The Spectrum of Primordial Radiation, its Distortions and their Significance

The idea of a hot Universe, with an overwhelming role of the radiation at the early stage, was accepted with enthusiasm in the last years.

Measurements of the isotropic radio radiation confirmed, or at least did not disprove, the equilibrium black-body spectrum (for a review see, for example, Ref. 1).

In the past year there has been disagreement between bolometric measurements on rockets[2] and indirect evaluations of the radiation at the short-wavelength end,[3] the last being consistent with the equilibrium spectrum.

The time has come to ask what theoretical cosmology has to say. To what extent is the prediction of the equilibrium spectrum rigorous? Would any small deviation of the spectrum from a Planck distribution destroy the hot-Universe model?—if one admits, of course, that the effect is proved without doubts not due to discrete sources.

As shown below, if energy is injected in the primeval plasma, one should obtain distortions of the Planckian spectrum.

The problem is not new, the relevant processes (Compton scattering, free-free emission, recombination and line radiation) were carefully listed by Weymann[4] and others. But in the last year we succeeded in giving an approximate physical picture which does not need a computer to relate the time of injection and the quantity of energy injected to the form of the ensuing spectrum distortions.[5,6]

In principle, the spectrum can give information about the annihilation of antimatter, about the amplitude of cosmological perturbations, about the recombination of hydrogen and helium in primeval matter.

It is difficult to estimate the reliability and accuracy of existing observational data. Further measurements are needed to resolve the controversy. Perhaps the full Planckian distribution will emerge. Still, it is possible that the bolometric measurements, even if not precise, indicate a strong deviation from a Planckian distribution. An overall, integrated over the spectrum, energy density which is 20–100 times greater than $\mathscr{E}_{\mathrm{eq}} = \sigma T^4_{\mathrm{R-J}} = \sigma(2.7\ ^\circ\mathrm{K})^4$ is not excluded by Ref. 3.

Yet, even if our calculations are premature and do not tell much when applied to known data, they will be perhaps of interest to experimentalists who are making more exact measurements.

We must warn at once that no explanation of the bolometric measurements[2] is given, which possibly means that these results are due to some kind of discrete sources. On the other hand, we feel that a full denial of the hot-Universe model is not warranted.

There are attempts to explain with discrete sources the radiation which is commonly believed to be primeval, but then the exact Rayleigh–Jeans frequency dependence at long wavelength is a mere coincidence. Measurements of the small-scale isotropy also give arguments against discrete sources.

However, if gross departures from an equilibrium spectrum are confirmed, the hot-Universe model must be extended by a tremendous injection of energy at some early, pregalactic stage of evolution, somewhere at temperatures between 30000 °K and 300 °K. The idea of a hot plasma dominated by radiation before this time span, $T > 30\,000$ °K, is not changed—therefore we are speaking of extension (not denial) of the hot model. Detailed assumptions on the energy sources are postponed.

There is a more subtle question: is it basically correct to speak about energy injection in the hot model? Isn't all the energy balance of the hot model given by the equations of the cosmological expansion?

Of course it is to some extent a matter of definition. It is clear that only a cosmological model with perturbations (as compared with the uniform and isotropic unperturbed Universe) can give the real picture with its structure, as given by galaxies, etc. It is the real world with perturbations that is considered and called the hot model; the extra energy associated with the perturbations is what is "injected" in the background represented by the idealized uniform unperturbed Universe.

After these preliminary remarks we will describe two principal cases of spectrum distortions: (1) energy injection before recombination, at $T > 4000$ °K, and (2) the same after recombination, at $T < 4000$ °K. After this some applications of the formulae to cosmological problems are shown.

1. *Processes in a fully ionized plasma*

This period is characterized by good thermal contact between electrons and radiation, due mainly to Compton scattering of quanta on electrons moving with thermal velocity. The free-free emission is small because the density of electrons and protons is much smaller than the density of quanta.‡ Therefore the rapid process is the adjustment of the equilibrium with a given density of electrons and quanta. The spectral distribution of the quanta is adjusted because their frequency is shifted by scattering ($\sim n_e$). The density of the quanta is not adjusted because that requires a slow free-free process ($\sim n_e^2$). Such a partial equilibrium is described by a Bose–Einstein distribution,

$$\begin{aligned} q &= (e^{(h\nu+\mu)/kT} - 1)^{-1}, \\ F_\nu &= \text{const}\ \nu^3 (e^{(h\nu+\mu)/kT} - 1)^{-1} = \text{const}\ \nu^3 (e^{x+m} - 1)^{-1}, \end{aligned} \tag{1}$$

which coincides with the Planck distribution only in the particular case $\mu = 0$. Here q is the occupation number of the quanta, F_ν is the spectral energy density, μ is the chemical potential; $x = h\nu/kT$, $m = \mu/kT$ are the dimensionless frequency and chemical potential.

The cosmological expansion preserves this form of spectrum, $m = \mu/kT = \text{const}$, $T \sim (1+z) \sim [a(t)]^{-1}$ during expansion (where a is the scale factor of the Universe, z the redshift). The result of the energy injection at a given quantum density is an increase of the temperature T and also of the chemical potential m. Therefore starting with full equilibrium, $m = 0$, after energy injection we obtain $m > 0$. This picture is overidealized, the free-free emission works to establish full equilibrium: it diminishes m. So for m an equation is written of the type

$$\frac{dm}{dt} = bQ - Cm \tag{2}$$

with Q proportional to the rate of energy injection and $-Cm$ describing the action of free-free emission. At large z, C is also large and no departure from a Planck distribution is possible. Free-free emission has a spectrum very different from a Planckian one; the adjustment of the equilibrium is not uniform. Detailed calculation gives the value of C in (2) and also the corrected form of the spectrum:

$$F_\nu = \text{const}\ \nu^3 [\exp(x + me^{-x_0/x}) - 1]^{-1}. \tag{3}$$

The Rayleigh–Jeans spectrum is restored at $\nu \to 0$, maximal departures are expected in the middle of the R–J part, at $x = x_0$ (Fig. 1).

The value of x_0 does not depend on the quantity of energy injected, but only on the rates of the free-free and Compton processes, that is

‡ The ratio $\gamma : n_e$ remains constant after the complete annihilation of e^+, e^-, that is at $T < 10^8$ °K. This ratio $\gamma : n_e$ is equal to $4 \times 10^7 \Omega^{-1}$, where Ω is a non-dimensional matter density:

$$\rho = \Omega \rho_{\text{crit}} = \Omega 3H_0^2/8\pi G = 2 \times 10^{-29} \Omega \text{ g/cm}^3$$

with $H_0 = 100$ km/sec-Mpc, ρ = present-day matter density.

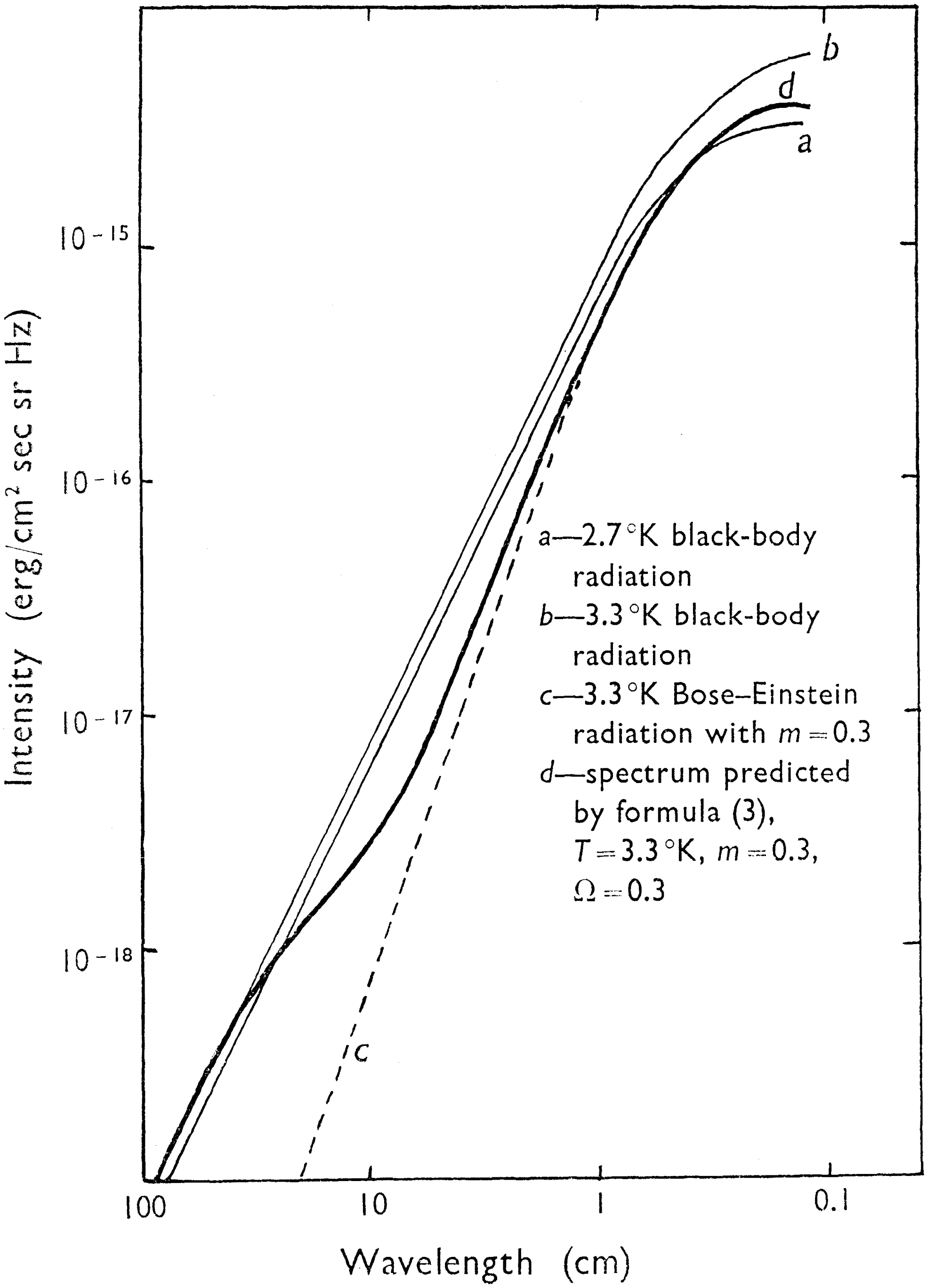

FIGURE 1.

only on the mean density of matter in the Universe. Calculations show that x_0 corresponds today to $\lambda_0 = 2.5\ \Omega^{-7/8}$ cm (2.5 cm with $\Omega = 1$, flat Universe, and 70 cm in the open model with $\rho_m = 4.5 \times 10^{-31}$ g/cm^3, $\Omega = 1/45$).

This region has now been fairly well investigated. No departures from the R–J law have been found. One can give a conservative estimate:

$$m < 5 \times 10^{-2}\ \Omega^{7/8}.$$

In turn, using (2) one can give a limit for the energy injected:

$$\int \frac{Q(t)}{\mathscr{E}(t)}\,dt = \int \frac{Q(t)}{\sigma T^4(t)}\,dt < 10^{-2}\,\Omega^{7/4},$$

where $\mathscr{E}(t)$ is the energy density of the radiation and Q is the rate of energy injection. So the observations of the radio spectrum can be interpreted as a proof of the smallness of the perturbations: the energy injected is less than 1% of the background radiation energy. The simple formula is written for the region where the decay of m due to free-free emission—the $-Cm$ term in (2)—is negligible. Thus the integral is taken from $z_1 = 10^4\,\Omega^{-1/5}$ (at redshifts greater than z_1 the Compton process can give the Bose–Einstein distribution of quanta in cosmological time) to $z_2 = 5.4 \times 10^4\,\Omega^{-6/5}$. The effect of an earlier energy injection is strongly damped by the simultaneous influence of Compton scattering and free-free emission. It is fair to say that z_2 gives the earliest moment for which the hot-model picture is proved by observations.‡

The connection between z and t at $z > 4 \times 10^4\,\Omega$ is $t = 3 \times 10^{19}\,z^{-2}$ sec; if $\Omega = 1/45$, $z_2 = 5.4 \times 10^6$, $t_2 = 10^6$ sec, $T_2 = 1.5 \times 10^7$ °K. Today, when relict neutrinos are not observable, the investigation of the Rayleigh–Jeans part of the relict radio spectrum is the best method to obtain information about the remote past of the Universe.

2. *Spectrum distortion due to energy injection after recombination*

We believe that the period of neutral matter starts after the recombination of helium and hydrogen; the residual ionization is small ($\sim 10^{-4}$).[7–9]

The process of recombination itself gives about one quantum per atom; this quantum belongs to the far Wien region of the spectrum ($x = h\nu/kT \approx 30$). Therefore the relative distortion of the spectrum in this region is not small—but its absolute intensity is low, so that infra-red sources, dust, etc., make observation very difficult. Neutral matter practically does not interact with radiation.

Now the contemporary situation is not that of a uniformly distributed neutral gas. Obviously at some time between recombination and the present day the formation of galaxies, quasars, radiosources, etc., took place. It is likely that these processes were accompanied by catastrophic events, and by injection of energy. Let us investigate the effect of this on the relict radiation.

The injection of energy in the form of shock waves, or hard radiation,

‡ To illustrate the weakness of bremsstrahlung: the optical depth of the horizon, ct, for free-free absorption of quanta with $h\nu \sim kT$ is of the order of unity only at $z > 10^8$, $t < 10^3$ sec.

or fast particles, first ionizes matter and makes electrons hot. We are led to consider the Compton scattering of relict radiation, with T_r varying from < 4000 °K ($z < 1500$) up to 2.7 °K ($z = 0$), on electrons whose temperature is of the order $10^4 < T_e < 10^8$ °K, so that $T_e \gg T_r$. Really important is only the last assumption $T_e \gg T_r$. The distortion of the spectrum consists in an increase more frequently than a decrease of the frequency of the quanta. Therefore the intensity in the Rayleigh–Jeans region decreases, the maximum is shifted to higher frequencies. Detailed calculations show that the resulting spectrum depends only on one parameter,

$$y = \int \frac{kT_e}{m_e c^2}\, d\tau \approx 10^{-11}\, \Omega^{1/2}\, T_e\, \bar{z}^{3/2},$$

where τ is the optical depth for Thompson scattering.

A Planckian spectrum which would give today T_0 if undisturbed, is deformed as a result of Compton scattering so that at low frequencies the effective temperature decreases,

$$T_{\mathrm{R-J}} = T_0 e^{-2y}$$

(but the frequency dependence remains $F_\nu \sim \nu^2$) and the overall energy of the radiation increases,

$$\mathscr{E} = \mathscr{E}_0 e^{4y} = \sigma T_0{}^4 e^{4y} = \sigma T_{\mathrm{R-J}}^4 e^{12y}.$$

For details see Ref. 5.

In principle, two measurements at different wavelengths are enough to calculate $T_{\mathrm{R-J}}$ and y and all the spectrum (Fig. 2). If distortions are found, precise measurements of the spectrum either agree with the predicted form and give y, or disprove the initial assumption about Compton scattering on hot electrons.

What do we obtain from observations? The bolometer results, taken literally at the lowest bound, call for $y \geqslant 0.5$. Measurements of CN excitation at $\lambda = 0.254$ cm (Ref. 3) give, with one standard deviation, $y \leqslant 0.15$. Finally, CH excitation[3] requires $y \leqslant 0.35$. If $\mathscr{E} = k\sigma T_{\mathrm{R-J}}^4$, $k = e^{12y}$,

$$k = 400,\ 6,\ 65 \qquad \text{for} \qquad y = 0.5,\ 0.15,\ 0.35.$$

Perhaps $y = 0.35$ (but not $y > 0.5$) could be reconciled with cosmic-ray observations.[10,11] The need for further precise and reliable measurements is obvious.

A minimal $z_i > 100$ is necessary for $y = 0.35$ because T_e must not exceed the upper limit given by x-ray measurements. The quantity of

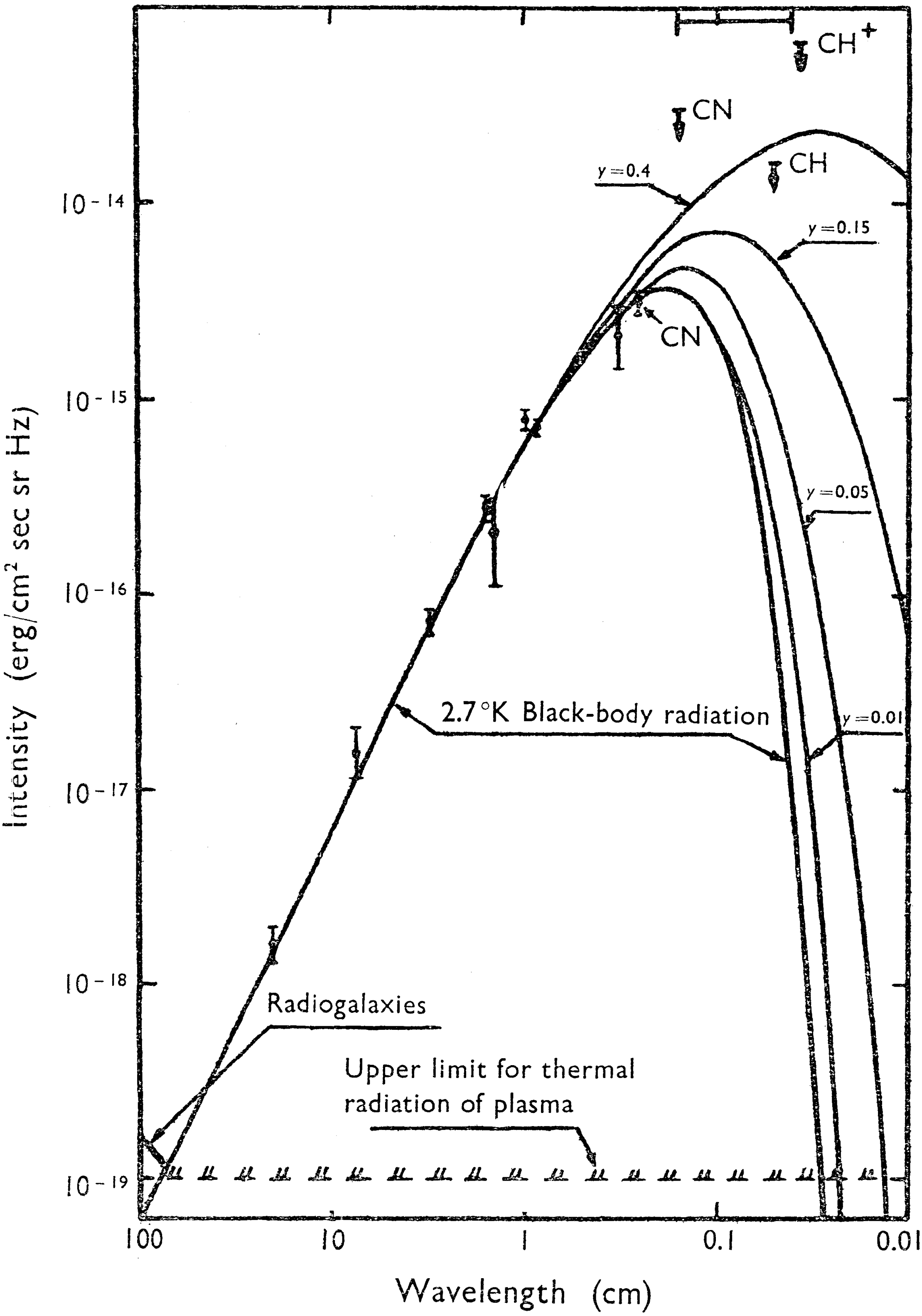
CH+
CN
CH
y = 0.4
y = 0.15
CN
y = 0.05
2.7 °K Black-body radiation
y = 0.01
Radiogalaxies
Upper limit for thermal radiation of plasma
10 - 14
10 - 15
10 - 16
10 - 17
10 - 18
10 - 19
Intensity (erg/cm² sec sr Hz)
100
10
1
0.1
0.01
Wavelength (cm)

FIGURE 2.

energy injected is given by

$$\sigma T^4_{\text{R-J}} (e^{12y} - e^{8y}) \overline{(1 + z_i)}$$

and depends not only on y, but also on the moment of injection $\overline{z_i}$.

We state briefly (see also Refs. 5 and 6) which are the cosmological implications of the spectrum measurements.

(1) Turbulence of the primeval ylem before recombination is often assumed; one can give the upper limit for the energy dissipated.

(2) The macroscopic motion of plasma with radiation leads to Doppler shift of the spectrum and after Thompson scatterings to a spectrum of the same kind as on Fig. 2.

(3) The gravitational instability at small wavelength is transformed into standing sound waves which are dissipated by photon viscosity[12] and shock generation.[13] Here also the upper limit is given.

(4) In Harrison's picture of matter and antimatter in the Universe[14] one should predict annihilation before recombination. The absence of spectrum distortions needs special explanation.

(5) The energy injection after recombination (leading to $y > 0$) does not remove the characteristic Bose–Einstein ($m > 0$) distortions. If real distortions at the short-wave end of the spectrum are established, the detailed picture of the catastrophic energy injection must be cleared up. Perhaps not only the heating of the electrons and the Compton effect are important, but also the motion of the plasma, the nonuniformity of its temperature, and the direct radiation of sources.

R. A. Sunyaev
Ya. B. Zeldovich

References

1. R. B. Partridge, Am. Scientist **57,** 37 (1969).
2. K. Shivanandan, J. R. Houck, and M. O. Harwit, Phys. Rev. Letters **21,** 1460 (1968).
   J. R. Houck and M. O. Harwit, Astrophys. J. **157,** L 45 (1969).
3. V. J. Bortolot, S. F. Clauser, and P. Thaddeus, Phys. Rev. Letters **22,** 307 (1969).
4. R. Weymann, Astrophys. J. **145,** 566 (1966).
5. Ya. B. Zeldovich and R. A. Sunyaev, Astrophys. Space Sci. **4,** 285 (1969).
6. Ya. B. Zeldovich and R. A. Sunyaev, preprint, 1969, Astrophys. Space Sci. (in press).
7. R. A. Sunyaev, Dokl. Akad. Nauk SSSR **179,** 45 (1968) [Soviet Phys.—Dokl. **13,** 183 (1968)].
8. Ya. B. Zeldovich, V. G. Kurt, and R. A. Sunyaev, Zh. Eksp. Teor. Fiz. **55,** 278 (1968) [Soviet Phys.—JETP **28,** 146 (1969)].
9. P. J. E. Peebles, Astrophys. J. **153,** 7 (1968).
10. M. V. K. Apparao, Nature **219,** 709 (1968).
11. P. Encrenaz and R. B. Partridge, Astrophys. Letters **3,** 161 (1969).
12. J. Silk, Astrophys. J. **151,** 459 (1968).
13. P. J. E. Peebles, preprint, 1969.
14. E. R. Harrison, Phys. Rev. Letters **18,** 1011 (1967).