1968, *Bull. Astr. Inst. Netherlands* **19** 421–431;
Communication from the Netherlands Foundation for Radio Astronomy and the Kapteyn Astronomical Laboratory at Groningen

THE INTERSTELLAR RADIATION DENSITY BETWEEN 912 Å AND 2400 Å

H. J. HABING

Received 29 January 1968

Recent observations of the fluxes of O and B stars and of the interstellar extinction at $\lambda < 2000$ Å indicate the necessity to recalculate the interstellar density of radiation. In this paper such calculations are carried out. The direct star light, averaged over space, has an energy density comparable to the one computed by DUNHAM (1939) and significantly lower than that by LAMBRECHT and ZIMMERMANN (1956a, b). In addition there exists the diffuse galactic light, caused by scattering by interstellar grains. Its energy density is 50 to 100% of that of direct star light, if the grain albedo exceeds 0.9.

It is suggested that the radiation density is constant throughout space. Important increases occur in the neighbourhood of associations, but only 10% of interstellar space is involved; field stars create such an increase in 0.5% of space.

A recommended value of the radiation density is given in table 8.

1. Introduction

Interstellar photons are probably the most important source of ionization in H I regions for potentials below 13.5 eV. Usually of less interest are collisional ionizations by particles of the gas and by low-energy cosmic rays. Only photons at $\lambda < 3000$ Å have sufficient energy to ionize; knowledge of their density is therefore of great interest. For the immediate neighbourhood of the Sun the photon density was calculated by DUNHAM (1939), by LAMBRECHT and ZIMMERMANN (1956a, b) and by ZIMMERMANN (1965a, b)—see table 1.

TABLE 1

Interstellar radiation density near the Sun u (erg·cm^{-3}·Å^{-1})

	DUNHAM* (1939)	LAMBRECHT and ZIMMERMANN (1956b)	ZIMMERMANN (1965b)
$\lambda = 1000$Å	34×10^{-18}	200×10^{-18}	110×10^{-18}
$\lambda = 1400$Å	43×10^{-18}	87×10^{-18}	83×10^{-18}
$\lambda = 2200$Å	47×10^{-18}	35×10^{-18}	96×10^{-18}

* We used the table for which no (extra) correction for interstellar reddening was made; i.e., the extinction was taken to be grey. It is this table that has principally been used in the literature [STRÖMGREN (1948), SEATON (1951), HOWARD, WENTZEL and MCGEE (1963)].

Zimmermann's work contains also estimates at different distances from the galactic plane near the Sun.

A number of reasons suggest a reconsideration of the problem:

1. Recent observational work, supported by theoretical models, predicts far UV fluxes from O and B stars which differ considerably from what was assumed previously.
2. Observations down to 1300 Å make it appear likely that the interstellar extinction is much stronger than has been assumed in the earlier calculations.
3. Scattering of star light by interstellar grains creates a so-called "diffuse galactic radiation". Observations indicate that its energy density is comparable to that of the direct star light and that it should be taken into account.
4. The variation of the radiation density with the position in the plane of the Galaxy has never been considered quantitatively although it is very important in the discussion of interstellar line ratios.

In this paper we present new calculations. They were restricted to the interval 912 Å $< \lambda <$ 2400 Å. Since H^0 is opaque at $\lambda < 912$ Å, no photons below this wavelength exist in H I regions. Originally we were interested in the ionization of Ca^+ and Na^0 and chose the upper limit 2400 Å. Because all elements (except K^0, Rb^0, Cs^0) have thresholds at smaller wavelengths, an extension did not seem important. This, in turn, implied that the calculations can be restricted safely to O and B stars.

2. The energy emitted by O and B stars; interstellar extinction

2.1. *The intrinsic spectrum of O and B stars*

Let $Q(\lambda)$ be the energy emitted by a star at wavelength λ expressed in erg · sec^{-1} · Å^{-1}. Let $f(\lambda) \equiv Q(\lambda)/Q_v$, where $Q_v = Q$ (5480 Å), v = visual. Q_v is given by the relation

$$\log Q_v = -0.4\, M_v + 31.681\,. \qquad (2.1)$$

ERRATUM

S. R. POTTASCH, The infrared lines and the temperature and ionization of the interstellar medium 469

Under table 3 (page 473) the following line should be added:

To obtain the emission per square degree of sky these intensities must be reduced by a factor of 10^3.

Here the constant follows from the calibration by CODE (1960) (cited by SMITH, 1967), according to which a star of $m_v = 0.0$ yields a flux density near the Earth of 4.0×10^{-9} erg · cm^{-2} · sec^{-1} · Å^{-1}.

In this section we discuss values of f derived from existing data and averaged over a few wavelength intervals (912 to 1040 Å; 1300 to 1500 Å; 1900 to 2400 Å). First we used satellite observations of OB stars by SMITH (1967) in order to obtain f (1300–1500 Å). Next the values of $f(\lambda)$ at the other two wavelength regions were found by extrapolations based on model atmospheres. The results are presented in table 2. They are larger by roughly a factor 5 than those used by ZIMMERMANN (1965b); from his table 1 we read for an O, a B0-B4, a B5-B9 star, respectively f (1500 Å) = 16, 4.5, 1.2. If the extinction were taken the same, our radiation densities would be higher than those found by Zimmermann by the same factor.

The following remarks apply to the details of the construction of table 2.

a) Strong absorption lines at $\lambda < 1500$ Å reduce the flux of B stars considerably, notably below 1050 Å. The O stars are too hot to show this effect. Only three models with absorption lines were available: O6 (ADAMS and MORTON, 1968), B1.5V (MIHALAS and MORTON, 1965), B4V (HICKOK and MORTON, 1968). The values of f (912 Å – 1040 Å) have been based on extrapolation by means of these three models; for stars of type B8 and B9 they are very uncertain.

b) For $\lambda > 2000$ Å the flux reduction due to line absorption is considerably less important: according to ELST (1967) at most 15% between 2000 and 2200 Å. We used the three models mentioned, and those by UNDERHILL (1960) and by MIHALAS (1966).

c) For reasons of simplicity we assumed that $f(\lambda)$ is independent of surface gravity. Models with different surface gravity at constant temperature are not available.

d) The three above mentioned model atmospheres appeared consistent with Smith's observations; both give approximately the same values for f (1380 Å) for O6, B1.5V and B4V stars: resp. 87, 36, 17 compared to 90, 32, 12.

2.2. *Extinction*

DUNHAM (1939) calculated two tables of interstellar energy densities. In the one that has been used extensively in the literature it was assumed implicitly that the interstellar extinction is grey, i.e. independent of wavelength. LAMBRECHT and ZIMMERMANN (1956a, b) and ZIMMERMANN (1965b) adopted a theoretical extinction law by Van de Hulst which at $\lambda < 3000$ Å shows a decrease in extinction as the wavelength decreases. However, BOGGESS and BORGMAN (1964) concluded from rocket observations that the extinction at 2200 Å is much stronger and that the λ^{-1} law still holds approximately. SMITH (1967) concluded that the extinction is still increasing even at 1300 Å.

In the following we assume that the extinction in magnitudes $A(\lambda)$ is proportional to A_v. From Boggess and Borgman's table 2 we adopted A (2200 Å) = 2.6 A_v; from Smith: A (1380 Å) = 3.5 A_v. For the region (912 Å–1040 Å) no observations were available; we adopted two extremes: $A = 4.5\ A_v$ (consistent with the λ^{-1} law) and $A = 2.0\ A_v$ (consistent with a smaller extinction as predicted in the Van de Hulst curve).

2.3. *Observational checks*

The values of $f(\lambda)$ shown in table 2 and the extinction estimate discussed in section 2.2 enabled us to predict fluxes at any of the three wavelength regions of any star with known V, B, and spectral type assuming $A_v = 3.1\ E_{B-V}$. We compared the predictions with observations.

First we compared with SMITH's (1967) observations. Over the 84 cases studied the average ratio (predicted flux/observed flux) equaled 1.12 with a r.m.s. deviation of 0.4. An average of 1.00 was expected. Smith had given the following two arguments, why in individual

TABLE 2

The spectral function $f(\lambda)$ for different spectral types

λ	O5, O6, O7	O8	O9.5	B0	B0.5	B1	B1.5	B2	B2.5	B3	B4
912–1040 Å	180	170	152	106	60	27	17	13	10	7	4
1300–1450 Å	90	85	76	59	43	35	32	29	22	18	12
1900–2400 Å	27	23	18	17	16	15	14	13	10	7	6.4

λ	B5	B5.5	B6	B7	B8	B9	Source:
912–1040 Å	2.5(:)	2.3(:)	2.1(:)	1.9(:)	1.1(:)	0.8(:)	(model atmospheres)
1300–1450 Å	8.2	7.7	7.2	6.0	3.7	2.8	(satellite observations)
1900–2400 Å	4.5	4.2	4.0	3.3	2.0	1.6	(model atmospheres)

cases there would be a difference between prediction and observations.

1) The $f(\lambda)$ values apply to slowly rotating main-sequence stars, not to supergiants.

2) The extinction is estimated by putting $R = A_v/E_{B-V} = 3.1$; however, in groups of young, hot stars R may be much larger.

A priori it was not clear whether these two arguments would lead to predictions deviating systematically, but apparently they do. It is of interest to note that the strongest deviations are ε Ori (O9.5Ia; ratio 2.1) and ζ Ori (B0Ia; ratio 3.0).

Secondly BOGGESS and BORGMAN (1964) published flux measurements of six stars at wavelengths around 2600 Å and 2200 Å. The following ratios between predicted and observed fluxes at 2200 Å are: α Cam: 1.76; δ Sco: 1.39; β Sco: 1.12; π Sco: 1.26; τ Sco: 1.12; ζ Oph: 1.24. The mean is 1.33 and again we overestimated the fluxes systematically. Here the cause may be partially the absence of absorption lines in the model atmospheres that gave us f(2200 Å). The strongest deviation (α Cam) is a supergiant (O9.5Ia).

Thirdly CHUBB and BYRAM (1963) made observations at wavelength regions around 1310 Å and around 1430 Å. Their results differ substantially from those of Smith. It seems probable that one or both sets were calibrated incorrectly. We preferred Smith's observations; his data are consistent with the model atmospheres and they have larger weight because of the longer observing times.

We conclude that our model for predicting stellar fluxes is more accurate than those used in earlier determinations of the interstellar radiation density. Two important, remaining sources of error, which will be discussed in section 5.2, are

1) abnormal extinction, like in Orion;
2) the assumption that $f(\lambda)$ within one spectral class is independent of luminosity class.

3. The diffuse galactic light

Do important, far UV photon sources exist, other than the OB stars? The lifetime of the UV photons is very short; either they leave the Galaxy or are absorbed by interstellar grains. The interstellar energy density present in other forms (gas motions, magnetic fields) is insufficient for the production. Therefore the interstellar matter plays only a negative role; it absorbs UV photons and converts them to lower-energy ones. Absorption by individual atoms and ions is negligible (see, e.g., WEIGERT, 1956), but interstellar grains are very effective in decreasing the stellar flux. We cannot, however, exclude the possibility that the grains have a high albedo (e.g. dielectric particles, Platt particles). Most of the colliding photons are then scattered, remain part of the interstellar radiation field and form the diffuse galactic light. According to HENYEY and GREENSTEIN (1941) and WITT (1968) this diffuse light is in the visible part of the spectrum comparable to that of the direct star light.

Let us make this reasoning somewhat more quantitative. Consider the energy density generated in the origin of a coordinate system by a star labeled j, which is at a distance r. Suppose that it emits $Q_j(\lambda)$ $\mathrm{erg \cdot sec^{-1} \cdot Å^{-1}}$ and that the interstellar extinction in its direction is $A(\lambda)$. The direct contribution to the radiation density at the origin equals

$$u_j(\lambda) = (4\pi c)^{-1} Q_j(\lambda) r^{-2} 10^{-0.4A(\lambda)} \ \mathrm{erg \cdot cm^{-3} \cdot Å^{-1}}. \tag{3.1}$$

Taking into account the photons that are scattered out of the beam, survive, and contribute to the interstellar radiation density, we get formally instead of eq. (3.1):

$$u_j(\lambda) = (4\pi c)^{-1} Q_j(\lambda) r^{-2} G. \tag{3.2}$$

G is a dimensionless function of the distribution, orientation and scattering properties of all interstellar grains. A priori G is restricted by the following requirements:

1) $0 \leq G \leq 1$.

2) If the albedo $a \to 0$ then $G \to 10^{-0.4A(\lambda)}$.

3) If $A(\lambda) \to 0$ then $G \to 1$.

We did not attempt to derive an expression for G in some realistic model, since at present very little about the grains is known by certainty. Instead we used a very simple, analytic expression derived under the following conditions.

1) All photons, emitted by the star in the direction of the Sun, will ultimately reach the Sun, unless they are absorbed. This condition is justified in a statistical sense, if the sphere with the Sun at its surface and its centre in the star, is homogeneously filled with grains. Since the layer of grains is only a few hundred parsecs

thick, this is definitely not the case for stars farther away than a few hundred parsecs. However, if the photons are scattered in directions in the plane (because the grains are all aligned), the assumption may be justifiable for distances several times the thickness of the layer.

2) On its way from the star to the Sun a photon will have a chance P to collide for the first time. We assume that P is equal to the chance to collide for the second time, etc. In reality, if the scattering is mainly forward, the second chance will be smaller than the first.

Under conditions 1 and 2 we get

G = chance of a photon not to be absorbed,
= chance to collide 0 times+chance to be scattered only 1 time+...,
$= (1-P)+(1-P)aP+(1-P)a^2P^2+\dots$
$= (1-P)/(1-aP)$.

Considering that $P = 1-10^{-0.4A(\lambda)}$, the result was:

$$G[a, A(\lambda)] = [a+(1-a)10^{0.4A(\lambda)}]^{-1}. \quad (3.3)$$

G is shown in figure 1 as a function of a for $A(\lambda) = 0.5$; 1.0; 2.0; 4.0. It is clear that G has the three required limits.

4. Calculation of the radiation density

In this section we used a model distribution of stars and dust (section 4.1) to derive

a) the radiation density in the plane of the Galaxy, averaged over a considerable area, say within 1 kpc from the Sun (section 4.2);

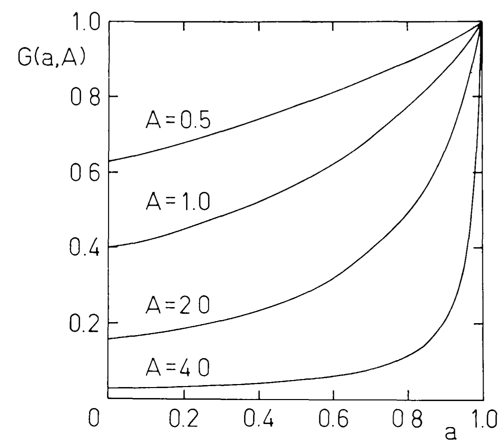

Figure 1. G as a function of the albedo a for various values of A.

b) the magnitude of local deviations from this average (section 4.3 and Appendix).

A cylindrical coordinate system (r, l, z) was adopted.

4.1. *A model distribution of O and B stars and dust*

The O and B stars were divided in a) field stars and b) stars belonging to associations. In c) we discuss the distribution of dust.

a) Field stars. For the field stars of spectral class j we adopted a density distribution $\rho_j(r, l, z)$ of the form

$$\rho_j(r, l, z) = \sigma_j(\beta_j\sqrt{\pi})^{-1} \exp(-z^2/\beta_j^2). \quad (4.1)$$

We took $\beta_j = 120$ pc, in agreement with SCHMIDT (1959); σ_j is a surface density (pc^{-2}). Note that ρ_j (and σ_j) are independent of r and l. This assumption seemed justified in the northern hemisphere [cf. figure 8 in BLAAUW (1956)]. Since we had only northern data available we extended this conclusion to the southern hemisphere, although such an extension is questionable; in the south the interarm region is close by and σ may be lower, especially for the field stars of the earliest types (B0, B1).

With this reservation in mind we determined values of σ from a list of O-B5 stars in the northern hemisphere (*I.A.U. Symp.* **1** 38). Blaauw has determined spectroscopic distances of these stars and he very kindly made his unpublished list available to us. First all stars with $m-M > 9.0$ and all known members of O-B2 associations were removed. Among the latter group are all O stars and supergiants, a feature noted before by BLAAUW (1956). The remaining stars were considered to be field stars. Five per cent were stars of luminosity class III and 95% stars of class IV and V. These were all grouped together. Next σ_j was determined in subsequent rings at increasing distances from the Sun. When σ_j started to decrease, incompleteness was supposed to set in; more distant rings were neglected and σ_j was averaged over the complete rings. It turned out that for all spectral classes incompleteness sets in at m_v between +6 and +7.

The values of σ_j finally deduced are shown in table 3, which also contains values of $Q(\lambda)$, calculated according to eq. (2.1).

b) Associations. A complete list of associations within 1 kpc with all required references has been given by BLAAUW (1964). Considering associations as point sources of UV radiation we calculated $Q(\lambda)$, the energy

TABLE 3

Data about associations and field stars

	σ (pc^{-2})	$Q(\lambda)$ (erg·sec^{-1}·Å^{-1}) $\lambda \approx 1000$ Å	$\lambda \approx 1400$ Å	$\lambda \approx 2200$ Å	r_{100} (pc)	r_s (pc)
Field stars (no luminosity class I and II)						
B0; B0.5	1.2×10^{-5}	2300×10^{32}	1600×10^{32}	450×10^{32}	20	70
B1; B1.5	2.5×10^{-5}	290×10^{32}	440×10^{32}	200×10^{32}	11	45
B2; B2.5	$11\ \times10^{-5}$	60×10^{32}	140×10^{32}	60×10^{32}	6	20
B3; B3.5	$20\ \times10^{-5}$	16×10^{32}	39×10^{32}	16×10^{32}	3.3	12
B5; B5.5	$23\ \times10^{-5}$	3×10^{32}	10×10^{32}	5×10^{32}	1.7	6
Associations						
Ori OB1	(0.032×10^{-5})	$12\ \times10^{36}$	6.9×10^{36}	2.0×10^{36}	184	240
Cep OB1	(0.032×10^{-5})	$10(:)\times10^{36}$	$6(:)\times10^{36}$	$2(:)\times10^{36}$	167	200
Cep OB3	(0.032×10^{-5})	6.3×10^{36}	3.7×10^{36}	1.2×10^{36}	133	160
Per OB2	(0.032×10^{-5})	2.7×10^{36}	1.8×10^{36}	0.7×10^{36}	83	150
Sco OB2	(0.032×10^{-5})	1.6×10^{36}	1.2×10^{36}	0.4×10^{36}	67	120
Lac OB1	(0.032×10^{-5})	0.9×10^{36}	0.8×10^{36}	0.3×10^{36}	47	82
Mon OB1	(0.032×10^{-5})	1.1×10^{36}	0.6×10^{36}	0.2×10^{36}	58	90
IC 2602	(0.032×10^{-5})	0.5×10^{36}	0.2×10^{36}	0.1×10^{36}	37	70
NGC 1502	(0.032×10^{-5})	0.3×10^{36}	0.2×10^{36}	0.1×10^{36}	29	74

emitted by the total association, in the same way as for the field stars. The results are given in table 3. It is seen that Q varies considerably from one association to the other. To get an average value we assumed that each association represents a unique subclass of associations each possessing its own probability distribution ρ for the position in space. In the following we represent ρ again by eq. (4.1); $\beta_j = 120$ pc and $\sigma_j = (\pi\times10^6)^{-1}$ pc^{-2}.

c) Dust. We assumed that the dust forms a homogeneous, plane-parallel layer of thickness 150 pc. The interstellar extinction $A(\lambda)$ is proportional to the distance traversed through the dust; the proportionality factor (to be called α) was assumed to range from 1 mag/kpc to 10 mag/kpc; the latter value may be valid around 1000 Å.

4.2. *The average radiation density in the plane of the Galaxy*

4.2.1. Formulae

Consider a point source (star or association) at (r, l, z). The amount of extinction $A(\lambda)$ between the star and the origin (0, 0, 0) of the coordinate system is given by $\alpha(\lambda)$ times the distance through the dust layer; the contribution to the radiation density in (0, 0, 0) is given by eq. (3.1):

$$u(\lambda) = Q[4\pi\, c\,(r^2+z^2)]^{-1} G(a, A). \qquad (4.2)$$

The function G describes the influence of the grains and has been given in section 3. Besides the extinction $A(\lambda)$ it contains as parameter the grain albedo a.

TABLE 4

Table of the function $H(a, \alpha, \beta)$ for $\beta = 120$ pc

α (mag/kpc)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
$a = 0.00$	16.9	13.1	11.0	9.5	8.5	7.6	6.9	6.4	5.9	5.5
$a = 0.50$	19.3	15.5	13.2	11.7	10.6	9.6	8.9	8.4	7.7	7.2
$a = 0.70$	20.9	17.1	14.8	13.2	12.0	11.0	10.2	9.8	9.1	8.6
$a = 0.90$	23.6	20.2	17.8	16.1	14.8	13.8	12.9	12.7	11.9	11.3
$a = 0.98$	25.6	23.5	21.1	19.3	18.0	16.9	16.0	16.0	15.2	14.6
$a = 1.00$*	(26.4)	(26.4)	(26.4)	(26.4)	(26.4)	(26.4)	(26.4)	(26.4)	(26.4)	(26.4)

* If $a = 1$, then $H = \infty$ (see text). The value 26.4 is obtained by stopping the integration at 3000 pc.

Now quite generally

$$\langle u(\lambda)\rangle = (4\pi c)^{-1} \sum_j Q_j(\lambda) \times$$
$$\iiint G(a, A)\,(r^2+z^2)^{-1} \rho_j(r, l, z) r \,\mathrm{d}r\, \mathrm{d}l\, \mathrm{d}z\,. \qquad (4.3)$$

The summation is over all spectral types, the integration over the whole of space. Inserting eq. (4.1) into eq. (4.3), we obtain

$$\langle u(\lambda)\rangle = (4\pi c)^{-1} \sum_j Q_j(\lambda) \sigma_j H(a, \alpha, \beta_j)\,, \qquad (4.4a)$$

where the dimensionless function H is defined by

$$H(a, \alpha, \beta_j) \equiv 2\,\pi^{\frac{1}{2}}\,\beta_j^{-1} \int_{-\infty}^{+\infty}\int_{0}^{+\infty} G(a, A)$$
$$\exp\,(-z^2/\beta_j^2)\,(r^2+z^2)^{-1} r\,\mathrm{d}r\,\mathrm{d}z\,. \qquad (4.4b)$$

Note that H is no longer a function of $A(\lambda)$ but of $\alpha(\lambda)$.

$H(a, \alpha, \beta_j)$ was evaluated by numerical integration out to a distance of 3 kpc. The results for $\beta_j = 120$ pc are shown in table 4. The cutoff at 3 kpc leads to an underestimate of 1% or less if $\alpha \geq 2$.

We make the following remarks.

1) If $a = 1$ and/or $\alpha = 0$ (no extinction or complete scattering), $G = 1$ and H is infinite. This is a logarithmic infinity, much less serious than Olbers' three-dimensional paradox. For $a < 1$ and $\alpha > 0$ the divergence is removed.

2) For a given albedo a the exact value of α, the extinction in magnitude per kpc, does not influence H very much; a change in α of a factor of 2 introduces about 30–50% change in H.

3) If we call I_0 the direct star light and I_1 the diffuse galactic radiation [see eq. (3.1)], then $I_0 \propto H(0, \alpha, \beta_j)$ and $I_0+I_1 \propto H(a, \alpha, \beta_j)$. This enables us to calculate the ratio I_1/I_0 for a number of albedos—see table 5. It is seen that albedos $a \geq 0.9$ are required to get I_1 of the same magnitudes as I_0—as it is observed (WITT, 1968).

TABLE 5

Ratio between the diffuse galactic light I_1 and the direct star light I_0

	$\alpha = 1$ mag/kpc	$\alpha = 7$ mag/kpc
$a = 0.00$	0.00	0.00
$a = 0.50$	0.14	0.29
$a = 0.70$	0.24	0.48
$a = 0.90$	0.40	0.87
$a = 0.98$	0.51	1.30

4) $H(a, \alpha, \beta_j)$ was calculated by dividing space in concentric shells around the origin of the coordinate system. For distant shells the following approximation appeared useful:

$$2\,\pi^{\frac{1}{2}}\beta_j^{-1} \iint_{\tau > \tau_1} G(a, A)\,(r^2+z^2)^{-1}$$
$$\exp\,(-z^2/\beta_j^2) r\,\mathrm{d}r\,\mathrm{d}z \approx 2\pi \int_{\tau_1}^{\infty} G(a, A) r^{-1}\,\mathrm{d}r\,.$$

Here $\tau \equiv \sqrt{(r^2+z^2)}$ and τ_1 is any number such that $\tau_1 \gg \beta_j$. We adopted $\tau_1 = 500$ pc $\approx 4\beta_j$.

4.2.2. Results

Using σ and Q from table 3 we found $\langle u\rangle$ by means of eq. (4.4a). For H we used values corresponding to extremes of both parameters α and a: strong versus

TABLE 6

Average radiation density u

	λ	α	(u in erg·cm^{-3}·Å^{-1}) $a = 0.00$ (direct star light)	$a = 0.90$ (diffuse light included)
Moderate extinction				
$\alpha_V = 1$ mag/kpc	1000 Å	2.0 mag/kpc	57×10^{-18}	88×10^{-18}
	1000 Å	4.5 mag/kpc	39×10^{-18}	67×10^{-18}
	1400 Å	3.5 mag/kpc	34×10^{-18}	56×10^{-18}
	2200 Å	2.5 mag/kpc	14×10^{-18}	23×10^{-18}
Strong extinction				
$\alpha_V = 2$ mag/kpc	1000 Å	4.0 mag/kpc	41×10^{-18}	70×10^{-18}
	1000 Å	9.0 mag/kpc	26×10^{-18}	52×10^{-18}
	1400 Å	7.0 mag/kpc	23×10^{-18}	42×10^{-18}
	2200 Å	5.0 mag/kpc	10×10^{-18}	18×10^{-18}

moderate extinction ($\alpha_v = 2$ resp. 1 mag/kpc); small versus large albedo ($a = 0.00$ resp. 0.90).

The results are shown in table 6. They are somewhat lower than ZIMMERMANN's (1965b) results (see table 1), which is in contradiction to the expectations formulated in section 2.1 and has to be attributed partially to our stronger extinction and partially to an error in Zimmermann's calculation (see section 5).

4.3. *Local deviations from the average density*

4.3.1. The occurrence of densities much larger than the average

Take a nearby star (or association) and move it away. At what distance will the stellar contribution to the radiation density equal the average value? We took as a standard 100×10^{-18} erg $\cdot$ cm^{-3} $\cdot$ Å^{-1} and defined r_{100} by the relation $(4\pi c)^{-1} Q(1000\text{ Å}) r_{100}{}^{-2} = 100 \times 10^{-18}$. In table 3 some r_{100} values are shown. We make the following remarks.

a) r_{100} as well as r_s are considerably larger than the radius of the associations as projected on the sky, except in the case of Lac OB1 (see BLAAUW, 1964). This justifies our consideration of associations as point sources.

b) Compare r_{100} to the Strömgren radius r_s (defined for a hydrogen density $n_H = 1$). Taking r_s values from GOULD, GOLD and SALPETER (1963, table 2), the ratio (r_{100}/r_s) is found to be approximately 0.3; for associations it is somewhat larger $[0.4 < (r_{100}/r_s) < 0.8]$. This is to be expected; for an association the total value of r_s depends on the sum of the cubes of the individual r_s values, whereas the total value of r_{100} depends on the sum of the squares.

If $n_H > 1$ the actual Strömgren sphere has a radius $r < r_s$, since $r = r_s n_H{}^{-\frac{2}{3}}$. Still it is obvious that ionization conditions within a distance r_s should never be considered as typical for H I regions.

c) As shown in the Appendix, a negligible fraction (0.5%) of interstellar space is contained by the spheres with radii r_{100} around the field stars. Therefore the chance is negligible that fortuitously a field star is so near to a gas cloud that only its radiation determines the ionization conditions. For associations the fraction is 10% (see Appendix) and nearby associations have to be taken into account when ionization conditions are calculated. Since, within 1 kpc, there are only a few, well known O-B2 associations, this can be done quite well in actual situations.

4.3.2. Regions with small radiation densities

Small radiation densities can be expected a) in places where there are fewer stars than on the average (a "hole" in the stellar distribution); b) inside dust clouds.

a) Consider the case of large extinction ($\alpha = 9$ mag/kpc) and large albedo ($a = 0.9$). From the integration of the function $H(0.9, 9, 120)$ it follows that 70% of the average radiation density originates in stars within 200 pc. Suppose, as an extreme situation, that this contribution is reduced by a factor 5, because of the presence of a "hole" as described. The radiation density then reduces to 30%+14% = 44% of the average value.

The situation is somewhat worse if $a = 0.0$; then there is no diffuse light and one finds a reduction to 30%.

b) Consider the inside of a dust cloud with in every direction a visual extinction $A_v = 0.50$. At 1000 Å, $A(\lambda) = 4.5\ A_v = 2.3$. If scattering by grains does not occur, the radiation density is reduced by a factor $10^{-0.4A} = 0.13$. If the scattering is considerable (say the grain albedo $a = 0.9$), the reduction factor is 0.60 according to eq. (3.3).

We conclude that a reduction of the radiation density by factors exceeding 3 can occur only inside dust clouds; but the value depends strongly on the grain albedo. Fluctuations in the star density have probably no important consequences.

4.3.3. "Average values" and "typical values"

In section 4.3.1 it has been suggested that the 10% of space in the neighbourhood of associations cannot be considered as typical for interstellar space. Excluding this 10% we cut out the denser parts of the radiation field and the average over the remaining part of space is lower. In the Appendix it is shown that at 1000 Å this results in a reduction of at most 25%. Probably the 75% of what has been defined by eq. (4.3) is a more typical value for normal interstellar space.

5. The radiation density in the immediate solar neighbourhood

5.1. *Calculations*

In the preceding section a model was used for calculations. Such a model is based necessarily on several assumptions, one of the most discussable being that of a smooth distribution of the dust. No assumptions have to be made in finding the radiation density in the immediate solar neighbourhood. It is therefore of interest to compare the local value with the average found in section 4.2.2.

The earlier calculations, summarized in table 1, refer to the local solar neighbourhood. Repetition seems desirable, since better basic data are available and we want approximate inclusion of the diffuse galactic light as in section 4. The calculations were carried out by the same method as used by the previous investigators. The required formulae have never been published and therefore this will be done here. We start from eq. (4.3), replacing the summation and integration by one summation over all stars and using for the distance the symbol r instead of $\sqrt{(r^2+z^2)}$:

$$u(\lambda) = (4\pi c)^{-1} \sum Q_j(\lambda) G(a, A_j) r_j^{-2}. \qquad (5.1)$$

Let F_v be the flux near the Earth at visual wavelengths of a star with apparent, visual magnitude m_v, extinction A_v, and distance r. We define $F_0 = F_v$ for a star with $m_v = 0$; F_0 is a constant, independent of spectral type. Now we have the following relations: i) $Q(\lambda) = f(\lambda)\, Q_v$; ii) $F_v = (4\pi)^{-1} Q_v r^{-2}\, 10^{-0.4 A_v}$; iii) $F_v = F_0 10^{-0.4 m_v}$.

In the older literature $10^{-0.4 m_v}$ has been called the "equivalent number". We define the "reduced equivalent number" $N \equiv 10^{-0.4\,(m_v - A_v)}$. Substituting the three formulae in eq. (5.1) we obtain

$$u(\lambda) = (F_0/c) \sum_j f_j(\lambda) N_j G[a, A_j(\lambda)]. \qquad (5.2)$$

If the grain albedo $a = 0$ (no diffuse light), eq. (5.2) reduces to

$$u(\lambda) = (F_0/c) \sum_j f_j(\lambda)\, 10^{-0.4\, m_v}\, 10^{0.4[A_v - A_j(\lambda)]}. \qquad (5.3)$$

The previous investigators used this formula after replacing $F_0 f_j(\lambda)$ by $4\delta_j S_j(\lambda)$. Here $S_j(\lambda)$ is the flux at the stellar surface (erg · cm^{-2} · sec^{-1} · Å^{-1}). δ_j is the "geometrical dilution factor" defined by $\delta_j = R_j^2/(4r_j^2)$; R_j being the stellar radius and r_j the distance of a star of $m = 0.0$ in the absence of interstellar extinction.

We take the opportunity to point out a serious error in the dilution factors used by LAMBRECHT and ZIMMERMANN (1956a, table 6). Consider the flux density F received at 4200 Å at Earth from a star of $m_{pg} = 0.0$. Its value should be independent of spectral type, about 64×10^{-10} erg · cm^{-2} · sec^{-1} · Å^{-1}. On the other hand $F = 4\delta_j S_j$(4200 Å). Using data about δ and S, e.g. from ZIMMERMANN (1965b), one finds for the O stars $F = 6 \times (64 \times 10^{-10})$ and for the B0-B4 stars $F = 3 \times (64 \times 10^{-10})$. For the other spectral types the range of F is from 1.5 to $0.6 \times (64 \times 10^{-10})$. This implies that in Zimmermann's computations the contribution by the O stars is too large by a factor 6 *over the whole wavelength range*, including the far UV. Dunham's δ's are, however, correct; his fluxes approach within 15% the value 64×10^{-10}.

TABLE 7

Radiation density u in the immediate solar neighbourhood (in parentheses the values obtained when, for the weaker stars, one takes A_v = 2 mag/kpc instead of A_v = 1 mag/kpc)

	(u in erg·cm^{-3}·Å^{-1}) $a = 0.00$ (direct star light)	$a = 0.90$ (diffuse light included)
$\lambda \approx 1000$ Å: $A(\lambda) = 2.0 A_v$	$52(50) \times 10^{-18}$	$91(90) \times 10^{-18}$
$A(\lambda) = 4.5 A_v$	$33(32) \times 10^{-18}$	$61(58) \times 10^{-18}$
$\lambda \approx 1400$ Å: $A(\lambda) = 3.5 A_v$	$42(39) \times 10^{-18}$	$75(71) \times 10^{-18}$
$\lambda \approx 2200$ Å: $A(\lambda) = 2.6 A_v$	$22(21) \times 10^{-18}$	$34(34) \times 10^{-18}$

Eq. (5.2) was used to calculate u. We took albedo's $a = 0.00$ and $a = 0.90$ representing extreme cases. f_j was given in table 2; the extinction $A_j(\lambda)$ is calculated as described in section 2.2. The results are given in table 7. They are based on O and B stars, making the following detailed assumptions.

a) Stars with $m_v < 6.5$.

All stars of type O-B9 were taken from the *Bright Star Catalogue*. Possible incompleteness of this catalogue was ignored. For each star an individual value of A_v was calculated from the relation $A_v = 3.1\, E$(B-V) with the following exceptions.

1. The B8 and B9 stars with $5.5 < m_v < 6.5$ were assumed to have absolute visual magnitudes of +0.1 and +0.6 respectively (BLAAUW, 1963). The extinction in front of these stars was found in the way shown by BOK (1937, p. 13) by adopting an extinction A_v = 1 mag/kpc or A_v = 2 mag/kpc.

2. Stars for which no B-V measurement exists were taken to have $A_v = 0$. This leads to an insignificant overestimate of the flux.

b) Stars with $m_v > 6.5$.

LAMBRECHT and ZIMMERMANN (1956a) give extensive tables of numbers of B0-B4 and B5-B9 stars between m(pg) = 6.5 and m(pg) = 12.0 in steps $\Delta m = 0.5$. Using these tables, we proceeded in the following way. We adopted absolute magnitudes (B0-B4: $M_v = -3.1$; B5-B9: $M_v = -0.2$) and an extinction α_v of resp. 1 and 2 mag/kpc. We then calculated A_v for the stars in one Δm interval. A_v and a yield G; m and A_v yield N.

The counts were made in the photographic magnitude system, but we used the visual system. Our results would have been correct, if the colour indices (pg-vis) of the B0-B9 stars were exactly zero; in fact they are −0.2 (B0-B4) and −0.1 (B5-B9). This means that we overestimate the B0-B4 flux by a factor antilog (0.2/2.5) = 1.20 and the B5-B9 flux by a factor 1.09. We corrected our results accordingly.

5.2. *Discussion*

First compare the results in table 7 for $a = 0.0$ (no diffuse light) to those of table 1. Rather close agreement exists with Dunham's values, quite large disagreement with those of LAMBRECHT and ZIMMERMANN (1956b) and of ZIMMERMANN (1965b). Reasons for disagreement have been presented in sections 1 and 5.1. The agreement with Dunham is fortuitous.

Secondly we compare tables 6 and 7. Good agreement exists if in table 6 we take the case "moderate extinction" ($\alpha_v = 1$ mag/kpc). There is some difference at 2200 Å, which may be due partly to the neglect in section 4 of B6-B9 stars.

It is of some importance to analyse the different contributions as follows.

1) As was predicted in section 4.3, there is no field star that contributes 50% or more of the total. At 1400 Å the two most important photon sources are β Cen (HD 122451; $m_v = 0.59$; B1 II) and Spica(HD 116658; $m_v = 0.96$; B1V). They contribute resp. 6% and 4%.

2) Stars with $m_v > 6.5$ contribute moderately: at most 26% (2200 Å, diffuse light included), at least 3% [1000 Å, $\alpha(\lambda) = 9$ mag/kpc, no diffuse light included].

3) With increasing wavelength the cooler stars contribute relatively more. If we divide the stars in three groups, O, B0-B3, B4-B9, we find the following contributions: at 1000 Å, respectively 35%, 60%, 5%; at 1400 Å, 15%, 69%, 16%; at 2200 Å, 9%, 71%, 19%. Even at 2200 Å the contributions of the B8 and B9 stars were unimportant. This supports the neglect of A-type stars in the calculation.

4) As stated in section 2.3, the fluxes from the supergiants of classes Ib, Iab, Ia may be considerably overestimated. This will have only a small influence on the results, since in total the supergiants contribute quite modestly, because they are very rare. At 1000 Å their contribution is 18%, at 1400 Å 12% and at 2200 Å about 10%. Similarly the uncertainty of the extinction in the Orion region will not be of too much importance; the stars contribute about 25% of the radiation at 1000 Å; ι Ori, δ Ori, ε Ori and ζ Ori contribute together about 20%.

6. Conclusions

In the preceding sections we presented calculations on the average radiation densities in the plane of the Galaxy. In our opinion the main sources of uncertainty are i) the interstellar extinction, notably below 1100 Å; ii) the albedo of the interstellar grains (and consequently the importance of the diffuse galactic light); iii) the stellar fluxes below 1000 Å (only three adequate stellar models were available). It should be noted, however, that neither in section 4 nor in section 5 the calibration of absolute, monochromatic magnitudes has any influence on the results; in eq. (4.4a) the product σQ is invariant for chances in M_v, in eq. (5.2) M_v does not enter at all.

The two most important conclusions are the following.

a) If the albedo of the grains is negligible, i.e. if only direct star light counts, we obtain average radiation densities that agree closely with those of DUNHAM (1939), but are much smaller than those of LAMBRECHT and ZIMMERMANN (1956b) and of ZIMMERMANN (1965b). If the albedo is considerable, we obtain roughly 1.5 × Dunham's values.

TABLE 8

Recommended values for the radiation density u in typical interstellar space

(u in erg·cm^{-3}·Å^{-1})	$\langle u \rangle$	min.	max.
1000 Å	40×10^{-18}	25×10^{-18}	140×10^{-18}
1400 Å	50×10^{-18}	25×10^{-18}	150×10^{-18}
2200 Å	30×10^{-18}	15×10^{-18}	130×10^{-18}

b) Field stars are negligible as causes for large deviations from the average density (section 4.3). Associations are quite important, but in only 10% of space do they cause an increase of the radiation density by a factor 3. Very low densities occur only inside of dense dust clouds; outside these clouds the background light is relatively constant, hardly ever falling below 40% of the average.

Finally we tried to summarize all considerations and calculations into one set of values that shall represent the conditions in "typical" interstellar space, i.e. space outside the immediate neighbourhood of associations. Stressing the uncertainties outlined above and drawing attention to the approximations in section 4.3, we present it in table 8.

Acknowledgements

I like to thank Professor S. R. Pottasch for suggestions, interest and encouragements; Dr. J. Borgman, Professor H. C. van de Hulst and Dr. H. van Woerden for very useful comments on the manuscript; Professor A. Blaauw for lending me his list of OB stars; Dr. T. S. van Albada and Dr. H. Zimmermann for discussions; Mr. L. Lammers and Mr. M. Klomp for programming and Mr. K. Stuit for assistance in computing.

Appendix (see section 4)

1. Consider a collection of point sources (stars and associations) intermingled with dust. Suppose that the position (r, l, z) of each is a stochastic variable; all variables are independent but they have an identical probability density

$$f(r, z) = 2(R_0^2\beta\sqrt{\pi})^{-1} r \exp(-z^2/\beta^2). \qquad \text{(A1)}$$

Here $0 \leq r \leq R_0$ and $-\infty < z < +\infty$. For R_0 we will take 1000 pc and for β 120 pc.

All sources together generate at the origin a radiation density

$$u = \sum u_j = (4\pi c)^{-1} \sum Q(r^2+z^2)^{-1} G(r, z). \quad \text{(A2)}$$

$G(r, z)$ is given by eq. (3.3); the summation is over all sources individually.

u_j as well as u are stochastic variables; both take only positive values. In principle we should like to have the distribution of u. This will not be obtained in this Appendix. It is easily proved that in deriving the distribution of u the central limit theorem is of no use because of the small number of stars involved (a necessary requirement is $\sigma \gg 10^{-4}$ pc^{-2}).

2. We now ask the same question as in section 4.2: What is the chance that a given point source is so nearby that its individual contribution exceeds the average? For convenience ask for the chance that $t \equiv \sqrt{(r^2+z^2)} < r_{100}$. It follows from eq. (A1) that t has the following probability density,

$$p(t) = 2\, R_0^{-2} t \,\{\text{erf}\,(t/\beta) - \text{erf}\,[\psi(t)/\beta]\}, \quad \text{(A3)}$$

where $\psi(t) = 0$ if $t \leq R_0$ and $\psi(t) = \sqrt{(t^2 - R_0^2)}$ if $t \geq R_0$.

Consider a group of N identical sources (e.g. B0 stars). The chance P that at least one of these is within a distance r_{100} from the origin, equals

$$P = 1 - \left\{1 - \int_0^{r_{100}} p(t)\,dt\right\}^N. \qquad \text{(A4)}$$

In the case of the *field stars*, $r_{100} \ll \beta$ and we approximate

$$\int_0^{r_{100}} p(t)\,dt \approx 4r_{100}^3/(3R_0^2\beta\sqrt{\pi}). \qquad \text{(A5)}$$

Since $\beta \ll R_0$, the integral is small compared to 1 and eq. (A4) reduces to

$$P = N \int_0^{r_{100}} p(t)\,dt = (4\,\pi^{\frac{1}{2}}\,\sigma r_{100}^3)/(3\beta) \approx 2.12\sigma r_{100}^3\beta^{-1}. \qquad \text{(A6)}$$

Here σ is again the surface density of the stars (number per pc^2).

For the B0 stars [$\sigma = 1.2 \times 10^{-5}$ pc^{-2}, $r_{100} =$ 20 pc (see table 3), $\beta = 120$ pc], we find $P = 0.1\%$; for the field stars of later type even a lower percentage is found.

In the case of the *associations*, $N = 1$ and eq. (A4) reduces to

$$P = \int_0^{r_{100}} p(t)\,dt.$$

For Ori OB1, $r_{100} \approx 180$ pc so $r_{100} > \beta$. Eq. (A5) does not hold and we have to carry out the integration of $p(t)$ numerically; for the results see table A1. In the case of Ori OB1, $P = 5\%$.

TABLE A1

r_{100} (pc)	$\int_0^{r_{100}} p(t)dt$
10	0.0033
30	0.032
100	1.5
200	5.5
300	11.5
500	29.5
700	55.5
900	89.5
1000	99.5
1005	99.8

3. If one cuts out of interstellar space the r_{100} spheres around the associations and the stars, $\langle u\rangle^*$, the average radiation density over the rest of space, will

be lower than $\langle u \rangle$, the average over all space. Will the difference be significant?

Again

$$\langle u \rangle^* = \sum_j \langle u_j \rangle^*,$$

where the summation is over all photon sources. $\langle u_j \rangle^*$ is the contribution of stars (or associations) of type j, averaged over all space outside a sphere with radius r_{100}, centred at the origin. We obtain therefore

$$\langle u_j(\lambda) \rangle^* = (4\pi c)^{-1} Q_j(\lambda) \sigma_j H_j^*(a, \alpha, \beta_j),$$

where

$$H_j^*(a, \alpha, \beta_j) = 2\pi^{\frac{1}{2}} \beta_j^{-1} \iint_{\tau \geq r_{100}} G(a, A) \exp(-z^2/\beta_j^2)(r^2+z^2)^{-1} r \, dr \, dz,$$

where

$$\tau = (r^2+z^2)^{\frac{1}{2}}.$$

For field stars r_{100} is small and $H^*(a, \alpha, \beta_j) \approx H(a, \alpha, \beta_j)$. For associations H^*/H is significantly smaller than 1. E.g., for Ori OB1, $r_{100} = 180$ pc; taking $a = 0.9$ we obtain $H^*/H = 0.43$ and $H^*/H = 0.52$ for $\alpha = 9$ respectively 5 mag/kpc. In total we estimate that at 1000 Å, $\langle u \rangle^*/\langle u \rangle = 0.75$, at 2200 Å, $\langle u \rangle^*/\langle u \rangle = 0.85$.

References

T. F. ADAMS and D. C. MORTON, 1968, to be published

A. BLAAUW, 1956, *Ap. J.* **123** 408

A. BLAAUW, 1963, *Basic Astronomical Data*, ed. K. A. Strand; *Stars and Stellar Systems*, ed. G. P. Kuiper and B. M. Middlehurst (Univ. of Chicago Press, Chicago) **3** 383

A. BLAAUW, 1964, *Ann. Rev. Astr. Ap.* **2** 213

A. BOGGESS and J. BORGMAN, 1964, *Ap. J.* **140** 1636

B. J. BOK, 1937, *The distribution of the stars in space* (Univ. of Chicago Press, Chicago)

T. A. CHUBB and E. T. BYRAM, 1963, *Ap. J.* **138** 617

A. D. CODE, 1960, *Stellar Atmospheres*, ed. J. L. Greenstein; *Stars and Stellar Systems*, ed. G. P. Kuiper and B.M. Middlehurst (Univ. of Chicago Press, Chicago) **6** 50

TH. DUNHAM, 1939, *Proc. American Phil. Soc.* **81** 277

E. W. ELST, 1967, *Bull. Astr. Inst. Netherlands* **19** 90

R. J. GOULD, T. GOLD and E. E. SALPETER, 1963, *Ap. J.* **138** 408

L. G. HENYEY and J. L. GREENSTEIN, 1941, *Ap. J.* **93** 70

F. HICKOK and D. C. MORTON, 1968, to be published

W. E. HOWARD, D. G. WENTZEL and R. X. MCGEE, 1963, *Ap. J.* **138** 988

H. LAMBRECHT and H. ZIMMERMANN, 1956a, *Mitt. Univ. Sternw. Jena* **13**

H. LAMBRECHT and H. ZIMMERMANN, 1956b, *Mitt. Univ. Sternw. Jena* **14**

D. MIHALAS, 1966, *Ap. J. Suppl.* **13** 1

D. MIHALAS and D. C. MORTON, 1965, *Ap. J.* **142** 253

M. J. SEATON, 1951, *Mon. Not. Roy. Astr. Soc.* **111** 368

M. SCHMIDT, 1959, *Ap. J.* **129** 243

A. M. SMITH, 1967, *Ap. J.* **147** 158

B. STRÖMGREN, 1948, *Ap. J.* **108** 242

A. B. UNDERHILL, 1960, *Pub. Dom. Ap. Obs. Victoria* **11** 433

A. WEIGERT, 1956, *Mitt. Univ. Sternw. Jena* **16**

A. N. WITT, 1968, to be published

H. ZIMMERMANN, 1965a, *Astr. Nachr.* **288** 95

H. ZIMMERMANN, 1965b, *Astr. Nachr.* **288** 99