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Anomalous Excitation on 57 Cygni

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An attempt is made to interpret the observations of the binary 57 Cygni (Ovenden, 1963) in terms of the thermal reflection effect. It is found that the components cannot heat each other to the degree apparently required by the presence of ions appropriate to an O-type star in the B3 spectrum. A correlation, between the velocity amplitudes of individual absorption lines and their wavelengths, is also shown to be inexplicable in terms of a conventional reflection effect.

It is tentatively suggested that the high-temperature lines are formed in a chromosphere. The latter might arise from the dissipation of turbulence over the heated hemispheres: turbulent motions are shown to be expected as a consequence of the mutual heating of the stars.

Key words: reflection effect — spectroscopic binaries — HD 199081

The Observations

The system is a double-lined spectroscopic binary of Harvard type B3 (B5V on the MK system) with period $2^{\mathrm{d}}.854822$. It was investigated by Ovenden (1963) in an attempt to determine empirically the effect of reflection on the radial velocity curves of spectroscopic binaries. Separate curves were derived for the individual lines. These were roughly sinusoidal but the amplitudes K_Δ were found to vary substantially from one to another. Thus the derived mass-function $(M_1 + M_2)\sin^3 i$ ranges from 1.29 ⊙ for N III to 4.78 ⊙ for Si III.

The K_Δ were found to be correlated with the spectral type at which a given line reaches maximum strength on the main sequence, and with wavelength. For example:

(i) N III and Si IV are found predominantly in O9 — B1 stars and have $K_\Delta \sim 170$ km/s, whereas Mg II, which is associated primarily with stars later than B5, has $K_\Delta \sim 250$ km/s. The high excitation lines appear only weakly in the spectrograms and do not affect the B5V classification.

(ii) at 3964 Å, He I has $K_\Delta \sim 180$ km/s, while at 4500 Å Si III has $K_\Delta \sim 250$ km/s. A linear K_Δ/λ correlation in this sense exists over the wavelength range investigated.

The Reflection Hypothesis

Ovenden (1963) attempted to account for both correlations in terms of the reflection effect, using qualitative arguments.

Because of the mutual illumination of the components, lines of higher excitation potential will be formed largely over the inner, heated hemispheres. Assuming that the same hemispheres permanently face each other, these "hot" lines will have smaller K_Δ than absorption lines corresponding to the cooler, averted surfaces. This is in the sense of the observed $K_\Delta/$ ionisation correlation.

As one goes to shorter wavelengths, the heated hemispheres become progressively brighter relative to the averted ones: the effective centre of light is shifted towards the hotspots. One therefore expects a statistical tendency for short wavelength lines to be more displaced towards the hotspots. For synchronous rotation this implies smaller K_Δ. At longer wavelengths the reduced contrast between day and night hemispheres lessens the tendency for lines to arise largely over the inner hemispheres; their K_Δ are statistically larger. The observed wavelength correlation is in this sense.

The High Excitation Lines

The presence of N III and Si IV ions (albeit weakly) in the spectra implies that the inner surface of each binary must be heated to a temperature appropriate to an O 7.5 — B0 star, although the spectral classification of 57 Cygni is B3 on the HD system. A temperature rise of 10000° — 20000° over the heated hemisphere must be provided by the reflection effect if Ovenden's 1963 hypothesis is to be accepted.

Let T_E represent the effective temperature over the averted hemisphere of either star (the components may be taken to be identical from the similarity of their spectra) and T_H that at the sub-stellar points. One requires $T_H/T_E > 1.7$; probably $T_H/T_E \sim 2$.

In the absence of multiple reflections, and if the components were in contact, we would have only $T_H/T_E = 2^{1/4} = 1.19$. To attain the desired temperature the components would have to be so close that secondary and higher order reflections predominate.

A lower limit to the separation may be obtained from light curves of the binary which have been obtained in several colours (Ovenden, 1968, unpublished). A variation with phase of less than $0\overset{\mathrm{m}}{.}03$ is indicated, and eclipses, if they occur at all, must be grazing. Let a represent the radius of each star and s the separation of the sub-stellar points. Then from the condition of marginal eclipse we have

$$s/a \geqq 2\,(\sin i - 1)\,. \tag{1}$$

In addition

$$K_\Delta = V \sin i \tag{2}$$

for a circular orbit, where V is the orbital velocity of one star with the other at rest. Eliminating i from (1) and (2) one obtains $s/a \geqq f\,(K_\Delta, V)$. V may be obtained from the known period and an assumed mass, and a probable range of K_Δ (freed from the effects of reflection) may be derived from the observations. The results are shown in Table 1.

Since, on the reflection hypothesis, N II, N III, Si IV and probably O II are formed solely over the heated hemispheres, a perusal of Fig. 3 Ovenden (1963) indicated that $K_\Delta \geqq 200$ km/s. Likewise it is improbable that the mass of a B5V star will exceed 10 $\odot$. Then from Table 1 $s/a \geqq 0.294$. If one adopts a "typical" mass of 6.60 $\odot$ then $s/a \geqq 0.43$, and if one further drops the assumption of a grazing eclipse and assumes that the components have typical radii for their spectral type, $s/a \doteqdot 2$.

Unfortunately the temperature distribution over the stellar surfaces when multiple reflections are present cannot be found as yet because of the mathematical complexity and because the phase law of multiply-reflected radiation from extended sources is not known. However the effect of *secondary* reflection on T_H can be found approximately: at the hotspot of either star, energy is incident from an overhead disc of known angular radius β_1. An annulus of radius β and area $2\,\pi \sin\beta\,.\,\Delta\,\beta$ emits intrinsic and reflected light whose intensity is reduced by a factor $\cos\beta$, since β is the zenith distance of each element of the annulus.

Table. *Minimum s/a assuming grazing eclipse*

K_Δ (km/s) / $M(\odot)$	150	200	250
5.0	0.255	0.538	1.131
7.5	0.188	0.376	0.712
10.0	0.130	0.294	0.528

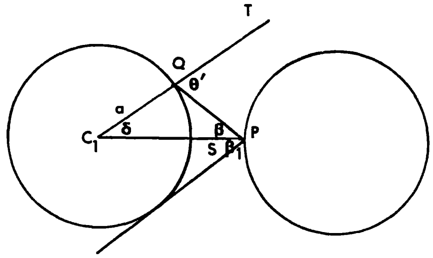

Fig. 1. Geometry of the binary system

The reflected component of this incident radiation has intensity $\mathscr{I}(\beta, \theta')$ where θ' represents the usual limb-darkening angle (see Fig. 1) and is given by

$$\sin\theta' = \left(1 + \frac{s}{a}\right)\sin\beta\,. \tag{3}$$

Thus the energy flux $\mathscr{F}'$ incident at the hotspot, due to radiation which has been reflected from the illuminating disc, is

$$\mathscr{F}' = \int_0^{\beta_1} 2\pi \sin\beta \cos\beta\; \mathscr{I}(\beta, \theta')\, d\beta\,. \tag{4}$$

The phase law of the reflected radiation will be taken in the form

$$\mathscr{I}(\beta, \theta') = \mathscr{I}(\beta, 0)\,(1 - u_r + u_r \cos\theta') \tag{5}$$

where u_r represents the limb-darkening coefficient of the reflected radiation. $\mathscr{I}(\beta, 0)$ is the intensity of light reflected along the local normal QT of the illuminating atmosphere.

From (3), (4), and (5),

$$\mathscr{F}' = 2\pi \int_0^{\beta_1} \mathscr{I}(\beta, 0)\left(1 - u_r + u_r \sqrt{\left\{1 - \left(1 + \frac{s}{a}\right)^2 \sin^2\beta\right\}}\right) \cdot \sin\beta \cos\beta\, d\beta\,. \tag{6}$$

The intensity distribution $\mathscr{I}(\beta, 0)$ was found from the theory of non-multiple reflection1), adopt-

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1) It is given by $\mathscr{I}(\beta, 0) = \mathscr{F}_{\mathrm{inc}}/\pi\left(1 - \frac{1}{3}u_r\right)$ in the notation of Napier (1968).

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ing $s/a = 0.294$, $T_E = 15000°$ and a limb-darkening coefficient 0.5 for the illuminating star. Then $\mathscr{F}'$ was found from (6), assuming also $u_r = 0.5$.

Allowing for the additional flux $\mathscr{F}'$ calculated in this way, one obtains $T_H/T_E = 1.17$, as compared with $T_H/T_E = 1.12$ neglecting secondary reflection. (5) may not be a good representation of the phase law; but the derived T_H is insensitive to large variations in u_r (e. g. for diffuse reflection, $u_r = 0$, one still obtains $T_H/T_E = 1.17$). Consequently it is unlikely that for any reasonable phase law of the re-emergent radiation, T_H/T_E will remotely approach the required value of about 2, even for a separation $s/a = 0.294$ as close as is consistent with the non-eclipsing nature of the system.

Taking $K_\Delta = 220$ km/s and $M = 6.6 \odot$ one finds $i = 38°$ from (2). It is unlikely that $i \leqq 30°$. Over each star there is therefore a polar cap of angular radius $> 30°$ which is concealed from the observer once per revolution. But the highly excited lines are visible over *all* phases of the binary and must therefore arise from a polar cap whose angular radius is appreciably greater than 30°, or 0.06 of the stellar surface. In units of the total energy/second emitted in the absence of reflection this would require each star to emit more than $0.06\ (T_H/T_E)^4$ units of reflected light, or approximately as much reflected as proper energy. But in fact at a relative separation $s/a = 0.294$ each star would intercept less than 6% of the light from its neighbour.

Thus both the degree of heating and the angular extent of the heated regions are inexplicable in terms of the thermal reflection effect.

The Wavelength Correlation

An absorption line of equivalent width W with an adjacent continuum intensity I_λ, removes energy $W I_\lambda$ from the continuum; then the mean velocity corresponding to the measured displacement of the line is given by

$$\overline{V} = \frac{\iint V W I_\lambda \, d\sigma}{\iint W I_\lambda \, d\sigma} \tag{7}$$

where V is the velocity of an element of area $d\sigma$ on the reflecting disc and the integration is carried over the visible surfaces. If $W I_\lambda$ is not symmetrical about the disc centre, as in a reflecting star, $\overline{V}$ does not generally correspond to the velocity of the mass centre, since $\overline{V}$ includes a rotation term.

Consider 57 Cygni at the nodes, taking the x-axis along the line of centres of the components, the y-axis being parallel to the direction of rotation, and the origin at the centre of the reflecting disc, which has unit radius. The isotherms in this configuration are parallel to the y-axis, and therefore the energy removed from a strip of width dx is $2\,W(x)\,I_\lambda(x)\sqrt{1-x^2}\,dx$. Relative to the mass centre the strip has radial velocity $V_{eq} \sin i\, x$, hence at the nodes

$$V = K\ (\text{mass centre}) - V_{eq} \sin i\, x$$

on neglecting the systemic velocity. Hence

K (light centre)

$$= \frac{\int_{-1}^{1} \sqrt{1-x^2}\, W(x)\, I_\lambda(x)\, [K(\text{mass centre}) - V_{eq} \sin i\, x]\, dx}{\int_{-1}^{1} \sqrt{1-x^2}\, W(x)\, I_\lambda(x)\, dx},$$

$$\therefore K\ (\text{light centre}) = K(\text{mass centre}) - V_{eq} \sin i\, \overline{x} \tag{8}$$

where

$$\overline{x} = \frac{\int_{-1}^{1} x \sqrt{1-x^2}\, W(x)\, I_\lambda(x)\, dx}{\int_{-1}^{1} \sqrt{1-x^2}\, W(x)\, I_\lambda(x)\, dx}$$

$\overline{x}$ is the fractional displacement of the effective centre of the absorption line towards the hotspot.

Consider two absorption lines at wavelengths λ_1 and λ_2 and for simplicity suppose them to be insensitive to changing atmospheric conditions: $W =$ constant.

Since

$$K_\Delta = K(\text{star } 1) + K(\text{star } 2) = 2K(\text{either star}),$$

$$K_\Delta(\lambda_1) - K_\Delta(\lambda_2) = 2\, V_{eq} \sin i (\overline{x}(\lambda_2) - \overline{x}(\lambda_1))\,. \tag{9}$$

If I_λ may be roughly represented by the Planck function, one finds that over the (λ, T) range of interest I_λ increases linearly with temperature:

$$I_\lambda = 1 + b_\lambda \left(\frac{T}{T_E} - 1\right)$$

where I_λ is normalised to unity over the averted hemispheres, and

$$b_\lambda = 2.08 \text{ at } \lambda = 3900\ \text{Å}\,,$$

$$b_\lambda = 1.85 \text{ at } \lambda = 4700\ \text{Å}\,.$$

The wavelength range observed is encompassed within (3900, 4700) Å.

Now

$$\overline{x} = \frac{\int_{-1}^{1} x \sqrt{1-x^2}\, I_\lambda(x)\, dx}{\int_{-}^{1} \sqrt{1-x^2}\, I_\lambda(x)\, dx} \tag{10}$$

$$= \frac{-\frac{1}{3} + \int_{0}^{1} x \left[1 + b_\lambda \left(\frac{T(x)}{T_E} - 1\right)\right] \sqrt{1-x^2}\, dx}{\frac{\pi}{4} + \int_{0}^{1} \left[1 + b_\lambda \left(\frac{T(x)}{T_E} - 1\right)\right] \sqrt{1-x^2}\, dx}$$

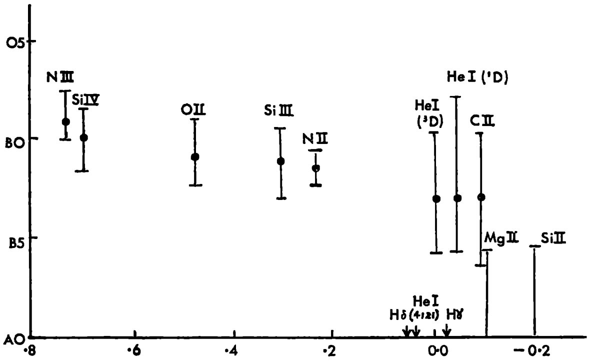

Fig. 2. Theoretical K_Δ/spectral type correlation for $T = 15000\,(1+x),\ 0 \leqq x \leqq 1$

and since

$$\int_0^1 x\sqrt{1-x^2}\,T(x)\,dx \leqq T_H \int_0^1 x\sqrt{1-x^2}\,dx = \frac{1}{3}\,T_H\,,$$

$$\int_0^1 \sqrt{1-x^2}\,T(x)\,dx \geqq T_E \int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}\,T_E$$

then

$$\bar{x} \leqq \frac{2}{3\pi}\left(\frac{T_H}{T_E}-1\right) b_\lambda\,.$$

Thus

$$K_\Delta(\lambda_1) - K_\Delta(\lambda_2) < \left|\frac{4}{3\pi}\left(\frac{T_H}{T_E}-1\right) V_{eq}\sin i\,\{b(\lambda_1)-b(\lambda_2)\}\right|$$

$$< \frac{1}{10}\left(\frac{T_H}{T_E}-1\right) V_{eq}\sin i$$

over the wavelength range required.

Over this range, $K_\Delta(\lambda_1) - K_\Delta(\lambda_2) \sim 100$ km/s from Ovenden (1963), Fig. 10. Taking V_{eq} sin i = 50 km/s and $T_H = 1.1$, one obtains only $K_\Delta(\lambda_1) - K_\Delta(\lambda_2) < 0.5$ km/s. Even for $T_H/T_E = 2$ the difference in velocity amplitudes is < 5 km/s (i. e. < 5% of that required), and therefore no reasonable degree of heating can account for the observed wavelength correlation.

Discussion and Conclusion

It has been argued that because ions appropriate to O-type stars are visible and have small velocity amplitudes, then there must be a variation in temperature from about 15000° over the night hemispheres to 25000° or 30000° at the hotspots. To test this some $T(x)$ distributions satisfying this condition were adopted, theoretical K_Δ were derived from (8) and compared with those observed. Figure 2 is a theoretical diagram so constructed for $T(x) = 15000\,(1+x)$ over the heated hemispheres, the equivalent widths $W(x)$ in (8) being taken from the W (spectral type) data of Rudnick (1936) and Williams (1936) and a modern T (spectral type) scale. $I_\lambda(x)$ was taken equal to the Planck function for simplicity. There is a fairly good resemblance to the observed correlation (Ovenden, 1963, Fig. 3) if one has K_Δ (mass centre) $\doteqdot 220$ km/s and V_{eq} sin $i \doteqdot 42$ km/s. In comparing the diagrams one must keep in mind that the observed K_Δ are affected by a wavelength correlation not due to the heating. Satisfactory diagrams were produced by $T(x)$ distributions rising smoothly from $T_E = 15000°$ to $22000° \leqq T_H \leqq 30000°$.

It is conceivable that the reflection effect would operate indirectly, by inducing circulation currents which might transport hot sub-photospheric material to the surface; but while there is evidence that such currents occur in 57 Cygni (Ovenden, 1969) the cooling time of the material is only about a second in a B-type atmosphere (Napier, 1966), and it is not to be expected that the material will have retained its sub-photospheric temperature on reaching the surface.

The expected magnitude variation Δm_λ of the binary can be calculated for any prescribed effective temperature distribution (Napier, 1968). Empirical $T(x)$ curves rising smoothly from $T_E = 15000°$ over the night hemispheres to $T_H = 25000°$ or more were usually found to give $\Delta m_\lambda \sim 0^{\mathrm{m}}15$ or $\sim 0^{\mathrm{m}}20$, whereas a preliminary reduction of the light curves gives $\Delta m_\lambda < 0^{\mathrm{m}}03$. Compatibility with the photometry was obtained only for the case where the inner surfaces had a uniform temperature T_H; then $\Delta m_\lambda = 0^{\mathrm{m}}0$. Such a temperature distribution is improbable and could not in any case produce the observed correlation of excitation potential with K_Δ.

If a thermal gradient is unacceptable the high-excitation lines must be formed in a strongly non-LTE environment over each hemisphere, induced in some way by the radiation of the neighbouring component. In the solar case it has frequently been suggested that the chromosphere arises by dissipation of shock waves formed from noise generated by granules or photospheric turbulence. A significant turbulence around the hotspots is therefore a conceivable source of the required chromosphere.

Strong turbulent velocities ($u \sim 20$ km/s) are theoretically expected in early-type stars (Smith, 1967); they arise from a shearing effect generated by meridional circulation currents. The observational evidence is very scanty, although a microturbulent velocity of about 15 km/s was discovered in the late O-type main sequence star 10 Lacertae by De Groot and Underhill (1964). A horizontal stream of velocity v gives a new turbulent velocity $(u^2 + v^2)^{1/2}$, the current being driven by a pressure difference $\Delta P = \varrho \mathcal{R}\, \Delta T/\mu$ arising from the temperature difference ΔT over the heated hemisphere. The horizontal component of

$$\varrho \frac{dv}{dt} = \varrho \boldsymbol{g} - \operatorname{grad} P$$

gives

$$\frac{v}{t} = O\left(\frac{\Delta P}{l}\right)$$

or

$$\Delta P \sim \varrho v^2 .$$

Thus

$$v \sim 0.36\,(\Delta T)^{1/2}\ \text{km/s} .$$

A temperature gradient $\Delta T \sim 1000°$ gives rise to currents with $v \sim 10$ km/s. Taking $u = 20$ km/s the new turbulent velocity is 2.5 km/s in excess of that expected in the absence of reflection. An increase in acoustic wave energy by a factor of 4 to 6 results, the conversion from turbulent to noise energy being proportional to u^x where $6 \leqq x \leqq 8$ (Biermann and Lüst, 1960). Unfortunately no detailed models of the outer layers of B-type stars are available as yet and one cannot say whether N III and other highly excited lines would in fact be observable in a chromosphere enhanced by reflection-induced turbulence.

Although the wavelength correlation cannot be explained in terms of the Planck function, an arbitrary $I_\lambda(x)$ could be formally inserted in (10) to give the required correlation. However this intensity distribution would give rise to a large change in magnitude of the binary as it revolves, which is inconsistent with the slight variability of the star (Napier, 1966): a non-thermal continuum emission cannot be postulated to account for the correlation.

Qualitatively the correlation is to be expected on the chromosphere hypothesis as fluorescence occurs there due to the over-populating of high energy levels; this strengthens short wavelength lines around the sub-stellar points giving a K_Δ/λ correlation in the required sense.

To conclude:

the hypothesis that the observations of 57 Cygni can be explained in terms of the thermal reflection effect is untenable. Nor can the effect operate indirectly by bringing hot material to the surface. In any case a thermal gradient over the surfaces (steep enough to explain the presence of N III) predicts too large a magnitude variation, except for the implausible case of uniformly hot inner hemispheres. One is led to the possibility that the high-excitation lines originate in a chromosphere occurring around each sub-stellar point. It has been shown that one expects turbulence to be generated from the reflection effect; and that a significant increase in the acoustic energy flux arises because of this additional turbulence.

Fluorescence in the enhanced chromosphere may account for the K_Δ/λ correlation. The alternative of a suitable non-thermal continuum emission conflicts with the photometry.

Progress in the interpretation of the observations appears to require a more detailed knowledge of the turbulent velocity fields of early-type stars, and a greater understanding of the physics of their upper atmospheres, in particular with regard to shock wave formation and dissipation.

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