to determine the mean density σ the dimensions of the earth are also needed. The mass of the earth is specified simply as the ratio of the geocentric and Cavendish gravitational constants. Since the geocentric gravitational constant, as pointed out above, is now determined to a relative accuracy of order 10^{-6}, our knowledge of the mass of the earth is entirely limited by the low accuracy of our knowledge of the Cavendish gravitational constant.

Using the value for the geocentric gravitational constant adopted at the 12th General Assembly of the IAU, $fM_\oplus = 3.980603 \cdot 10^{20}\ \mathrm{cm}^3 \cdot \mathrm{sec}^{-2}$, and the value found by Heyl and Chrzanowski for the Cavendish gravitational constant, $f = (6.673 \pm 0.003) \cdot 10^{-8}\ \mathrm{cm}^3 \cdot \mathrm{g}^{-1} \cdot \mathrm{sec}^{-2}$, we may compute the mass of the earth to be

$$M_\oplus = (5.973 \pm 0.003) \cdot 10^{27}\ \mathrm{g}.$$

In order to obtain the mean density of the earth its mass must be divided by its volume Ω. Representing the earth by an ellipsoid of revolution whose volume $\Omega = {}^4\!/_3\,\pi a^3(1 + e^2)^{1/2}$, where a is the major semiaxis of the ellipsoid and e^2 is the eccentricity of a meridional cross section, we find

$$\sigma_\oplus = \frac{M_\oplus}{\Omega} = \frac{3}{4}\,\frac{M_\oplus}{\pi a^3 (1 - e^2)^{1/2}}.$$

According to a recommendation of the 12th General Assembly of the IAU, the true value lies in the range 6,378,080 m < a < 6,378,240 m. The accuracy of this value, like that of the quantity $e^2 = 0.0067$, cannot limit the accuracy with which the mean density of the earth is determined. Again the error in deriving $\sigma_\oplus$ depends entirely on the uncertainty in the determination of the Cavendish gravitational constant.

If we substitute into the last equation the values given above for $M_\oplus$, a, and e^2, we obtain a value for $\sigma_\oplus$ which, like the value of $M_\oplus$, cannot pretend to take its place among the most reliable modern values for these constants. The values given here have been called upon merely to illustrate the reason for their low accuracy:

$$\sigma_\oplus = (5.513 \pm 0.003)\ \mathrm{g/cm}^3.$$

Thus one of the fundamental constants of science, the Cavendish gravitational constant, is in need of a new experimental determination.

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