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THERMAL STRUCTURE OF JUPITER*

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ABSTRACT

The observed net flux of energy from the surface of Jupiter is assumed to derive from the thermal energy of the protons in the interior of the planet The thermodynamics and transport properties of metallic hydrogen at finite temperature are discussed, and the temperature distribution in the interior is calculated for various assumed luminosities and atmospheric boundary conditions. The resulting models possess deep convective envelopes for surface fluxes in excess of $\sim 10^2$ ergs cm^{-2} sec^{-1} and are wholly convective for surface fluxes exceeding $\sim 10^3$ ergs cm^{-2} sec^{-1}.

I. INTRODUCTION

Recent observations of Jupiter in the infrared (Low 1966) indicate that the planet radiates somewhat more energy than it receives from the Sun and may possess an internal heat source. If the heat source is real, it is desirable to know whether it is in the deep interior or is an atmospheric phenomenon. The simplest model for an internal heat source would be residual thermal energy in the hypothetical proton lattice of the deep interior, and it is the purpose of this paper to examine whether such a cooling model agrees with all known parameters of Jupiter.

In order to construct thermal models, we assume that the metallic hydrogen of which Jovian matter largely consists can be described by the ordinary Debye theory of metals, with the run of density and pressure fixed once and for all by the zero-temperature model of De Marcus (1958). Since the pressure in the outer 20 per cent of the radius of the planet, which is comprised largely of molecular hydrogen, appears to be temperature-sensitive (Peebles 1964), we define the outer 20 per cent of the radius to be the atmosphere. To a sufficient approximation, the luminosity passing through the base of the atmosphere is equal to the total luminosity, and thus we take the temperature and luminosity at the base of the atmosphere as the boundary conditions for the cooling model.

In § II below we discuss the observational limits which may be placed on the intrinsic luminosity, and some of the properties of the Jovian atmosphere; in § III we discuss the thermodynamics of metallic hydrogen; in § IV we discuss the opacities; and in § V, cooling models of Jupiter are presented.

II. THE INTRINSIC LUMINOSITY OF JUPITER

Öpik (1962) discussed the data then available on the brightness temperature under various assumptions about the spectrum of Jupiter, and concluded that there must be a net flux of energy from the surface in excess of 10^3 ergs cm^{-2} sec^{-1}, and possibly as high as 10^4. Öpik's speculations have possibly been confirmed by Low (1966), who estimates a net flux of energy from the surface of Jupiter of more than twice the solar input, yielding a surface flux of 10^3–10^4 ergs cm^{-2} sec^{-1}. Low's result is based upon measurements of the brightness temperature at 1 mm and 20 μ, both yielding about 150° K (at the equator); however, this is subject to the validity of using the brightness temperature as a measure of the effective temperature.

The brightness temperature increases considerably with wavelength, with $T_b =$ 144° K at 8.35 mm (Thornton and Welch 1963), 224° K at 6 cm (Dickel 1967), and

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260° K at 21.2 cm (Berge 1966). This phenomenon is usually interpreted as being due to a decrease in opacity at longer wavelengths, exposing hotter layers at greater depths. If this interpretation is correct, we may reject the possibility of a cold, isothermal atmosphere.

Trafton (1967) has computed model atmospheres for Jupiter and has shown that the atmosphere is convectively unstable in the deep layers. With observed He/H_2 ratios, Trafton concludes that Jupiter probably radiates more energy than it receives from the Sun, with an effective temperature greater than 130° K.[1]

The figure of 10^3–10^4 ergs cm^{-2} sec^{-1} for the surface flux of Jupiter is therefore indicated by the observations but cannot be regarded as a reliable value at present. The evidence seems to suggest, however, that the flux cannot be very much less than 10^3 ergs cm^{-2} sec^{-1}.

III. THERMODYNAMICS OF METALLIC HYDROGEN AT FINITE TEMPERATURE

In the Debye theory of metals, it is assumed that the Helmholtz free energy of the ions can be written in the form

$$F = N\epsilon_0 + NkT[3 \ln (1 - e^{-\Theta/T}) - D(\Theta/T)] \tag{1}$$

(Landau and Lifshitz 1958), where N is the number of ions, ϵ_0 is the interaction energy per ion, depending only on the density, k is Boltzmann's constant, Θ is the Debye temperature, and D is the usual Debye function; thus,

$$D(x) = \frac{3}{x^3} \int_0^x \frac{z^3 dz}{e^z - 1} . \tag{2}$$

In the present case, Θ must be calculated theoretically and depends upon the dominant vibrational mode of the lattice. For the case of a coulomb lattice, Van Horn (1968) obtains $\Theta = 1.74 \times 10^3 (2Z/A) \rho^{1/2}$ in degrees Kelvin, where ρ is the density in g cm^{-3}, Z is the effective atomic number, and A is the atomic weight. In the case of hydrogen, this result is in approximate agreement with an estimate by Critchfield (1942). For terrestrial alkali metals, $Z \sim 1$ and $A \gg 1$, giving Debye temperatures of the order of a few hundred degrees Kelvin. On the other hand, since the ions in a metallic hydrogen lattice are merely protons, their vibrational frequency is much higher, and at Jovian densities ($\rho \sim 1$) the Debye temperature is several thousand degrees. Such a high Debye temperature gives indirect evidence that the internal temperature of Jupiter is not much less than a thousand degrees, since the heat capacity drops markedly as the temperature falls well below the Debye temperature, and the planet would then be able, at the presently estimated luminosity, to radiate all its available energy in an unacceptably short time.

Adding to the free energy (eq. [1]) the free energy of a degenerate electron gas, we can then calculate the adiabatic relation between temperature and pressure. The calculation is simplified by assuming that the temperature-dependent parts of the internal energy and pressure represent small perturbations. Since the temperature-dependent part of the entropy is a function only of Θ/T, we evidently have

$$\left(\frac{\partial \ln T}{\partial \ln \rho}\right)_s = \tfrac{1}{2}\left(\frac{\partial \ln T}{\partial \ln \Theta}\right)_s = 0.5 . \tag{3}$$

Thus,

$$\nabla_s \equiv \left(\frac{\partial \ln T}{\partial \ln P}\right)_s = \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_s \Big/ \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_s \approx 0.5 \Big/ \frac{\partial \ln P_0}{\partial \ln \rho} . \tag{4}$$

[1] The maximum effective temperature that Jupiter could have without an intrinsic luminosity is about 120° K.

Here P is the pressure, and P_0 is the pressure at zero temperature. In the limit of high density $P \propto \rho^{5/3}$, and thus $\nabla_s \rightarrow 0.3$. For $\rho \sim 1$, ∇_s can be calculated using the equation of state given by De Marcus (1958), which in turn is taken from the work of Wigner and Huntington (1935). We find that ∇_s can be fitted to within a few per cent by the formula

$$\nabla_s = 0.3 - 0.161\rho^{-1} + 0.012\rho^{-2} . \tag{5}$$

Note that pressure ionization tends to depress the adiabatic gradient, in a manner somewhat analogous to the effect of temperature ionization.

The melting temperature of metallic hydrogen can be estimated in a variety of ways. One method is to postulate that the lattice melts when the mean-square vibrational displacement of the protons, $\langle x^2 \rangle$, exceeds a given fraction of the mean-square interionic distance, a^2. The critical fraction, Γ_x^{-1}, can be written for a coulomb lattice as

$$\Gamma_x = a^2/\langle x^2 \rangle = Z^2e^2/akT_M , \tag{6}$$

where T_M is the melting temperature and

$$a = (3m_{\rm H}/4\pi)^{1/3}\rho^{-1/3} , \tag{7}$$

where $m_{\rm H}$ is the mass of a hydrogen atom.

Estimates of the value of Γ_x vary from about 10 to as high as 125 (Van Horn 1968). In the case of Jupiter, if Γ_x were taken to be 125, the melting temperature would fall so far below the Debye temperature that an ionic lattice phase might not exist at all; instead the ions would form a quantum liquid. However, an explicit thermodynamic calculation by Van Horn (1967) of the melting density of hydrogen suggests that Γ_x is of the order of 10, for high densities at least The adopted value of Γ_x is not crucial; we merely wish to be certain that it is possible for the proton lattice phase to exist in Jupiter. For the purposes of this paper we therefore adopt $\Gamma_x = 40$. Since this value is considerably higher than the value of $\Gamma_x \sim 1$ chosen by Critchfield (1942), conditions seem somewhat less favorable for the existence of metallic hydrogen in Jupiter than has been previously thought.

Above the melting temperature, the adiabatic relation (3) changes to some extent. In particular, the latent heat of fusion must be included in the thermodynamic relations. However, in the case of Jupiter, the melting temperature is not much greater than the Debye temperature, and quantization of the protons is important even in the liquid phase. Since no theory is available for the thermodynamics of a strongly coupled quantum liquid, it is difficult to estimate the adiabatic gradient in such a case. As a preliminary estimate, however, we assume that the quantum effects are negligible and that adiabats above the melting temperature can be calculated using the numerical results of Brush, Sahlin, and Teller (1966) for the thermodynamics of a strongly coupled plasma. For T slightly greater than T_M, ∇_s can be approximated by formula (4) to within 10 per cent, assuming that the electron gas can still be considered highly degenerate.

For the Debye model at temperatures above the Debye temperature, the heat capacity is given by the law of Dulong and Petit, $3k$ per proton. For temperatures well below the Debye temperature, the heat capacity falls strongly to zero. However, even at a temperature of $\frac{1}{2}\Theta$, the heat capacity is still about 80 per cent of the limiting value of $3k$ (Landau and Lifshitz 1958), and, since we will be interested in temperatures ranging from T_M to about $\frac{1}{2}\Theta$, we take $3k$ to be an adequate approximation to the heat capacity.

Finally, we estimate the degeneracy temperature of the metallic hydrogen by setting the pressure of an ideal gas of ionized hydrogen at a density of 1 g cm^{-3}, $1.6 \times 10^8\ T$ dynes cm^{-2}, equal to the pressure of metallic hydrogen at zero temperature and the same density, 2.2×10^{12} dynes cm^{-2}. The degeneracy temperature is then about $1.4 \times$

10^4 ° K. The estimated melting temperature at $\rho = 1$ is about 40 per cent of the degeneracy temperature.

IV. OPACITY

The main carriers of energy in Jupiter are probably electrons rather than photons, since the opacity given by Kramer's law for free-free scattering of photons is much higher than the calculated conductive opacity for temperatures below about 10^5 ° K. Two calculations of the conductivity of metallic hydrogen exist in the literature: Critchfield (1942) obtains

$$K \sim 4 \times 10^9 \rho^2 (1 - 0.25\rho^{-1/3})^4 \text{ ergs cm}^{-1} \text{ sec}^{-1} \,°\,\text{K} . \tag{8}$$

Abrikosov (1964) has calculated the electrical conductivity of metallic hydrogen, and upon applying the Wiedemann-Franz law to his result, we obtain

$$K \sim 10^8 \rho^{4/3} \text{ ergs cm}^{-1} \text{ sec}^{-1} \,°\,\text{K} . \tag{9}$$

Both expressions (8) and (9) are calculated in the limit $T > \Theta$. Expression (9) is in agreement with a result obtained independently by Hubbard (1967).

The great discrepancy between results (8) and (9) needs careful consideration. First, it must be noted that expression (9) is an exact result in the limit of high density where the electron wave functions can be taken to be plane waves and the Born approximation is valid. The failure of Critchfield's expression to yield the correct asymptotic result is due to neglect of an important contribution to the electron-scattering cross-section. The scattering cross-section is approximately proportional to the quantity C^2, where

$$C = \frac{\hbar^2}{3m_e} \int |\nabla u_k|^2 d^3r + \int V |u_k|^2 d^3r \tag{10}$$

(Wilson 1953). Here m_e is the mass of an electron, u_k are the usual Bloch functions where the electron wave functions ψ_k are proportional to $u_k \exp(-i\, \boldsymbol{k} \cdot \boldsymbol{r})$, V is the ionic potential, and the integrals are taken over the Wigner-Seitz sphere. The first term in C represents the interaction of an electron with the screening cloud of electrons around each ion, while the second term represents the interaction of an electron with the ionic potential itself. In the limit of high density, u_k becomes constant, and the second term dominates the first term. Since Critchfield's calculation uses only the first term in C, it gives an incorrect asymptotic limit. In order to estimate the relative importance of the two terms for Jovian densities, the quantity

$$\int |u_k|^2 V d^3r$$

has been evaluated using the wave functions given by Wigner and Huntington. The result is that plane waves (u_k = const.) give a satisfactory approximation, and therefore the formula (9) may be extrapolated to $\rho \sim 1$ without serious error. We conclude therefore that the Critchfield formula greatly overestimates the thermal conductivity.

Expression (9) gives the thermal conductivity in the range between the Debye temperature and the melting temperature. It is independent of temperature because the heat capacity of the electrons, which is proportional to T, is exactly compensated by the scattering cross-section which is also proportional to T. Below the Debye temperature, the number of phonons which are able to scatter electrons decreases sharply, and the mean free path becomes large. In the case of hydrogen, umklapp processes are not possible, and following Abrikosov, we assume that the thermal conductivity is given approximately by

$$K = K^0 \frac{T}{\Theta} [\exp(\Theta/T) - 1] , \tag{11}$$

where K^0 is the thermal conductivity given by expression (9). Expression (9) or (11) actually gives the so-called ideal conductivity, which is the value the conductivity would have if electrons were scattered by phonons only. In reality, the conductivity is much lower due to the effects of random impurities such as helium ions, lattice defects, etc. The effect of dilute impurities can be estimated in the following way: the thermal conductivity can be written in the form $K \propto \rho TG$ (Hubbard 1966), where G is a dimensionless scattering integral. In the case of phonon scattering in metallic hydrogen above the Debye temperature, we obtain $G \equiv G_P = 1.5 \times 10^4 \rho^{1/3}/T$, in agreement with expression (9). At the melting temperature, $G_P \sim 3$, so we can write $G_P = 1/(\frac{1}{3} T/T_M)$. In the case of dilute random impurities, G takes the form $G \equiv G_I \simeq 1/\ln(1 + 4 k_F^2/k_s^2)$, where k_F is the Fermi wavenumber and k_s is the inverse of the screening length (Hubbard 1966). In the case of Jupiter, we estimate that the impurities are screened out in roughly the radius of a Wigner-Seitz sphere and thus k_s should be of the order of k_F, and hence $G_I \sim 1$. Since the scattering cross-section is proportional to the number of scatterers, G_I must be multiplied by the reciprocal of the impurity concentration. We may suppose that the impurities in Jupiter are mainly helium ions, roughly 10 per cent by number. Hence the effective G is given by

$$G \simeq 1/(G_P^{-1} + G_I^{-1}) = 1/(\tfrac{1}{3}\frac{T}{T_M} + 0.1) . \tag{12}$$

The impurity scattering dominates after T falls below about $\frac{1}{3}$ the melting temperature, and the very large conductivity predicted by equation (11) will never be realized. Thus expression (9) represents an upper limit for the conductivity for all temperatures below T_M.

All the above expressions for the thermal conductivity presume the validity of the Born approximation, and we must therefore verify that the Born approximation is valid at densities under consideration. In order for the Born approximation to be valid, it is necessary for the perturbing potential to be small compared with the kinetic energy of the electrons. It should be noted that a perturbing potential results when an ion is displaced from its ideal lattice site, and therefore the Born approximation is valid at a sufficiently low temperature. A criterion for the validity of a transport equation based on the Born approximation is

$$\hbar/\tau \ll E_F \tag{13}$$

(Van Hove 1955), where τ is the mean free time of the electrons and E_F is the Fermi energy. For a density of 1 g cm^{-3}, this becomes $T \ll 10^5$ ° K. Hence, at the lattice melting temperature and below, the Born approximation is probably valid. We therefore adopt expression (9) as a satisfactory estimate of the thermal conductivity of metallic hydrogen for the temperatures and densities of interest in the present discussion.

V. THERMAL MODELS OF JUPITER

In the following discussion, spherical symmetry is assumed. It is clear that in the case of convective equilibrium, the rather rapid rotation of the planet will have a profound effect on the distribution of the convective currents. In addition, interaction of the pressure-ionized convective elements with magnetic fields may be important. Nevertheless, the results presented may provide a starting point for more elaborate models.

First, it is necessary to note that, with the observed limits on the luminosity and the adopted thermal conductivity, the central temperature is given by $K(T_c - T_0)/R_J \sim H$, where T_c is the central temperature, T_0 is the temperature at the base of the atmosphere, R_J is the radius of Jupiter, and H is the net surface flux. With $K \sim 10^8$ ergs cm^{-1} sec^{-1} ° K, $T_0 \sim 0$, $R_J = 7 \times 10^9$ cm, $H \sim 3 \times 10^3$ ergs cm^{-2} sec^{-1}, we obtain $T_c \sim 10^5$ ° K, an improbably high temperature. This temperature is so high that the proton lattice would be completely destroyed, and the electrons would only be partially degenerate. It would be difficult to reconcile this result with the success of the zero-temperature models of Jupiter if the possibility of convection were not admitted. With an adiabatic temperature gradient, relatively large fluxes of energy may be transported through the planet with a relatively low central temperature.

For the conductive portion of the planet, the usual equation of heat flow is employed:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K \frac{\partial T}{\partial r}\right) = \rho C \frac{\partial T}{\partial t}, \tag{14}$$

where r is the radius, C is the heat capacity per gram, and t is the time. Since we assume K independent of temperature and take $C = 3k/m_{\mathrm{H}}$, equation (14) possesses a solution of the form $T(r)\exp(-\omega t)$ and becomes a simple eigenvalue problem. In reality we have an initial-value problem, but since information on the earlier evolutionary configurations of Jupiter is lacking, we take the fundamental eigensolution to be the physically interesting one, since it dominates in the limit of large t. In the same spirit, for regions in convective equilibrium, we assume the energy release to be governed by the same time constant ω, so that the total luminosity passing through a shell of radius r is given by

$$L_r = \int_0^r 4\pi r'^2 dr' 3(k/m_{\mathrm{H}})\omega\rho(r')T(r'), \tag{15}$$

whether conductive or convective equilibrium obtains. Thus for a given atmospheric boundary temperature T_0, we obtain a different luminosity for each different value of ω, and the boundary of the convective envelope is fixed by the condition

$$\frac{d\ln T}{d\ln P} \equiv \nabla = \nabla_s \tag{16}$$

in the usual manner.

Since equation (14) is homogeneous in T and ∇_s is independent of T, it is clear that the position of the conductive-convective interface is a function of the eigenvalue ω only and is independent of T_0. Thus a solution for a boundary temperature T_0 can be obtained from another solution for a boundary temperature T_0' by multiplying temperatures and luminosities by T_0/T_0'.

For an adiabatic atmosphere with a temperature of 150° K at a pressure of 1 atm, Peebles (1964) obtains a temperature at the base of the atmosphere of about 2000° K, using a Debye model for the molecular hydrogen equation of state. Since an adiabatic atmosphere seems more consistent with observation than an isothermal one, we adopt 2000° K as the probable boundary temperature T_0. The corresponding adiabat is given by $\Theta/T \sim 2$. As an extreme (and possibly unrealistic) case, we also consider a boundary temperature of 7000° K, which yields an entirely molten interior. The adiabat is calculated from the data of Brush *et al.* (1966) and corresponds to an entropy per proton $S = 6.6\ k$, with the entropy zero point determined by the condition that the proton entropy tends to the perfect-gas value

$$\frac{S_0}{k} = \tfrac{5}{2} + \ln\left[\left(\frac{kT}{2\pi}\right)^{3/2}\frac{2m_{\mathrm{H}}^{5/2}}{\hbar^3\rho}\right] \tag{17}$$

as $T \rightarrow \infty$.

In Figure 1 are plotted three adiabats for the Jovian interior, together with two partially conductive models for $T_0 = 7000°$ K (curves 1 and 2). Curve 1 corresponds to a surface flux of 180 ergs cm^{-2} sec^{-1}, while curve 2 corresponds to a surface flux of 130 ergs cm^{-2} sec^{-1}. The conductive-convective interface is at 0.35 R_J and 0.60 R_J, respectively. For lower values of the boundary temperature, the corresponding surface fluxes are proportionately lower. In Table 1 we present the adiabatic models for $T_0 = 7000°$ and

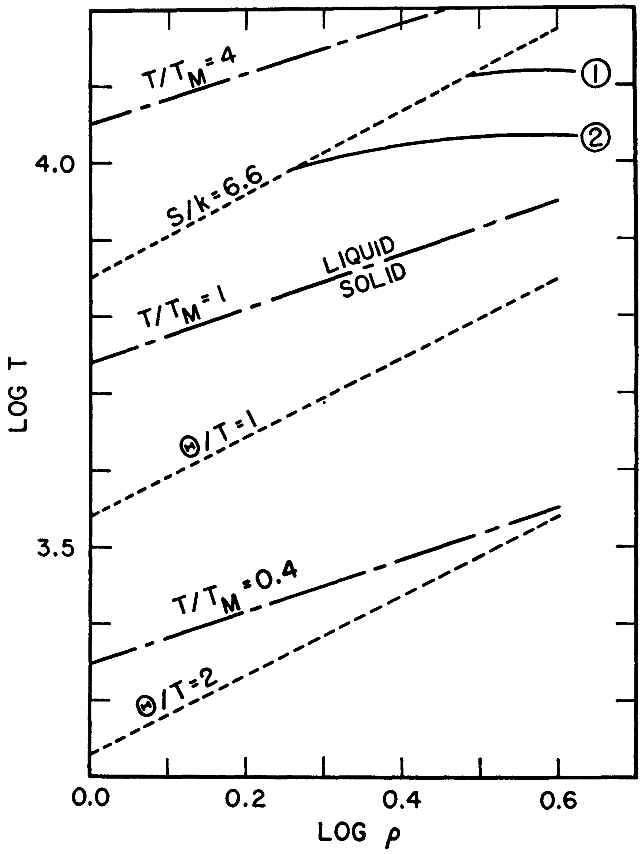

FIG. 1.—Adiabats for T_0 = 7000°, 3500°, and 1700° K. Numbered curves are models with conductive cores and are discussed in the text.

TABLE 1

TEMPERATURES OF ADIABATIC MODELS AS A FUNCTION OF RADIUS FOR T_0 = 7000° AND 2000° K

r/R_J	ρ* (g cm^{-3})	T(° K) (T_0=7000° K)	T(° K) (T_0=2000° K)	Θ(° K)	T_M(° K)
0 0..	4 1	15100	4000	7050	8650
2...	3 7	14400	3800	6700	8350
4..	2 9	12600	3400	5900	7700
5...	2 5	11500	3200	5500	7300
6....	2 1	10500	2900	5050	6900
7..	1 63	9100	2500	4450	6350
0 8....	0 96	7000	2000	3400	5300

* From Peebles (1964).

2000° K, together with assumed melting temperatures and Debye temperatures. The position of the conductive-convective interface is given as a function of H in Table 2, together with cooling times.

To conclude this section, we verify that convective motions in Jovian matter are capable of transporting the observed flux without a significant departure from the adiabatic gradient. According to the mixing-length theory of convection, the convective flux is given by

$$H_c = 3\frac{k}{m_{\mathrm{H}}}\left[\frac{T}{P}(\nabla - \nabla_s)\right]^{3/2}\left|\left(\frac{\partial\rho}{\partial T}\right)_P\right|^{1/2}\left(\frac{dP}{dr}\right)^2\left(\frac{l}{2}\right)^2, \tag{18}$$

TABLE 2

RADIUS AT WHICH CONVECTIVE INSTABILITY STARTS AS A FUNCTION OF SURFACE FLUX

(Includes Cooling Times)

H (ergs cm^{-2} sec^{-1})	T_0=7000° K		T_0=2000° K	
	r/R_J	ω^{-1} (years)	r/R_J	ω^{-1} (years)
50	Conductive	3×10^{12}	0 35	10^{12}
130	0 6	10^{12}	Convective	4×10^{11}
180	0 35	10^{12}	Convective	3×10^{11}
220	Convective	9×10^{11}	Convective	$2\ 5\times10^{11}$
1000	Convective	2×10^{11}	Convective	6×10^{10}
2000	Convective	10^{11}	Convective	3×10^{10}
10^4	Convective	2×10^{10}	Convective	6×10^{9}

where l is the mixing length (Schwarzschild 1958). We assume that the pressure can be calculated from the Debye model according to

$$P = P_0 + \frac{k}{m_{\mathrm{H}}}\rho T[\tfrac{3}{2}qe^{-q}(1 - e^{-q})^{-1} - \tfrac{1}{2}qD'(q)], \tag{19}$$

where P_0 is the pressure at zero temperature and $q = \Theta/T$. To lowest order in T we then obtain

$$\left|\left(\frac{\partial\rho}{\partial T}\right)_P\right| = 3\frac{k}{m_{\mathrm{H}}}\rho^2\frac{\nabla_s}{P_0}\left[4D(q) - \frac{3qe^{-q}}{1 - e^{-q}}\right]. \tag{20}$$

For $q \sim 1$, $\rho \sim 1$, we obtain

$$\left|\left(\frac{\partial\rho}{\partial T}\right)_P\right|^{1/2} \sim 10^{-3}. \tag{21}$$

For a perfect gas, the value would be about 10^{-2}. Solving equation (18) for $\nabla - \nabla_s$ for $H_c = 10^4$, we find

$$\nabla - \nabla_s = \frac{10^{-6}}{Tl^{4/3}}, \tag{22}$$

where l is now expressed in units of R_J. For any plausible value of l, the difference between ∇ and ∇_s is clearly negligible. Although one might expect convection to be suppressed in a degenerate gas due to the lower thermal expansivity, the large heat capacity at higher densities more than compensates.

In the preceding analysis, we have assumed that the metallic hydrogen possesses no strength but can be treated essentially as a fluid under the high pressures which occur in the interior of Jupiter. An indication of strength in Jovian matter might be a lack of coincidence between the equipotential surfaces and surfaces of constant density, which would be indicated by a disagreement between the measured gravitational moments J and K, and the J and K calculated from fluid models.[2] Unfortunately, this test is not a sensitive one because the calculated J and K vary considerably with the choice of atmospheric structure and chemical composition (Peebles 1964). A fit to the observed J and K can be made for a plausible choice of parameters, however, and we conclude that there is no indication of strength in Jupiter from such considerations.

VI. DISCUSSION

One of the more puzzling features of Jupiter is the presence of a rather strong magnetic field. It can easily be verified that the field is not a primordial one, for if we compute the electrical conductivity of the Jovian matter, we find

$$\sigma \sim 3 \times 10^{20} \rho^{4/3}/T \text{ esu} \tag{23}$$

using expression (9) and the Wiedemann-Franz law. The decay time in seconds for a magnetic field is given by the usual expression

$$\tau \sim 4\pi L^2 \sigma / c^2 , \tag{24}$$

where c is the velocity of light in centimeters per second and L is a typical dimension of the planet. Taking $L \sim 10^9$ cm, one finds a decay time of 10^7 years, which implies that the field is continuously generated. The magnetic field could, of course, be generated by some suitable atmospheric mechanism for transforming solar energy into magnetic energy, but deep convection in pressure-ionized matter would seem to offer at least as attractive a mechanism.

If Jupiter is in fact largely convective, its present state may be a remnant of a primordial luminous phase during which the planet evolved down a Hayashi track but failed to start hydrogen burning. The immediate predecessor of the present Jupiter may have been an object similar to a white dwarf but completely convective and with a rather low effective temperature. If Jupiter's predecessor were such an object, the planet must now be completely mixed and cannot possess a core of higher-density elements. According to the models of De Marcus (1958) and Peebles (1964), if such a core exists, it is quite small, and there is no great difficulty in fitting observed parameters without a core. It should be noted, however, that the present thermal model does not exclude a small conductive core.

It is of course possible that the present energy radiated by Jupiter comes from the atmosphere only rather than from the deep interior. This would be possible if the planet were able to store energy from an earlier epoch when it received more energy from the Sun, which might occur in the case of a variable albedo. The model herein presented is therefore only one of several possible interpretations of the observed intrinsic luminosity of Jupiter. Observations of Jupiter at additional wavelengths may help to clarify the situation.

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[2] The author acknowledges a very helpful discussion with Dr. Bruce Murray on this point.

REFERENCES

Abrikosov, A. A. 1964, *Soviet Phys.—J. Exper. and Theoret. Phys.*, **18**, 1399.
Berge, G. L. 1966, *Ap. J.*, **146**, 767.
Brush, S. G., Sahlin, H. L., and Teller, E. 1966, *J. Chem. Phys.*, **45**, 2102.
Critchfield, C. L. 1942, *Ap. J.*, **96**, 1.
De Marcus, W. C. 1958, *A.J.*, **63**, 2.
Dickel, J. R. 1967, *Ap. J.*, **148**, 535.
Hubbard, W. B. 1966, *Ap. J.*, **146**, 858.
———. 1967, unpublished Ph.D. thesis, University of California, Berkeley.
Landau, L. D., and Lifshitz, E. M. 1958, *Statistical Physics* (Reading, Mass.: Addison-Wesley Publishing Co.), p. 188.
Low, F. J. 1966, *A.J.*, **71**, 391.
Öpik, E. J. 1962, *Icarus*, **1**, 200.
Peebles, P. J. E. 1964, *Ap. J.*, **140**, 328.
Schwarzschild, M. 1958, *Structure and Evolution of the Stars* (Princeton, N.J.: Princeton University Press), p. 48.
Thornton, D. D., and Welch, W. J. 1963, *Icarus*, **2**, 228.
Trafton, L. M. 1967, *Ap. J.*, **147**, 765.
Van Horn, H. M. 1967, *Phys. Rev.*, **157**, 342.
———. 1968, *Ap. J.*, **151**, 227.
Van Hove, L. 1955, *Physica*, **21**, 517.
Wigner, E., and Huntington, H. B. 1935, *J. Chem. Phys.*, **3**, 765.
Wilson, A. H. 1953, *The Theory of Metals* (Cambridge: Cambridge University Press), p. 256.