DERIVATION OF THE EAS SPECTRUM

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Measurements of the size or energy spectrum of extensive air showers require detecting arrays of widely different dimensions, and the shower sizes obtained with different arrays are not always directly comparable, as different methods are used for deriving "N" from the observations. An attempt is made to put the results of different experiments on a common scale, and it is pointed out that for very large showers there are several advantages in using the signal size at a certain fixed distance from the axis (perhaps 400—500 m) as the initial measure of a shower size. An empirical, calorimetric method is then used to relate shower sizes to primary energies, and good agreement is found between the energy spectra up to $>10^{19}$ eV obtained from various experiments by the empirical method and from model calculations.

Measurement of shower size

Measurements on the largest air showers, from 10^{17} eV upwards, are typically made at distances of hundreds of metres from the shower axis, so until the way in which the lateral distribution of particles changes with shower size is well known the actual number of particles, N, in the shower may be incorrectly estimated. The practical advantages of using the apparent particle density (or detector signal) at some distance $R \approx 400-500$ m from the shower axis as at least a preliminary classification of shower size are pointed out; then it is shown that this is actually useful in determining the primary energy spectrum.

First, starting with the raw data of particle densities, a particular distribution has to be assumed in order to determine the shower size and location of the axis. In the Haverah Park analysis, for example, the detector signal is assumed $\sim r^{-n}$ (200 m $< r <$ 1000 m, say), where n varies with shower size, and has to be determined empirically. Procedures in other experiments differ, but Haverah Park will serve as an example. In this case, results have been quoted in terms of E_{100}, the total energy deposition which would have been observed in a continuous detector covering the area from 100 m to 1000 m from the axis (the original array having three outer detectors spaced at 500 m from a central detector). For a given set of observed particle densities in a shower, a different choice of n will result in a slightly different best-fit core position, and a different inferred value of E_{100}. In a sample of 50 observed showers it was found that analysing them with assumed values of n differing by 0.6 typically gave values of E_{100} differing by a factor ≈ 1.7. (The "normal" n is probably known to an uncertainty much less than this, but if for each shower n were chosen to give an exact fit to the four densities in the original array, the values of n would fluctuate much more.) However, because of the geometry of the array, the alternative fits to the data obtained with the different structure functions are found usually to give the same answer for the density ρ_{500} at 500 m from the axis: in the sample examined ρ_{500} was usually altered less than 12% by the different assumptions. A similar effect will arise in other large arrays, the exact distance R for which the density is well-determined depending on the detector spacing.

It may be thought that since most of the particles lie within about 40 m of the axis, the density at 500 m is a poor measure of the energy of the shower. In several respects, however, it is better than a near-axis measurement. Briefly, (a) it is less subject to fluctuation in shower development, for the particle density at large axial distance R reaches its maximum lower in the atmosphere than does the total number N of particles, and at sea level showers of $10^{17}-10^{19}$ eV are favourably placed for observation around 500 m; (b) in the case of 120 cm water detectors, the signal at 400 m is proportional to primary energy and independent of primary mass over this energy range [1]. So not only is ρ_{500} for large showers more accurately measurable than N, but it is more closely related to primary energy. (Showers could also be grouped over a somewhat greater range of zenith angles, as follows from (a).)

Comparison of shower size spectra measured in different experiments

The more closely packed arrays such as those at Moscow and Agassiz obtained a more direct measure of shower size N. The widely spaced Volcano Ranch and Haverah Park arrays did not, so their shower size observations have been related to those obtained in the Agassiz experiment and others by attempting to join smoothly the density distribution curves at around 300 m, as the uncertainty in the density at 300 m on the smaller array is less serious than the uncertainty of extrapolating the data of the larger arrays in to 40 m.

Thus to relate the Haverah Park measure "E_{100}" to N, the signal obtained at 500 m was calculated (knowing the value of n used in the analysis), this was then converted to "scintillator density" using ratios determined by KELLERMANN and TOWERS (to be published) and joined to an MIT curve to find N as measured on the MIT scale. To continue this N calibration to larger shower sizes, a continuous change in the structure function was assumed, using Nishimura – Kamata – Greisen curves for the soft component, in which s changed smoothly to 1.0 at $N \approx 10^{11}$, starting from the observed shape at 10^6. The form of the muon distribution was supposed to contract steadily as the height of production fell, but the effect of this assumption was marginal. The integral spectrum of "shower size" N thus derived is shown in Fig. 1: for clarity the common procedure of plotting $N^{1.5} \times$ Rate is adopted.

When the Volcano Ranch data are treated in a similar way, working back from the quoted shower sizes, via the published assumed structure function to find the actual densities at 300 – 500 m in the showers detected, and then re-estimating N from a synthetic structure function as described earlier (allowing for changes in scale and s with altitude) the values of N obtained were around twice the sizes quoted in the original reports. (I understand that the muon densities would also support a similar conclusion, according to private communication from TURVER.) Replotting point by point the Volcano Ranch N spectrum [2] with the revised sizes, one sees that the long-standing discrepancy with the Chacaltaya spectrum disappears (Fig. 1). The fact that the size spectra at Chacaltaya and Volcano Ranch appear continuous, though taken at different altitudes, is deceptive: really they should cross at their common point, where showers reach maximum mid-way between the two altitudes and have the same size at both depths.

The size spectrum observed at sea level and at 3 mountain elevations are shown on Fig. 1. The small triangles continue the expected spectrum at the Volcano Ranch altitude to smaller shower sizes, and were obtained from the rate of inclined showers at Chacaltaya [3] which had penetrated 820 g cm^{-2} of atmosphere.

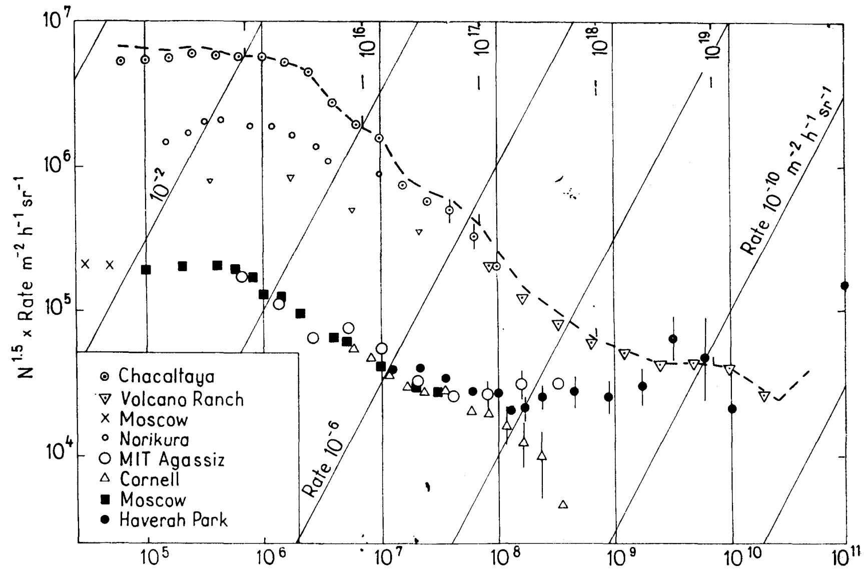

Fig. 1. Rate R of showers of more than N particles (plotted as $N^{1.5}R$) observed at various altitudes, and derived energy spectrum (see text). Data: Haverah Park: HOLLOWS, HUNTER, SURI (in press); ANDREWS et al. [12]; Volcano Ranch: LINSLEY [2], Moscow: VERNOV and KHRISTIANSEN [7], MISHNEV and NIKOLSKY [8]; Chacaltaya: BRADT et al. [3]; Cornell: DELVAILLE et al. [9]; MIT: CLARK et al. [10]; Norikura: KATSUMATA [11]

Derivation of the primary energy spectrum

It is useful first to deduce the primary energy spectrum empirically from the N spectrum at different altitudes, applying the calorimetric method. Fig. 2 shows the number of charged particles, N, as a function of depth, t, for showers (at the lower limit of a group) which arrive at the rate of 10^{-7} m^{-2} sec^{-1} sr^{-1} and also for those corresponding to a rate 10^3 times smaller, deduced from the Chacaltaya observations on inclined showers. The curves also agree well with sea level measurements. To obtain the total energy loss to ionization above sea level by a track-length integral the curve must be extrapolated above 520 g cm^{-2}: the shading indicates roughly the uncertainty in this if the extrapolation is guided by different theoretical models which fit the later part of the curve. Fluctuations in the longitudinal development somewhat distort the $N(t)$ curve, but tests suggest that they affect the $\int N(t)dt$ integral no more than does the extrapolation error. The total energy content of these two groups of showers then appears to be as given in Table 1.

(An energy loss of 2.2 MeV per g cm^{-2} was assumed, as presumably used in deriving N from scintillator signals. The 1965 Chacaltaya results agree better with rates observed in other experiments, and so have been preferred. However, scaled down points from the 1968 paper are shown as crosses in Fig. 2 and used as additional guides to the shape of the second curve.)

Table 1

Energy content of showers of two selected sizes

Size N at shower maximum	3.9×10^6	7.8×10^7
Size N at sea level	6.3×10^5	2.0×10^7
Ionization above sea level	4.4×10^{15} eV	8.8×10^{16} eV
Energy of soft component at sea level	0.14×10^{15} eV	0.4×10^{16} eV
Energy of hadrons at sea level	0.08×10^{15} eV	0.2 ?
Energy of muons at sea level	0.64×10^{15} eV	1.3×10^{16} eV
Energy of neutrinos (estimated)	0.33×10^{15} eV	0.6×10^{16} eV
Total energy E_p	5.6×10^{15} eV	11.3×10^{16} eV

From the two cases it appears that the primary energy is 1.4 GeV times the size N_{max}, to be compared with 1.6 GeV and 1.4 GeV, for the two energies in question, obtained by LaPointe et al. [4] from a more detailed model. As energies approach 10^{20} eV the conversion factor may start to rise a little, as the increasing width Δt of the shower outweighs the increasing efficiency of conversion of energy to particles.

If shower size spectra were obtained at all possible altitudes, the envelope of the resulting curves would be the graph of the rate of showers as a function of their size at maximum, neglecting the relatively small effects which fluctuations in development have on this conclusion. The dashed curve in Fig. 2 thus represents the N_{max}

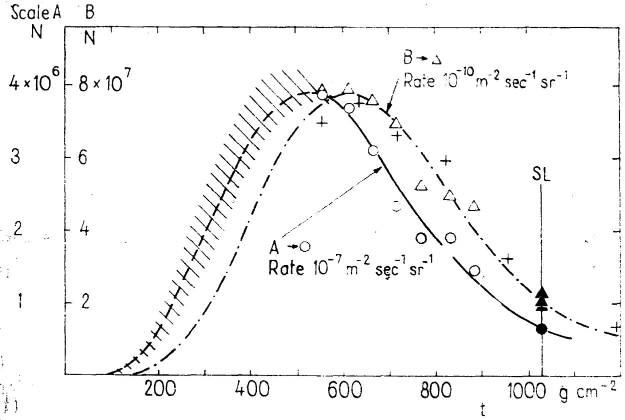

Fig. 2. The shower size N, above which the rate of detection is (A) 10^{-7} $m^{-2}sec^{-1}sr^{-1}$, (B) 10^{-10} $m^{-2}sec^{-1}sr^{-1}$, under various thicknesses, t, of atmosphere

spectrum of showers, touching the Chacaltaya and Volcano Ranch curves around sizes 10^6 and 10^9.

Multiplying the N scale by 1.4 GeV, the dashed line gives very closely the primary energy spectrum (though the structure-function-joining and calibration errors may increase around 10^{20} eV).

Having derived this spectrum empirically, it is useful to compare it with the primary energy spectrum obtained by a shower model calculation. Fig. 3 shows the integral energy spectrum already derived (in the form $E^{1.5} \times$ Rate), together with that obtained from the theoretical analysis of the Chacaltaya results [4] and from the

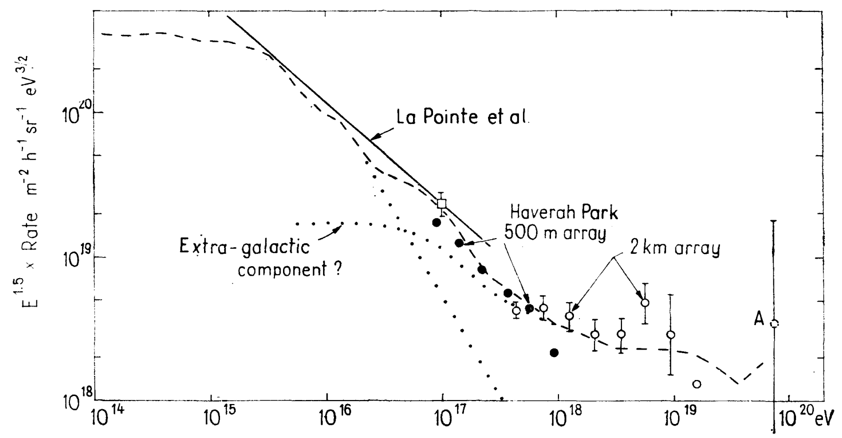

Fig. 3. The primary integral energy spectrum $R(E)$ plotted as $E^{1.5}R$. The dashed line is derived from Fig. 1 (see text), and the continuous line and points from theoretical models applied to particular shower groups observed at Chacaltaya and Haverah Park. Dotted line shows a possible separation of components

Haverah Park results. In view of the comments made in the first part of this paper, the energies calculated here for the Haverah Park showers (from the data presented by ANDREWS et al. [12] and data kindly supplied by Dr. WATSON) were obtained from the detector signal at 500 m from the axis using the conversion factor given in [1]. (Energy values a little higher are obtained if E_{100} is used as the measure of shower size because of the form of structure function assumed in the data reduction to derive E_{100}.)

It appears that there is quite good agreement between the different experiments in respect of the primary energy spectrum to beyond 10^{19} eV, and the theoretical and experimental estimates are in good accord.

A comparison of the N_{max} and $N_{sea\ level}$ size spectra may give a clue to the atomic mass of the primary particles. Fig. 4 shows the ratio N_{max} to $N_{sea\ level}$, reading from Fig. 1 the sizes of showers with equal rates. If to the right of the "knee" in the spectrum at $N \approx 10^6$ there is a constant energy per nucleon, as in the case of a rigidity cut-off, the height of shower maximum should remain steady, and the start of a fall is surprising, but the apparent rapid fall in the curve at 10^{17} eV, showing that showers suddenly seem to penetrate much further, suggests that this is where the extragalactic proton component takes over, and possible curves for the galactic and extragalactic components are shown dotted on Fig. 3. One might have expected this change to show in the proportion of muons observed in the showers, contrary to the observations at Haverah Park [5], but it is noted in [1] that the proportionality of ρ_{400} (almost the most effective region in judging shower size, as remarked above) to E_p makes the ratio of muon density to apparent shower size almost independent of the number of nucleons amongst which the primary energy is shared.

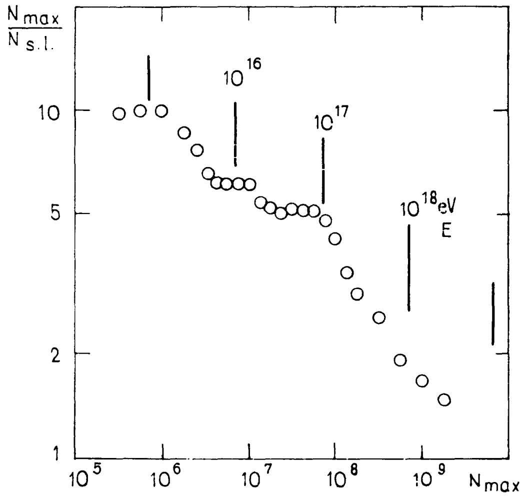

Fig. 4. Ratio of shower size at maximum to shower size at sea level, for different primary energie

The largest shower observed at Haverah Park (*A* on Fig. 3) might perhaps just be reconciled with the degradation expected from the 3° K black body radiation, but the largest Volcano Ranch shower raises the question as to whether there is also another primary radiation component, such as the neutrinos suggested by BEREZINSKY and ZATSEPIN [6].

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