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EVOLUTION OF THE STARS AND GAS IN GALAXIES

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ABSTRACT

A numerical computation of evolution starts from gas with Population I composition; then stars are formed at all times, at rates which are functions of stellar mass and mass of gas in the galaxy. Discrete time steps of 10^9 years are used, and 13 stellar masses. The stars are placed on the H-R diagram according to their masses and ages; each star ends as a white dwarf, while its excess mass enriches the interstellar gas. Different evolutionary sequences are constructed by adjusting four parameters of a stellar birth-rate function. Then "galaxies" resulting from each sequence of 10–12 $\times$ 10^9 years are compared with observed local galaxies with respect to colors, mass-to-light ratio, relative mass of gas, and types of stars contributing to the light.

"Galaxies" closely resembling all normal types, Im to E, can be formed with a stellar birth rate proportional to the inverse square of stellar mass and to the mass of gas in the galaxy; the types differ in initial rate of gas consumption and in the birth rate of very low-mass stars. These types can all have the same age, and do not form an evolutionary sequence.

It is shown that giant elliptical galaxies may have been so much brighter at short wavelengths a few billion years ago that the observed magnitude-redshift relation can be interpreted in terms of cosmological models that do not suffer from the high density and small age of the conventionally preferred model.

I. INTRODUCTION

This paper describes a method for studying the past history of galaxies by numerical computation of the evolution of their stars and gas.

The original aim of this investigation was to construct histories of giant elliptical (gE) galaxies for use in the cosmological magnitude-redshift relation, but in order to develop a theory for the evolution of gE galaxies, it was necessary to study spirals and irregulars as well. It has been found that all normal types of galaxies may have arisen in the same length of time (12 $\times$ 10^9 years) since the start of Population I star formation, with very similar stellar birth-rate functions. No evidence has been found for regarding the Hubble sequence of galactic types as an evolutionary sequence. Dynamical evolution has not been considered, so that, for example, reference to a "spiral galaxy" is to a system with the over-all gas and light characteristics associated with normal spiral galaxies.

The next section describes the basic assumptions used and the method of calculation. The results are given in § III, an application to cosmology is presented in § IV, and the main conclusions are summarized in § V.

II. CONSTRUCTION OF GALACTIC EVOLUTION SEQUENCES

a) Starting Point

The evolution of a galaxy should, ideally, be deduced by following its history either backward in time from a known population of stars and gas, or forward from primeval gas. Neither of these methods is feasible because of great uncertainties as to the present or initial constituents of galaxies. However, although the composition of the primeval gas in our Galaxy is unknown, evidence of early, rapid enrichment with heavy elements is provided by the apparently normal composition of very old galactic clusters (e.g., Johnson and Sandage 1955; Sandage 1962*a*). The composition of most galaxies also appears to be similar to that of local Population I stars (Roberts 1963), and metal-poor stars are not important contributors to the light.

Therefore, the starting point chosen here was gas with a Population I composition. The fractions by weight of hydrogen, helium, and heavier elements were taken as $X =$

0.708, $Y = 0.272$, $Z = 0.020$. The mass of gas used was 5×10^{11} solar masses ($M_\odot$), but the observable integrated properties are independent of this scale factor. A rough estimate was made of the time required for stars of mass 30 $M_\odot$ to increase the value of $Y + Z$ for the gas from zero to 0.292, the result being 10^8 years with 10^{10} stars having evolved. To allow for these, 10^{10} white dwarfs were added to the initial gas, but these white dwarfs were found to be of no consequence in the subsequent history.

b) Nature of a Computed Sequence

A machine-computed evolutionary history of a galaxy from the above starting point to 12×10^9 years later will be referred to as a "sequence." Many sequences were computed, using the same program but changing four parameters of a stellar birth-rate function.

Calculations were made in twelve discrete time steps of 10^9 years. At each time t_j, $j \times 10^9$ years from the start, the number of stars, n_{ij}, of mass m_i born in the preceding time interval were calculated, 13 masses m_i being chosen to represent the whole range. These and the already existing stars were placed at appropriate positions in the H-R diagram, with the number at each point proportional to the time spent there. All stars at the end of their evolutionary tracks were assumed to become white dwarfs of mass 0.5 $M_\odot$, returning the remainder of their mass to the interstellar medium. The mass of gas, $m_{g,j}$, remaining at t_j, the fraction by mass of elements heavier than hydrogen in the gas, $Y + Z$, and the ratio of mass of hydrogen gas to mass of the galaxy, $m_{\rm H}/m_{\rm tot}$, were found at each time. The total mass was assumed constant. The star numbers at 136 positions in the H-R diagram, with known bolometric corrections (B.C.) and colors in the *UBVRIJKL* system for these positions, gave the integrated colors and luminosity of the galaxy at each time. Other quantities computed for comparison with observed galaxies were the mass-to-light ratios in the V and B bands, $f(V)$ and $f(B)$, and the ratio of mass of hydrogen gas to luminosity in the B band, $f_{\rm H}(B)$. The positions in the H-R diagram from which stars contributed more than 1 per cent or 10 per cent of the total light in any filter band, and their relative contributions, were listed, for qualitative estimates of spectral features. Also the numbers of stars at each position were listed, so that further details of spectra, colors in other photometric systems, etc., could be calculated. Finally, for use in cosmology, the absolute spectral energy at the effective wavelength of each filter band was listed at each time.

Details of the stellar birth rate and evolutionary tracks and compilation of B.C. and colors will now be given.

c) Stellar Birth Rate

The stellar birth rate is obviously the most important function determining the history of a galaxy. It is unlikely that the initial luminosity function of the solar neighborhood is widely applicable: Limber (1960) and Salpeter (1965) note that it is not compatible with the observed mass-to-light ratios and colors of most galaxies; also, the present solar neighborhood is not typical of any whole galaxy because of the large numbers of A–F stars which evidently are not important in the integrated light of galaxies (de Vaucouleurs and de Vaucouleurs 1958, 1959; de Vaucouleurs 1961; Roberts 1963). The stellar birth rate must depend on local physical conditions in the interstellar gas, but no definitive theory is available to give it in terms of these conditions (E. M. Burbidge 1962; G. R. Burbidge 1962; Spitzer 1965). Therefore, a simple function was adopted here with four parameters; this function appears to be sufficiently general, since, as will be shown, it can give rise to the main present galactic types.

The birth-rate function adopted here gives n_{ij}, the number of stars of mass m_i formed in the time interval 10^9 years before t_j:

$$n_{ij} = C\xi_i(m_{g,j-1}/m_{g,0})^{1+q_i}m_{g,0} \, ; \tag{1}$$

$m_{g,j-1}$ is the mass of gas in the galaxy at the start of the interval (t_{j-1}), and $m_{g,0}$ is the initial mass of gas ($5 \times 10^{11}\ M_\odot$). C is a constant chosen so that

$$m_{g,1} = \beta m_{g,0}\ ; \tag{2}$$

ξ_i depends on star mass, and is calculated from a power law:

$$\xi_i = x^{-1}\int_{\sqrt{(m_i m_{i-1})}}^{\sqrt{(m_i m_{i+1})}} m^{-(x+1)} dm = (m_i m_{i-1})^{-x/2} - (m_i m_{i+1})^{-x/2}\ . \tag{3}$$

For ξ_i of maximum i, the value of m_{i+1} is unimportant but was taken to be 100 $M_\odot$. The value of ξ_1 was taken as

$$\xi_1 = D\xi_2\ . \tag{4}$$

In equation (1), q_i was either taken to be zero for all masses, or assigned the values

$$q_i = \begin{cases} 0 & \text{if } m_i < M_\odot\ , \\ 2\log\langle m_i\rangle & \text{if } m_i > M_\odot\ , \end{cases} \tag{5}$$

where $\langle m_i\rangle$ is the mass of the star with mean absolute visual magnitude in the range m_{i-1} to m_{i+1}. Equation (5) thus gives a greater proportion of massive stars earlier in the galactic history. The dependence of n_{ij} on gas mass corresponds to the law used by Schmidt (1963), with his parameter $n_1 = 1$ and his parameter $q = 0$ (if all q_i in equation [1] are zero) or $q = 2$ (if eq. [5] is used). These two sets of values of q_i will be referred to by the value of Schmidt's "q," 0 or 2, respectively.

The four parameters of the birth-rate function, with their values used to obtain different sequences given in parentheses, are: β (0.0001, 0.01, 0.1, 0.3, 0.5, 0.7, 0.9, 0.99), x (0.5, 1.0, 1.5, 2.0, 2.5), D (10, 100, 1000, 10000), and q (0,2). Many of the possible combinations of parameters were not tried, because results from others made it clear that they would lead to totally unrealistic "galaxies."

d) Stellar Evolution Tracks

The 13 stellar masses listed in Table 1 were found to represent the range suitably. Solar units will be used for mass throughout. Stars of masses 1.00–1.25 evolve from the main sequence (MS) into red giants in times of 5 to 11×10^9 years, so many masses were needed to give smooth changes in time of the properties of galaxies. All stars were given the composition $X = 0.708$, $Y = 0.272$, $Z = 0.020$ at birth, for simplicity in calculation, even though the interstellar medium becomes enriched in heavy elements. The probable effects of changing the initial composition, or allowing for changes in time, will be discussed in § III*e*.

The evolutionary tracks on the theoretical H-R diagram used are shown in Figure 1, while the times at various stages and the sources of data are summarized in Table 1.[1] For the final stages of stellar evolution, the approximation used was that every star forms a white dwarf of mass 0.5 and the same composition, corresponding to the cooling line shown on Figure 1. The rather arbitrary choice of composition is not important because white dwarfs never contribute significantly to the light of the galaxy. A different choice of mass would affect the quantity and composition of the gas returned to the interstellar medium, but by less than the uncertainty of observations of gas in galaxies, so would not affect any present conclusions.

For stages of evolution beyond the MS for stars of masses 1 to 2, there are no adequate theoretical calculations; so, following a suggestion by Woolf (1966), these were represented by the red-giant branches of old galactic clusters. These stages of evolution are so rapid that the cluster red-giant branches can be regarded as tracks of one

[1] Further details are available in Tinsley (1967).

star. The four clusters and sources of data used were: NGC 188 (Sandage 1962*a*, *b*), M67 (Johnson and Sandage 1955), NGC 7789 (Burbidge and Sandage 1958; Arp 1962), and NGC 2158 (Arp and Cuffey 1962). On comparison of the cluster H-R diagrams with the calculated MS tracks for stars of masses 1 to 2, it was decided to represent the further evolution of those stars by the clusters as listed in Table 1; for mass 1.25 a track between M67 and NGC 7789 was drawn. The cluster red-giant branches and theoretical MS tracks were joined by freehand sketches. For masses 1.0–1.5, the tracks were extrapolated to the luminosity of the helium flash, $\log L/L_\odot = 3.14$ (Hayashi, Hōshi, and Sugimoto 1962); NGC 2158 has stars more luminous than this, consistent with theory which indicates that a star of mass 2 would not undergo a helium flash.

The times on the red-giant branches were estimated theoretically and from counts of stars in the richer clusters (see Tinsley 1967).

In constructing most sequences, it was assumed that when a star reaches the end of its track shown in Figure 1 it becomes a white dwarf in negligible time. However,

TABLE 1

STELLAR EVOLUTION DATA

Mass of Star ($M_\odot$)	Times (10^9 years)			References* and Sources			Notes
	Pre-MS	MS	Later	Pre-MS	MS	Later	
0 05	>12	....		*a*			1, 2
0 26 .	0 11	>12	..	*a*, *b*	*a*	..	2, 3
0 8. . .	0 07	>12	. .	*c*	*d*	....	4, 5
1 00	0 050	10 80	1 54	*c*	*d*	NGC 188	5, 6
1 03..	0 046	9 50	1 54	*c*	*d*	NGC 188	4, 5, 6
1 06 .	0 043	8 50	1 34	*c*	.	M 67	4, 6, 7
1 09	0 040	7 50	1 24	*c*		M 67	4, 6, 7
1 18	0 033	5 50	0 94	*c*		M 67	4, 6, 7
1 25	0 030	4 40	0 54	*c*		Clusters	6, 7
1 5	0 018	1 40	0 41	*c*	*e*	NGC 7789	5, 6
2 0	0 0083	0 78	0 34	*c*	*f*	NGC 2158	4, 5, 6
3 0	0 0025	0 22	0 10	*c*	*g*	*g*	. .
15 0 .	6.2×10^{-5}	0 011	0 001	*c*	*h*	*h*, *b*	8

* References: *a*, Hayashi and Nakano (1963); *b*, Hayashi *et al* (1962); *c*, Iben (1965*a*); *d*, Demarque and Larson (1964); *e*, Henyey, LeLevier, and Levée (1959); *f*, Auman (1965); *g*, Iben (1965*b*); *h*, Iben (1966).

NOTES TO TABLE 1

1. The final point is discussed in § II*e*.
2. An adjustment was made in $\log T_e$ to allow for composition differences: in accord with the authors' discussion of opacity, $\log T_e$ was reduced by 0.01 at each luminosity.
3. The tracks were interpolated in Fig. 2 of Hayashi and Nakano (1963), ending at their zero-age main sequence (ZAMS) point. Above $\log L/L_\odot = 0.5$, the tracks were interpolated in Table 4-4 of Hayashi *et al.* (1962).
4. Pre-MS tracks were interpolated for intermediate masses between those given by Iben (1965*a*).
5. The tracks were shifted, because of composition differences, by the difference between the author's and Iben's (1965*a*) ZAMS values of $\log L$ and $\log T_e$, and the times were multiplied by the change in ratio X/L. The changes were consistent with the effects of composition found by Demarque and Larson (1964) and Kelsall (1965).
6. The use of galactic cluster giant branches is described in the text.
7. For masses 1.06 to 1.25, the ZAMS luminosities were calculated from the relation $L \propto m^5$, indicated by the results of Demarque and Larson (1964), and then values of $\log T_e$ were found from Iben's (1965*a*) ZAMS. The times on the MS were assumed proportional to m/L at the ZAMS. Subsequent parts of the MS were drawn similar to that for mass 1 00, for masses 1.06 and 1.09, but for 1 18 and 1.25 the track was drawn similar to that for 1 5, since these stars all have convective cores Unfortunately, no more accurate tracks were available in this critical range where the core of the star changes from radiative to convective. The uncertainties were probably not important for the integrated light of galaxies, however.
8. The carbon-burning stage was taken from the calculations of Hayashi and Cameron for mass 15 6 (quoted in Hayashi *et al.* 1962), with $\log T_e$ adjusted for the mass difference as in that paper.

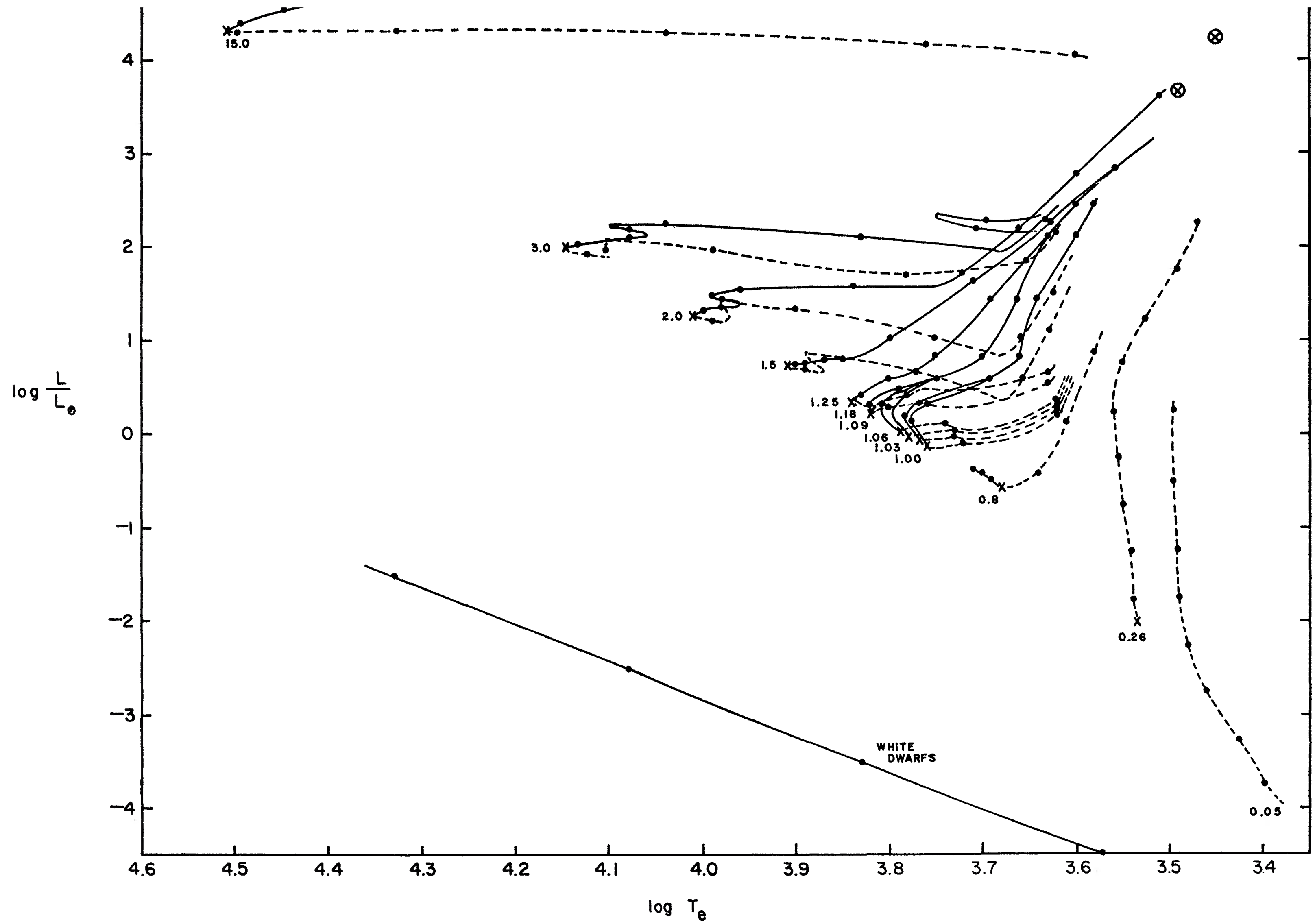

FIG. 1.—Hertzsprung-Russell diagram showing stellar evolution tracks used. Star masses, in solar units, are shown on the main sequence. Dots mark positions at which stars were placed in the computations; crosses mark zero-age main sequence positions; crosses in circles show alternative final positions. Broken lines show pre-main-sequence tracks.

there is evidence (summarized by Deutsch 1961; Weymann 1963) that very red giants are losing mass at a rate which could slow down the later stages of evolution considerably. Stars in the region of the H-R diagram where mass loss is observed have been found in some old galactic clusters (Walker 1958; Walker and Bidelman 1960), while the small numbers of stars in the clusters used here make it statistically possible that such red giants as these are part of the evolutionary history of stars with masses 1 to 2. Therefore, some sequences were computed with alternative final positions for the stars. These positions, shown on Figure 1, had to be chosen rather arbitrarily (one is a typical long-period variable, another the very red star observed by Walker [1958] in NGC 6940), but they should provide a reasonable upper limit to the effect on whole galaxies. This effect was noticeable only in the J, K, and L bands, where there are observations only for E galaxies (Johnson 1966*a*). For these, the differences between sequences with alternative star tracks were 0.01–0.05 mag, which is much less than the scatter in observations. Thus the comparison of computed with observed galaxies is not affected by these uncertainties in the final stages of stellar evolution.

e) Bolometric Corrections and Colors of Stars

The greatest wavelength range for which there are adequate observations of stars and galaxies is given by the *UBVRIJKL* wide-band photometry of Johnson and his collaborators, at the effective wavelengths 0.36, 0.44, 0.55, 0.70, 0.90, 1.25, 2.2, and 3.4 μ. Graphs were drawn of B.C. and the seven colors $U - V$ to $V - L$ versus log T_e, separately for luminosity classes I, III, and V, using data from Johnson (1964, 1965*a*, *b*), Mendoza V. and Johnson (1965), and Harris (1963). Then for each of the 136 points in Figure 1, the B.C. and colors were read for the appropriate log T_e, interpolating between luminosity classes where necessary.[2] For the absolute spectral energy at the effective wavelength of each filter band, the absolute calibration given by Johnson (1965*c*) was used.

The final point for mass 0.05 was taken at log $L/L_\odot = -3.75$, log $T_e = 3.33$, with colors for that T_e (M7 III) given by Mendoza V. and Johnson (1965). The position was somewhat arbitrary as the star continues cooling beyond the last point calculated by Hayashi and Nakano (1963). Since this star dominates the light in the J, K, and L bands, several sequences were computed using alternatively the colors of M9 III. Changes in the integrated colors of galaxies were all less than the errors in observation, so this uncertainty is not important.

After the present computations were completed, a slight revision of the stellar data was published (Johnson 1966*b*), showing that differences of several hundredths of a magnitude should be expected between observed and computed galactic colors, because of uncertainties in stellar data.

III. RESULTS AND COMPARISON WITH OBSERVED GALAXIES

A large number of sequences was computed, using different combinations of stellar birth-rate parameters and alternatives in star tracks, and then the resulting galaxies were compared with galaxies observed nearby in time and space.

a) Properties of Observed Galaxies

The parameter β determines the mass of gas remaining in the galaxy, and this property was chosen to give the approximate Hubble type represented by the sequence. Then the combinations of other parameters were found which gave suitable other properties: computed galaxies were not required to have gas and light characteristics corresponding to the mean observed galaxy of any type, but to have consistent relations between these characteristics. Table 2 gives observed properties which the computed final galaxies were required to match.

[2] The values adopted are tabulated in Tinsley (1967)

b) Properties of Good Final Galaxies

Table 3 shows the *only* sets of parameters, out of all combinations of the values listed in § II*c*, that gave sequences with acceptable final galaxies. Column (1) gives the name by which the sequence will be called subsequently, column (2) its approximate Hubble type, columns (3)–(6) the stellar birth-rate parameters, and column (7) the ages at which the galaxy had properties within the range observed. Columns (8)–(12) give computed values, at age 12×10^9 years, of quantities of which the observed values are given in Table 2. Column (13) gives the relative mass of elements heavier than hydrogen in the interstellar gas, to be compared with the initial value, 0.292.

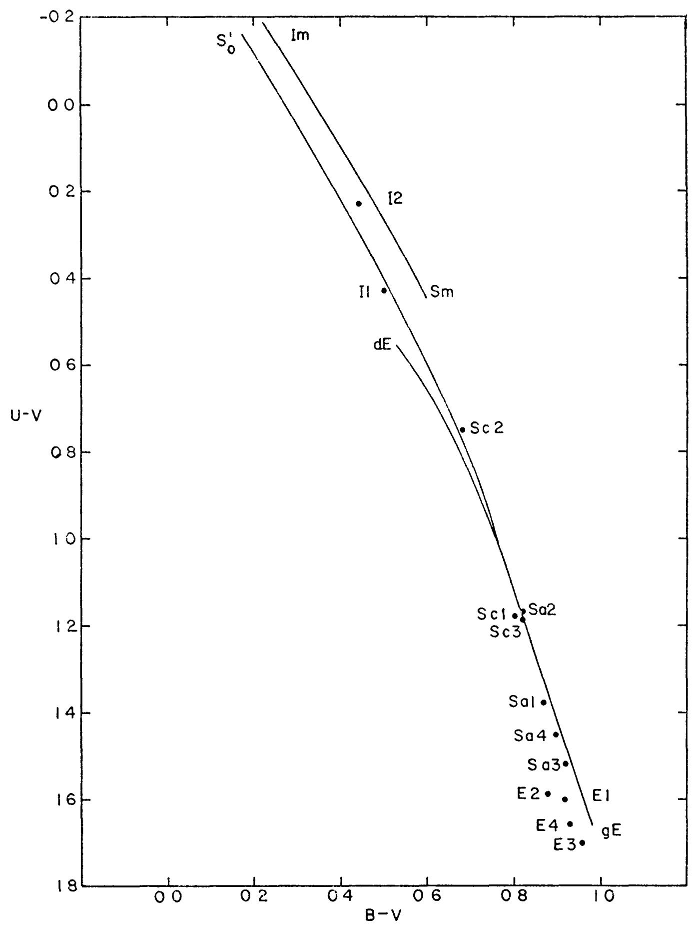

FIG 2 —Color-color relation for mean observed galaxies and computed galaxies at 12×10^9 years. The curves labeled Im–Sm, S_0', dE–gE are observed means for Magellanic types, emission-free spirals, and dwarf-giant ellipticals, respectively, corrected for effects of galactic latitude and redshift (de Vaucouleurs 1961). Points for computed galaxies are numbered as in Table 3.

Figures 2, 3, and 4 show how closely the properties of the computed final galaxies agree with those observed. In Figure 2, all the computed galaxies have colors well within the observed range (de Vaucouleurs 1961), except for sequences Sc1 and Sc3 which are rather too red. The observational data in Figure 3 are from Holmberg (1964), his color index and mass-to-light ratio being converted to $B - V$ and $f(B)$ by the relations given by de Vaucouleurs (1961). The agreement with computed relations is seen to be excellent within the ranges observed (which do not include the E systems), except that sequences Sa1 and Sc1 have too large $B - V$. Figure 4 compares the computed colors of E systems with the mean and range observed by Johnson (1966*a*).

TABLE 2

OBSERVED PROPERTIES OF GALAXIES

Type	gE and S0 (E)		Sa–Sb	Sc–Sm	Im	References and Notes (1)
m_{H}/m_{tot} . . .	$\lesssim$0.001		0.0001–0.01	0.01–0.1	$\geq$0.1	Epstein (1964) Holmberg (1964) Roberts (1963) Robinson and Koehler (1965) de Vaucouleurs (1959, 1966) (2)
$f(B)=m_{tot}/L(B)$	20–200, or greater		10–20	5–10	1–5	
$f_{H}(B)=m_{H}/L(B)$	0 03 (only observation: NGC 4472: 0.007)		0.005–0.05	0.05–0.1	0.23–2	
Range of $B-V$ and corresponding $U-V$	0.75–0.95 0.95–1.6 (means: 0.89, 1.35)		0.70–1.00 0.75–1.55	0.45–0.75 0.25–0.85	0.2–0.6 −0.25–0.4	de Vaucouleurs (1961) (3)
Colors and spectral energy:	Mean E	Revised gE				Johnson (1966*a*) de Vaucouleurs (1966) (4, 5)
$U-V$	1.56	1.30				
$B-V$	1.02	0.91				
$V-R$	0.92	0.85				
$V-I$	1.67	1.58				
$V-J$	2.23	1.92				
$V-K$	3.04	2.80				
$V-L$. .	3.48	3.43				
Contributing types of stars	<50% of V light in red dwarfs (probably 40%); 40–50% of R light in red dwarfs		G8–early M giants, not dwarfs, dominant in *UBV* (<50 % dwarfs in *R*; few A or early F stars	Few A or early F stars. For Sm (part of LMC): spectral type changes B0 to F5 in U to B; order in V: B0–B5, K, M, G	From NGC 4449: types A–F in B wavelengths	Code (1959) Morgan (1959*a*, *b*) Morgan and Mayall (1957) Roberts (1963) Spinrad (1962) Tull (1966) de Vaucouleurs and de Vaucouleurs (1958, 1959)

NOTES TO TABLE 2

1. Dwarf elliptical types were not considered; gE and S0 were grouped together and will be referred to as "E" systems.

2. Mass-to-light ratios for E and S0 galaxies are usually quoted as less than 100, but mass estimates are so uncertain that ratios of several hundred are consistent with the observations (de Vaucouleurs 1966).

3. The colors listed are corrected for interstellar absorption in this Galaxy, and for redshift.

4. The first set of colors are means of Johnson's (1966*a*) observed colors for 5 E galaxies; the range of observations is indicated in Fig. 4. His observations of spirals refer to nuclear regions only, so are not relevant here. After the present calculations were completed, de Vaucouleurs (1966) communicated to the author a revision of the reduction of Johnson's observations of E galaxies: to obtain colors more appropriate for comparison with a synthetic stellar population, de Vaucouleurs applied corrections to the observed colors, individually to each galaxy, for the effects of aperture, internal color distribution, and redshift (using the results of de Vaucouleurs 1961), and for interstellar absorption in this Galaxy; NGC 4168 was not included in the mean since it is a dwarf E galaxy. The revised means for the 4 gE galaxies are the second set of colors listed. Computed galaxies were required to match the original observations, but the differences are less than the error and scatter in observation, except for $B - V$ and $U - V$. The revised $B - V$ agrees better than the original with the computed galaxies, and the possibility for reducing the computed $U - V$ is discussed in § III*e*.

5. The photometry of Wood (1966) was published after the present calculations were completed, but comparison of the final E galaxies with this photometry is given in the text and Table 4.

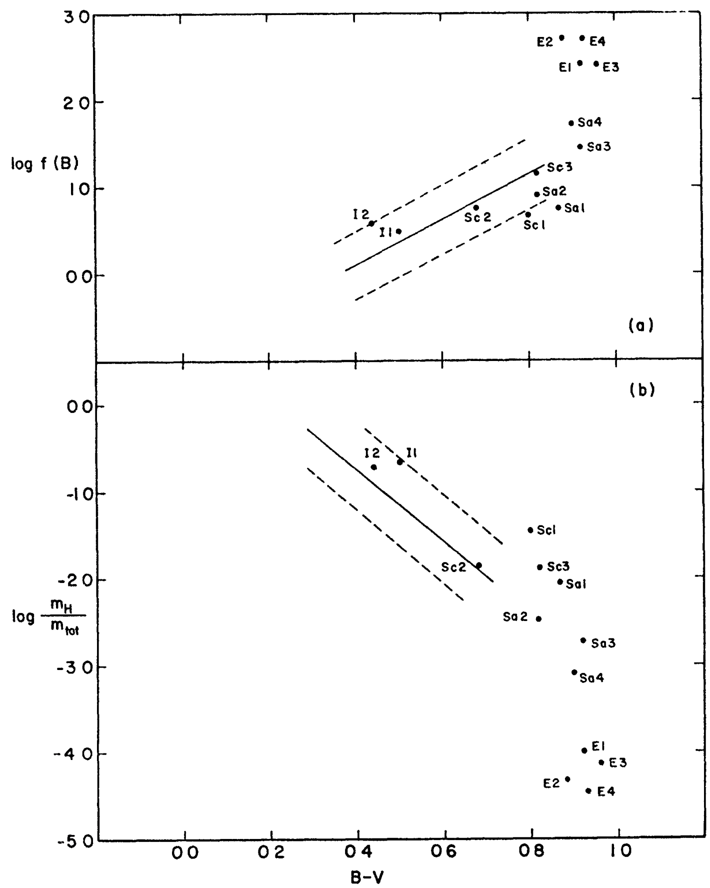

FIG. 3.—Relations between color and (*a*) mass-to-light ratio in *B* light (solar units), and (*b*) fraction of total mass in hydrogen gas, for observed galaxies and computed galaxies at 12 × 10^9 years. Solid lines are the mean linear relations of Holmberg (1964), and broken lines indicate the range of values he observed. Points for computed galaxies are numbered as in Table 3.

TABLE 3

PROPERTIES OF COMPUTED GALAXIES

Seq. (1)	Type (2)	β (3)	x (4)	D (5)	q (6)	Final Age (7)	m_H/m_{tot} (8)	$B-V$ (9)	$U-V$ (10)	$f(B)$ (11)	$f_H(B)$ (12)	$Y+Z$ (13)
I1 . .	Im	0 9	0 5	10	2	10, 11, 12	0 21	0 50	0 43	3 1	0 65	0 48
I2. ..	Im	9	1 0	10	0	10, 11, 12	0 19	44	0 23	3 7	.70	.41
Sc1	Sc–Sm	7	0 5	10	2	10, 11, 12	0 035	80	1 18	4 8	17	56
Sc2..	Sc–Sm	7	1 0	10	0	10, 11, 12	0 014	68	0 75	5.7	08	57
Sc3	Sc–Sm	.7	1 5	10	0	10, 11	0 013	82	1 19	14	18	38
Sa1 ..	Sa–Sb	5	0 5	10	2	12	0 0089	87	1 38	5 6	050	59
Sa2....	Sa–Sb	5	1 0	10	0	12	0.0032	.82	1 17	8.0	026	.53
Sa3 .	Sa–Sb	5	0 5	100	2	12	0 0018	92	1 52	28	050	39
Sa4. .	Sa–Sb	5	1 0	100	0	12	0 0008	.90	1 45	52	042	38
E1 .	E	.3	0 5	1000	0,2	12	$1\ 0\times10^{-4}$	92	1 60	262	026	31
E2 ...	E	3	1 0	1000	0,2	12	$4\ 5\times10^{-5}$	88	1 59	516	023	30
E3	E	1	0 5	1000	0,2	12	$7\ 4\times10^{-5}$	96	1 70	266	020	30
E4	E	0 1	1 0	1000	0,2	12	$3\ 4\times10^{-5}$	0 93	1 66	520	0 018	0 30

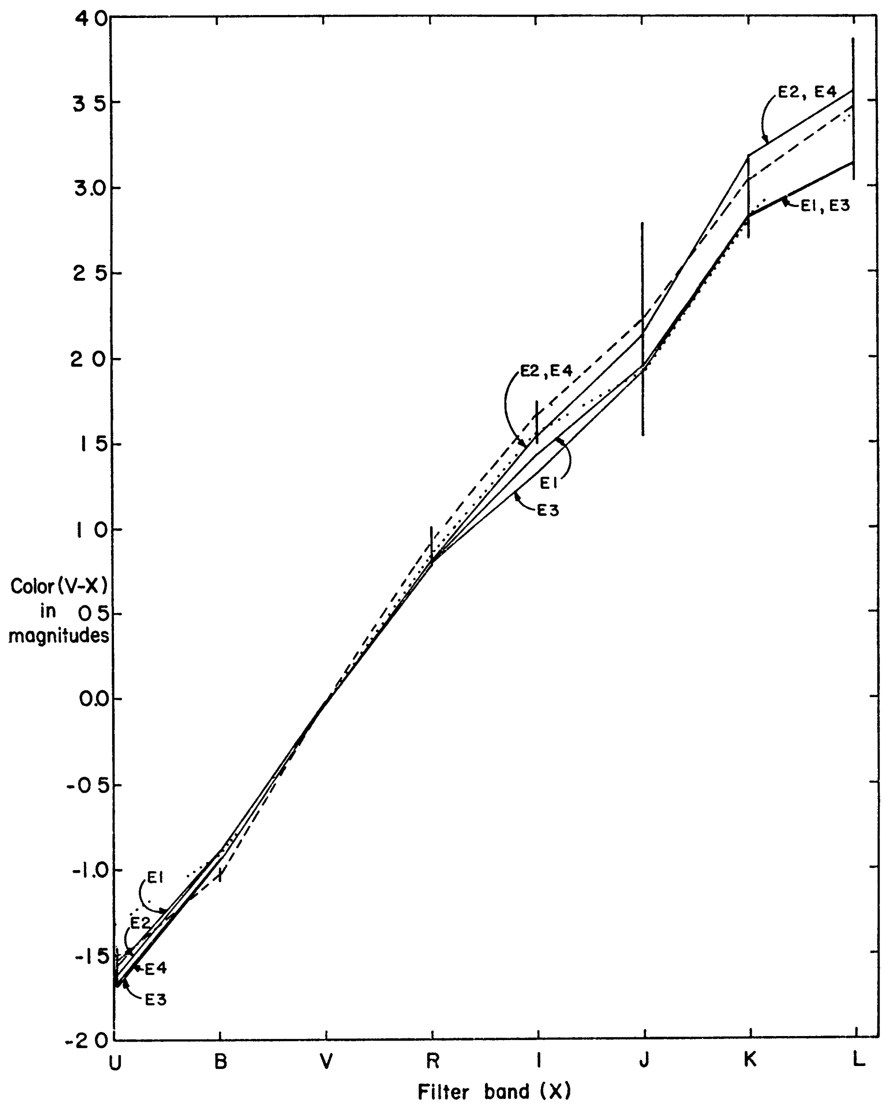

FIG. 4.—Colors of observed E and S0 galaxies and computed galaxies at 12×10^9 years. The broken line is the mean of 5 E galaxies observed by Johnson (1966*a*), and vertical lines show the ranges observed. Solid lines show computed galaxy colors, the sequences being numbered as in Table 3. The dotted line shows de Vaucouleurs' (1966) reduction of Johnson's data.

After the present calculations were completed, narrow-band photometry between 3459 and 7331 Å, for twenty-two galaxies and twenty-five star types, was published by Wood (1966). Because sequence E2 is to be used (in § IV below) in cosmological calculations, integrated colors and line indices on Wood's system were found for its stellar population at 12×10^9 years. The data enable six integrated color differences and five line indices to be calculated. These are given in Table 4, and compared with the mean values for the seven E and S0 galaxies observed by Wood. The result shows agreement: the mean observed-minus-computed index is 0.00 mag, and its rms is 0.04, compared to the rms probable error of observations, 0.06. The population of sequence E2 at 12×10^9 years is rather different from any synthesized by Wood to match E or S0 galaxies, which is possible because he had twenty-five star types to match to twelve colors. The main differences are the absence of horizontal-branch stars from the computed population, and the presence here of many very red dwarfs, which give a much larger mass-to-light ratio than Wood could obtain but are too red to contribute appreciably at 7000 Å.

TABLE 4

COMPARISON WITH WOOD'S PHOTOMETRY

Index	Computed Seq E2	Wood's Mean Observed E and S0	O−C
C_{35}–C_{55}	+2 45	+2 45	0 00
C_{41}–C_{55}	+1 16	+1 16	00
C_{47}–C_{55}	+0 44	+0 48	+ 04
C_{60}–C_{55}	−0 20	−0 21	− 01
C_{67}–C_{55}	−0 38	−0 39	− 01
C_{73}–C_{55}	−0 57	−0 66	− 09
L_{52}	+0 10	+0 08	− 02
L_{59}	−0 09	−0 05	+ 04
L_{66}	−0 04	−0 08	− 04
L_{62}	+0 05	+0 07	+ 02
L_{71}	−0 07	0 00	+0 07

c) *Age and Stellar Birth Rate*

The final age cannot be less than 10×10^9 years after the start of Population I star formation, or no stars have evolved up the NGC 188 track, which would be inconsistent with our own Galaxy. For sequences with $\beta = 0.7$ or 0.9, the changes in time are slow enough that agreement with observed properties is hardly altered from 10 to 12×10^9 years, except that Sc3 (the only good sequence with $x = 1.5$) is definitely too red after the twelfth time interval. In Sa–Sb and E sequences, the evolution of stars near 1 solar mass from the MS into the red-giant region controls the light of the galaxy in bands U–R, and it is only after 12×10^9 years that there are enough red giants contributing in U, B, and V. However, at greater ages there would be too many giants contributing in the R band. Thus only at 12×10^9 years after the start of Population I star formation are there final galaxies representing all the types considered.

None of the sequences studied passes through stages representing other galactic types in all properties. For example, the sequences finally giving E systems pass through $B - V$ values of other types but have always too large a mass-to-light ratio for them. While these calculations cannot prove that the sequence irregular-spiral-elliptical is not an evolutionary order, they show that all the principal galactic types *may* have originated at the same time, but with some differences in physical conditions that led to different stellar birth rates.

The stellar birth-rate laws also may be surprisingly uniform. Table 3 shows that it is possible to have all types with the same age, x, and q, in two ways: age 12×10^9 years, and either $x = 0.5$, $q = 2$, or $x = 1.0$, $q = 0$. For E systems q makes little difference because the massive stars are unimportant. It does not seem possible to make a good Sc–Sm galaxy with $q = 2$: sequence Sc1 is the worst of any listed in Table 3 for agreement with observed galaxies. Thus it seems necessary to reject Schmidt's conclusion that $q = 2$, which was derived for the solar neighborhood only (Schmidt 1963), and the most reasonable conclusion is that q should be taken as zero for all galactic types. Therefore, while other laws of star formation are not excluded, all the types of galaxies considered *can* be formed with a stellar birth rate proportional to the mass of gas present and to the inverse square of star mass per unit mass. The types of galaxies differ in the initial rate of gas consumption to form stars and the relative number of stars formed less massive than 0.1 $M_\odot$, the latter determining mainly the mass-to-light ratio and the infrared luminosity of the final galaxy.

The elliptical galaxies formed here have most of their strong infrared radiation from late red *dwarfs*. A similar result was recently found by Spinrad (1966) for the nucleus of M31, from an analysis of narrow-band photometry including luminosity-sensitive regions in the infrared. If the spiral nucleus is similar to an elliptical galaxy in population as well as in color (Johnson 1966*a*), those observations provide good support for the present calculations.[3]

d) Time Changes in Galaxies

Some representative features of the history of each type of galaxy are shown in Figure 5, *a–d*, for the sequences with $x = 1.0$, $q = 0$. The data for E types had to be smoothed in time to some extent because the use of discrete star masses instead of a continuum resulted in some irregularities. In Figure 5, *b*, the luminosities are all for the galactic mass $5 \times 10^{11} M_\odot$, but would vary in proportion to mass if this were different.

It can be seen from Figure 5, *d*, that changes in the composition of the interstellar gas should not be neglected for irregular and spiral systems if an accurate account of their history is required. But for comparison of observed and final computed galaxies, the changes are not very important: the rate of evolution would be appreciably altered, as discussed in § III*e*, but for the later types only the stars formed early are important contributors at the end, and for the earlier types the evolution of the galaxy is slow enough that such changes would not critically affect the comparison with observed properties. For E systems, composition changes are probably quite negligible. The rise and fall of $Y + Z$ for sequence E4 is probably greatly exaggerated, being due to the gap between the lifetimes of stars of masses 1.5 and 1.25 (1.83 and 4.94×10^9 years).

The absolute spectral energy distribution at different times, for sequence E2, is shown in Figure 6. A shift of the energy maximum toward longer wavelengths occurs, and a rapid early drop in luminosity; these are due mainly to the increasing number of red giants and the decreasing number of massive blue stars as the rate of star formation decreases. For comparison with the final computed galaxy, the energy distributions corresponding to Johnson's (1966*a*) mean color for E galaxies and de Vaucouleurs's (1966) revision of these are also shown.

e) Effects of Initial Composition

The initial composition used here was chosen somewhat arbitrarily, partly because of the availability of many theoretical stellar evolution tracks. While this was found by Sears (1964) to be the initial composition of the Sun, there is no reason to believe that all Population I stars in the Galaxy had this composition at birth. Approximate homol-

[3] A referee has made the interesting suggestion that some of the far-infrared radiation from galaxies could be from a source similar to that in quasi-stellar sources Clearly, if any of the galactic infrared radiation is of non-stellar origin, the number of late red dwarfs inferred here must be too large.

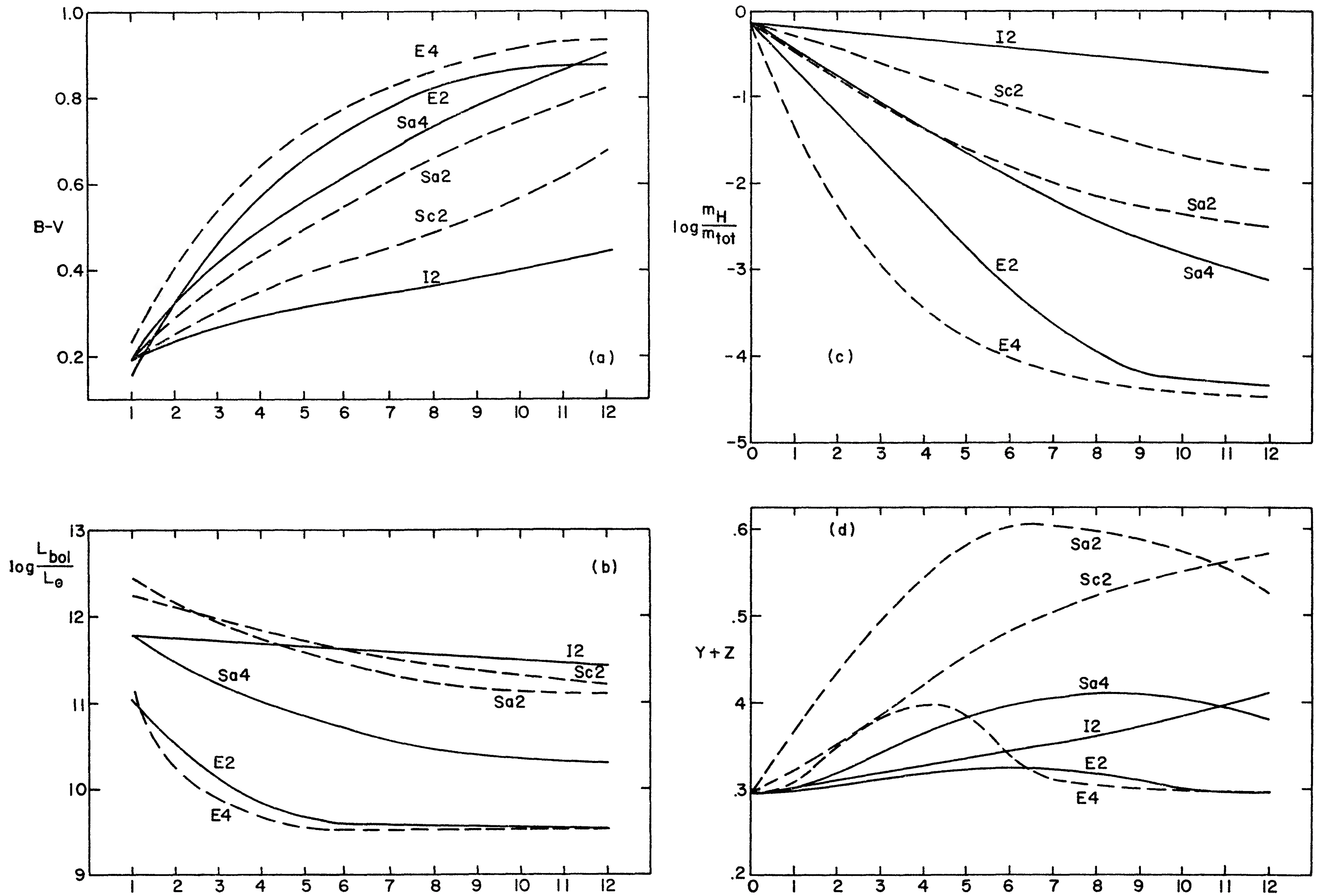

FIG. 5.—Changes in time of computed galaxy properties: (*a*) $B - V$; (*b*) bolometric luminosity; (*c*) fraction of mass as hydrogen gas; (*d*) proportion by weight of heavy elements in the gas. The abscissa in each case is time in units of 10^9 years. Sequences are numbered as in Table 3.

ogy relations indicate that, if a different Population I composition were chosen, the luminosities of all MS stars would be changed and their times spent at equivalent stages of evolution, in such a way that $\int L dt \propto X$ at any stage, since this would give the same increase in core mass. The contribution to integrated luminosity from stars at a given stage is proportional to the luminosity $\times$ number of stars, or to luminosity $\times$ time at the stage, which is therefore proportional to X for hydrogen-burning stars.

Log T_e would also be affected, and hence the B.C. and colors of the stars. For the observed cluster giants, there would be no color changes, B.C. changes would be small, and times would be adjusted so that the contributions would be proportional to X. In computed final E galaxies, the B.C. and colors of the important MS stars near 1 $M_\odot$ would change, as well as their contributions changing in proportion to X. For example, a reduction in X or Z would make the galaxy bluer: a rough calculation indicated that an increase in log T_e of these stars by 0.06, corresponding to a reduction of 0.1 in X or 0.015 in Z, would reduce $U - V$ considerably more than is required to give agreement with the corrected mean value for gE galaxies (cf. note 4 to Table 2).

These effects would probably not be great enough to alter the stellar birth-rate parameters found to give good galaxies. It seems that the most important effect of initial com-

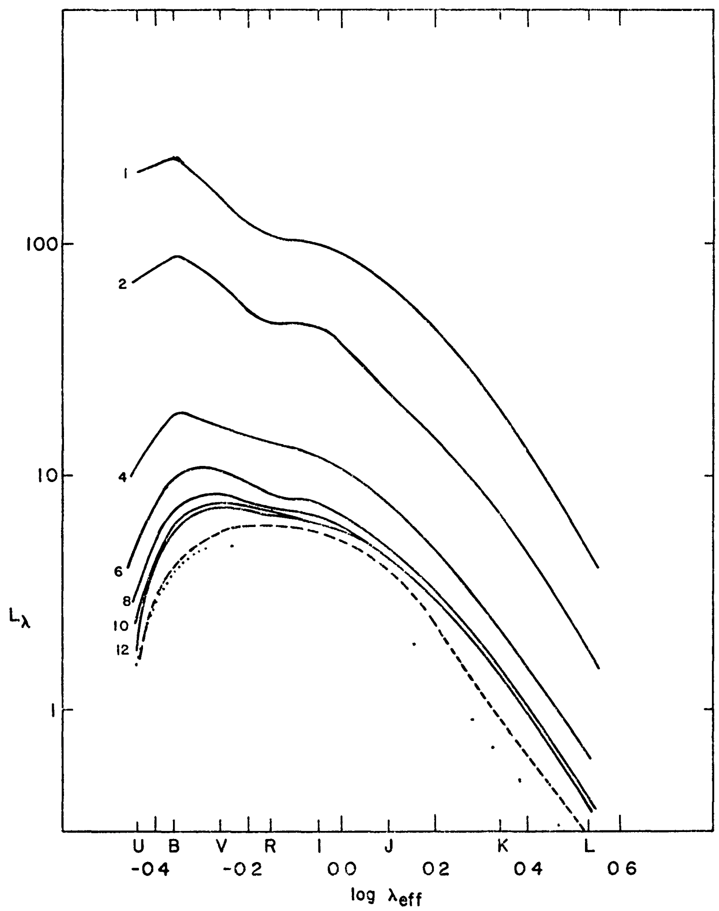

FIG. 6.—Absolute spectral energy distribution in sequence E2 at different times, compared with observed E galaxies. Times are shown on the curves in units of 10^9 years. L_λ is plotted on a logarithmic scale, in units of 10^{42} ergs sec^{-1} μ^{-1}; effective wavelength is in microns, and positions of the filter bands are shown. The broken curve is the energy distribution for the mean of 5 E galaxies observed by Johnson (1966*a*) on an arbitrary vertical scale. Similarly, the dotted curve is for de Vaucouleurs' (1966) reduction of Johnson's data.

position is on the time scale of galactic evolution. Changes in MS luminosity are rather small, so the time for a star of given mass to become a red giant is approximately proportional to X. Therefore, the integrated properties found at 12×10^9 years with $X = 0.708$ would be approximately reproduced for a different X (but same stellar birth-rate parameters) at time $12\ (X/0.708) \times 10^9$ years. It would not be possible to reproduce all the observable properties of galaxies at the same time with a different X by changing the stellar birth-rate parameters.

IV. APPLICATION TO COSMOLOGY

The possible importance of galactic evolution for cosmology will be illustrated in this section, by using one of the computed gE sequences, E2, in relations between V magnitude and redshift (m_V-z relations).[4] The observations used to choose between cosmological models extend now to such great distances that light travel times of several billion years are involved. Thus if gE galaxies have histories resembling E2 (cf. Fig. 6) they must be seen as they were when much more luminous than at present, especially at short wavelengths such as that of the U band, which will be observed at the V band at a redshift of 0.53.

a) Choice of Cosmological Models To Study

Homogeneous, isotropic, pressure-free models of general relativity can be specified by three parameters (Robertson 1955; Stabell and Refsdal 1966), which will here be taken as the Hubble constant, H_0, the age of the model, t_0, and the density parameter $\sigma_0 = 4\pi G\rho_0/(3H_0^2)$ (where G is the Newtonian gravitational constant and ρ_0 the local mean density), since some limits can be placed on these. For consistency in the present discussions, it is necessary that t_0 is greater than 12×10^9 years. A recent review by Abell (1965) gives limits for ρ_0 if $H_0 = 75$ km sec^{-1} Mpc^{-1}, but these are derived in such a way that for general H_0 they show that probably $0.015 \leq \sigma_0 \leq 0.5$. Estimates of H_0 vary widely (e.g., Sandage 1962*c*), so three values have been considered: 75, 98, and 120 km sec^{-1} Mpc^{-1}.

Figure 7 shows the past histories of the models that will be used for illustration. The ordinate is the scale factor R in terms of its present value, or the equivalent redshift z, and the abscissa is the light travel time τ in units of H_0^{-1}. The time 12×10^9 years ago is marked for the three values of H_0 considered. The broken line shows the model with cosmological constant $\Lambda = 0$ and deceleration parameter $q_0 = \sigma_0 = +1$, which is seen to be too young, and too dense by the rather conservative estimate above. Model I is typical of the possible models with zero Λ and within the above range of parameters; these are all open, and I has $q_0 = \sigma_0 = 0.03$. Model II, with $q_0 = 0.14$, $\sigma_0 = 0.015$, is one of the very small possible range of open oscillating models with Λ negative; they are not distinguishable from model I. Model III is an Eddington-Lemaître model, with $q_0 = -1.3$, $\sigma_0 = 0.05$, and a minimum radius corresponding to maximum redshift 2.0 (to which attention was drawn by Petrosian, Salpeter, and Szekeres 1967). Model IV is a Lemaître model, with $q_0 = -2.0$, $\sigma_0 = 0.50$. All of models I through IV are old enough if $H_0 = 75$, but only III and IV are old enough if $H_0 = 98$ or 120 km sec^{-1} Mpc^{-1}.

b) The m_V-z Relations

Figure 8 shows the calculated m_V-z relations, obtained by the method of Davidson (1959) with the transmission functions for the U, B, and V filters approximated by transmission only at their effective wavelengths (λ_{eff}) and with the usual equations for cosmological models (e.g., Refsdal, Stabell, and de Lange 1967). The use of λ_{eff} is not expected to affect the qualitative features found on comparison of m_V-z relations of different models; indeed, since the gradients of the galactic energy distributions do

[4] A fuller discussion of the cosmological implications of the present study of galactic evolution is given in Tinsley (1967).

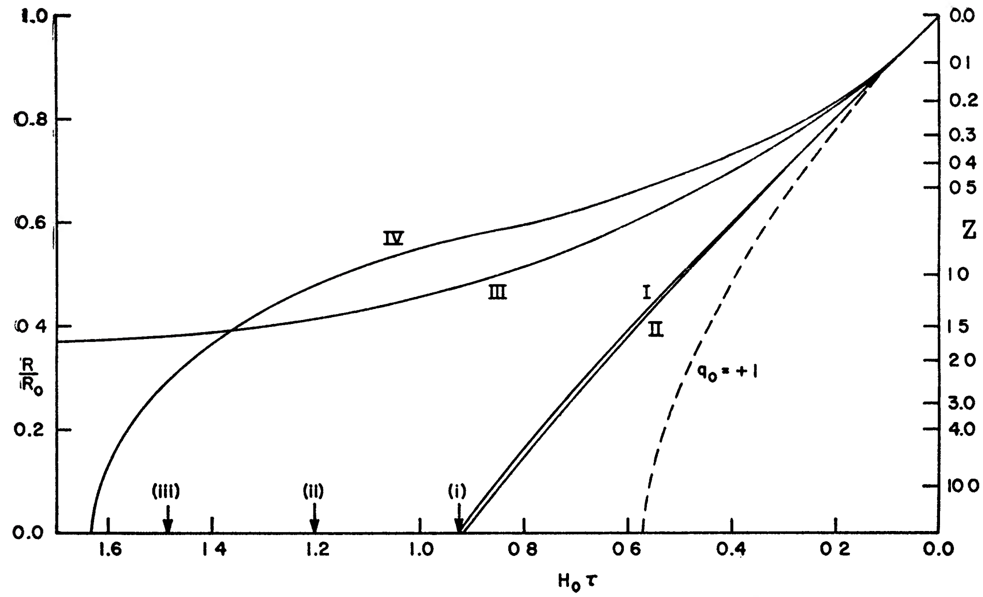

FIG. 7.—Past behavior of cosmological models. R is the scale factor and R_0 its present value; z is the redshift; τ is the light travel time; H_0 is Hubble's constant, in reciprocal time units on the abscissa ($H_0\tau$). The arrows mark the time 12×10^9 years, for 3 values of Hubble's constant, H_0 = (i) 75; (ii) 98; (iii) 120 km sec^{-1} Mpc^{-1}. Models are described in text.

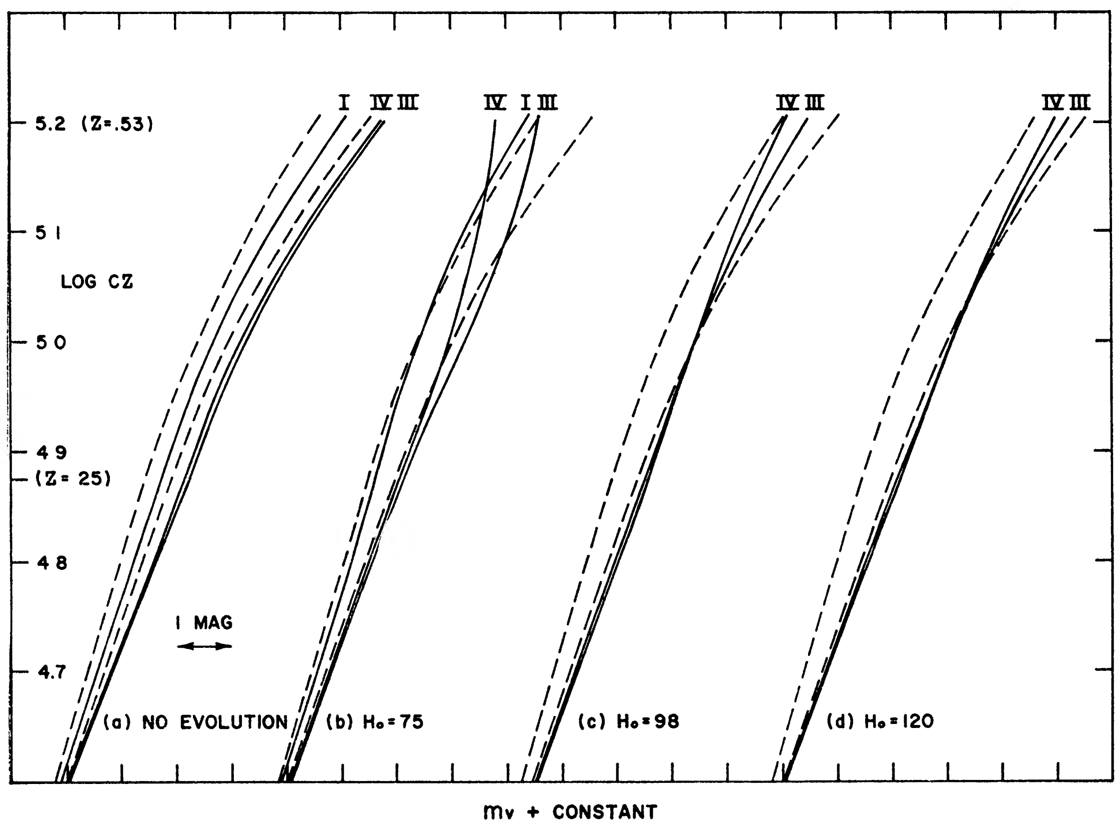

FIG. 8.—Magnitude-redshift relations for V magnitudes. The abscissa is apparent visual magnitude, plus a different arbitrary constant for each of the 4 graphs; the ordinate is $\log_{10} cz$, where c is the speed of light and z the redshift. In Fig. 8, a, there is no evolution of galaxies. In Fig. 8, b, c, and d, the effects of evolution are included for the numbered cosmological models, with time scales given by H_0 = 75, 98, and 120 km sec^{-1} Mpc^{-1}, respectively. Models are described in text.

not increase so sharply toward shorter wavelengths at early times (cf. Fig. 6), the evolutionary corrections are underestimated by this approximation. The ordinate in Figure 8 is $\log_{10} cz$, where c is the speed of light; the redshifts 0.25 and 0.53 are those at which emission is at the wavelengths of the B and U bands, respectively. One could not interpret any data further out until emission by local stars and galaxies shortward of U is better known. The abscissae for the four parts are displaced horizontally, and give the apparent magnitude V, plus an arbitrary constant. This can be compared directly with observation without the usual addition of the "K term," since that term is incorporated into the theoretical calculations. In all four parts of Figure 8, the broken lines are for the models $\Lambda = 0$; $q_0 = +1$ (*upper curve*) and the steady-state model (*lower curve*), with no galactic evolution. Figure 8, *a*, shows models I, III, and IV, also with no evolution. Figure 8, *b*, shows that if $H_0 = 75$ km sec^{-1} Mpc^{-1}, and evolution is considered, the greater luminosity in the past brings the curves up to brighter magnitudes. In Figure 8, *c* and *d*, H_0 is larger so the light travel time is smaller at a given redshift, and evolution is not so important. However, the curves are still at rather brighter magnitudes than without evolution.

c) Discussion

These figures show that with galactic evolution given by the present sequence E2, models I, II, III, and IV have m_V-z relations which differ from that for the model with $\Lambda = 0$, $q_0 = +1$ by less than about half a magnitude out to redshift 0.53. Consequently, consideration of galactic evolution may enable the observed magnitude-redshift relation (Sandage 1967) to be interpreted in terms of models that do not suffer from the small age and high density of those with $\Lambda = 0$ and high q_0.

However, the possibility of choice between various models is made more remote because evolution reduces the differences between their m-z relations, for the following reason: The models with fainter m-z relations without evolution have the greater light travel time at a given redshift. The galaxies are therefore seen as they were longer ago, and the evolutionary effect means they had greater absolute luminosities, so their m-z relations are brought up to brighter magnitudes.

The evolutionary corrections to m-z relations found here are much greater than those obtained in the well-known paper by Sandage (1961), which can be explained as follows. The model for galactic evolution used by Sandage does not differ importantly from that found here for gE galaxies, but the difference lies in the fact that Sandage found the evolutionary correction to the bolometric magnitude, so in effect assumed that the evolution of the K term is negligible. The K term was not explicitly calculated above, but the difference between evolutionary corrections to the bolometric and to the V magnitudes is equivalent to evolution of the K term. For nearby galaxies, the observed V magnitude depends (in the $\lambda_{\rm eff}$ approximation) on the absolute monochromatic luminosity at the effective wavelength of the V band (0.55 μ), while at $z = 0.53$ it depends on that at the effective wavelength of the U band (0.36 μ), as would be evaluated at the galaxy. The proper evolutionary correction to V, at $z = 0.53$, is therefore the change in absolute U magnitude of the galaxy. This is much greater than the bolometric change because the U light comes mainly from the brightest MS stars, which are not only bolometrically fainter later in time but are also emitting a smaller fraction of their light at 0.36 μ. The evolutionary correction to V at $z = 0.53$ in model I, for example, is 0.68 mag, of which 12 per cent is due to the bolometric change and 88 per cent to the change in the K term.

V. SUMMARY AND CONCLUSIONS

Evolutionary histories of galaxies have been constructed, from the time at which stars with Population I composition started to form, up to the present. It has been shown that systems with characteristics of irregular, normal spiral, and E–S0 galaxies all may have arisen in the same length of time (about 12×10^9 years) since that starting point. The different types have not been found to form an evolutionary sequence, in the sense that irregulars evolve through spirals into ellipticals, or vice versa. A four-parameter stellar birth-rate function has been used, but all the types could be formed with the same power law for dependence on stellar mass, inverse square, and with the same dependence on mass of gas in the galaxy, linear. The two parameters determining the type of galaxy eventually formed were the initial rate of gas consumption to form stars, and the relative birth rate of stars less massive than 0.1 $M_\odot$.

The significance of the stellar birth rate has not been discussed, but it would clearly be of interest to obtain some physical understanding of these results. Holmberg (1964) has suggested that one parameter, density, possibly determines the position of a galaxy in the Hubble sequence, the density increasing from irregular to S0 types. With the present results, this implies that an initially denser system would have a greater fractional rate of gas consumption to form stars, which is certainly to be expected, and a greater fraction of extremely low-mass stars, which is very plausible since the critical mass for Jeans's instability decreases as density increases.

It should be possible to make greatly refined calculations along the present lines, most importantly by increasing the number of criteria used for comparison of final galaxies with observation, and by trying a variety of initial compositions and allowing for changes in stellar evolution as the composition of the gas is enriched. The most sensitive criteria for comparison with observation will be the equivalent widths of spectral lines, which give much more information on the spectral and luminosity classes of contributing stars than is contained in broad-band colors. Narrow-band photometric criteria will also be of importance. At present, there are not enough observational data on galaxies to justify extensive calculations of these quantities for the computed galaxies. Different choices of initial composition should make possible better agreement between observed and computed colors, but construction of sequences must await further theoretical stellar evolution tracks for other compositions, while the observed colors of galaxies are still too uncertain for attempts to improve the present agreement to be justified yet.

The application of the computed past history of a gE system to cosmology has shown that evolution may be of great importance to the interpretation of m-z relations. Evolutionary effects on broad-band magnitudes have been found to be much greater than those on bolometric magnitudes, a result which seems to be of general significance even if the present calculations are too schematic to provide a realistic model of galactic evolution.

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