SOVIET ASTRONOMY — AJ VOL. 10, NO. 4 JANUARY-FEBRUARY, 1967

THE HYPOTHESIS OF CORES RETARDED DURING EXPANSION AND THE HOT COSMOLOGICAL MODEL

Ya. B. Zel'dovich and I. D. Novikov

Translated from Astronomicheskii Zhurnal, Vol. 43, No. 4, pp. 758-760, July-August, 1966
Original article submitted March 14, 1966

The existence of bodies with dimensions less than $R_g = 2GM/c^2$ at the early stages of expansion of the cosmological model leads to a strong accretion of radiation by these bodies. If further calculations confirm that accretion is catastrophically high, the hypothesis on cores retarded during expansion [3, 4] will conflict with observational data.

Ambartsumyan has long held the view that stars and galaxies formed from hypothetical superdense D-objects, whose material never passed through a low-density state before conversion into ordinary celestial bodies [1, 2]. In 1964 one of the authors of this note advanced the hypothesis [3] that the D-objects might be individual pieces of matter, or cores, in an expanding Friedmann universe, which for reasons that we shall not review here have been retarded in their expansion for an external observer, are located inside their Schwarzschild spheres of radius $R = R_g = 2GM/c^2$, and emerge from these spheres only after a long period has passed since the beginning of the general expansion. Subsequently, and independently, a similar hypothesis was advanced by Ne'eman [4].

This hypothesis was treated within the scope of a cosmological model in which the density of all forms of radiation was considered negligibly small.

The discovery of general cosmic radiation with $T \approx 3°K$ would appear to argue for a hot cosmological model. This circumstance radically changes the situation regarding the retarded-cores hypothesis. If these bodies had existed at an early epoch in the expansion, when the mean radiation density ρ_r was large and much greater than the baryon density ρ_b, then accretion (gravitational capture) of radiation by the retarded cores might have been catastrophically strong.[1] Let us estimate this effect.

Stationary accretion of relativistic gas by a gravitating center of mass M may be regarded either as the motion of noninteracting particles (if the mean free path is greater than R_g) or as the motion of a continuous medium. Both approaches yield the same expression, in order of magnitude, for the mass variation of the body. The formula for noninteracting particles is

$$\frac{dM}{dt} = \frac{27}{4}\pi R_g^2 c\rho_r. \tag{1}$$

Substituting $\rho_r = 3/32\pi G t^2$, as is the case for the Friedmann model, and integrating over the interval from $t = t_0$ to time $t = t_1$, when $\rho_r = \rho_b \simeq 10^{-17}$ g/cm^3, with $t_1 \approx 3 \cdot 10^{11}$ sec (at subsequent times ρ_r will decline more rapidly, like $t^{-8/3}$, and its contribution will be small), we obtain

$$M_1 = M_0 \Big/ \left(1 - \frac{81}{32}\frac{GM_0}{c^3}\left(\frac{1}{t_0} - \frac{1}{t_1}\right)\right). \tag{2}$$

Equation (2) may yield infinite values of M_1, for

$$t_0 \leqslant \left(\frac{81}{32}\frac{GM_0}{c^3}\right) \Big/ \left(1 + \frac{81}{32}\frac{GM_0}{c^3 t_1}\right). \tag{3}$$

To a certain extent this singularity is a formal one since Eq. (2) is applicable only for a stationary flux, and it is clear that the mass of a core within the Schwarzschild sphere cannot increase because of accretion by more than the value of the mass that a perturbation wave can penetrate up to time t_1 at the velocity of light (approximately equal to the sound

[1] In a radiation-free model there is a solution in which a retarded core is surrounded by a region of lower density; the mean density does not change, and there is no accretion. Obviously, the reason for the difference in the hot-model case is the motion of quanta, equivalent to a quantum-gas pressure $p = \varepsilon/3$, unlike the dust for which $p = 0$. Accretion has been considered in [5-7].

velocity), beginning at the time t_0 of collapse of the core, or at $t_0 = 0$ if the core has existed from the start of the expansion.

In the first place, then, M_1 should definitely be less than the mass of the radiations in the metagalaxy out to the optical horizon at time t_1;[2] that is, $M_1 \leq \sim 10^{16} M_\odot$. This limit is very high; captured radiation will not participate in the cosmological expansion.[3]

Secondly, if the core has existed since the start of the expansion, then until the time when the optical horizon encompasses a mass of order M_0 of surrounding material the mass of the core will not grow appreciably; it is this time that should be used in Eq. (2) in place of t_0.

This epoch is determined by the relation

$$t_c = \frac{GM_0}{c^3}, \tag{4}$$

and corresponds to the mean density of the cosmological model, being equal to the characteristic density of gravitational self-closure for a body of mass M_0, namely $\rho_c = M_0/(\frac{4}{3}\pi R_g^3)$.

Of course these considerations give only the order of magnitude of t_0. From formulas (3) and (4) it follows ($t_1 \gg t_0$), that t_c is of just the order of the critical value t_0, giving a formal singularity, or more explicitly implying an accretion of radiation mass out to the current optical horizon.

Thus the answer depends essentially on the numerical coefficient in the exact nonstationary expression for dM/dt. If up to the time when the mean density equals the gravitational self-closure density $\rho_c = M_0/(\frac{4}{3}\pi R_g^3)$ the expression for dM/dt differs from Eq. (1) by a factor significantly less than unity (more accurately, $< 32/81$), then the increase in the mass of the retarded cores due to accretion of radiation will be of the same order as the mass of the cores themselves;[4] otherwise the captured mass would tend to be catastrophically large.

The value of the factor should be determined by more detailed calculations, and it may depend on the particular assumptions made. One might show that the hypothesis of nuclei retarded in expansion and existing from the early stages of expansion is contrary to observation in certain respects.

Finally, we note that if gravitational collapse of any body occurs at a time when the mean radiation density is less than the gravitational self-closure density of the body, then accretion of radiation will already be unimportant.

Apart from the question of the accretion of radiation, the fraction α of all the material that suffered collapse at an early stage (when $\rho_r \gg \rho_b$) should be small. In fact, ρ_r/ρ_b will decline during the expansion, while the radiation in the cores will not participate in the cosmological expansion but will have $\rho_c/\rho_b = \text{const}$, where ρ_c is the density of the collapsed masses averaged over all space. The observations now imply $\rho_c/\rho_b < 160$. One can thereby show that $\alpha < 10^{-6}(M/M_\odot)^{1/2}$, where M is the mass of the collapsed bodies. Similar considerations provide bounds on the initial spectrum of inhomogeneity fluctuations.

LITERATURE CITED

1. V. A. Ambartsumyan, Scientific Works [in Russian] (Erevan, 1960), Vol. 2.
2. V. A. Ambartsumyan, Rapport 13, Conseil de physique Solvay, Brussels (1964).
3. I. D. Novikov, Astron. zh., 41, 1075 (1964) [Soviet Astronomy – AJ, Vol. 8, p. 85].
4. Y. Ne'eman, Astrophys. J., 141, 1303 (1965).
5. E. E. Salpeter, Astrophys. J., 140, 796 (1964).
6. Ya. B. Zel'dovich, DAN SSSR, 155, 67 (1964) [Soviet Physics – Doklady, Vol. 9, p. 195].
7. Ya. B. Zel'dovich and I. D. Novikov, Uspekhi Fiz. Nauk, 86, 433 (1965).
8. J. A. Wheeler, Phys. Rev., 97, 511 (1955).

[2] Or at the present time. The important thing is that the limit on M_1 is very high.

[3] Such a body is similar to a geon [8]: its mass is much greater than the rest mass of the baryons comprising it. The distinction from specific Wheeler models is that a nonstationary collapsing formation is considered essential here.

[4] In this event it would be interesting to study the observable effects associated with collisions between expanding cores and relativistic gas falling in from outside.