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THE COLLAPSE OF THE UNIVERSE: AN ESCHATOLOGICAL STUDY

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Introduction. The simplest cosmological models—those which are homogeneous and isotropic, and have zero cosmical constant—predict that the universe will expand indefinitely only if the deceleration parameter $q_0 \leqslant \frac{1}{2}$: if $q_0 > \frac{1}{2}$, the mutual gravitational attraction between different regions will eventually halt the expansion, and cause a universal contraction back to a compressed state. Although observational estimates of q_0 are still somewhat unreliable, the best evidence[1] yields a value $q_0 \simeq 1$, and thus indicates that the universe is indeed fated to collapse. All structural features of the cosmic scene would be destroyed during this devastating compression. If the universe is perpetually oscillating, and this contraction is merely a prelude to a subsequent re-expansion, then plainly stars, galaxies and clusters must form anew in each cycle. The catastrophic processes whereby the contracting cosmos reverts to primæval chaos—together with some speculations on how "genetic" information may survive from one cycle to the next—are the subject of this article.

Dynamics and collapse of closed model. In a closed universe with zero pressure, the scale factor $R(t)$ is given parametrically by

$$\left.\begin{aligned} R &\propto 1-\cos 2\psi \\ t &\propto 2\psi-\sin 2\psi \end{aligned}\right\} \qquad (1)$$

During the complete cycle, ψ goes from 0 to π, and the deceleration parameter is given by

$$q = \frac{-R\ddot{R}}{\dot{R}^2} = \frac{1}{1+\cos 2\psi} \qquad (2)$$

If the true present value of q were $q_0 = 1$, about 9 per cent of the cycle would already have elapsed, and R would increase to a value $R_{\max} = 2R_0$ before (at a time $\sim 3 \times 10^{10}$ years hence, for a Hubble constant $H_0 \simeq 75$ km sec^{-1} Mpc^{-1}) the expansion would halt and a collapse ensue. A value of q_0 less than 1, but $>\frac{1}{2}$, still implies a closed universe, but one with a longer time-scale for a complete cycle, and a larger $R_{\max}$.

The neglect of pressure is valid for almost the whole of the time. However, the "black body" radiation would have been dynamically important during the first ~ 1000 years of the expansion (when $R/R_0 \lesssim 3 \times 10^{-5}$ and the temperature $T \gtrsim 10^5$ °K), and instead of (1) we then have

$$R \propto t^{\frac{1}{2}} \qquad (3)$$

The pressure builds up during the collapse, and must again be taken into account when $R \ll R_0$. However, the eventual contraction will not precisely "mirror" the expansion because

(*a*) the radiation field will have been augmented by emission throughout the history of the universe, and

(*b*) the universe may be more irregular during the contraction phase, and random material motions may give rise to a substantial kinetic pressure.

Both these effects generate additional pressure, and speed up the collapse, relative to the expansion phase. We shall consider them in turn.

As regards (*a*), the emission from galaxies and other sources will produce (in addition to the contribution aT^4 from the black body radiation) an energy density

$$\epsilon(t) \propto \frac{1}{(R(t))^3}\int_{t'=0}^{t'=t} \mathscr{J}(t') \left[\frac{R(t')}{R(t)}\right] \mathrm{d}t' \tag{4}$$

where $\mathscr{J}(t)$ is the excess of emission over absorption per unit mass*. In fact absorption is negligible, except for very low frequencies, until the rapid late stages of collapse which contribute only a small amount to the integral.

At present, ϵ is $\sim 10^{-2}$ eV cm^{-3} (ref. 2) (or perhaps somewhat higher if there is a large unobserved infra-red background flux), compared with $\sim 0{\cdot}4$ eV cm^{-3} for the black body radiation. It is clear from (1) and (4) that the main contribution to ϵ is likely to come from the long epoch when $R \simeq R_{\text{max}}$. The luminosity of galaxies is likely to decrease as they age; and the inferred evolution of radio galaxies and quasars indicates that these, too, make a decreasing contribution to $\mathscr{J}(t)$. It is therefore unlikely that ϵ will ever exceed the black body energy density by a substantial factor (although this conclusion would be modified if q_0 were close to $\frac{1}{2}$, instead of ~ 1). When $R \lesssim 10^{-3} R_0$, this radiation will be thermalized.

Discussion of (*b*) involves considering how clusters, galaxies and stars lose their identity during the collapse.

The mass within observed galaxies is little more than ~ 1 per cent of the total mass density required in a cosmological model with $q_0 = 1$ (ref. 3). The remainder may be either in faint, low-luminosity galaxies, in intergalactic gas, or in collapsed objects. The well-known fact that the constituent galaxies appear to have insufficient mass to bind the great clusters such as those in Coma and Virgo suggests that this "missing matter", whatever form it may take, is concentrated in clusters.

One may crudely estimate that clusters fill ~ 1 per cent of space. Evidently by the time the universe has contracted by a factor ~ 5 (*i.e.* $R/R_0 \simeq 0{\cdot}2$) clusters will have merged into one another; galaxies would then have peculiar velocities ~ 500 km sec^{-1}, comparable with their present random speeds within clusters. These velocities thereafter tend to increase as R^{-1}, although collisions between galaxies would by this time be frequent. In such collisions, a proportion of stars may be dislodged from their parent galaxies, or a pair of galaxies may form a single more extended bound system. Direct encounters between stars would be exceedingly rare. By the time the universe has contracted by a further factor ~ 10, and its *mean* density has become comparable with the present density *within* a typical galaxy, no galaxies will

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*It has been suggested that the sense of "thermodynamic time" may reverse when $R = R_{\text{max}}$, in which case it would be meaningless to consider radiation being emitted, and entropy being generated, during the collapse phase.

It is assumed here that there is no such intimate connection between thermodynamics and cosmology, and that an observer would not be aware of any peculiar local phenomena at the epoch when the expansion stops. (Very soon after this epoch, nearby external galaxies would begin to display blue shifts, though more distant galaxies would retain red shifts for a longer time. As time goes on, the blue shift of a given galaxy would increase, and a greater fraction of the galaxies in the universe would become blue shifted.)

survive as separate gravitationally bound entities. The stars will form the molecules of a free "gas", with random speeds ~ 3000 km sec^{-1}.

We now investigate the eventual fate of individual stars. Those surviving from the present time will be almost entirely low mass main sequence stars or white dwarfs, but there may also be younger stars which formed after the onset of the collapse. If the fraction of matter in stellar form (which is likely to be at least 1 per cent) were f, and the stars had masses $\sim M_\odot$ their mean separation would be

$$d_s \simeq 5 \times 10^{20} \frac{R}{R_0} f^{-\frac{1}{3}} \text{ cm.} \tag{5}$$

We can easily show that it is very improbable for stars to collide with one another until a very late stage indeed; before collisions can occur, most stars will in fact have been destroyed by a completely different process. In the absence of collisions, the typical random velocities v (in cm sec^{-1}) are given by

$$v\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \simeq 3 \times 10^6 \left(\frac{R}{R_0}\right)^{-1} \tag{6}$$

$$(\text{for } R \lesssim 10^{-2} R_0).$$

The collision time-scale is

$$t_{\text{coll}} \simeq \frac{d_s^3}{\sigma v}, \tag{7}$$

σ being the cross-section. When the velocities are as high as those given by (6) the appropriate value of σ is of the order of the geometrical cross-section—or perhaps somewhat smaller if the stars are centrally condensed. (If the non-stellar matter is in diffuse form, the time-scale for braking will be f^{-1} times shorter than t_{coll}.) The contraction time-scale for the universe, neglecting pressure, is

$$t_{\text{contr}} \simeq 2 \times 10^{17} \left(\frac{R}{R_0}\right)^{\frac{3}{2}} \text{ sec.} \tag{8}$$

The inclusion of pressure further reduces t_{contr}*. However, even using the time-scale given by (8), and taking $\sim 10^{21}$ cm^2 for σ we find that collisions are unlikely (*i.e.* $t_{\text{coll}} \gtrsim t_{\text{contr}}$) until $R \lesssim 10^{-9} R_0$.

Stars would first "sense" the universal collapse via the intense radiation background given by (4): it is the influence of this radiation which actually causes their destruction.

During the contraction, the radiation temperature rises in proportion to R^{-1} (or slightly faster when one allows for the contribution made by ϵ when it has been thermalized). When $R \simeq 10^{-3} R_0$, T_{rad} will approach the surface temperature T_s of the stars. This will not affect the energy production rate in the central regions, but the radii r and surface temperature T_s will have to adjust themselves to new values r' and T_s' so that

$$r^2 T_s^4 = r'^2 (T_s'^4 - T_{\text{rad}}^4). \tag{9}$$

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*The stars attain relativistic velocities at roughly the same time as the energy density of radiation becomes dynamically significant. Thus their kinetic pressure will appreciably accelerate the final collapse, and (8) will only apply until R becomes $\lesssim 3 \times 10^{-5} R_0$.

There may also be rapid "evaporation" from the outer layers of the star. As T_{rad} rises above T_s, the external radiation pressure will squeeze the surface inwards and drastically modify the star's internal structure. Eventually, when T_{rad} becomes comparable with the central temperature, an explosive ignition of the remaining nuclear fuel can disrupt the star*. This stage is reached long before collisions occur, and while the separation of neighbouring stars is still large compared with their individual dimensions.

A cyclic universe? One may speculate whether the expansion and re-contraction of the universe approximated by (1) is merely a single episode of a cyclic universe—whether, in other words, the collapse presages another expansion, and the present expansion represents a "rebound" from a previous collapse phase. Such a concept, even in so far as it is meaningful, probably cannot be handled adequately by conventional physics. It is nonetheless interesting to consider two problems that it entails—posed respectively by gravitation theory and by thermodynamics—and to suggest how they may be surmounted.

(*a*) General relativity requires there to have been a singularity in the past of our universe, and requires also that a singularity should develop during the collapse phase. Such singularities can always be regarded as a signal that "new physics" must be invoked, and this may well permit a "bounce" to occur. One need not, however, resort to this in order to rescue the concept of a cyclic universe, for the following reason. It is only in a *strictly uniform* universe that *all* matter is engulfed in a singularity. The deviations of the actual universe from strict homogeneity may be sufficient to ensure that only isolated singularities develop: most of the matter may escape these, and re-expand.

The characteristic scale of the singularities may be tentatively estimated. The collapsing universe is unstable to the growth of very large-scale perturbations. The pressure of the "stellar gas", however, stabilizes scales below the Jeans' mass, and only those perturbations whose mass exceeds $\sim 10^{14}\,M_{\odot}$ grow uninterruptedly during the collapse. (This corresponds to the Jeans' mass at the time when the stars attain velocities $\sim c$. Note that the "pressure" of the stars is sufficient to prevent any bound systems from collapsing before they become radiation-dominated. Consequently the collapse cannot be so inhomogeneous as to invalidate our preceding discussion.) Being guided by the observation that the universe is at present *less* perturbed on the *larger* scales, we may predict that the most marked irregularities in the collapse would have scales $\sim 10^{14}\,M_{\odot}$.

Theories of the nature of singularities in the inhomogeneous case are as yet undeveloped. However, we may expect some with this characteristic mass—remnants of inhomogeneities in previous cycles—to be potentially detectable. The spectrum of density perturbations present at the start of the expansion phase may be peaked at a scale corresponding to $\sim 10^{14}\,M_{\odot}$. This scale—corresponding to the mass of a cluster of galaxies—may therefore

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*A fuller discussion of how stars can be affected by an intense external radiation field will, I hope, appear elsewhere. This topic is relevant to the behaviour of stars in dense galactic nuclei, where there may be so much gas that light cannot escape freely. It may be related to the quasar phenomenon.

In the case of the collapsing universe, a detailed treatment of the problem is complicated by the fact that the stars may be moving relativistically, and the incident external radiation is hotter in the forward direction.

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represent the primary condensations which separate out (it would not be affected by radiative damping during the expansion[5]). It may also be the scale which is "remembered" from cycle to cycle.

(*b*) The radiation given by (4) will eventually be thermalized, but not until it has been blue-shifted by a huge factor. In no sense is there symmetry between emission during the expansion and absorption during the contraction. Assuming that thermodynamics can be applied through the "bounce", it thus appears that the entropy per baryon must increase significantly from one cycle to the next. This raises difficulties for the view that there has been an infinite number of oscillations in the past[6].

There are two ways of evading this conclusion. First, the singularities, if time-like, may inject new information (*i.e.* negative entropy) during the "bounce"[7] or may violate ordinary thermodynamics in some other way. A second solution to this problem seems possible in the context of a theory in which the overall net baryon number of the universe is zero, and in which galaxies or clusters have evolved from primordial baryon inhomogeneities[8]. Provided that the same fractions of particles and anti-particles fail to annihilate during each re-expansion*, there then seems no reason why the gross structure of the universe, and the average ratio of photons to baryons, should not repeat itself. (Because of the efficient mixing on scales up to $\sim 10^{14}\,M_{\odot}$ provided by stars in the collapse phase, the scale of the baryon inhomogeneities would probably need to be even larger.)

Singularities and collapsed objects. The bulk of the "missing matter" may take the form of singularities of mass $\sim 10^{14}\,M_{\odot}$ which formed either during the "bounce" or from dense bound systems which collapsed within their Schwarzschild radii during the present expansion phase. Such an object would be almost impossible to detect directly, except conceivably by the gravitational lens effect, or as a black disk against the background of a bright galaxy. Some slight support for their existence is provided by small systems of galaxies such as Stephan's quintet and VV 172, in which the galaxies have very large velocity dispersions[4,9]. Such systems would be bound if they contained a collapsed object of mass $\sim 10^{14}\,M_{\odot}$. The tidal effects of such a massive object on the galaxies may be detectable.

Conclusions. If $q_0 > \frac{1}{2}$, as the observations suggest, then the simplest appropriate cosmology is a closed model which will eventually collapse. During the contraction clusters of galaxies, and then the galaxies themselves, will merge with one another. When $R/R_0 \lesssim 10^{-2}$, the stars will behave like molecules of a gas with $\gamma = \frac{5}{3}$ until, at a stage when $R/R_0 \simeq 10^{-4}$ and the mean density of the universe has risen to $\sim 10^{-17}\,\mathrm{gm\,cm^{-3}}$, they attain relativistic speeds. The stars are eventually destroyed by the influence of the external radiation (comprising the present black body background together with blue shifted emission from galaxies) which eventually becomes hotter and more intense than the radiation in stellar interiors.

The only instabilities which enjoy uninterrupted growth during the collapse are those with scales $\gtrsim 10^{14}\,M_{\odot}$. This suggests that, if the universe is cyclic, irregularities on the scale of clusters of galaxies may perpetuate

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*Any "cyclic" cosmology requires that most of the matter should be heated to $\gtrsim 10^{10}$ °K during the bounce, in order that all nuclei of heavy elements should be broken down. In this model, the matter must attain a temperature $\gtrsim 10^{12}$ °K in order that baryon pairs should form.

themselves from one cycle to the next. The theoretical difficulties associated with an oscillating universe are not necessarily fatal to all such models. The singularities—in our past and in our future—predicted by general relativity may have characteristic scales $\sim 10^{14} M_{\odot}$. Collapsed objects of this size may constitute the missing mass in large clusters, and may be responsible for binding small (and apparently unstable) groups of galaxies.

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NOTES FROM OBSERVATORIES

Light Curve of the N-type Galaxy 3C 371

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Since the discovery of long-term secular changes in the optical magnitude of the quasi-stellar source 3C 273B by Smith[1] and Smith and Hoffleit[2] three more compact systems, 3C 371[3], 3C 390.3[4] and 3C 120[5], have been observed to undergo long-term fluctuations of large amplitude. Recent short-term observations of a number of compact galaxies made at Herstmonceux over two to three years will soon be submitted for publication[6] and the purpose of the present communication is to record the photographic data prior to 1968 January 1 that have become available on 3C 371 since the original study[3]. The new data confirm all aspects of the light-curve of 3C 371 that were previously observed; they were in part supplied to us by A. Bohrmann

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