

THE PECULIAR VARIABLE FG SAGITTAE*

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ABSTRACT

The brightening of FG Sge from $m_{pg} = 13.6$ in 1894 to $B = 9.6$ in 1965 is examined in terms of the continuous outflow of matter from a hot, subluminescent star. FG Sge is centered in an apparently normal planetary nebula of $18''$ radius, but this envelope predates the present activity. Coudé spectrograms are available for the years 1960–1967, while information from low-dispersion plates begins in 1955. Over this period, the spectral type has changed progressively from B4 I to A5 Ia, the colors (1960–1965) have altered in the same sense, and strong P Cygni-like emission fringes at the Balmer lines have faded out. The high-luminosity spectrum is interpreted as being due to gas lying above a false photosphere which itself is rising very slowly in the rapidly outflowing material, at approximately 26 stellar radii (in 1965) above the invisible central star. There has also been some variation in radial velocity and spectral characteristics that may be linked with the erratic *UBV* fluctuations observed at Sonneberg. The observational data are discussed on the assumption that FG Sge is in process of producing another planetary nebula as it did once before, about 3000 years ago.

I. INTRODUCTION

The earlier photometric history of FG Sge¹ has been reviewed by Richter (1960) and by Kukarkin (1960). The remarkable feature of the light curve is that the star has risen steadily from about $m_{pg} = 13.6$ in 1894 to $B = 9.6$ in 1965 (see Fig. 1). There are also irregular fluctuations superimposed upon this nearly linear brightening; the photographic observations indicated a range of about 0.5 mag and a duration of from a few days up to 100 days. Fürtig and Wenzel (1962) reported rapid changes of smaller amplitude on the basis of photoelectric observations; their second paper (Wenzel and Fürtig 1967) gives details and contains a more complete account of the photometric behavior of FG Sge than we give here.

One can find an example of almost every imaginable type of photometric behavior among the peculiar variables. Most of these are of interest only to the specialist, in the present state of understanding of such phenomena. In the case of FG Sge, however, the implication of some type of *progressive* activity together with the presence of a planetary-nebula-like envelope around the star suggest that this object may be of somewhat wider interest.

II. THE NEBULOSITY AND THE COMPANION

It was noticed early in 1960 that the image of FG Sge was suspiciously diffuse on small- and medium-scale direct photographs.² Plates taken in June and July 1960 with the 120-inch reflector showed (Fig. 2, Pl. 8) that the variable is surrounded by a faint, nearly circular nebulous envelope having a mean radius in the 3500–5000 Å region of

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¹ FG Sge = 377.1943 Sge, = S 3702, = KZP 5066, = Iriarte-Chavira 78. $\alpha(1900) = 20^h07^m5$, $\delta(1900) = +20^\circ02'$.

² This was the time that the nebulosity was noted by Herbig, but it had already been found by Henize (1961) in 1955 and was described briefly as entry No. 5 in his list of new planetary nebulae. It was found independently also by Fürtig and Wenzel (1962).

PLATE 8

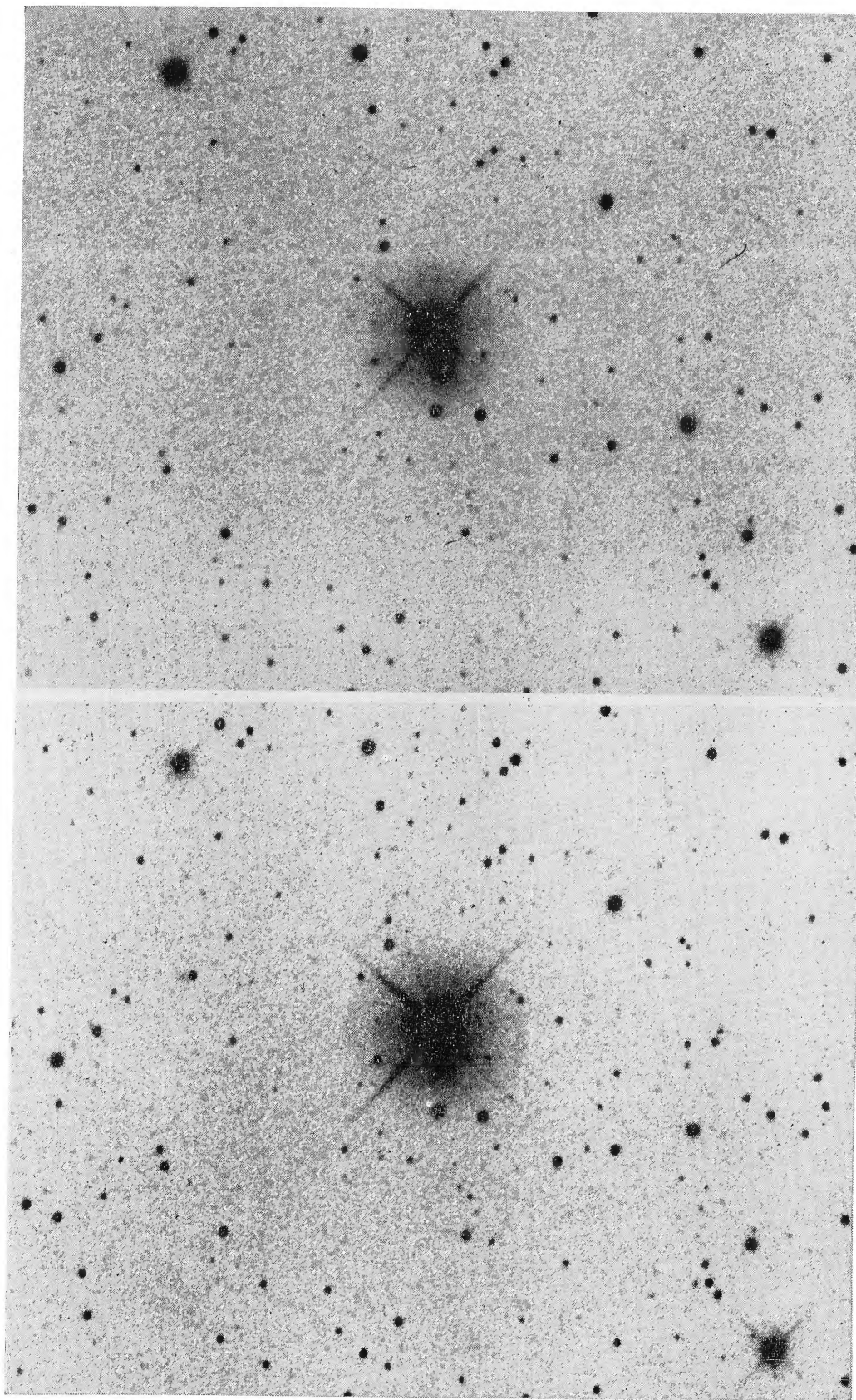


FIG. 2.—Direct photographs of the nebula surrounding FG Sge, taken with the 120-inch reflector in (*left*) photographic light ($\lambda\lambda 3600-5000$) on July 28, 1960, and (*right*) in the red ($\lambda\lambda 6000-6700$) on July 1, 1963. North is at the top and east to the left; the scale is $1''.6 \text{ mm}^{-1}$. The $8''$, mag 14 companion appears on the left edge of the overexposed image of the variable.

HERBIG AND BOYARCHUK (*see* page 397)

about 18". The nebula appears as a featureless disk over the inner 12"–14"; outside this radius, the peripheral region is fainter. This "limb darkening" gives the impression that the nebula is not a thin shell of gas with an empty interior. FG Sge is projected as nearly as one can determine on the center of symmetry of the nebula. The only other possible candidate for identification as the central star lies halfway to one edge. Therefore, there is no reason to doubt that the variable and the nebula are organically associated.

The spectrum of the nebula was obtained in 1960 and in 1967 in the blue-ultraviolet with the nebular spectrograph of the Crossley reflector, and in 1966 in the visual region

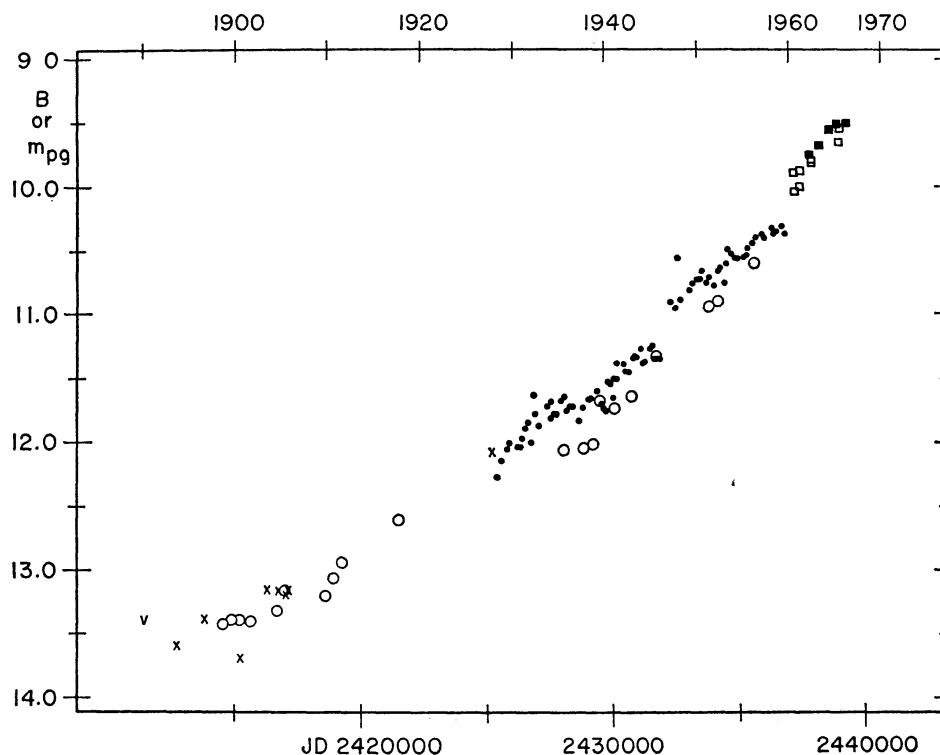
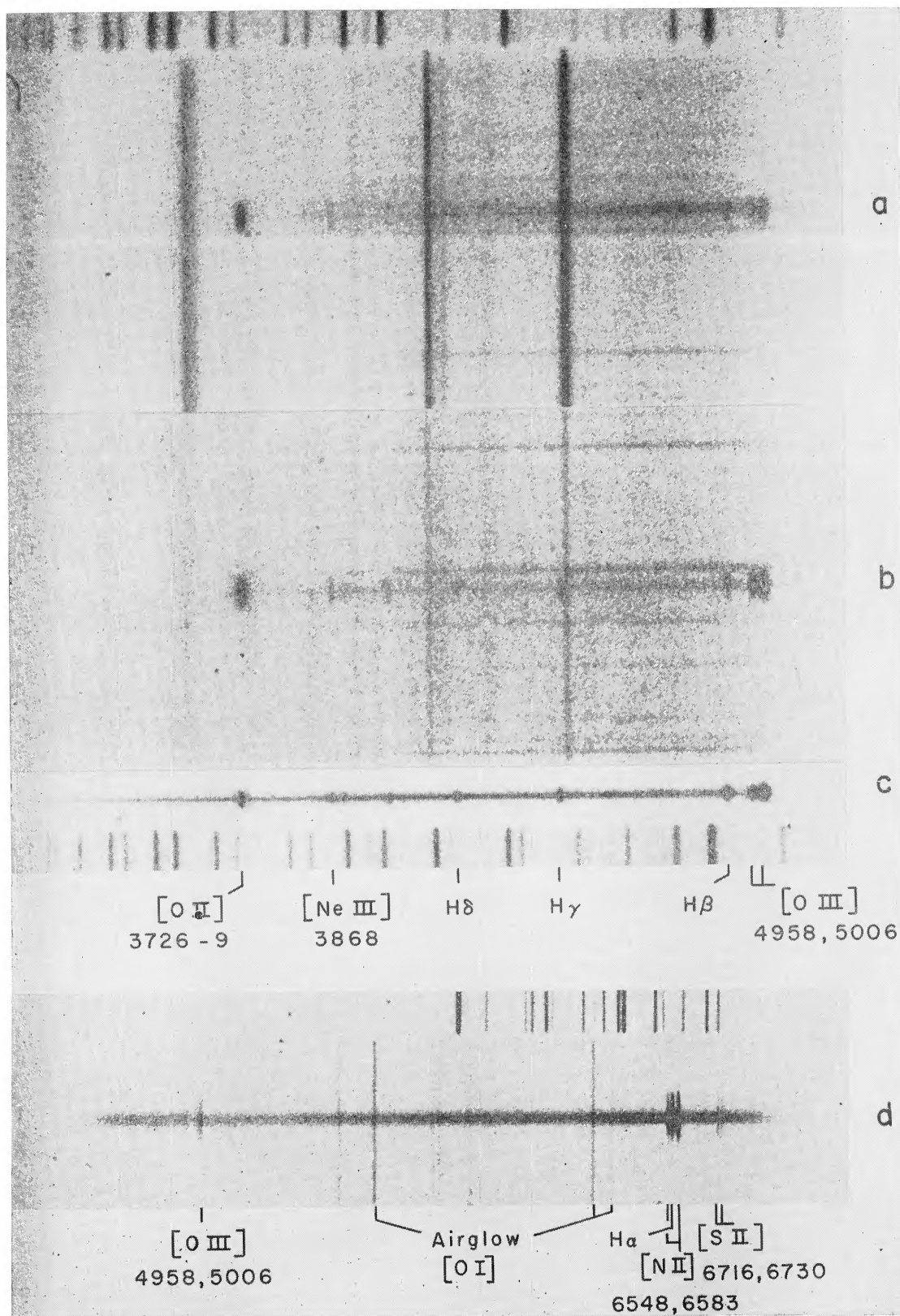


FIG. 1.—Light curve of FG Sge between 1894 and 1966. Open circles represent the Moscow photographic observations of Kukarkin (1960). Small filled circles are the Sonneberg photographic 100-day means published by Richter (1960), while crosses and a "fainter-than" symbol indicate the early Heidelberg (Jackisch) and Harvard (Ashbrook) observations reported by Richter. The boxes (1960 and later) represent photoelectric B magnitudes: the open squares are individual observations from Table 2, while the filled squares are the Sonneberg annual means of Wenzel and Fürtig (1967). No attempt has been made to reduce these various series of observations to a common photometric system.

at the prime focus of the 120-inch telescope (Fig. 3, Pl. 9). The plates of the photographic region were exposed with the slit across the nebula about 10" south of the variable and its companion. The spectrum shows strong emission lines of [O III] $\lambda\lambda 4959, 5007$, and [O II] $\lambda 3726 + \lambda 3729$, the Balmer lines through H8, and [Ne III] $\lambda\lambda 3868, 3967$. The spectrograms were not calibrated and no analysis has been attempted, but the emission spectrum is quite similar to that of a gaseous nebula of Aller's emission class 4 (Aller 1956). A Crossley spectrogram of IC 2149, a typical member of that class, is also shown in Figure 3. The value of the electron density determined for IC 2149 by Aller (1956, p. 149) is $n_e = 3 \times 10^3 \text{ cm}^{-3}$. However, the [O II] pair $\lambda\lambda 3726, 3729$ in the nebula of FG



[Legend for Fig. 3 on p. 420]

HERBIG AND BOYARCHUK (see page 398)

Sge is resolved on an $80 \text{ \AA mm}^{-1}$ spectrogram taken by Henize in 1955.³ The plate is not calibrated, but certainly $I(\lambda 3729) > I(\lambda 3726)$. For $T_e \approx 10^4 \text{ }^\circ \text{K}$, this corresponds to $n_e < 700 \text{ cm}^{-3}$ (Seaton and Osterbrock 1957). A value of $n_e \approx 500 \text{ cm}^{-3}$ would appear reasonable.

It is certain that this emission nebula around FG Sge was present before the current brightening of the star. Two plates of the region taken with the so-called Willard lens (aperture 6 inches, focal length 31 inches) in 1894 and 1910 are available in the Lick files. On the 1894 plate, the image of FG Sge is suspiciously soft, while on the 1910 exposure it is clearly more diffuse than nearby stars. The scale of these plates is such that a nebulosity of the present dimensions and brightness would be expected to have such a marginally diffuse appearance. Furthermore, on September 19, 1916, the planetary nebula NGC 6886, which lies about $23'$ southeast of FG Sge, was photographed with the Crossley reflector (focal length 206 inches) by H. D. Curtis. This plate shows the heavily comatic and weakly nebulous image of the variable about halfway to one corner. Within the uncertainties of such comparison, the nebula has the same dimensions as on the 120-inch blue exposures of 1960. This fact makes it very probable that, in turn, the dimension was not greatly different in 1894. Therefore, it is clear that the present photometric activity of FG Sge cannot be associated with the ejection of this old envelope.

The variable has a companion of about $m_{pg} = 14$ in p.a. 100° , distance $8''.1$. (There is also a much fainter star at $5''.3$ in 34° .) This companion is rather red (Wenzel and Fürtig give a color index of $+1.5 \text{ mag}$), but the spectral type is only G, on the basis of a single Crossley spectrogram. More precise classification is not possible because of interference by the emission lines of the nebulosity. No mention of the companion was made by Richter or Kukarkin. In response to an inquiry, Dr. J. Ashbrook kindly informed us that on the plates used by him for the three early Harvard observations plotted in Figure 1, an $8''$ double would probably not have been resolved. It seems likely, therefore, that these early observations represent the combined light of the pair. In 1907 (Paris Astrographic Chart, $20^h 08^m + 20^\circ$) and in 1910 (Franklin-Adams print), the variable was equal or slightly fainter than the companion. According to Figure 1, the total light was then about 0.3 mag brighter than in 1895–1900, so the variable itself in those 10–15 years must have brightened by 1.0 – 1.5 mag . This is a rate of increase that is comparable with that followed after 1930. On this basis, we are inclined to regard the flattening-out of the light curve prior to about 1910 as due to the dominance of the companion. Therefore, the full light range of FG Sge and the duration of its present activity are unknown. A similar conclusion was, in effect, drawn also by Wenzel and Fürtig (1967, Abb. 1).

III. THE STELLAR SPECTRUM

Henize (1961) described the spectrum of FG Sge on $80 \text{ \AA mm}^{-1}$ plates taken in 1955 as having “sharp emission lines at $H\alpha$, $H\beta$, and $H\gamma$, with $H\gamma$ showing a distinct P Cygni profile. Higher members of the Balmer series through H_{16} are visible as narrow absorption lines.” The result of our re-examination of one Mount Wilson plate is described later in this paper. Richter (1960, 1961) on slit spectrograms of 145 and $297 \text{ \AA mm}^{-1}$ noted the same features as did Henize, and reported, as well, rapid changes in the strength and structure of the more prominent lines. The forbidden emission lines that he found on these plates probably arise in the nebulosity, however, not in the star. A single Haute Provence $19 \text{ \AA mm}^{-1}$ spectrogram exposed in August 1960 was reproduced by Bertaud (1961), who described the P Cygni structure of the lower Balmer lines and the presence of a large number of narrow He I and metallic lines in absorption.

³ We are indebted to Dr. Henize and to Dr. H. W. Babcock, Director of the Mount Wilson and Palomar Observatories, for permission to re-examine the 1955–1956 Mount Wilson spectrograms.

The series of Lick spectrograms began in 1960 and have been obtained throughout with the 20-inch camera of the coudé spectrograph, 120-inch telescope. A 600-groove mm^{-1} Babcock grating gives a dispersion of $16 \text{ \AA mm}^{-1}$ in the second-order blue-ultraviolet. The individual spectrograms are listed in Table 1 and most of them are shown in Figures 4 and 5 (Plates 10 and 11). The spectrum of the nebulosity does not appear on these coudé plates.

During the period 1960–1967, the spectrum of FG Sge has slowly changed from that of a B9 Ia star with longward emission fringes on the lower Balmer lines, into that of an A5 Ia with no emission in the photographic region except for a weak, bright fringe on $H\beta$. This change has been continuous in the sense that the star passed through the intermediate spectral types: the C II and He I lines faded out gradually, the Ti II, Sr II, Sc II, Mg I, Fe I, and Fe II lines slowly strengthened, the Balmer jump increased, while Si II and Mg II (as would be expected in this spectral interval) showed little change. H and K are marginally double on the best plates; the shortward component is probably interstellar. Small fluctuations in the line strengths probably also took place on a shorter time scale during 1960–1965, but the coverage was not complete enough to say more. It was the presence of these rather minor variations in line strength that caused us to think originally (Herbig and Boyarchuk 1962) that a shell spectrum of variable intensity was present. Possibly these spectral variations are only the natural changes in “spectral type” that accompany the rapid activity in *UBV* observed by Wenzel and Fürtig.

There is no indication of dilution in the spectrum: the sensitive lines of Si II and Mg II originating in non-metastable lower levels were of normal strength. $H\alpha$ was observed only once at coudé dispersion; a $32 \text{ \AA mm}^{-1}$ plate taken in September 1960 shows a strong, moderately wide emission line with a narrow absorption component superimposed, somewhat shortward of the center. The structure of $H\alpha$ at that time was thus of the type found in shell stars, rather than of the classical P Cygni type. Except for the emission fringes, on these plates the star would unhesitatingly be classified as a normal supergiant except for the fact that the radial velocity varies from line to line, especially along the Balmer series. These velocity anomalies are real; velocities measured from the same features on single $16 \text{ \AA mm}^{-1}$ spectrograms obtained with the same equipment of two normal supergiants, HD 39970 (A0 Ia) and 164514 (A5 Ia) do not show the effect. This comparison data is given at the foot of Table 1.

During this same period, *UBV* observations of FG Sge were made by a number of observers. Table 2 contains those that were available to us, except for those published by Wenzel and Fürtig. Between 1960 and 1965 the value of $B - V$ has changed progressively by a very small amount, if at all. On the other hand, $U - B$ has changed from -0.40 to about -0.02 . This fading of the ultraviolet, also noted by Wenzel and Fürtig, is due to the great increase in the Balmer jump which accompanied the change in spectral type from B9 Ia to A3 Ia. The normal colors for those types according to the data of Johnson are shown in Table 3. The ratio of $E(U - B)/E(B - V) = 0.4$ in Table 3 is not normal for interstellar reddening; a value between 0.7 and 0.8 is expected in this part of the sky (Johnson 1965). The discrepancy could be intrinsic to the star, but it may be due entirely to a systematic error in the $U - B$ colors tabulated here.⁴ We assume that the $E(B - V)$ is the more reliable value because $U - B$ is not only suspect for photometric reasons, but $E(U - B)$ is more susceptible to effects of surface gravity and misclassification. With $A_v/E(B - V) = 3.5$, we obtain $A_v = 1.3$ mag. In another investigation, Herbig *et al.* (1965) have plotted the increase of $E(B - V)$ with distance as determined from reddened stars in this general area. This curve, entered with $E(B - V) = +0.38$, gives a true distance modulus of 12.0 mag for FG Sge, a value which we

⁴ The 1965 $U - B$ observations in Table 2 and those of Wenzel and Fürtig disagree by about 0.1 mag. Perhaps the two series are not on the same color system. Until this question can be clarified, we have felt it best not to combine the two sets of data. But as pointed out to us by Dr. W. Wenzel in private correspondence, the ratio $E(U - B)/E(B - V)$ is normal if the Sonneberg data alone are used.

PLATE 10

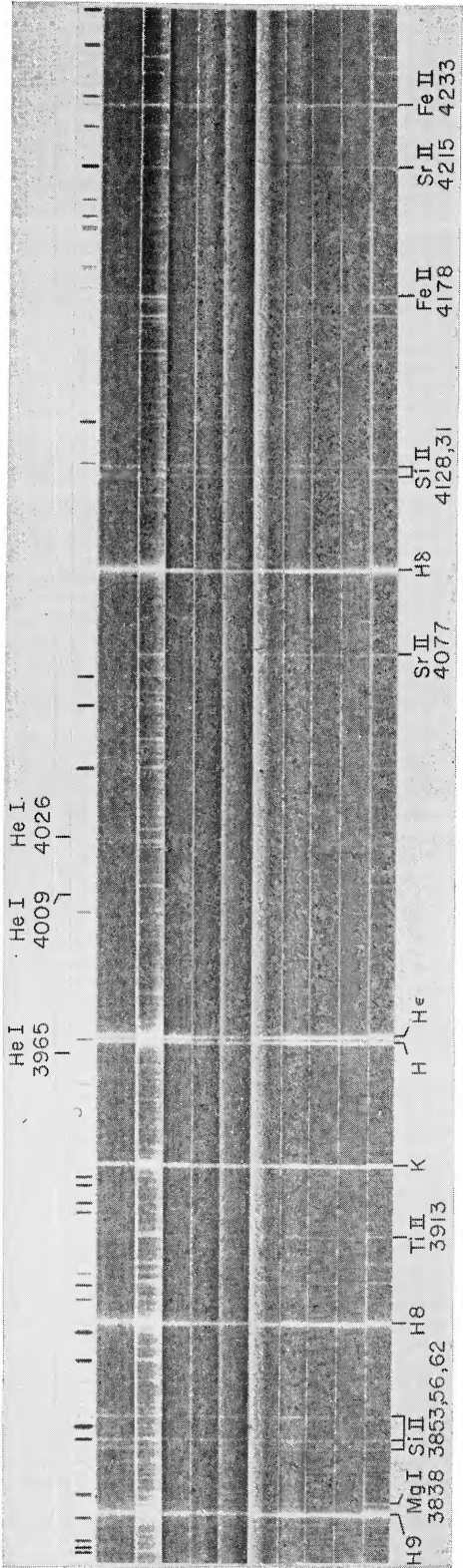
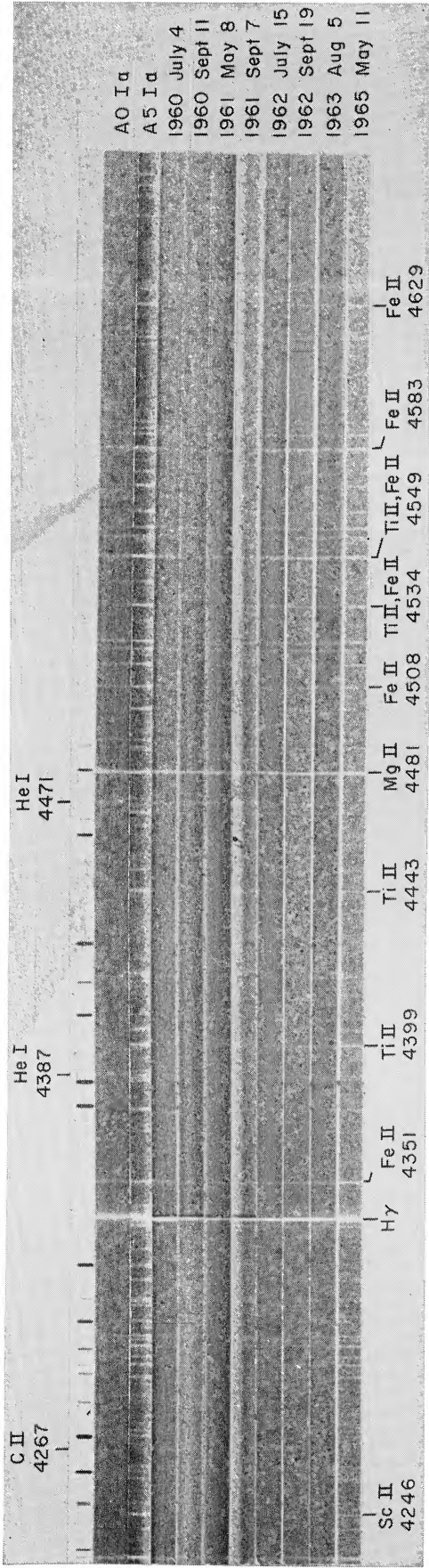


FIG. 4.—*Above*: 120-inch coude spectrograms of FG Sge taken between 1960 and 1965, together with two normal supergiants for comparison (AO Ia: HD 39970, and A5 Ia: HD 164514). The region shown is $\lambda\lambda 4230-4700$. The gradual disappearance of the longward emission edge on H_γ is apparent, and the change of spectral type shown by the fading of He I and C II and the strengthening of the lines of the ionized metals.
Below: the same spectrograms in the $\lambda\lambda 3800-4250$ region. The same progressive changes can be seen in this spectral region, but there is no variation in the intensity of the Si II lines, as is to be expected in this spectral-type interval.
HERBIG AND BOYARCHUK (see page 400)

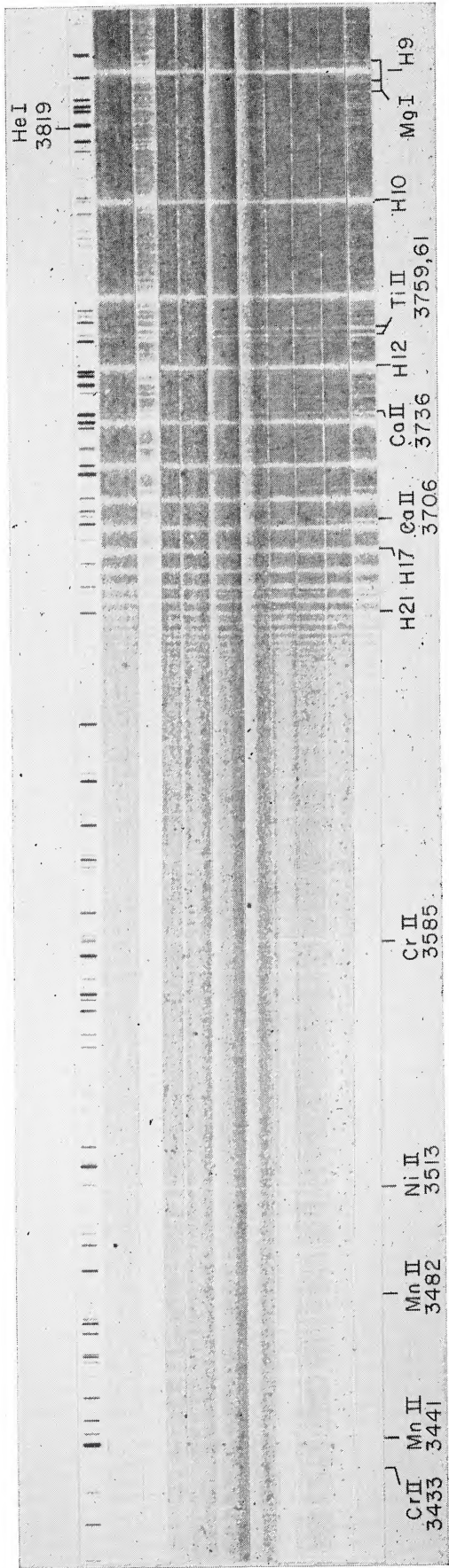


FIG. 5.—The same spectrograms as shown in Fig. 4 (Pl. 10), but in the $\lambda\lambda 3400\text{--}3850$ region. The increase in the magnitude of the Balmer jump in FG Sge causes the ultraviolet of $\lambda < 3650$ to become progressively fainter during this interval.

HERBIG AND BOYARCHUK (see page 400)

TABLE 1
RADIAL VELOCITIES FROM COUDÉ SPECTROGRAMS OF FG SGE*

Plate Number EC-	Date (UT)	Line or ion									
		He I, C II	Ca II [H,K]	Si II, Mg II	Fe II	Ti II (3685, 3759/61)	Mg I, Sr II	Hγ	Hδ	H8	H12
165	1960 July 4	+44. (8)	+13.	+44. (5)	+39. (3)	+33. (3)	+42. (2)	- 2.	- 3.	+ 3.	+24.
266	1960 Sept. 11	+37. (6)	+14.	+40. (5)	+20. (2)	+32. (1)	--	+ 8.	+ 7.	+13.	+21.
657	1961 May 8	+43. (3)	+10.	+36. (6)	+35. (2)	+31. (3)	+36. (3)	+ 7.	+ 8.	+17.	+23.
870	1961 Sept. 7	+62. (5)	+ 6.	+46. (6)	+ 6. (2)	+34. (3)	+43. (5)	+10.	+10.	+12.	+24.
1279	1962 July 15	+62. (5)	+10.	+41. (6)	+20. (5)	+33. (3)	+36. (5)	+10.	+10.	+19.	+27.
1474	1962 Sept. 19	+33. (2)	+10.	+26. (6)	+20. (18)	+18. (3)	+22. (5)	+ 8.	+11.	+14.	+20.
2152	1963 Aug. 5	+27: (1)	+ 6.	+35. (6)	+29. (17)	+35. (3)	+26. (4)	+16.	+20.	+29.	+34.
4221	1965 May 11	+16: (1)	+10.	+39. (6)	+40. (24)	+36. (3)	+38. (5)	+22.	+26.	+38.	+40.
5668	1967 June 27	--	+ 9.	+31. (4)	+34. (19)	+37. (3)	+34. (4)	+45.	+42.	+42.	+33.

Comparison data for normal supergiants:

Star	MK Type	Cat. † vel. (km/sec)									
HD 39970	A0 Ia	+0.5	+ 9. (4)	+ 2.	+ 1. (6)	+ 4. (12)	+ 4. (3)	--	+ 6.	+ 3.	+ 5.
HD 164514	A5 Ia	-1.1	--	- 5.	0. (6)	0. (21)	+ 7. (2)	- 2. (5)	0.	- 3.	- 4.

* The mean velocities are in km/sec, and are followed (in parentheses) by the number of lines included in the mean.
† The "catalogue velocity" for the two supergiants are from the General Catalogue of Stellar Radial Velocities. Both are of quality b.

shall use henceforth. These values lead to $M_v = -4.1$ for the variable in 1965. Note that although we have adopted the normal colors of a Ia supergiant for this discussion, the resulting luminosity of FG Sge is about 3 mag fainter than the mean for normal stars of class Ia.

We have also examined an 80 Å mm⁻¹ Mount Wilson spectrogram of FG Sge obtained by Henize in October 1955, and described by him (Henize 1961). Aside from the longward emission edges on the early Balmer lines, the spectrum resembles that of a supergiant slightly earlier than B5. We have assigned the type B4 I.

TABLE 2
INDIVIDUAL PHOTOELECTRIC OBSERVATIONS OF FG SGE*

Date	V	B-V	U-B	Observer	Observatory	Note†
Sept. 18, 1960	9 49±0 02	+0 40	-0 40	Chugainov	Crimea	1
Oct. 11, 1960	9 64± 02	+ 40	- 40	Chugainov	Crimea	1
Apr 25, 1961	9 45± 15	+ 43±0 03	- 38±0 03	Klemola	Lick	6
May 2, 1961	9 58±0 02	+ 41±0 01	- 30±0 01	Klemola	Lick	
Sept. 19, 1962	9 40	+ 41	- 20	Paczynski	Lick	2
Sept. 19, 1962	9 33	+ 44	- 17	Paczynski	Lick	3
Sept. 23, 1962	9 42	+ 38	- 20	Paczynski	Lick	4
Aug. —, 1965	9 21	+ 44	00	Kohoutek		5
Sept. 17, 1965	9 12	+0 42	-0 04	Chugainov	Crimea	1

* Annual means of a large number of Sonneberg *UBV* observations made in 1962–1966 have been published by Wenzel and Fürtig (1967)

† Notes: 1, includes companion: diaphragm diameter about 30"; 2, mean of four observations, with companion excluded; 3, companion included; 4, companion excluded; 5, Kohoutek (1966); 6, very poor night

TABLE 3
UBV COLOR EXCESSES OF FG SGE

Spectral Type	B-V	U-B	FG Sge in:	E(B-V)	E(U-B)
B9 Ia	-0 01	-0 57	1960	+0 41	+0 17
A3 Ia	+0 08	-0 18	1965	+0 35	+0 16
			Mean	+0 38	+0 16

IV. ANALYSIS OF THE ABSORPTION SPECTRUM

Equivalent widths of absorption lines measured from direct-intensity tracings of 6 coude spectrograms of FG Sge are given in Table 4; the column for July 3 and 4, 1960, contains the means from two plates. The equivalent widths of the hydrogen lines H8–H30 are listed in Table 5, the values for earlier members of the Balmer series not being given because of contamination by emission. These data show both the progressive and the erratic changes in line strength that took place in FG Sge during the period of observation, which have already been mentioned.

The value of N_2H , the number of H I atoms cm⁻² in the level $n = 2$ above the “photosphere” can be obtained by Unsöld’s method, wherein

$$W_\lambda = \frac{\pi e^2 \lambda^2}{m c^2} f N_2 H \tag{1}$$

TABLE 4
VALUES OF $\text{LOG } 10^6 W_{\lambda} / \lambda$ FOR ABSORPTION LINES

λ	Ion; RMT Multiplet Number	1960 July 3,4	1960 Sept. 11	1962 Sept. 19	1963 Aug. 5	1965 May 11
3318.02	Ti II 7			1.35		
22.94	Ti II 7	1.64		1.70		
29.46	Ti II 7	1.44		1.81		
32.11	Mn II	1.18				
35.19	Ti II 7	1.44		1.84	1.52	
36.33	Cr II 4	1.64				
39.80	Cr II 4	1.41		1.86	1.60	
40.34	Ti II 7					
41.88	Ti II 16	1.66		1.92	1.45	
42.51	Cr II 4	1.54		1.66		
49.40	Ti II 1	1.88		2.21	1.65	
58.50	Cr II 4	1.63		1.56		
60.10	Fe II 105	1.70		1.68		
60.30	Cr II 21					
61.21	Ti II 1	1.63		2.21		
68.05	Cr II 4	1.77		2.08	1.63	
72.80	Ti II 1	1.65		2.20	1.55	
80.28	Ti II 1	1.68		1.72	1.37	
82.68	Cr II 3	1.30		1.49		
83.76	Ti II 1	1.78		1.94	1.47	
87.83	Ti II 1	1.49		1.64	1.37	
91.43	Cr II 3	1.31		1.40		
94.32	Cr II 21	1.58		1.64	1.25	
94.57	Ti II 1					
3403.32	Cr II 3	1.61		1.81	1.40	
07.30	Ni II 4	1.45		1.34	1.07	
08.77	Cr II 3	1.64		1.92	1.54	
21.20	Cr II 3	1.55		1.77	1.34	
22.74	Cr II 3	1.66		1.79	1.48	
33.30	Cr II 3	1.54		1.68	1.44	
41.98	Mn II 3	1.48		1.80	1.34	
44.31	Ti II 6	1.26		1.54	1.34	
60.31	Mn II 3	1.36		1.70	1.44	
61.50	Ti II 6			1.58		
68.68	Fe II 114	1.34		1.52	1.34	1.64
74.04	Mn II 3	1.46		1.74	1.27	1.98
74.12	Mn II 3					
77.18	Ti II 6			1.61	1.16	1.92
82.90	Mn II 3	1.43		1.60	1.41	2.08
87.99	Fe II 4			1.65	1.46	
88.68	Mn II 3					
91.05	Ti II 6			1.51	1.00	1.85
93.47	Fe II 114			1.63	1.48	1.78
95.37	Cr II 2			1.57		
95.83	Mn II 3					
97.54	Mn II 3	1.10		1.36	0.94	

TABLE 4 (Continued)

λ	Ion; RMT Multiplet Number	1960 July 3,4	1960 Sept. 11	1962 Sept. 19	1963 Aug. 5	1965 May 11
3504.89	Ti II 88			1.62	1.20	
10.84	Ti II 88	1.61		1.66	1.30	
13.93	Ni II 1			1.57	1.33	1.52
17.30	V II 6			1.15	1.11	1.63
20.25	Ti II 98			1.38		
35.41	Ti II 98			1.45	1.33	1.45
56.80	V II 5			1.38	1.00	1.67
72.47	Zr II 1			1.49	1.05	1.74
72.52	Sc II 3					
76.76	Ni II 4			1.45	1.38	1.46
81.20	Fe I 23	1.36		1.12		
85.31	Cr II 13	1.54	1.19	1.65	1.22	1.52
89.75	V II 4			1.10	1.18	
3603.80	Cr II 13	1.17		1.52	1.37	
08.86	Fe I 23			0.98	1.09	1.22
13.26	Cr II 13			1.26	1.14	1.62
13.64	He I 6	1.22				
18.77	Fe I 23			1.10		
21.22	Co II 1	1.40	1.19	1.37	1.25	
24.83	Ti II 52			1.28	1.22	1.14
31.49	Cr II 12	1.40		1.63		1.66
41.33	Ti II 52			1.37	1.14	1.72
43.22	Cr II 1			1.05		1.50
51.68	Cr II 1			0.92		1.48
77.86	Cr II 12	1.28		1.79		1.77
85.19	Ti II 14	1.64		1.80	1.51	1.96
3705.07	He I 25	1.30				
06.03	Ca II 3			1.80	1.62	1.83
15.19	Cr II 20			1.63	1.54	1.80
27.35	V II 21					
27.37	Cr II 117	1.10		1.47	1.30	1.63
27.62	Fe I 21					
36.90	Ca II 3	1.08		1.83	1.57	1.83
38.38	Cr II 20			1.30	1.27	1.55
41.63	Ti II 72	1.12	0.70	1.80	1.54	1.95
45.56	Fe I 5			1.42	1.27	1.68
45.81	V II 15					
54.59	Cr II 20	1.14		1.30	1.30	1.51
57.68	Ti II 72			1.57	1.51	1.83
58.24	Fe I 21					
59.29	Ti II 13	1.64	0.79	2.08	1.78	2.05
61.32	Ti II 13	1.56	0.70	2.02	1.78	2.04
63.79	Fe I 21					
64.09	Fe II 29	1.10		1.38	1.35	1.61
74.33	Y II 7			1.08	0.72	1.57
76.06	Ti II 72			1.08	0.82	1.37
78.36	V II 21					1.20
79.42	Fe I 222				0.82	1.25
79.58	Fe II 23					

TABLE 4 (Continued)

λ	Ion; RMT Multiplet Number	1960 July 3,4	1960 Sept. 11	1962 Sept. 19	1963 Aug. 5	1965 May 11
3783.35	Fe II 14			1.49	1.38	1.53
88.70	Y II 7			0.96	0.82	1.42
3805.76	He I 63	1.00				
13.39	Ti II 12			1.03		1.32
14.58	Ti II 12	1.26		1.32	1.32	1.57
15.84	Fe I 45	0.93		1.12		1.37
19.64	He I 22	1.37	1.57:			
20.43	Fe I 20	0.92		1.46	1.23	1.57
21.92	Fe II 14			0.89		1.26
24.91	Fe II 29			1.29	1.19	1.54
25.88	Fe I 20	0.88		1.26	1.19	1.54
27.08	Fe II 153 }			1.11	1.11	1.27
27.83	Fe I 45 }					
29.35	Mg I 3			1.34	1.20	1.53
32.30	Mg I 3	1.20	1.04	1.61	1.37	1.59
38.29	Mg I 3	1.23	1.04	1.60	1.46	1.71
40.44	Fe I 20			0.89	0.96	1.18
41.05	Fe I 45					1.20
49.58	Ni II 11	0.97		1.02	1.06	1.49
53.66	Si II 1	1.37	1.04	1.53	1.55	1.60
56.02	Si II 1	1.69	1.43	1.85	1.72	1.87
59.91	Fe I 4			1.19	0.96	1.36
62.59	Si II 1	1.67	1.47	1.79	1.74	1.77
65.53	Fe I 20 }	0.90	0.79	1.06	0.89	1.36
65.69	Cr II 167 }					
67.50	He I 20	0.88	0.79			
71.82	He I 60	0.92	0.79			
72.50	Fe I 20 }			1.11	0.81	1.34
72.76	Fe II 29 }					
78.02	Fe I 20 }			1.06	0.89	1.25
78.58	Fe I 4 }					
3900.55	Ti II 34	1.11		1.75	1.45	1.91
02.95	Fe I 45 }			0.95	0.71	1.26
03.27	V II 11 }					
05.53	Si I 3 }	1.15	0.60	1.19	1.28	1.52
06.04	Fe II 173 }					
13.46	Ti II 34	0.93		1.77	1.40	1.82
14.48	Fe II 3			1.19	0.88	1.48
16.42	V II 10				0.88	1.06
18.98	C II 4	0.91	1.03		0.78	0.58
20.68	C II 4	0.98	1.00	0.72	0.78	0.78
26.53	He I 58	0.93:	1.00			
27.92	Fe I 4					0.68
30.31	Fe II 3			1.25	1.06	1.45
32.01	Ti II 34			1.18	1.00	1.34
33.66	Ca II 1	2.31	2.16	2.38	2.24	2.49
35.94	Fe II 173			1.11	1.31	1.46
38.29	Fe II 3				1.06	1.48
44.01	Al I 1			0.58		0.92

TABLE 4 (Continued)

λ	Ion; RMT Multiplet Number	1960 July 3,4	1960 Sept. 11	1962 Sept. 19	1963 Aug. 5	1965 May 11
3945.21	Fe II 3			0.88	1.00	1.33
47.50	O I 3			0.58	0.95	0.88
50.35	Y II 6			0.58		0.88
51.97	V II 10			0.87		1.10
58.24	Zr II 16			0.87		1.38
61.52	Al I 1			0.80	1.18	1.28
64.73	He I	1.05	1.19			
74.16	Fe II 29			1.05		1.24
79.51	Cr II 183			0.94		0.87
82.00	Ti II 11			0.87		1.05
82.59	Y II 6					0.80
98.98	Zr II 16			0.87	0.80	1.26
4002.07	Fe II 29			0.87:		1.04
02.55	Fe II 190 }			0.87	0.87	1.27
02.94	V II 9 }					
05.71	V II 32			0.94	1.00	1.18
09.27	He I 55	1.05	1.14			
12.37	Ti II 11 }					
12.47	Fe II 126 }			1.18	1.18	1.59
15.50	Ni II 12			0.79	0.79	1.05
23.39	V II 32			1.10	1.20	1.24
24.55	Fe II 127			1.17	1.24	1.24
25.14	Ti II 11					0.94
26.20	He I 18	1.26	1.33			
28.33	Ti II 87			1.17		1.33
32.95	Fe II 126			0.79	0.87	0.87
35.63	V II 32			1.04	1.00	1.10
45.82	Fe I 43			1.30	1.24	1.50
48.68	Zr II 43 }					
48.83	Fe II 172 }			1.00	1.17	1.37
51.97	Cr II 19			0.83	0.69	0.93
53.45	Cr II 19 }					
53.81	Ti II 87 }			0.99	0.79	1.34
63.60	Fe I 43			0.99	0.79	1.20
67.03	V II 9 }					
67.05	Cr II 193 }			1.08	1.20	1.36
67.05	Ni II 11 }					
71.74	Fe I 43			1.04	0.87	1.36
77.71	Sr II 1	1.10:		1.70	1.59	1.76
82.30	Cr II 165					0.70
4111.01	Cr II 18,26			0.86	0.56	
4120.90	He I 16	0.89	1.08			
22.64	Fe II 28	0.70		1.16	1.08	1.36
28.05	Si II 3	1.52	1.36	1.51	1.53	1.50
30.88	Si II 3	1.52	1.40	1.53	1.53	1.56
32.06	Fe I 43 }					
32.41	Cr II 26 }			0.84	0.78	0.88
43.76	He I 53	0.84				

TABLE 4 (Continued)

λ	Ion; RMT Multiplet Number	1960 July 3,4	1960 Sept. 11	1962 Sept. 19	1963 Aug. 5	1965 May 11
4143.42	Fe I 523 }			0.88	0.99	
43.87	Fe I 43 }					
45.77	Cr II 162	0.70		0.68	0.55	0.75
49.22	Zr II 41			0.98	0.98	1.34
50.97	Zr II 42 }			0.68	0.68	1.04
51.00	Cr II 163 }					
56.24	Zr II 29			0.86	0.56	1.08
61.05	Cr II 162 }			1.04	0.78	1.28
61.20	Zr II 42 }					
63.64	Ti II 105	0.60		1.28	1.04	1.41
71.90	Ti II 105			1.23	1.19	1.40
73.45	Fe II 27	1.17		1.60	1.50	1.67
77.54	Y II 14			1.38	0.98	1.46
78.86	Fe II 28	1.18	0.76	1.60	1.57	1.70
83.44	V II 37				0.69	0.88
99.10	Fe I 522			0.57		0.67
4205.05	Eu II 1			0.45	0.74	0.87
15.52	Sr II 1	0.78		1.56	1.33	1.66
27.43	Fe I 693			0.60	0.77	0.87
33.17	Fe II 27	1.65	1.50	1.81	1.69	1.82
42.38	Cr II 31	0.97		1.32	1.02	1.40
46.41	Cr II 31 }			1.40	1.15	1.48
46.83	Sc II 7 }					
50.12	Fe I 152 }			0.54	0.54	0.85
50.79	Fe I 42 }					
52.62	Cr II 31			0.67	0.54	0.77
54.35	Cr I 1			0.83		0.76
58.16	Fe II 28			1.02	0.91	1.37
60.48	Fe I 152			0.67	0.77	0.84
61.92	Cr II 31	0.82		1.14	1.02	1.28
67.15	C II 6	1.17	1.35	0.80		
69.28	Cr II 31			0.67	0.36	0.78
71.76	Fe I 42			0.91	0.85	1.07
73.32	Fe II 27			1.08	0.97	1.27
75.57	Cr II 31			1.02	0.85	1.24
78.10	Cr II 161 }			0.76		1.02
78.13	Fe II 32 }					
84.21	Cr II 31			0.85	0.76	1.14
90.22	Ti II 41			1.45	1.07	1.50
94.10	Ti II 20			1.56	1.26	1.54
96.57	Fe II 28	0.50		1.43	1.21	1.43
4300.05	Ti II 41	0.85		1.64	1.32	1.70
01.93	Ti II 41			1.24	0.76	1.20
03.17	Fe II 27	1.18		1.57	1.53	1.67
07.90	Ti II 41 }			1.45	1.17	1.41
07.91	Fe I 42 }					
12.86	Ti II 41			1.36	0.91	1.42

TABLE 4 (Continued)

λ	Ion; RMT Multiplet Number	1960 July 3,4	1960 Sept. 11	1962 Sept. 19	1963 Aug. 5	1965 May 11
4314.08	Sc II 15			1.20	1.02	1.41
14.29	Fe II 32					
14.98	Ti II 41			1.38		1.38
16.81	Ti II 94			0.73		1.12
20.75	Sc II 15					
20.97	Ti II 41			1.06		1.32
25.01	Sc II 15			0.94		1.05
25.77	Fe I 42			1.06	0.85	1.14
51.76	Fe II 27	1.45	1.14	1.69	1.62	1.77
67.66	Ti II 104			0.84	0.90	1.06
69.40	Fe II 28	0.70:				0.96
74.94	Y II 13			1.17	0.83	1.38
83.55	Fe I 41	1.00		1.14		1.20
84.64	Mg II 10	1.00	0.50			1.20
85.38	Fe II 27	1.22	0.79	1.43	1.38	1.50
86.86	Ti II 104			0.65		1.00
87.93	He I 51	1.08	1.06			0.50
90.59	Mg II 10			0.96	0.96	0.96
95.03	Ti II 19	0.50		1.63	1.31	1.69
99.77	Ti II 51			1.17	0.83	1.25
4411.08	Ti II 115				0.76	0.96
16.82	Fe II 27	1.11		1.48	1.13	1.52
17.72	Ti II 40			1.05		1.35
43.80	Ti II 19	0.58		1.50	1.26	1.56
50.49	Ti II 19			1.00	1.65	1.13
64.46	Ti II 40			0.95		1.05
68.49	Ti II 31			1.57	1.23	1.56
70.86	Ti II 40			1.16	1.00	1.05
71.48	He I 14	1.46	1.51			
72.92	Fe II 37			0.86	0.89	0.89
81.23	Mg II 4	2.03	1.87	2.00	1.95	2.03
88.32	Ti II 115			0.80	0.75	1.13
89.19	Fe II 37	0.58		1.30	1.25	1.43
91.40	Fe II 37	0.65		1.48	1.13	1.47
4501.27	Ti II 31	1.10:		1.52	1.28	1.55
08.28	Fe II 38	1.10		1.56	1.39	1.61
15.34	Fe II 37	1.04		1.42	1.30	1.59
20.23	Fe II 37	0.85		1.47	1.34	1.38
22.63	Fe II 38	1.40	0.93	1.60	1.46	1.67
33.97	Ti II 50			1.60	1.38	1.69
34.17	Fe II 37					
41.52	Fe II 38			1.30	1.28	1.38
49.47	Fe II 38					
49.62	Ti II 82	1.70	1.33	1.76	1.62	1.86
54.03	Ba II 1			1.04	0.82	1.34
55.89	Fe II 37	1.16:		1.52	1.46	1.57
58.66	Cr II 44	1.26	0.78	1.52	1.36	1.63
63.76	Ti II 50			1.38	1.22	1.56
65.78	Cr II 39			0.88		0.94
71.97	Ti II 82	0.97		1.47	1.27	1.62

TABLE 4 (Continued)

λ	Ion; RMT Multiplet Number	1960 July 3,4	1960 Sept. 11	1962 Sept. 19	1963 Aug. 5	1965 May 11
4576.33	Fe II 38			1.27	1.04	1.36
80.06	Fe II 26			0.82	0.52	0.99
83.83	Fe II 38	1.54	1.00	1.66	1.48	1.78
88.22	Cr II 44	1.19		1.45	1.27	1.54
89.89	Cr II 44					
89.96	Ti II 50			0.88	0.82	1.18
92.09	Cr II 44	0.54:		1.04	1.11	1.34
4616.64	Cr II 44			0.94	0.51	0.94
18.83	Cr II 44	0.99		1.38	1.03	1.40
20.51	Fe II 38			1.03	0.63	0.99
25.91	Fe II 186			0.90		
29.34	Fe II 37	1.12		1.42	1.36	1.52
34.11	Cr II 44	0.99		1.29	0.94	1.26
35.33	Fe II 186	0.83		1.00		0.81
56.97	Fe II 43					1.17
63.70	Fe II 44			0.63		0.67
66.75	Fe II 37			0.93		1.10
70.17	Fe II 25			0.63		0.81
4731.44	Fe II 43			0.96		
4824.13	Cr II 30			1.46	1.22	
48.24	Cr II 30			1.31	1.04	
84.57	Cr II 30			1.12		
4923.92	Fe II 42			1.71	1.48	

is plotted against upper quantum number. This dependence of $\log N_2H$ upon line number in the Balmer series is shown in Figure 6 for the five dates of observation, with data from α Cyg for comparison. Apparently this method sets only a lower limit on N_2H ; the decrease of N_2H for the high-series members in FG Sge might be due partially to overlapping, but there may also be a contribution from stratification (§ V). The effect was noted in shell stars by Searle (1958). A value of N_2H can also be obtained from the magnitude of the Balmer jump, D , by (Unsöld 1955, p. 481)

$$\frac{1}{R} = \frac{1}{k_2 N_2 H} + \frac{1}{R_c}, \tag{2}$$

TABLE 5
EQUIVALENT WIDTHS (IN Å) OF BALMER ABSORPTION LINES

Line	July 3, 4, 1960	Sept 11, 1960	Sept 19, 1962	Aug. 5, 1963	May 11, 1965
H8 .	1 02	0 85	1 49	2 01	2 52
9	1 50	89	1 72	1 87	2 55
10	1 50	87	1 64	1 94	2 44
11	1 55	85	1 72	2 28	2 46
12 .	1 22	83	1 57	2 14	2 58
13 .	1 20	74	1 66	1 90	2 50
14.	1 12	74	1 57	1 79	2 22
15	1 12	71	1 51	1 70	1 97
16 .	0 98	66	1 28	1 42	1 71
17	0 91	62	1 24	1 34	1 45
18 .	0 71	.51	1 00	0 95	1 34
19	0 54	44	1 00	1 00	1 19
20 .	0 46	36	0 96	0 75	0 85
21	0 36	23	0 72	0 48	0 65
22	0 32	18	0 40	0 49	0 52
23	0 25	22	0 45	0 37	0 43
24	0 18	.14	0 32	0 19	0 34
25	0 13	.10	0 17	0 14	0 23
26	0 10	.10	0 15	0 15	0 19
27	0 06	04	0 11	0 09	0 11
28..	0.04	02	0 10	...	0 10
29	...	0 01	0 06	.	.
H30 .	0 02	..	0.06	.	.

where $R = 1 - 10^{-D}$, $k_2 = 1.38 \times 10^{-17}$, and the central depth of the line R_c is given by the observations. The values of $\log N_2H$ resulting from equation (2) are also in Table 6.

The electron density in the atmosphere n_e can be obtained either from the number m of the last resolvable Balmer line by the Inglis-Teller formula

$$\log n_e = 23.26 - 7.5 \log m \tag{3}$$

or by Unsöld's (1941) expression

$$W_\lambda^{5/2}(n) = 1.54 \times 10^{-13} C_n N_2 H n_e. \tag{4}$$

Here, $W_\lambda(n)$ is the equivalent width of the Balmer line having upper quantum number n , and C_n is a parameter obtainable from Stark broadening theory. For the line H11, Houziaux (1958) gives $C_{11} = 0.182 \times 10^{-16}$. The results obtained from equations (3)

and (4) are given in Table 6 as $\log n_e(\text{I.T.})$ and $\log n_e(\text{H11})$, respectively. The values of $\log N_2H$ and $\log n_e$ determined for α Cyg by Groth (1961) are also shown.

The lines of other elements have been analyzed by Unsöld's (1955, p. 412) curve-of-growth method, in which $\log (W_\lambda c / 2\lambda R_c v_t)$ is plotted against $\log X = \log (NHf\lambda / 2\pi R_c v_t)$, where c is the velocity of light, f the oscillator strength, and v_t a velocity. The curves of growth were constructed by first estimating a value of $\theta_{\text{exc}} = 5040/T_{\text{exc}}$ from the spectral type and then improving it by means of a first-approximation curve of growth. The adopted values of θ_{exc} are listed in Table 6. Two representative final curves of growth (from the spectrogram of September 19, 1962, for the regions shortward and longward of the Balmer limit) are shown in Figure 7. The value of the damping parameter $a = \gamma/\Delta\omega_D$ is 0.03.

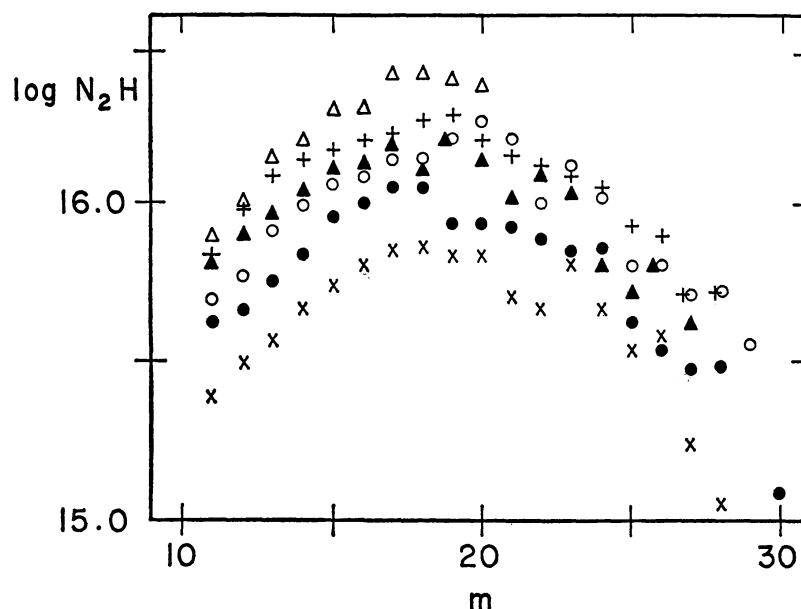


FIG 6 — The dependence of $\log N_2H$, the number of H I atoms (in $n = 2$) cm^{-2} above the photosphere on the assumption of an optically thin absorbing layer, as a function of the principal quantum number of the upper state (m). The symbols have the following significance: *open triangles*, data from α Cygni, while the remaining data are for FG Sge: *filled circles*, July 4, 1960; *crosses*, September 11, 1960; *open circles*, September 19, 1962; *filled triangles*, August 5, 1963; *plus marks*, May 11, 1965.

Values of θ_{ion} were obtained from the relative abundances of Mg I/II, Fe I/II, and Cr I/II with the aid of $n_e(\text{H11})$. Groth's (1961) equivalent widths for α Cyg (type A2 Ia) have been analyzed in the same way as the FG Sge data, and these results also are given in Table 6. It can be seen that the physical conditions in the atmosphere of FG Sge (n_e , v_t , θ_{exc} , θ_{ion}) are similar to those in this standard supergiant.

With the values of n_e and θ_{ion} , allowance can be made for unobservable stages of ionization, and the resulting values of the total abundances are listed in Table 6. The results for Fe, Cr, and Ti have been used to connect the results from the two sides of the Balmer limit. The chemical composition of FG Sge can then be compared with that of α Cyg. Define for each element $A \equiv \log NH(\text{FG Sge})/NH(\alpha \text{ Cyg})$; $\langle A \rangle$, the mean value of A for Si, Ti, Cr, and Fe is given for each date in the last line of Table 6. It can be seen that in 1960 there were about 0.06 as many atoms above the photosphere of FG Sge as in the case of α Cyg. The mean value for 1962–1965 was about 0.3.

The values of $A_i - \langle A \rangle$ for the different elements observed are given in Table 7. There appear to be some small differences in chemical composition between the two

TABLE 6
log NH VALUES FOR VARIOUS ELEMENTS, AND ATMOSPHERIC PARAMETERS*

Element	1960 July 3,4	1960 Sept. 11	1962 Sept. 19	1963 Aug. 5	1965 May 11	α Cyg	n
H (n = 2)	16.53	16.44	16.87	17.15	17.20	17.21	25
H	23.03	22.85	23.38	23.50	23.62	23.65	
He I	25.48	22.89			24.72	25.12	4
He	25.48	22.89			24.72	25.12	
C II	22.05	20.92	22.84	22.56	23.06	22.43	3
C	22.05	20.92	22.84	22.56	23.06	22.43	
O I			20.04	20.16	20.34	19.98	1
O			20.43	20.72	20.67	20.68	
Mg I	13.02	12.67	13.86	13.59	13.99	13.94	3
Mg II	17.89	17.60	18.81	18.93	18.82	19.05	2
Mg	18.45	18.80	18.88	19.06	18.88	19.28	
Si II	18.21	17.54	19.14	19.11	19.09	19.36	5
Si III	17.46	17.58					
Si	18.28	17.86	19.14	19.11	19.09	19.36	
Ca II	14.85 [†]	14.97 [†]	15.47	15.15	15.55	15.70 [†]	1
Ca	17.23 [†]	18.03 [†]	17.00	16.86	16.92	17.75	
Sc II			13.51	13.21	13.10	13.26	
Sc			14.07	13.92	13.91	14.21	
Ti II	14.29	13.44	15.50	15.11	15.66	15.83	27
Ti	15.30	15.19	15.80	15.54	15.88	16.37	
V II			14.87	14.95	15.09	15.00	4
V			15.09	15.27	15.28	15.48	
Cr I			12.90		12.80	13.00	1
Cr II	14.73		15.57	15.21	15.67	15.96	13
Cr	16.30		16.74	16.41	16.82	17.25	
Mn II	15.02		15.74	15.32		16.09	5
Mn	15.39		15.75	15.40		16.22	
Fe I	13.48		13.92	13.82	14.17	14.21	11
Fe II	16.82	16.47	17.70	17.52	17.82	18.36	25
Fe	17.02	17.12	17.71	17.55	17.83	18.42	
Co II	15.67		15.91	15.88		15.95	1
Co	15.77		15.91	15.88		15.97	
Ni II			16.36	16.25	16.49	16.72	3
Ni			16.36	16.26	16.49	16.73	

TABLE 6 (Continued)

Element	1960 July 3,4	1960 Sept. 11	1962 Sept. 19	1963 Aug. 5	1965 May 11	α Cyg	n
Sr II	11.94		12.89	12.73	13.03	12.38	2
Sr	14.28		14.41	14.46	14.44	14.34	
Y II			13.22	13.09	13.59		2
Y			14.66	14.73	14.80		
Zr II			14.16	14.10	14.58	14.86	2
Zr			14.52	14.59	14.84	14.97	
Ba II			11.90	11.67	12.24	11.81	1
Ba			14.07	13.97	14.27	14.35	
Eu II			12.19	12.49	12.63	12.73	1
Eu			14.1	14.5	14.4	15.1	
$\log 2R_c v_t$	1.28	0.92	1.18	1.02	1.22	1.22	
R_c	0.75	0.65	0.80	0.72	0.83	0.75	
$\log n_e$ (I.T.)	12.16	12.16	12.16	12.36	12.30	12.05	
$\log n_e$ (H II)	13.14	12.74	12.91	13.02	12.94	12.89	
θ_{exc}	0.58	0.50	0.66	0.63	0.66	0.62	
θ_{ion}	0.52	0.50	0.60	0.58	0.61	0.57	
D	0.14	0.12	0.29	0.32	0.40	0.36	
$\langle A \rangle$	-1.12	-1.33	-0.50	-0.70	-0.46	0.00	

*The upper figure is the log NH value for the ion indicated, the lower is that summed over all stages of ionization. The number of lines upon which the first value depends is given in the last column. All the available lines of Mn II and Co II have $\lambda < 3650 \text{ \AA}$. The f-values used in the analysis were taken from the following sources: H I, He I, C II, O I : Wiese, Smith, and Glennon 1966; Mg I : Aarts and Bosch 1964; Mg II : Biermann and Lübeck 1948; Al I : Penkin and Shobanova 1965; Si II : Jugaku, Sargent and Greenstein 1961; Ca II, Sr II, Ba II : Ostrovski and Penkin 1961; Sc II, Ti II, V II, Cr II, Mn II, Fe II, Co II and Ni II : Warner 1967; Ca II, Cr I, Y II, Zr II, Eu II : Corliss and Bozman 1962; Fe I : Corliss and Warner 1964.

[†]Ca II $\lambda\lambda 3706, 3736$ were not measured on these spectrograms.

[‡] ± 0.4

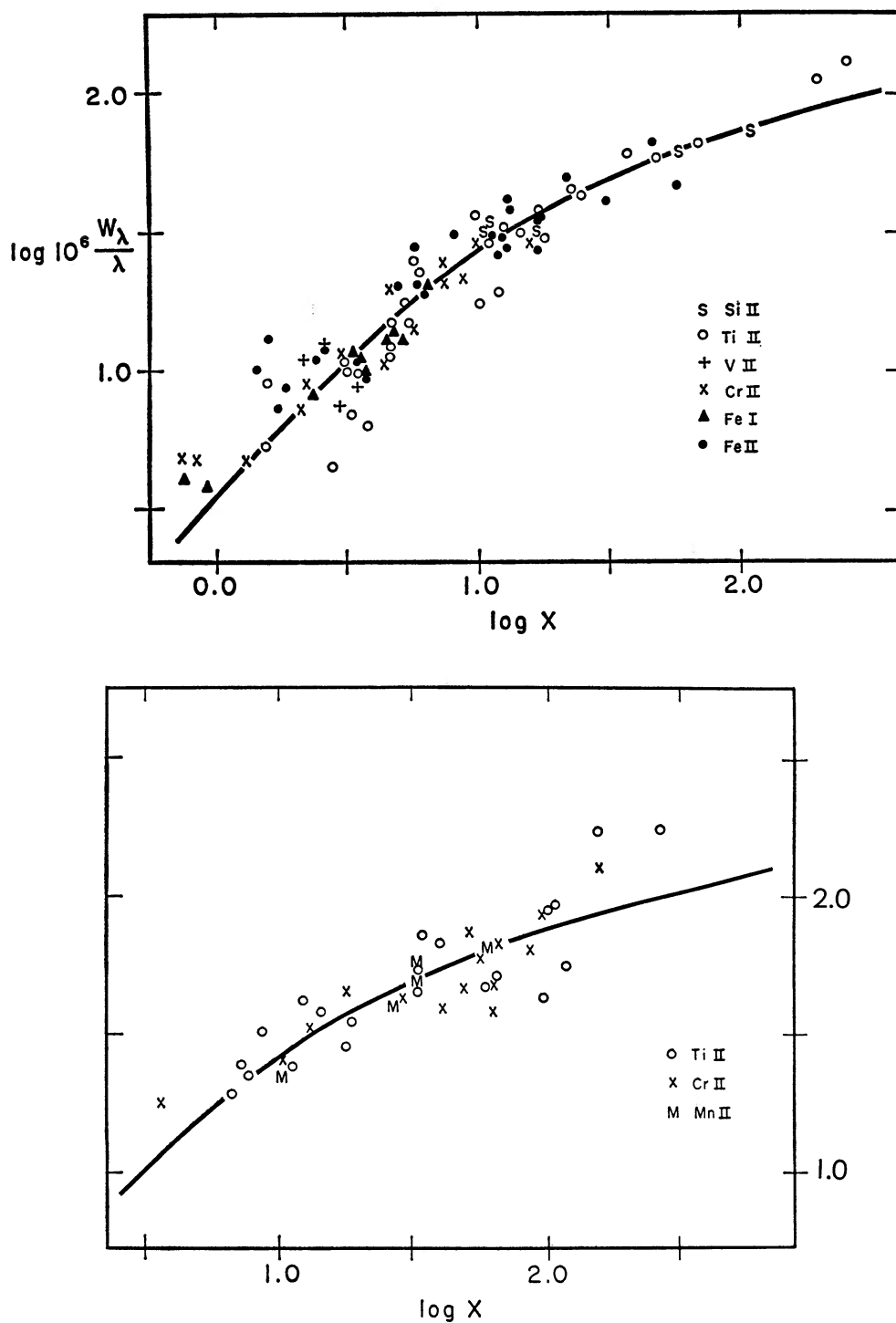


FIG. 7 —Representative curves of growth of FG Sge for the regions longward (*top*) and shortward (*bottom*) of the Balmer limit. This pair of curves is derived from the data of September 19, 1962.

stars, but these may not be real in the cases of H, He, and C because of the high excitation potentials of the accessible lines. For these elements, the apparent discrepancies in the last column of Table 7 could be explained either by deviations from thermodynamic equilibrium or by an error in θ_{exc} of about 0.05. The overabundances of Co and Sr may be real, but in the case of Co the result depends upon only one line. In general, however, the ratios of abundances in the two stars indicate that the chemical composition of the atmosphere of FG Sge does not differ significantly from those of normal Population I objects.

TABLE 7
VALUES OF $A_i - \langle A \rangle$: THE LOGARITHMS OF THE RELATIVE ABUNDANCES
IN FG SGE WITH RESPECT TO α CYG*

Element	July 3, 4, 1960	Sept 11, 1960	Sept 19, 1962	Aug 5, 1963	May 11, 1965	Mean
H	+0 50	+0 53	+0 23	+0 55	+0 43	+0.4
He	+1 48	-0 90	..	..	+0 06	+ 2:
C	+0 74	-0 18	+ 91	+ 83	+1 09	+ 7
O	..	..	+ 25	+ 74	+0 45	+ 5
Mg	+0 29	+0 85	+ 10	+ 48	+0 06	+ 4
Si	+0 04	-0 17	+ 28	+ 45	+0 19	+ 2
Ca	(+0 60)	(+1 61)	- 25	- 19	-0 37	- 3
Sc	..	..	+ 36	+ 41	+0 16	+ 3
Ti	+0 05	+0 15	- 07	- 13	-0 03	0
V	..	..	+ 11	+ 49	+0 26	+ 3
Cr	+0 17	..	- 01	- 14	+0 03	0
Mn	+0 29	..	+ 15	- 12	..	+ 1
Fe	-0 28	+0 03	- 21	- 17	-0 13	- 2
Co	+0 92	..	+ 44	+ 61	..	+ 7
Ni	..	..	+ 13	+ 23	+0 22	+ 2
Sr	+1 06	..	+ 57	+ 82	+0 56	+ 8
Zr	..	..	+ 05	+ 32	+0 33	+ 2
Ba	..	..	+ 22	+ 32	+0 38	+ 3
Eu	..	..	-0 5	+0 1	-0 2	-0 2

* The effect of the different *total* number of atoms above the photospheres of the two stars has been taken out by forcing agreement in the abundances of Si, Ti, Cr, and Fe

V. DISCUSSION

The existence of what appears to be a normal planetary nebula surrounding FG Sge shows that at some time in the past the variable has ejected a gaseous envelope. Our attempt to interpret the present activity of the star is based on the assumption that the same event is again taking place.

The differences in radial velocity between absorption lines indicate that there are large velocity differences between various levels in the atmosphere. The cores of the early members of the Balmer series are formed higher than are those of lines nearer the end of the series, and the observations (Table 1) show that the mean velocity of $H\gamma$ and $H\delta$ is always more negative than that of $H18$ and higher members. The amount of this difference was about 30 km sec^{-1} in 1960 and less than 20 km sec^{-1} in 1965. (This change was not due to the disappearance of the emission fringes on the low H lines because the effect is apparent along the entire Balmer series.) It is concluded that a velocity gradient exists in the atmosphere in the sense that material is accelerated outward as it rises. Presumably the differences between other lines are related in the same way to mean level of formation, with He I and C II on this basis being formed deepest. Such systematic changes in velocity along the Balmer series are of course well known in shell spectra.

Our model consists of a very deep expanding atmosphere surrounding a very hot star, out of which material began to flow sometime prior to 1900. The amount of material in the line of sight is now large enough that an optical depth in the continuum of the order of unity is reached at a level R_p high in this outflowing material, and the central star of radius R_* is thus invisible. Gas from below continues to stream through this level R_p , which in effect "floats" at a point in the density gradient that is determined by the continuous opacity of still higher material. R_p thus represents the photosphere above which the line spectrum of the Ia supergiant is formed. We consider separately (1) the outermost boundary of the expanding atmosphere and (2) the material in the vicinity of R_p .

1. The expansion velocity of the topmost levels of the atmosphere is not determined, since the center-of-mass velocity is unknown. The emission lines of the old planetary-nebula envelope might be trusted to give the center-of-mass velocity, but our spectrograms of the nebula are of too low dispersion to be useful for this purpose. The displacement of the absorption reversal in H α from the center of symmetry of the broad emission line is an index of the expansion velocity under the assumptions of spherical

TABLE 8
PHYSICAL PARAMETERS OF FG Sge

Date	Spectral Type	$\log T_e$	m_{pg}	M_{bol}	$R_p/R_\odot$
1955 8	B4 I	4.09	10.5	-3.6	10
1960 5 ..	B9 Ia	3.99	10.1	-3.9	18
1965 4 .	A3 Ia	3.93	9.7	-4.1	26
1967 5....	A5 Ia	3.92	.	..	

symmetry and radial expansion; this displacement on the coudé spectrogram of September 1960 is -70 km sec^{-1} . As pointed out, the Balmer lines show a differential displacement of $20\text{--}30 \text{ km sec}^{-1}$ along the series. Since they represent an integration with depth through the visible part of the atmosphere, the full differential expansion velocity must be still larger, so an expansion velocity of the order of 70 km sec^{-1} with respect to the center of mass is not unreasonable. (The expansion velocities observed in planetary nebulae range from very small values up to about 100 km sec^{-1} ; Wilson 1950.) In time, the expanding material should appear as a disk of finite size around the star. At the estimated distance of 2.5 kpc, if expansion at 70 km sec^{-1} began about 1895, the disk should now have a diameter of about $0''.8$. A disk of uniform surface brightness having that size would be photographically distinguishable with the 120-inch telescope under good conditions, but not in the presence of such a bright central star. Therefore, unless FG Sge should become very faint, this test of our proposal will not be practicable for many years.

2. Between 1955.8 and 1965.4, the spectral type and apparent magnitude changes of FG Sge can be converted, under the usual assumptions, to the M_{bol} and $R_p/R_\odot$ values contained in Table 8. The $\log T_e$'s are based on Johnson's (1966) calibration for supergiants.

The velocity of expansion of the star, dR_p/dt , indicated by these results is only 0.04 km sec^{-1} and is clearly not to be associated with the velocity of the out-streaming gas which is three orders of magnitude higher. The light curve of such an expanding surface can be represented by the product of surface area and surface brightness, the latter

being given by the Planck function. If $F(t)$ represents the photographic flux (effective wavelength 4250 Å) at time t , then

$$\frac{F(t)}{F(0)} = \frac{R_p^2(t)}{R_p^2(0)} \frac{\exp[3.384 \times 10^4/T(0)] - 1}{\exp[3.384 \times 10^4/T(t)] - 1}. \quad (5)$$

If t represents 1960 when $T \approx 10000^\circ \text{K}$ and $t = 0$ represents 1895 when the star was 4.0 mag (photographically) fainter, then $F(t)/F(0) = 40$, and from equation (5) one can obtain $R_p(t)/R_p(0)$ as a function of the starting temperature that is assumed: see Table 9, first and second columns. There is also a corresponding change in the bolometric absolute magnitude (third column). In the special case where M_{bol} is constant, and the material below the level R_p simply acts to rethermalize the radiation without loss of energy, the solution of equation (5) is $T(0) = 63300^\circ \text{K}$, $R_p(t)/R_p(0) = 40$, and since in this special case R_p and T are coupled through the condition $R_p^2 T^4 \approx R_p^2 T_e^4 = \text{constant}$, a light curve can be computed from equation (5) as a function of $R_p/R_p(0)$. Such

TABLE 9
THE CONSEQUENCES OF EQUATION (5)*

Starting Temperature: $T(0)^\circ \text{K}$	Radius Increase Required To Produce $\Delta m_{\text{pg}} =$ 4.0: $R_p(t)/R_p(0)$	Resulting Increase in Bolometric Ab- solute Magnitude: ΔM_{bol} (mag)
1×10^4	6.3	-4.0
2×10^4	16.0	-3.0
5×10^4	34.2	-0.7
6.33×10^4	40.1	0.0
1×10^5	53.2	+1.4
2×10^5	78.6	+3.5

* Note that T is, strictly, the monochromatic brightness temperature at the R_p level in the expanding atmosphere. T_e is the effective temperature at the same level, while T_e is the electron temperature of the outer nebulosity.

a light curve has the property, however, that an upper limit to Δm_{pg} is reached at $T = 8500^\circ$ because the bolometric correction for 4250 Å goes through a minimum at that temperature. Since FG Sge passed through $T = 8500^\circ$ about 1965 with no decrease in the rate of brightening, it is obvious that the condition $\Delta M_{\text{bol}} = 0.0$ does not hold. This is shown also, in another way, by the brightening of the "observed" M_{bol} values in Table 8. It seems entirely reasonable that the continuing activity in the underlying star of FG Sge that is responsible for the mass ejection should also cause the luminosity of the star to increase as well.

Consider now the present source of excitation of the emission nebulosity around FG Sge. Such a nebula cannot be excited by a normal star⁵ of spectral type B9 Ia to A3 Ia, as is clear from the spectra of galactic nebulae illuminated by such stars (Herbig 1958). Furthermore, if FG Sge did emit at the "photospheric" level sufficient energy below 912 Å to produce such a H II region, the first result would be that the expanding layers above R_p would become fully ionized. One would expect then to observe a recombination emission-line spectrum in the integrated light of the star, as in the post-maximum spec-

⁵ A similar problem to this exists in the case of the planetary nebula NGC 1514, in which the central star appears to be a normal object of type B9. The presumption in that case has been that there is an invisible, hot companion (Aller and Wilson 1954), although it has recently been reported that both the light and the radial velocity of the B9 star are variable, suggesting that it may itself be a peculiar star.

trum of a nova. No such spectrum is observed in the case of FG Sge, except the longward emission structure at the H lines which faded away during 1960–1965. Probably those emission lines were produced by such a region but one lying very low in the atmosphere, and their weakening probably was a direct consequence of the decrease in T over those years. The point is that there apparently is not now in FG Sge any source of energy to maintain the ionization of the nebulosity, and presumably there has not been one since the star fell below a value of $T \approx 25000^\circ \text{ K}$, which is approximately the exciting star temperature at which the transition between emission and reflection nebulae is observed to take place. When FG Sge passed through this transition temperature is not known.

Consider the rapidity of the change in the ionization of the nebula to be expected after the cutoff of excitation. After time t , at constant electron temperature T_e the original electron density $n_e(0)$ will be reduced to (Aller 1956, p. 160)

$$n_e(t) = n_e(0) / [1 + C_H(T_e) t n_e(0)], \quad (6)$$

where $C_H(T_e)$ is the hydrogenic recombination coefficient, which is $4.2 \times 10^{-13} \text{ cm}^3 \text{ sec}^{-1}$ at $T_e = 10000^\circ \text{ K}$ (Seaton 1959). If we assume that $n_e(0) = 10^3 \text{ cm}^{-3}$, then n_e will require about 100 years to fall by a factor 2. The surface brightness of the nebula in the Balmer lines, being proportional to $n_p n_e$ which is essentially n_e^2 , would decrease by a factor 4 in the same interval. Actually, the decay of brightness should proceed somewhat more rapidly than this, because of the decrease in T_e through the cooling effect of forbidden-line radiation. We estimate the magnitude of the effect by considering only the loss of energy by 1 cm^3 of the nebula through H^+ recombination and $[\text{O III}]$ radiation:

$$\frac{d}{dt} \left(\frac{3}{2} n_e k T_e \right) = - n_e n_3 b(T_e) h\nu - n_e^2 C_H(T_e) \frac{3}{2} k T_e, \quad (7)$$

where n_3 is the number density of O^{++} ions, $b(T_e)$ represents the probability of collisional excitation of the $[\text{O III}]$ lines by electrons, and $h\nu$ is the energy of a quantum of forbidden-line radiation. The integration of equation (7) leads to

$$1 - \frac{9}{2} \frac{k C_H(T_e)}{b h\nu} \frac{n_e(0)}{n_3(0)} F(T, T_0) = \frac{1}{[1 + n_e(0) C_H(T_e) t]^y}, \quad (8)$$

where

$$y = \frac{C_3(T_e)}{C_H(T_e)} - 1 \approx 3, \quad (9)$$

$C_3(T_e)$ being the recombination coefficient for O^{++} . The approximation of equation (9) means we have assumed C_3 to be hydrogenic. In equation (8), b is the temperature-independent factor in $b(T_e)$, and

$$F(T, T_0) = \int_{T_0}^T T^{1/2} \exp(h\nu/kT) dT. \quad (10)$$

If $T_0 = 10000^\circ \text{ K}$, $n_e(0)/n_3(0) = 10^3$, and $n_e(0) = 10^3 \text{ cm}^{-3}$, the decay of n_e with time that is predicted by equation (8) is somewhat more rapid than if T_e remained constant at 10000° K : at $t \approx 100$ years, n_e calculated from equation (8) together with the appropriate modification of equation (6) is about 30 per cent lower than from equation (6). These figures must not be taken literally, however, because we have not considered cooling by ions other than $[\text{O III}]$, and have ignored throughout the effects of higher ions of O. It would be difficult to demonstrate that there has been a decrease at this rate in the surface brightness of the nebula at FG Sge on the basis of the old, uncalibrated

photographs. A more favorable test would seem to be the detection of a progressive change in emission-line ratios, but the time interval available may still be too short. The two blue-ultraviolet Crossley spectrograms taken with the same spectrograph, emulsion, and exposure time in 1960 and 1967 (Fig. 3, Pl. 9) show no apparent change in either the absolute intensity of the nebular emission lines or their ratio. Unfortunately, however, the sensitive line [O III] $\lambda 4363$ is masked by terrestrial Hg I $\lambda 4358$ at this dispersion. Under our interpretation, unless there is some other means by which the ionization of the nebulosity can be maintained, such as by high-speed particles or "holes" in the atmosphere of FG Sge, the surface brightness and spectrum of the outer nebulosity can be expected to change detectably over the next few decades.

VI. FINAL REMARKS

We have suggested that the present brightening of FG Sge may be considered as part of the process of ejection of a shell by an invisible, hot, central star, a phenomenon which on an earlier occasion produced an apparently normal planetary nebula around the same object. The present radius of that nebula ($18''$) corresponds to 0.2 pc at our estimated distance of 2.5 kpc, a radius which at the roughly estimated nebular-expansion velocity of 70 km sec^{-1} would have been attained in about 3000 years. This does not seem an unreasonable value for the interval between ejections of two successive nebular shells (especially if one remembers the uncertainties involved) and adds to the plausibility of the interpretation.

If one then takes the position that this interpretation is in essence correct, he can ask: what new things can FG Sge tell us about planetary nebulae? It must be kept in mind that planetaries might be produced in more than one way, by more than one kind of star, and that there is now no basis to decide how typical a case is FG Sge. With this disclaimer of generality, we point out the following.

1. The star was probably fainter than m_{pg} about 15.0 prior to about 1895. This corresponds to an M_{pg} fainter than about $+1.3$ at our adopted distance. Such an upper limit on the luminosity is not far from the values estimated for the nuclei of planetary nebulae by Harman and Seaton (1964).

2. There is no evidence of a violent nova-like explosive event having taken place: the brightening of FG Sge has now been under way for over seventy years at essentially a constant rate, if allowance is made for the contribution of the companion to the early data. Furthermore, the inferred expansion velocity is characteristic of planetary nebulae, not of nova shells, which have expansion velocities one order of magnitude higher.

3. Except for certain minor peculiarities, the abundances in the atmosphere of FG Sge are those of a normal Population I supergiant.

We make no attempt to confront with these results the large number of hypotheses that have been advanced as to the origin and predecessors of the planetary nebulae. Such an attempt seems premature, especially since the development of FG Sge continues, and may itself make such speculations obsolete or unnecessary.

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LEGEND FOR PLATE 9

FIG. 3.—The spectrum of the nebulosity at FG Sge. Spectra *a* and *b* were taken in the photographic region on October 4, 1967, and September 19, 1960, respectively, with the nebular spectrograph, Crossley reflector. The original dispersion was 430 Å mm^{-1} at $H\gamma$, and both exposures were 3 hours in length, on baked Kodak IIaO emulsion. The slit was 6' long and crossed the nebula in p.a. 90° about $10''$ south of FG Sge. The narrow continuous spectrum centered on the nebular emission lines is due to scattered light from the star. The two strong emission lines crossing spectrum *b* are Hg I $\lambda\lambda 4047$ and 4358 , from artificial illumination of the sky over the Santa Clara Valley. Seven years later when spectrum *a* was taken under approximately the same conditions, the sky brightness in the Hg I lines was very much greater, and Hg I $\lambda\lambda 3650$ – 3663 and $\lambda 4077$ appear as well. No obvious change has taken place in the spectrum of the nebula. (Note that *a* is reversed vertically with respect to *b*.)

c is the spectrum of the planetary nebula IC 2149 taken with the same equipment as *a*, *b*. The relative intensities of its emission lines are quite similar to those in the nebula at FG Sge.

d is the spectrum of the nebulosity at FG Sge in the red-yellow region, taken with the prime-focus spectrograph of the 120-inch telescope on October 18, 1966. The original dispersion was 375 Å mm^{-1} . The slit was placed in p.a. 0° , crossing the nebula midway between FG Sge and its $8''$ companion. The strong continuum is due to scattered light from the variable. The region of the [O III] lines and $H\alpha$ is out of focus.