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Comets and Nongravitational Forces. III

B. G. MARSDEN

Smithsonian Institution, Astrophysical Observatory, Cambridge, Massachusetts

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Three particular problems encountered in Paper II of this series are explored further. The first concerns the difference between the nongravitational effects detected in the motions of the short- and the long-period comets. In the former case, one determines essentially the transverse component of the forces, and in the latter case the radial component: The radial component seems to depend on the inverse square of heliocentric distance, while the transverse component evidently depends on a much higher power of inverse heliocentric distance. A further study has been made of the second problem, that of the systematic residuals in the orbit of P/Encke, but we are still unable to resolve it completely. The longest section of the paper deals with the difficulties that seem to arise when a comet strongly affected by nongravitational forces makes a relatively close approach to Jupiter. We have not succeeded in uncovering precisely what went amiss in the cases of P/Schaumasse and P/Perrine-Mrkos, but we find that the problem is perhaps not so general as might be supposed. On the other hand, we have established that the secular variation in the motion of P/Pons-Winnecke changed sign around the turn of the century, a fact that is presumably connected with this comet's repeated encounters with Jupiter.

I. INTRODUCTION

IN Papers I and II of this series (Marsden 1968a, 1969a), we showed that, as a rule, the motions of short-period comets are affected by nongravitational forces that seem to act in an essentially continuous manner. It is, of course, hard to *prove* that the forces act continuously, for the same effects could be produced by discrete impulses—but these impulsive forces would have to be remarkably regular in their action. The numerical results obtained provide considerable evidence in favor of the Whipple (1950) icy-conglomerate model for cometary nuclei, provided that the model is modified so that the inner regions of the nucleus contain less volatile—perhaps even nonvolatile—material. In this third paper, we present the results of additional computations. In Secs. II–IV we explore further some of the problems encountered in Paper II: the possible detection of the effects of nongravitational forces in the motions of long-period comets, the systematic residuals remaining in the case of Encke's comet, and the question of the interaction of nongravitational and severe gravitational forces, particularly when a comet makes a close approach to Jupiter.

II. LONG-PERIOD COMETS

Comet 1960 II (Burnham)

Although this comet was under observation for only six months, it was shown in Paper II that if one solves for nongravitational parameters, not only can the systematic residuals be considerably reduced, but it

also becomes unnecessary to postulate that the comet originated outside the solar system. It was assumed that the auxiliary quantities C and α were equal to 2 and 3, respectively (the values used in the case of the short-period comets), and the mean residual of the 37 observations by Roemer *et al.* (1966) could thereby be reduced from 3".06 to 1".86. The residuals were still not completely satisfactory, however. A calculation has now been performed in which $C = \infty$, $\alpha = 2$ (i.e., the nongravitational forces vary according to the inverse square of the heliocentric distance). This caused the mean residual to be reduced to only 1".39, the individual values being shown in Table I. Comparison with the figures in Table IX of Paper II shows further significant improvement. The osculating elements and nongravitational parameters are:

Epoch	1960 Apr. 16.0 E.T.
T	= 1960 Mar. 20.99168 $\pm$ 0.00035 (m.e.) E.T.
ω	= 306°.63182 $\pm$ 0°.00114
Ω	= 251.95829 $\pm$ 0.00044
i	= 159.60182 $\pm$ 0.00017
q	= 0.5043310 $\pm$ 0.0000046 a.u.
e	= 1.0000592 $\pm$ 0.0000158
$1/a$	= -0.0001175 $\pm$ 0.0000313 a.u. ⁻¹
A_1	= (+9.2715 $\pm$ 0.5138) $\times 10^{-5}$
A_2	= (-1.8779 $\pm$ 0.3574) $\times 10^{-5}$ orbit III.

Little significance should be attached to the transverse component A_2 , but on dividing the radial component

TABLE I. Residuals for comet 1960 II (Orbit III).

1960 U.T.	$\Delta\alpha \cos\delta$	$\Delta\delta$	1960 U.T.	$\Delta\alpha \cos\delta$	$\Delta\delta$
Jan. 4.07	-0.6	-0.9	Apr. 25.41	+0.6	-0.7
4.07	-0.1	-0.7	25.41	+0.7	-0.4
18.07	-1.7	-1.0	29.36	+1.5	+0.6
18.08	-0.8	-0.3	29.36	+0.5	+1.1
28.08	-1.9	+1.0	May 1.38	-1.5	+0.2
28.09	-2.6	+0.5	1.39	-2.0	+0.5
Feb. 7.09	+1.3	0.0	6.31	-1.4	-0.4
7.10	+1.5	+0.2	6.31	-1.6	+0.3
15.09	+2.5	+0.3	11.19	+1.4	-1.0
15.10	+3.0	+0.8	11.19	+2.0	-0.9
17.10	+2.4	+1.4	16.25	+2.1	-0.2
17.10	+2.8	+1.5	16.25	+1.5	-0.1
Apr. 4.51	-3.3	-1.5	25.22	-0.8	+1.6
4.51	-1.2	-2.1	25.23	-0.3	+1.7
8.50	-0.9	+1.0	31.24	-0.1	-1.1
8.50	-0.4	+1.0	31.26	+0.1	-0.9
17.48	-3.2	+0.7	June 17.20	+0.5	+0.5
17.48	-3.7	+0.5	17.25	+1.0	+1.2
21.44	+0.2	+0.3			

A_1 by 0.473, we see that the nongravitational force away from the sun amounts to 20×10^{-5} that of the solar gravitational force of attraction (as in fact was estimated in Table X of Paper II). Since the osculating eccentricity of orbit III is even less than that of orbit II ($C=2$, $\alpha=3$; see Paper II), it is even more apparent that the original orbit of this comet was an ellipse.

Comet 1957 III (Arend-Roland)

Sekanina (1968) has suggested that this comet is another one for which one has to conclude either that the motion is strongly affected by nongravitational forces (acting essentially continuously) or that the original orbit was substantially hyperbolic. His gravitational solution from 563 observations covering the whole observed arc (1956 November 7 to 1958 April 11) gave a mean residual of $2''.04$ and noticeable systematic trends.

A gravitational solution by the writer, based on 150 selected observations, confirms the systematic trends (to about $4''$); the mean residual is $1''.69$. The failure to explain the motion by purely gravitational theory is certainly not so striking as with comet 1960 II, but it is nonetheless quite evident. A solution for the nongravitational parameters as well (with $C=2$, $\alpha=3$) showed an improvement, the mean residual being reduced to $1''.53$, but there are still systematic trends of some $3''$. Finally, a solution in which the nongravitational forces follow an inverse-square law of heliocentric distance gave an excellent result, with the mean residual only $1''.36$:

$$\begin{aligned}
 \text{Epoch } & 1957 \text{ May } 2.0 \text{ E.T.} \\
 T & = 1957 \text{ Apr. } 8.03201 \pm 0.00011^{\frac{1}{2}} (\text{m.e.}) \text{ E.T.} \\
 \omega & = 308.77725 \pm 0.00048 \\
 \Omega & = 215.15968 \pm 0.00008 \\
 i & = 119.94930 \pm 0.00006 \\
 q & = 0.3160361 \pm 0.000024^{\frac{1}{2}} \text{ a.u.}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} 1950.0$$

$$e = 1.0002015 \pm 0.0000067$$

$$1/a = -0.0006377 \pm 0.0000213 \text{ a.u.}^{-1}$$

$$A_1 = (+3.1987 \pm 0.3126) \times 10^{-5}$$

$$A_2 = (+1.5765 \pm 0.4960) \times 10^{-6}.$$

The transverse component could again have been neglected, while the radial component shows that the repulsive force amounts to 7×10^{-5} that of solar gravitational attraction.

The value of $1/a$ in the purely gravitational solution was $-0.0007798 \pm 0.0000058 \text{ a.u.}^{-1}$. Applying the increment (Marsden 1968b) to reduce this to the original value (referred to the center of mass of the solar system), we obtain the result $-0.0000976 \text{ a.u.}^{-1}$ —still hyperbolic by 17 times the mean error of its determination. Applying the same increment instead to the value of $1/a$ in the above nongravitational orbit, we find the original value to be $+0.0000445 \text{ a.u.}^{-1}$, and this comet, too, could clearly have originated in the solar system.

Remarks

The discovery that the nongravitational forces acting on these two long-period comets seem to obey something like an inverse-square law, whereas the forces on short-period comets definitely do not, may seem puzzling at first sight. What it probably means is that the radial component of the forces varies according to an inverse-square law, but that the transverse component goes as a much higher power of inverse heliocentric distance. [The quantities $C=2$, $\alpha=3$ for short-period comets (where it is essentially the transverse component that one determines) were adopted rather arbitrarily; but if one wants to use instead an inverse-power law ($C=\infty$), the power α has to be something like 12.] In terms of the Whipple model, this means that the lag angle λ depends on a high power of the heliocentric distance. A theoretical reason why this should be so is not obvious, and indeed the finding (Paper II) that for short-period comets the radial component is generally an order of magnitude larger than the transverse component suggests that λ is a small and therefore approximately constant angle. Possibly this means that cometary nuclei rotate very slowly. In spite of the fact that an inverse square law may be preferable for the radial component of the nongravitational forces, we shall continue to use here $C=2$, $\alpha=3$ for both force components for the short-period comets—the subject of the remaining sections of this paper.

One might imagine that comet 1962 VIII (Humason), which exhibited tremendous variations in its physical appearance and which was under observation for more than four years, would show in its motion the effects of nongravitational forces. A solution for A_1 gives a value less than half its mean error, however, and the resulting residuals were actually slightly worse than in the (excel-

lent) purely gravitational representation (Marsden 1967a). Perhaps the perihelion distance of this comet was too large (2.1 a.u.) for the forces with which we are concerned here to show. Of the recent comets with small perihelion distances, a good candidate for showing the forces would be comet 1962 III (Seki-Lines), having a perihelion distance of only 0.031 a.u. and observed quite extensively on each side of perihelion passage. The gravitational solution by Marsden (1969b) gives a mean residual of 1".79, but there were observational problems with this comet, and it is not at all improbable that the rather systematic trend in the residuals soon after perihelion is due to the difficulty in measuring positions from overexposed images of a bright comet at low altitude.

It is rather curious that comets 1957 III and 1960 II (apparently the only recent comets the motions of which are noticeably affected by nongravitational forces) should be physically so dissimilar. While the spectrum of the former comet showed a rather strong continuum, that of the latter was noteworthy for its almost complete lack of continuum (Liller 1961).

III. PERIODIC COMET ENCKE

The first solution in Paper II for which observations over the interval 1927–1967 were used (orbit III) gave for the nongravitational parameters:

$$\begin{aligned} A_1 &= (-2.0751 \pm 0.2428) \times 10^{-7}, \\ A_2 &= (-5.2655 \pm 0.0099) \times 10^{-8}, \\ B_2 &= +0.7647 \pm 0.0046. \end{aligned}$$

As already stated, we assume that $C=2$, $\alpha=3$, which means that the additional acceleration components in the equations of motion are $G_i \exp(-r^2/2)r^{-3}$, where $G_i = A_i \exp(-B_i \tau)$, the heliocentric distance r being measured in astronomical units and the A_i being consistent with a time unit of 40 d. The time τ is measured in units of 10^4 d, in this case from the epoch 1957 October 9.0 E.T. The subscript 1 refers to the radial component (positive outward from the sun); 2 refers to the transverse component; B_1 was set equal to zero.

Orbit III is not completely satisfactory, however, the mean residual of the 130 observations used being 2".6, with systematic trends in excess of 6". In Paper II we made the statement that the secular acceleration coefficients determined in the past (by several investigators) strongly suggested an exponential decrease with time. This was one of the reasons why we included the factor $\exp(-B_i \tau)$ in our equations of motion. A stronger motivation for using this exponential variation, rather than, say, a linear variation, was that it prevents the magnitude of the force from becoming larger with time (no reliable negative values of B_i were found) and the direction from changing sign (secular accelerations were not observed to change into secular decelerations). It is

apparent from the logarithmic plot by Sekanina (1969, Fig. 3), however, that the decrease of the secular acceleration coefficient κ for Encke's comet has not been exponential. This is particularly true if one considers that orbit III corresponds essentially to a straight line through the three closed circles in his figure. It is evident that orbit III "predicts" much larger values for κ in the nineteenth century than were actually observed; indeed, if one runs orbit III directly back into the past, one obtains perihelion times that were 3.6 days too early in 1868, 48 days too early in 1819, and 158 days too early in 1786.

In order to try to improve the "predictions" for these earlier perihelion times, we have now assumed that the B_i are *themselves* exponential functions of the time, namely that $B_i = B_{0i} \exp(-b_i \tau)$. We also took $B_{01} = B_{02}$, $b_1 = b_2$, which is perhaps preferable to adopting $B_1 = 0$, as was done in orbit III. The fit to the observations 1927–1967 gave the following result:

Epoch	1957 Oct. 9.0 E.T.
T	= 1957 Oct. 19.84878 $\pm$ 0.00011 (m.e.) E.T.
ω	= 185°.22573 $\pm$ 0°.00043
Ω	= 334.73196 $\pm$ 0.00042
i	= 12.37468 $\pm$ 0.00007
q	= 0.3381160 $\pm$ 0.0000007 a.u.
e	= 0.8474007 $\pm$ 0.0000003
A_1	= (-1.7010 $\pm$ 0.1548) $\times 10^{-7}$
A_2	= (-5.2346 $\pm$ 0.0123) $\times 10^{-8}$
$B_{01} = B_{02}$	= +0.8676 $\pm$ 0.0164
$b_1 = b_2$	= -0.1765 $\pm$ 0.0223 orbit IV.

Except for the fact that $B_1 = B_2$ now changes with time (and is in fact increasing), orbit IV does not differ very much from orbit III. The residuals of the observations improved only slightly, the mean residual falling to 2".4. What is important, however, is that orbit IV represents the earlier perihelion times very much more satisfactorily, as shown in Fig. 1. All the perihelion times back to 1832 are satisfied within 0.4 d, and by 1786 the error had increased only to 3 d. Since only two (rather discordant) observations were made in 1786 (and there was none in 1789 and 1792), one cannot be sure whether the inflection at the left-hand end of the figure is real. The broken curve in Sekanina's figure is essentially (but by no means exactly) the time derivative of our curve and shows how orbit IV represents the various determinations of κ .

Sekanina has suggested that the determinations of the secular acceleration coefficient seem to follow a law of the form $\kappa = \Lambda(1 - \alpha_0 t)^2$, where Λ and α_0 are constants and t is the time (α_0^{-1} is in fact the year of the "death date" of the comet, or at least the date when κ vanishes). Accordingly, we made a solution (again from the observations 1927–1967) with $G_i = A_i(1 - \beta \tau)^2$. The result for

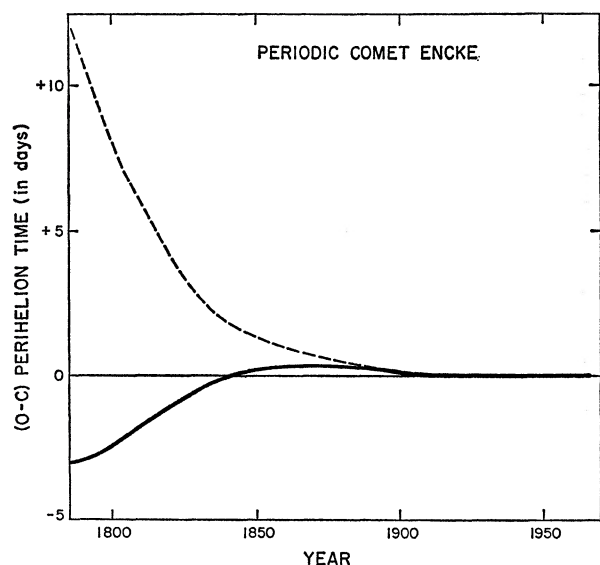


FIG. 1. (O-C) perihelion times: The solid curve represents the "double exponential" solution (orbit IV), the broken curve the solution with the quadratic form $(1-\beta\tau)^2$. Both orbits were fitted to observations over the interval 1927-1967.

the nongravitational parameters was

$$\begin{aligned} A_1 &= (-1.7255 \pm 0.1508) \times 10^{-7}, \\ A_2 &= (-5.2339 \pm 0.0029) \times 10^{-8}, \\ \beta &= +0.4393 \pm 0.0022. \end{aligned}$$

The residuals were very similar to those of orbit IV, but as can be seen from Fig. 1, the earlier perihelion times are not represented so well. Sekanina's long-scale result $\alpha_0^{-1} = 2036$ corresponds to our $\beta = +0.351$. A solution in which this value is assumed does not fit the observations since 1927 at all well, however, the residuals showing systematic trends in excess of $20''$.

We have not succeeded in selecting equations of motion that yield residuals over the interval 1927-1967 better than those of orbit IV. Our attempts to include a force contribution that is asymmetrical with respect to perihelion failed completely. The fact that orbit IV represents so well the perihelion times during the nineteenth century suggests that it may do so during much of the twenty-first century. On the other hand, it must not be presumed that it gives information about the magnitude of the nongravitational forces before the eighteenth century.

It is worth pointing out here that a slight over-generalization was made in Paper II. We stated that when the radial component of the nongravitational force acting on a comet was determined at all well, it was outward from the sun and about an order of magnitude larger than the transverse component. This certainly seems to be the case for the majority of comets. In the case of P/Encke, the various solutions for A_1 seemed to be rather inconclusive, but with an apparent preference for *negative* values. It is obvious,

however, that while the radial component may or may not be directed away from the sun, it is definitely *not* an order of magnitude larger than the transverse component *and* directed away from the sun.

IV. INTERACTION OF NONGRAVITATIONAL AND GRAVITATIONAL FORCES

Very serious difficulties were encountered in Paper II in the case of P/Schaumasse. It proved to be quite impossible to link the observations in 1911, 1919, and 1927-1928 with those in 1943-1944, 1951-1952, and 1959-1960. We surmised that the trouble was connected with the close approach to Jupiter (to within 0.37 a.u.) in 1937. The nature of the trouble was felt to be more mathematical than physical, i.e., due to the inadequacy of the equations of motion (which, after all, are empirical as far as the nongravitational terms are concerned), rather than to any direct physical disturbance from Jupiter or other objects in its vicinity. It is very necessary to establish whether similar difficulties arise with other comets when they pass near Jupiter. After all, one of the principal reasons put forth in Paper I for utilizing actual components of the nongravitational forces, rather than applying secular variations to the Keplerian elements, was the hope that one could in this manner handle cases where there is an encounter with Jupiter—and consequently large changes in the Keplerian elements.

P/Perrine-Mrkos

Sitarski (1968) linked the observations in 1955 and 1961-1962 and provided a prediction for the return of this comet in 1968. This prediction was closely confirmed by a similar computation by the writer (unpublished), who obtained a perihelion time $T = 1968$ November 2.31 E.T., some 0.04 d later than that predicted by Sitarski. The fact that this comet is unusually diffuse suggested that it is quite strongly affected by nongravitational forces, and indeed, on running the orbit back to 1909 and 1896 (the only other occasions when the comet has been observed), we found that it gave perihelion times some 3 or 4 d later than observed. This indicated that the comet had a secular acceleration of about 0.1 d per (period)². A further solution for the orbital parameters from the 1955 and 1961-1962 observations, *assuming* that the nongravitational parameter $A_2 = -3.354 \times 10^{-6}$ ($C=2$, $\alpha=3$), caused the 1896 perihelion time to be satisfied almost exactly and gave $T = 1968$ November 2.21 E.T. A discrepancy of almost 0.5 d still remained at the 1909 return, but we felt that the published 1909 elements (Kobold 1909) were less likely to be reliable than the 1896 elements by Osten (1898). Clearly, if we had instead fitted the 1909 perihelion time, the 1968 figure would not have changed by more than 0.01 d; it also seemed unnecessary at this stage to try to determine a

new orbit from all the observations 1896–1962 and solve for the nongravitational parameters directly.

But the *observed* perihelion time in 1968 was not 0.10 d earlier than given by our purely gravitational prediction, but 0.77 d earlier! At recovery, the comet was only 0.31 a.u. from the Earth, so its position in the sky was more than 2° from any of the predictions. It was not brighter than 15th mag. If the 122-cm Schmidt had not been used to recover the comet (Tammann 1968), it would undoubtedly have been missed (and presumed no longer to exist).

The reason for the error in the prediction evidently stems from the close approach to Jupiter (to within 0.38 a.u.) in June 1959. As with P/Schaumasse, the encounter seems to have considerably accentuated the nongravitational forces, this time even more drastically. We should reiterate that it is certain that there would have been no trouble in either case if the comets were not subject to nongravitational forces. It would require a very much closer approach to Jupiter to cause any instability in the gravitational contribution to the equations of motion.

We have attempted to represent the observations at all three recent returns (1955, 1962, and 1968) by solving for A_1 and A_2 as well as for the Keplerian elements. We obtained the result $A_2 = (-2.7014 \pm 0.0831) \times 10^{-5}$, which is not unexpectedly about 8 times larger than the earlier value and hence completely spurious. There were also systematic trends in the residuals, amounting to more than $15''$.

For the purpose of predicting the position of this comet at its next return, in 1975, it is obvious that we should ignore the 1955 observations and determine the orbit from those in 1961–1962 and 1968–1969 only, so that the result is free from the effects of the encounter with Jupiter. The elements in Table II are preliminary figures obtained in such a manner (they are preliminary because not all the 1968–1969 observations are available yet). This solution also gives $T = 1955$ September 27.91 E.T., which is 0.52 d later than was observed. We did not include any terms for nongravitational forces in this solution. In view of the difficulties with this comet, it is hard to say how large the nongravitational forces really are, although one ought to be able to determine them from the 1896–1897, 1909, and 1955 observations; we could suggest that our 1975 prediction will require a correction of $\Delta T = -0.1$ d.

P/d'Arrest

This is another comet strongly affected by nongravitational forces (Paper I; Recht 1939). It has in fact a secular deceleration. As mentioned in Paper I, this comet is being considered for a flyby in 1976. Since the comet passed only 0.41 a.u. from Jupiter in 1968, we should anticipate a similar situation in 1976 as with P/Perrine-Mrkos in 1968, whether observations are made at the 1970 return or not. It is all the more

TABLE II. P/Perrine-Mrkos (observations 1961–1968).*

62 Feb. 25.0	68 Oct. 31.0	75 Aug. 16.0
62 Feb. 12.7674	68 Nov. 1.5374	75 Aug. 2.8779
165.9606	166.0641	166.4950
240.2268	240.1590	239.9074
17.7666	17.7619	17.8103
1.268886	1.272212	1.295473
0.643141	0.642630	0.638226

* The successive lines give the epoch of osculation and the time of perihelion passage (in E.T. and with 1900 subtracted from the year), the argument of perihelion, the longitude of the ascending node and the inclination (in degrees and referred to the ecliptic and mean equinox 1950.0), the perihelion distance (in a.u.), and the eccentricity.

important to study the matter because we feel that Roberts' (1965) estimate that the comet can be recovered 100 days before the projected launch date (1976 April 21) is excessively optimistic, and there may in fact be little or no time in which to make any pre-launch correction. The error in the position of the comet could be too large for a midcourse correction.

The orbital elements and nongravitational parameters given below were determined from 60 observations at the 1923–1924, 1943, 1950–1951, and 1963–1964 apparitions:

Epoch	1943 Oct. 3.0 E.T.
T	$= 1943$ Sept. 22.48042 $\pm$ 0.00051 (m.e.) E.T.
ω	$= 174^\circ 38531 \pm 0^\circ 00047$
Ω	$= 143.62098 \pm 0.00026$
i	$= 18.01088 \pm 0.00009$
q	$= 1.3855313 \pm 0.0000035$ a.u.
e	$= 0.6107682 \pm 0.0000012$
A_1	$= (-2.8901 \pm 2.6290) \times 10^{-6}$
A_2	$= (+6.0830 \pm 0.0045) \times 10^{-6}$
B_2	$= +0.1172 \pm 0.0068$

The representation is very satisfactory, the mean residual being $1''.45$, and during the interval covered by the observations the comet never passed within 2.6 a.u. of Jupiter. It is also worth noting that, as in the case of P/Encke, A_1 is quite definitely not an order of magnitude larger than A_2 .

Comet P/d'Arrest approached Jupiter within 0.50 a.u. in March 1920. It is thus of importance to see how well the above orbit represents the observations at the preceding apparition, that of 1910–1911. Osculating elements back to 1910 are included in Table III, and it is most encouraging to find that, in spite of the approach to Jupiter, the residuals do not then amount to more than $100''$, indicating that the 1910 perihelion time requires a correction of about -0.02 d only. The table also gives elements forward to 1970 and 1976, and we can hope that any correction they will require will be of the same order. But we cannot be *sure* of this. It does not follow that because all was well in 1920, the same would be true in 1968, when the approach to Jupiter was somewhat closer. Even if the comet is

TABLE III. P/d'Arrest (observations 1923–1964).

10 Sept. 6.0	17 Feb. 21.0	23 Sept. 18.0	30 Apr. 14.0
10 Sept. 16.8964	17 Apr. 2.9429	23 Sept. 15.5478	30 May 10.4498
173.7764	173.8661	174.1140	174.2049
146.9266	146.8682	143.7701	143.7103
15.7883	15.8047	18.0546	18.0471
1.269789	1.266210	1.357281	1.360482
0.636971	0.637623	0.615716	0.615223
36 Dec. 18.0	43 Oct. 3.0	50 June 8.0	57 Feb. 11.0
37 Jan. 8.8014	43 Sept. 22.4804	50 June 6.3713	57 Feb. 12.9920
174.3646	174.3853	174.4233	174.4168
143.6364	143.6210	143.6064	143.6346
18.0419	18.0109	18.0538	18.0492
1.376434	1.385531	1.377555	1.376246
0.612276	0.610768	0.612359	0.612522
63 Oct. 18.0	70 May 14.0	76 Aug. 10.0	
63 Oct. 23.0194	70 May 18.4347	76 Aug. 13.0069	
174.5008	178.8357	178.9305	
143.5969	141.4087	141.3525	
18.0789	16.6776	16.6920	
1.368944	1.166801	1.164196	
0.613789	0.655559	0.656111	

close to its predicted position in 1970, this may not be true in 1976. One of the consequences of the current encounter with Jupiter is a change in the perihelion distance from 1.37 to 1.17 a.u. This 15% reduction and resultant increase in the solar radiation falling on the comet as it passes through perihelion in 1970 could well throw off just the 1976 prediction.

But this fear may be unfounded. Before an encounter with Jupiter in 1861 (to within 0.36 a.u.), the perihelion distance was only 1.17 a.u. The prediction by Leveau (1869) for the 1870 return, based on the observations in 1851 and 1857–1858 (the comet was missed at its 1864 return), required a correction of only $\Delta T = +0.11$ d. Actually, because of the secular deceleration, one would have expected an error of more like $\Delta T = +0.3$ d, so it seems plausible that the encounter with Jupiter did not contribute an additional uncertainty of more than 0.2 d (and most of the discrepancy was probably due to approximations made in the computation).

P/Pons-Winnecke

From the comparison of gravitational fits to two different sets of observations (Paper I), we deduced that this comet had a slight secular deceleration. The fact that we were able to obtain a purely gravitational fit to the observations at the 1939, 1945, 1951, and 1964 returns suggests that we ought to be able to determine A_1 and A_2 rather well if we also make use of the observations in 1933. Actually, this did not turn out to be the case. We obtained the values,

$$A_1 = (+2.9648 \pm 4.2327) \times 10^{-7},$$

$$A_2 = (+9.2001 \pm 0.2260) \times 10^{-8},$$

but there were systematic trends in the residuals in excess of 6". There was only a slight improvement when we solved for B_2 as well. We then felt that it was worthwhile to try to determine the nongravitational param-

eters from the 1939–1964 observations only. The purely gravitational solution had given a mean residual of 2".3, and this is quite a lot higher than what we now think of as an acceptable value. The nongravitational fit over 1939–1964 gave

$$A_1 = (-1.2726 \pm 0.3991) \times 10^{-6},$$

$$A_2 = (+6.4604 \pm 0.4483) \times 10^{-8}.$$

The mean residual was 1".67, which is *still* perhaps a little high, but one must remember that in 1939 this comet came particularly close to the Earth (to 0.11 a.u.); possibly the result is also being affected by the approach to Jupiter (to 0.64 a.u.) in 1942. The value of A_1 is again very uncertain, but the positive A_2 is indeed consistent with a secular deceleration. Presumably, a more definite value of A_2 will be obtainable from the observations over 1945–1970.

In Paper I we remarked that a "thorough study" of P/Pons-Winnecke by Haerdtl (1889) did not show evidence of nongravitational activity between 1858 and 1886, but that Dubiago's (1948) modification of his results suggested that there was indeed a very slight secular deceleration. We quickly dismissed the result by Oppolzer (1880), from the observations at the apparitions of 1858, 1869, and 1875, that the comet had a definite secular *acceleration*.

Michielsen (1968) remarks that his attempts to fit the nineteenth century perihelion times indeed indicate that P/Pons-Winnecke then had a secular acceleration. This led us to reexamine Haerdtl's work, and we found that we could not reproduce his results. We agreed with Oppolzer that the observations in 1858, 1869, and 1875 were consistent with a small but definite secular acceleration, the nongravitational parameters being

$$A_1 = (+3.0471 \pm 0.1992) \times 10^{-6},$$

$$A_2 = (-2.0294 \pm 0.0665) \times 10^{-7}.$$

The mean residual of the 59 observations used was 3".0, but this is not excessively high for visual observations.

We did not attempt to include the 1886 observations in the solution, for the comet passed only 0.44 a.u. from Jupiter in 1882. There was another approach to within 0.87 a.u. in 1870, but this was almost certainly too remote to affect the reliability of the solution over 1858–1875. In Table IV we give osculating elements from this solution for epochs near the times when the comet was observed. The residuals in 1886 amounted to a maximum of some 2', corresponding to a correction of $\Delta T = +0.03$ d and indicating that the 1882 approach to Jupiter in fact causes little difficulty. It is also of interest to see how well this orbit represents the observations in 1819. The observations were then extremely discordant, and it is apparent that the orbit determined from them by Encke (1820) was little more than a guess (which was why the comet was not again observed until accidentally found by Winnecke in 1858). The residuals from the 1819 elements in Table IV do not

TABLE IV. P/Pons-Winnecke (observations 1858-1875).

1819 July 25.0	1858 May 1.0	1869 July 2.0	1875 Mar. 13.0	1886 Sept. 11.0
1819 July 19.7279	1858 May 2.5386	1869 June 30.4414	1875 Mar. 12.5995	1886 Sept. 4.8555
161.9607	162.1750	162.4279	165.1861	172.0685
114.8143	114.7611	114.6237	112.4907	104.9312
10.7497	10.7970	10.8004	11.2796	14.5242
0.771881	0.768942	0.781521	0.829011	0.885497
0.754167	0.754855	0.751941	0.741004	0.726175

amount to more than 5', and if the perihelion time is changed by only $\Delta T = -0.05$ d, they can be reduced to not more than 1', which is about what they were from Encke's orbit.

But why should the secular variation of this comet change from an acceleration to a deceleration? In terms of the Whipple model, one can of course explain it by permitting the inclination of the comet's equator to its orbit to pass through the value 90° . It must be pointed out, however, that this comet is unique in that during the past century its mean motion has steadily been passing through the 2:1 resonance with Jupiter. Other comets pass through this resonance very quickly: An approach to Jupiter may put the mean motion into resonance, and another one 12 years later will cause it to get completely clear. Since P/Pons-Winnecke has been passing through the resonance slowly, there have been moderate encounters with Jupiter every 12 years. We have mentioned three of these. The minimum approach was 0.35 a.u. in November 1918, but there were also approaches to 0.45 a.u. in 1894, 0.42 a.u. in 1906, and 0.47 a.u. in 1930. On each occasion the effect on the Keplerian elements was in the same direction, and it is reasonable to suppose that the nongravitational parameters have been consistently changed in a similar manner. The precise mechanism is, of course, not at all clear. We should expect, at any rate, that the nongravitational parameters in the nineteenth century should be *different* from (and possibly larger than) those in the twentieth. From 1819 to 1880, the comet's perihelion distance was only about 0.8 a.u. Since 1945 it has been 1.2 a.u., about half as large again, and the comet is obviously never subjected to anything like the amount of solar radiation as previously.

P/Daniel

Owing to the paucity of reliable observations and the complication of an approach to Jupiter (0.53 a.u.

in 1959), we were not able in Paper I to estimate the secular variation in the mean motion of this comet. We suggested that there was in fact a secular deceleration, even though the projection of the gravitational fit to the 1937 and 1943-1944 observations gave a perihelion time in 1964 some 0.6 d *later* than was observed. At first sight it appears that we have the same situation as with P/Perrine-Mrkos and P/Schaumasse: that an encounter with Jupiter produced a spurious secular acceleration, even though in this case the comet normally has a secular deceleration. In the absence of other information, we should not want to infer that the change of sign is permanent, as it seems to be with P/Pons-Winnecke.

Logic dictated that we should try first to satisfy the observations in 1937, 1943-1944, and 1950-1951, using now the equations that include nongravitational terms. The result was rather successful, showing that there was indeed a secular deceleration (A_2 positive), but there were severe difficulties in representing the majority of the observations at the 1943-1944 apparition. Specifically, one could fit the Engelhardt observations (Martinoff 1946) of, say, 1943 October 26 and November 20, but the 1944 observations gave consistently positive residuals of some 40'' in right ascension. The important point was that this fit caused the 0.6-d discrepancy in 1964 completely to disappear, the residuals of the observations on 1964 February 6 and 13 (Roemer 1965) being less than 3''! The main trouble with the 1944 observations is evidently that they are referred to equinox 1944.0, not equinox 1943.0 as was stated. But the 1937 and 1943-1944 observations are generally so discordant that this serious discrepancy was not detected when the calculations given in Paper I were made!

Correction of the equinox allowed us to salvage one of the six 1944 observations. The following result is from a final differential correction using 26 observations

TABLE V. P/Daniel (observations 1937-1964).

1937 Jan. 27.0	1943 Nov. 12.0	1950 Aug. 27.0	1957 May 2.0	1964 May 5.0	1971 June 18.0
1937 Jan. 27.8501	1943 Nov. 22.5228	1950 Aug. 23.9104	1957 Apr. 25.1681	1964 Apr. 21.2489	1971 May 31.7337
6.0455	6.1091	7.2424	7.3533	10.7833	10.8291
70.4784	70.4405	69.7468	69.6815	68.5123	68.4832
19.8244	19.8520	19.7023	19.7019	20.1408	20.1113
1.535672	1.526828	1.464567	1.463983	1.661655	1.670504
0.573197	0.574614	0.586266	0.586500	0.549915	0.548622

TABLE VI. P/Wirtanen (observations 1948–1967).

47 Dec. 1.0	54 Aug. 6.0	61 Apr. 11.0	67 Dec. 16.0	74 July 12.0	80 June 10.0	86 Mar. 31.0
47 Dec. 2.9311	54 Aug. 13.5083	61 Apr. 15.2378	67 Dec. 15.6597	74 July 5.6655	80 May 22.9284	86 Mar. 19.7933
343.5195	343.5242	343.5062	343.6098	351.8301	351.9416	356.0822
86.5093	86.4847	86.4663	86.4018	83.5425	83.4951	81.6464
13.3558	13.3765	13.3888	13.4021	12.2677	12.2690	11.6709
1.634956	1.625235	1.618049	1.612334	1.256059	1.256016	1.084352
0.540490	0.542063	0.543305	0.544248	0.614209	0.614174	0.652110

(1937–1964):

Epoch 1943 Nov. 12.0 E.T.

 $T = 1943 \text{ Nov. } 22.52282 \pm 0.00223 \text{ (m.e.) E.T.}$ $\omega = 6^\circ 10911 \pm 0^\circ 00153$ $\Omega = 70.44046 \pm 0.00033$ } 1950.0 $i = 19.85201 \pm 0.00012$ $q = 1.5268279 \pm 0.0000082 \text{ a.u.}$ $e = 0.5746142 \pm 0.0000032$ $A_1 = (+8.6734 \pm 0.6283) \times 10^{-5}$ $A_2 = (+5.4463 \pm 0.1405) \times 10^{-6}$

The mean residual is 1".57, and the result can now be classed as satisfactory. Osculating elements for epochs near all the perihelion times 1937–1971 are given in Table V.

P/Wirtanen

The observations of this comet, like those of the remaining two considered here, do not span times of encounters with Jupiter, but since encounters will take place (or have taken place) before the comets return to perihelion, it is appropriate to discuss them in this section.

Comet P/Wirtanen has a fairly pronounced secular acceleration (Paper I), and the following result was obtained from 29 observations (mean residual 1".39) made during 1948–1967 (although not all the observations at the 1967 return are yet available):

Epoch 1954 Aug. 26.0 E.T.

 $T = 1954 \text{ Aug. } 13.50671 \pm 0.00170 \text{ (m.e.) E.T.}$ $\omega = 343^\circ 52316 \pm 0^\circ 00083$ $\Omega = 86.48478 \pm 0.00029$ } 1950.0 $i = 13.37640 \pm 0.00012$ $q = 1.6252347 \pm 0.0000064 \text{ a.u.}$ $e = 0.5420923 \pm 0.0000063$ $A_1 = (-2.0412 \pm 3.8196) \times 10^{-5}$ $A_2 = (-7.1500 \pm 0.0673) \times 10^{-6}$

Elements for various epochs from 1947 to 1986 are given in Table VI. The closest approach to Jupiter is 0.28 a.u., occurring in April 1972. This causes the period to be reduced from 6.7 to 5.9 yr. Another approach in February 1984 (to 0.46 a.u.) will reduce the period further, to 5.5 yr. The perihelion distance will then be only 1.08 a.u. (two-thirds of the present value), and

although the transition through the 2:1 mean-motion resonance is much more rapid than in the case of P/Pons–Winnecke, permanent substantial changes in the nongravitational parameters could well be incurred.

P/Arend

The pair of positions obtained on 1967 November 4 (Roemer 1969) makes it possible to determine that this comet has a secular acceleration. The following result is from 48 observations made at the 1951–1952, 1959–1960, and 1967 apparitions (the mean residual being 1".26):

Epoch 1951 Nov. 10.0 E.T.

 $T = 1951 \text{ Nov. } 23.40827 \pm 0.00262 \text{ (m.e.) E.T.}$ $\omega = 44^\circ 48672 \pm 0^\circ 00131$ $\Omega = 357.69508 \pm 0.00017$ } 1950.0 $i = 21.69974 \pm 0.00006$ $q = 1.8209745 \pm 0.0000041 \text{ a.u.}$ $e = 0.5355700 \pm 0.0000020$ $A_1 = (-1.2013 \pm 2.9170) \times 10^{-5}$ $A_2 = (-2.5067 \pm 0.3283) \times 10^{-6}$

The current approach to Jupiter (0.64 a.u. in July 1969) is not so close as the ones we have been considering for other comets, and it is quite possible that it will not cause any difficulty. The real reason for including this comet here is that the approach causes the mean motion to cross to the opposite side of the 3:2 mean-motion resonance with Jupiter. The situation is in many ways analogous to that of P/Pons–Winnecke, as is discussed elsewhere (Marsden 1970). Osculating elements 1951–1975 are given in Table VII.

P/Oterma

Previous investigations of this comet, by Oterma (1957) and Herget (see Marsden 1961), revealed no evidence of nongravitational activity. An important

TABLE VII. P/Arend (observations 1951–1967).

51 Nov. 10.0	59 Aug. 20.0	67 May 30.0	75 May 28.0
51 Nov. 23.4083	59 Sept. 1.2032	67 June 13.6615	75 May 24.6836
44.4867	44.5930	44.6852	46.9222
357.6951	357.6090	357.5273	355.6584
21.6997	21.6488	21.6625	19.9556
1.820974	1.831538	1.822121	1.846905
0.535570	0.533960	0.535166	0.537640

TABLE VIII. P/Oterma (observations 1942–1962).

19 Mar. 23.0	42 Aug. 29.0	50 July 18.0	58 June 6.0	83 July 5.0
18 Dec. 30.4208	42 Aug. 21.0859	50 July 16.4698	58 June 10.5008	83 June 18.2488
241.7969	354.6747	354.7892	354.8724	55.9373
35.1362	155.1714	155.1300	155.1093	331.1947
3.0721	3.9897	3.9887	3.9922	1.9418
5.789293	3.388893	3.404882	3.387823	5.470861
0.160392	0.144722	0.142763	0.144488	0.242961

additional series of observations (Roemer *et al.* 1966) is now available. It seemed desirable that we redo the calculation, incorporating these new observations and considering also the perturbations by those planets that were previously ignored.

After weeding out some four dozen observations of poor or doubtful quality, we were left with 222 observations covering the interval 1942 February 17 to 1962 August 7. Our final solution from these gave a mean residual of $1''.00$, and no single residual was larger than $3''$. It is quite clear that no detectable nongravitational forces were acting on this comet.

At the time of the final observation, the comet was only 0.3 a.u. from Jupiter. Eight months later it reached its minimum separation of 0.095 a.u. This prolonged encounter with Jupiter considerably increased the size of the comet's orbit, with the result that the comet is no longer observable. When the comet was under observation, its mean motion was in 3:2 resonance with that of Jupiter, and as is well known (e.g., Oterma 1958), there was a very similar encounter with Jupiter a few years before the comet was discovered.

In Table VIII we give new osculating elements for epochs near the observed perihelion times. We also give osculating elements for epochs near the previous perihelion passage in 1918 and the next one in 1983. An ephemeris from the latter should be very precise, and it is hoped that all efforts will be made to recover the comet at the surrounding oppositions: It is not expected to be much brighter than magnitude 21.

Remarks

The calculations presented in this section suggest that the grave difficulties arising in the cases of P/Schaumasse and P/Perrine-Mrkos from some interaction between nongravitational forces and large gravitational forces may not be general. Similar encounters between P/d'Arrest and between P/Daniel and Jupiter seem to have caused no difficulty, although the effect of a further approach of the former comet is not yet known. There are certainly other examples of comets that have passed close to Jupiter and for which very successful predictions have been supplied afterward. Two that come particularly to mind are P/Kopff (Kępiński 1961) and P/Grigg-Skjellerup (Sitarski 1969), although these are comets that are evidently affected only slightly by nongravitational forces. But even the multiple encounters of P/Pons-Winnecke

caused no outstanding difficulty: The orbit given earlier from the observations made during 1858–1875 represents the observed perihelion time as late as 1939 within 0.3 d.

We cannot say why P/Schaumasse and P/Perrine-Mrkos behaved the way they did and which other comets will cause similar trouble. There was nothing particularly unusual about the motions of these comets with respect to Jupiter: The former was slightly inside Jupiter's orbit and receding from the sun, while the latter was 5.7 a.u. from the sun and just past aphelion. One could describe the anomaly in the case of P/Schaumasse as a sudden decrease of 1.9 m/sec in its velocity when in the vicinity of Jupiter (the total heliocentric velocity of the comet was then 11.8 km/sec). For P/Perrine-Mrkos the decrease would be more like 3.5 m/sec (out of a total of only 7.4 km/sec).

The curious state of affairs concerning P/Pons-Winnecke warrants further study. The role of the 2:1 resonance in the change of sign of the secular variation is not apparent. No other comet has been observed at more than one apparition on both sides of the resonance. Unless the long-lost P/Tempel-Swift can be recovered (Marsden 1969c), we shall have to wait until the third apparition of P/Wirtanen in its inner orbit (and that will not be until 1997) before we can test whether the resonance causes a sign change for any other comet. P/Arend offers the opportunity of studying the same effect with respect to the 3:2 resonance.

The realization that there are no nongravitational forces affecting the motion of P/Oterma is no surprise. It merely suggests that such forces do not significantly affect comets at heliocentric distances of 3.4 a.u. and more. Confirmation of this is provided by P/Schwassmann-Wachmann 1, which does not come within 5 a.u. of the sun; Cunningham (1968) and Herget (1968) have determined that its motion since 1902 has been completely Newtonian—this in spite of the tremendous physical activity directly observed in this comet.

Evidently the forces involved in surges in brightness and out-pouring of material from comets are not the ones that affect cometary motions; note also the remarks earlier concerning comet 1962 VIII. It is difficult to specify the heliocentric distance beyond which cometary motions are not affected by nongravitational forces (if indeed there is a fairly definite cutoff). It is perhaps surprising that P/Schwassmann-Wachmann 2, with a

TABLE IX. Summary of nongravitational parameters.

Comet	q	$10^6 A_1'$	$10^6 A_2'$	B_2
Whipple	2.5	+0.3?	-0.3	?
Schwassmann-Wachmann 2	2.2	+1.9	-1.6	+0.3
Arend	1.8	<2?	-0.6	?
Wirtanen	1.6	<4?	-2.5	?
Daniel	1.5	+3.8	+2.4	?
Forbes	1.5	+1.2	+0.9	+1.2
d'Arrest	1.4	<0.5?	+3.5	+0.1
Perrine-Mrkos	1.3	?	-2.5?	?
Pons-Winnecke (new)	1.2	-0.1?	+0.06?	?
Schaumasse	1.2	+1.5	-1.7	?
Pons-Winnecke (old)	0.8	+0.6	-0.4	?
Honda-Mrkos-Pajdušáková	0.6	<3?	-2.1	?
1960 II	0.5	+20	?	?
Encke	0.3	-0.1?	-0.3	+0.8
1957 III	0.3	+7	?	?

perihelion distance as large as 2.2 a.u., should be so strongly disturbed by these forces. On the other hand, P/Whipple and also P/Wolf (Kamieński 1933), which have perihelion distances of 2.5 a.u., are really very little affected by them. Unfortunately, there are now no periodic comets known with perihelion distances between 2.5 and 3.4 a.u. After its approach to Jupiter in 1981, however, P/Whipple will have a perihelion distance of 3.1 a.u., and it will be interesting to see if its nongravitational activity then ceases entirely.

We give in Table IX an extension and revision of Table X of Paper II. The quantities A_1' and A_2' are again *roughly* the values A_1 and A_2 would have if the nongravitational forces followed an inverse-square law. We have omitted the column B_1 . The general trends noted in Paper II have been maintained. There is evidence that the value of B_2 for P/Encke (a very "old" comet) increases with time. There is perhaps a suggestion that comets known to be "new," such as P/Schwassmann-Wachmann 2, have low values of B_2 . In terms of the Whipple model, it seems rather plausible that as a comet nucleus ages and its surface-to-volume ratio increases, its volatile materials are depleted more and more rapidly. Studies of the evolution of the orbits of P/Forbes and P/d'Arrest (Marsden 1967b) do not show any evidence that their perihelion distances were in very recent times much larger than they are now, so they have physically been exposed to strong solar radia-

tion for rather longer than, say, P/Schwassmann-Wachmann 2 and P/Whipple. The conclusion that B_2 is so large for P/Forbes and so small for P/d'Arrest makes some sense when one notices that the latter comet has the largest value of A_2' in the table; P/d'Arrest may have been exposed to intense solar radiation for a long time, but it is still an exceptionally active comet.

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