OBSERVATIONS ON COLLISIONAL STARK BROADENING IN RADIO FREQUENCY DISCHARGES*

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ABSTRACT

The profiles of low Balmer lines emitted from a radio frequency discharge ($N_e \approx 10^{13}$ cm^{-3}) are expected to be in the domain of collisional electron Stark broadening, except for the far parts of the line wings. In order to accomplish a more accurate comparison with the modified generalized impact theory and with the quasi-static theory (including shielding and correlation effects) for the ions only, or the ions and electrons, the contributions other than Stark contributions of the observed Hδ-profile are quantitatively evaluated. Thus for the main body of the profile, the electron broadening contribution is found small compared to that of the ions; the half-width is almost accounted for by the ion broadening contribution. However, toward the far parts of the line wing, where a transition to quasi-static electron broadening can be expected, a slow buildup of a considerable electron broadening contribution is observed, finally equaling the ion contribution.

I. INTRODUCTION

During the last decade there has been considerable development in the theory of Stark broadening of atomic lines which is of great interest to laboratory plasma diagnostics and astrophysical work. Indeed the astrophysical applications of the theory for the lines of the hydrogen atom reflect this development. Early applications of the theory of Stark broadening of Balmer lines (Verweij 1936*a, b;* de Jager 1952, 1954, 1959; Odgers 1952; Traving 1955, 1957) employed the Holtsmark theory, taking into account the perturbations of slowly moving ions but neglecting the contributions of the faster moving electrons. In more recent investigations (Elste, Aller, and Jugaku 1956; Osawa 1956; Aller and Jugaku 1958, 1959; Cayrel and Traving 1960; David 1961) aiming at more accurate and consistent analyses, this omission has been remedied and the broadening caused by the electrons has also been taken into account in the generalized impact theory of Griem, Kolb, and Shen (1959, 1960). Since then the generalized impact theory has been refined further (Lewis 1961; Griem *et al.* 1962; Griem 1962). An extrapolation to the regime in which the electrons can also be treated in the quasi-static approximation has been incorporated into the theory.

Most experimental work on hydrogen profiles has been performed in the visible at the relatively high electron densities encountered in arcs and shock tubes (Jürgens 1952; Bogen 1957; Wiese, Paquette, and Solarski 1961, 1963; Kitayeva and Sobolev 1961; Berg, Ali, Lincke, and Griem 1962). As regards agreement with theory the different experimental findings are somewhat inconsistent, though the recent extensive experimental studies of Wiese *et al.* are in agreement with theory in its latest version within the limits of experimental errors and remaining theoretical uncertainties, which are estimated to be in the order of 15–20 per cent. High-density shock-tube observations are also available for the Lyman-α and β profiles (Elton and Griem 1964). A comparison of relative wing intensities of the Lyman-α line to theory, contained in the study of Elton and Griem, is complemented by a comparison on the basis of absolute absorption coefficients (Boldt and Cooper 1964). The investigations of Boldt and Cooper combine the advantage of a virtually homogeneous effective plasma zone with that of steady-state

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operation (as will the investigations reported here). These observations yield a broadening in the line wings reduced in comparison to the generalized impact theory.[1]

In many plasma experiments and astrophysical applications, electron densities are encountered much lower than those of the arc and shock-tube experiments. Therefore, experimental results at low electron densities are of direct interest. Moreover, comparisons to theory over an extended scale of parameters promises more insight into the applicability or shortcomings of the theoretical approximations presently available.

With a radio frequency discharge in a static magnetic field, considered here, Balmer lines can be studied at about the lowest electron densities at which Stark broadening of lower Balmer lines is still stronger than Doppler broadening (electron densities $N_e \approx 10^{13}$ cm^{-3}, gas and electron temperatures $t_g \approx t_e \approx 2 \times 10^{3}$ ° K). The Balmer series is emitted from a virtually homogeneous zone and can be studied under steady-state conditions. For the parameters considered, the aforementioned recent refinements of the generalized impact theory are very important. It has already been shown previously that the quasi-static approach, including shielding and correlation effects, is appropriate for large parts of the profiles in the case of high lines; complete quasi-static profiles for $2 \times N_e$ become good approximations toward higher lines (Ferguson and Schlüter 1963; Vidal 1964; Schlüter, Avila, and Durham 1965). These findings are consistent with the modified impact theory, since it incorporates a smooth transition to the quasi-static regime of the electrons. In the case of low Balmer lines, impact broadening of the electrons is appropriate for the whole profile (except for the line wings extremely far out from the line center). The previous observations mentioned have shown that the recent modifications of the generalized impact theory are necessary, but because of the presence of other than Stark-broadening contributions they do not establish quantitatively to which extent the modifications and the assumptions and numerical simplifications, inherent in the approximations presently available for the conditions considered, might be still unsufficient to achieve consistency with the experiments.

This paper reports the accurate determination of the non–Stark-broadening contributions of a low Balmer line and the quantitative evaluation of the actual Stark broadening. It concentrates on the now most interesting comparison to theory in a region for which collisional electron broadening is expected to be genuinely applicable.

II. THEORY

The theory of Stark broadening of spectral lines started out with two opposite concepts, with the impact concept and the quasi-static concept. Earlier work on these two concepts, on their theoretical connection, and on their regions of validity, is reviewed in detail by Margenau and Lewis (1959) and by Traving (1960). In 1958, Baranger and independently Kolb and Griem developed a generalized impact theory based on quantum mechanical treatments. Numerical calculations have been undertaken by the latter authors for several elements and a wide range of plasma parameters. These calculations take into account quasi-static broadening caused by the relatively slow ions combined with impact broadening of the faster electrons and include non-adiabatic collisions. The case of most practical interest, the linear Stark broadening of hydrogen, has been worked out in great detail (Griem *et al.* 1959, 1960, 1962; Griem 1962). Recently the theory has been improved further, and an interpolation to a quasi-static behavior of the electrons has been worked into the theory (Lewis 1961; Griem 1962), yielding a smooth transition between impact and quasi-static regime of the electrons.

The latest theoretical treatment relevant here (Griem 1962) suggests an electron

[1] These investigations and some refinements of the calculations will be discussed in an article by H. R. Griem (to be published).

broadening function (n being the upper principal quantum number of the Balmer transition; $\alpha = \Delta\lambda/F_0 = \Delta\lambda/2.61eN_e$; and e, electron charge in electrostatic c.g.s. units):

$$F_e(\alpha) = \frac{1}{\pi}\frac{\gamma}{\gamma^2 + (\alpha/K_n)^2}\frac{1}{K_n}, \tag{1}$$

where the damping constant γ is a function of $\Delta\lambda$ due to the recent modification of the theory:

$$\begin{aligned}\gamma = {} & 1.5\pi\left(\frac{\Delta\lambda_w}{F_0 K_n}\right)^{-1/2} \\ & + [5.6\times 10^{-6} N_e^{1/3} t_e^{-1/2} \log\left(\frac{4\times 10^6 t_e}{n^2 N_e^{1/2}}\right)\frac{n^5 + 2^5}{n^2 - 2^2}\frac{\ln(\Delta\lambda_w/\Delta\lambda)}{\ln(\Delta\lambda_w/\Delta\lambda_p)}.\end{aligned} \tag{2}$$

With the standard notations the following definitions are used:

$$K_n = 5.5\times 10^{-5}\frac{(n\times 2)^4}{n^2 - 2^2}, \tag{3}$$

$$\Delta\lambda_w = \lambda^2 k t_e / c h n^2, \tag{4}$$

$$\Delta\lambda_p = \lambda^2 (N_e e^2/\pi m c^2)^{1/2}. \tag{5}$$

The logarithmic reduction factor of the second term of equation (2) has only to be applied for $\Delta\lambda_p \leq \Delta\lambda \leq \Delta\lambda_w$. For $\Delta\lambda > \Delta\lambda_w$, instead of the impact broadening function (1), a quasi-static broadening function S_n shall be applied, as given in the following for the ion broadening. The dependence of γ on $\Delta\lambda$ according to equation (2), creates some ambiguity. Due to this dependence, the broadening function F_e of equation (1) is in general not a monotonic function of (positive) $\Delta\lambda$. Therefore, here a final electron broadening function $\bar{F}_e$ is constructed by fitting together with progressing $\Delta\lambda$ pieces of different curves $F_e[\gamma(\Delta\lambda)]$ which are made to have all the same top intensity but become narrower with increasing $\Delta\lambda > \Delta\lambda_p$; subsequently a normalization to 1 is performed (normalization factor f). This is equivalent to[2]

$$\bar{F}_e(\Delta\lambda/F_0) = f\,\frac{1}{\pi}\frac{\gamma(\Delta\lambda)}{\gamma^2(\Delta\lambda) + (\Delta\lambda/F_0 K_n)^2}\frac{1}{K_n}\frac{\gamma(\Delta\lambda)}{\gamma(0)}. \tag{6}$$

Around $\Delta\lambda_w$ a smooth transition to the quasi-static values of $S_n(\Delta\lambda/F\ )$ is used.[3] The modifications of the generalized impact theory outlined by Griem (1962) have to some extent the character of extrapolations, but an accuracy on the order of 20 per cent is expected for most cases.

In the unmodified form of the generalized impact theory (Griem 1960) the damping constant (2) did not contain the first term and the logarithmic reduction factor of the

[2] The procedure chosen here brings the reduction of electron broadening in the modified version of the generalized impact theory to full bearing. The half-widths thus obtained are considerably smaller than those calculated directly from the damping constant (2) (Vidal 1964) for a representative, not exactly defined, $\Delta\lambda$, e.g., for the measured half-width.

[3] This is performed in a manner such that not only the change of the broadening function with $\Delta\lambda$ in the far wings conforms to the quasi-static theory, but that also the intensity of a $\Delta\lambda$ band in the far wing constitutes the correct fraction of the total line intensity demanded by the quasi-static theory for electrons, as confirmed by experimental findings (Ferguson and Schlüter 1963; Vidal 1964; Schlüter *et al.* 1965). The latter demand for the asymptotic behavior in the wings, together with the requirement of renormalization after construction of the broadening function for the main body of the profile, may in general necessitate some interpolations and iterations, but here it does not lead to serious ambiguity and complication, since here $\Delta\lambda_w$ is far off the main part of the profile.

second term, whereas the remaining log contained a -0.125 in its argument. No transition to the quasi-static approximation was incorporated.

The electron broadening function $\bar{F}_e$ (or F_e in the unmodified theory) has to be folded with the ion broadening function which is almost always properly given in the quasi-static approximation:[4]

$$S_n(\alpha) = \sum_j \frac{I_{jn}}{c_{jn}} W\left(\frac{\alpha}{c_{jn}}\right). \tag{7}$$

The c_{jn} are the displacement constants and the I_{jn} the relative intensities of the components j of the Balmer line H_n (Underhill and Waddell 1959). Here the field distribution function W is not used in the Holtsmark approximation (Underhill and Waddell 1959) since shielding and correlation effects are important (Ecker 1957; Hoffmann and Theimer 1958; Ecker and Müller 1958). The recent improved calculations of Baranger and Mozer, i.e., their calculations for the low frequency field component, are used (Baranger and Mozer 1959; Mozer and Baranger 1960).[5] The profiles S_n thus obtained are noticeably narrower than Holtsmark profiles.

For the following investigations, the structure that purely quasi-static profiles exhibit in the region around the line center—here the narrow dip of the Hδ-profile—is not subjected to a simplifying smoothing procedure, as, for instance, used by Griem (1960). However, such a smoothing procedure (preserving, of course, the total area under the profile) would not cause much change in the final profile after incorporation of Doppler effect and other minor non-Stark effect contributions; there would be a change of the top intensity only of about 4 per cent, which is within the limits of experimental accuracy of the observed profile. Electron impact contributions tend to perform a smoothing procedure. Rather small modifications in a very narrow region of the line center may also be introduced by non-quasi-static ion perturbations. (In this connection for details on non-adiabatic collisions and induced transitions reference is made to Unsöld [1955] and Traving [1960]). The presence of effects with magnitudes of the order of the smoothing procedure (for the center dip of Hδ) cannot be proved or disproved by the following observations. It should be stressed that the presence or absence of much larger (electron) broadening contributions will be under discussion and that relatively minor details of the actual broadening contribution very close to the line center are not within the scope of this study.

For the asymptotic behavior of a profile in the far wings of a line, closed expressions are given by Griem (1962); these expressions contain a transition to the quasi-static broadening of both ions and electrons discussed above.

Equation (4) for $\Delta\lambda_w$ represents an estimate for the borderline between impact and quasi-static regime of the electrons. A similar estimate according to Unsöld (1955) shall be mentioned, since it proves useful in the following analysis of the observations (v is the relative velocity of electrons and atoms in cm/sec, $\Delta\lambda_L$ and λ are in cm, $n < 3$):

$$\Delta\lambda_L \approx 0.62 \times 10^{-12} v^2 \lambda^2 / n(n-1) . \tag{8}$$

Equation (8) has the same basic structure as the estimate of Griem's (4), but yields values of about a factor 2–3 lower.

[4] The intensity distribution $I_n(\Delta\lambda) = S_n(\alpha)d\alpha/d\Delta\lambda$ is normalized to 1, or in the case of Balmer lines with odd n, to $f_\pm/(f_\pm + f_0)$. The f-values are contained on the tables of Underhill and Waddell (1959).

[5] The use of the high-frequency component in the approximation of Baranger and Mozer (1959) in cases of quasi-static electron broadening instead of the low-frequency component applied for the ions would, for conditions as considered here, not appreciably alter the over-all profiles, as demonstrated previously (Ferguson and Schlüter 1963).

III. EXPERIMENTS

a) *Apparatus*

The hydrogen plasma is generated in a quartz vessel (6 cm in diameter and 50 cm in length) by a 30-Mc/s oscillator inductively coupled to the vessel by a copper strap centered between two solenoids. These solenoids provide an inhomogeneous axial static magnetic field shaping the plasma into a hollow double-cone configuration, touching the wall of the discharge tube only in the center of the arrangement. The observations of the Balmer series are made off the center in the region of strongest magnetic field strength inside one of the solenoids. There, for certain discharge conditions, "trapped" recombination controlled plasma "ellipsoids" (cigar-shaped plasmas) appear (Schlüter 1961, 1963) which are sharply separated from the neighboring—only weakly radiating—plasma. These ellipsoids emit extremely clear Balmer series virtually without molecular and impurity background provided ultravacuum conditions are maintained before the gas filling.

The plasmoid-like ellipsoids are highly homogeneous as has been shown by end-on and side-on observations through different layers. Virtually the same line profiles are obtained for different layers; also the temperature and density determinations from the Balmer continuum described below lead to practically the same results. (For side-on measurements of the continuum and of the lowest lines the [directly measurable] background from the weakly radiating plasma outside the ellipsoids is not negligible and has to be subtracted.) The good agreement reached between observed and theoretical line broadening whenever the relative simple case of purely quasi-static broadening of ions and electrons is approached (Ferguson and Schlüter 1963; Vidal 1964; Schlüter *et al.* 1965) may be considered as further support for the virtual absence of inhomogeneous layers and for the correctness of the value of the electron density determined below. The important feature of homogeneity has also recently been investigated and confirmed by Vidal (1964).

Plasma densities and temperatures are varied by changes in the magnetic field strength, geometry of the arrangement, oscillator power, oscillator tuning, and gas pressure. For the constancy of the plasma parameters and the accuracy of the following measurements the precise control of the gas pressure by an automatic regulator is very important.

The spectroscopic observations are made with a stigmatic grating spectrometer of high resolving power in first order (Bausch and Lomb grating with 180000 lines). A precision scanning device with a 1P28 photomultiplier is used. The light beam is chopped and the multiplier signal is amplified by a phase and frequency-sensitive unit. Radio frequency perturbations are eliminated by the use of a shielded measuring cabin. A tungsten filament lamp and a carbon arc are used for absolute and relative intensity calibration. With this arrangement the wing intensities of low Balmer lines are measurable down to less than 10^{-4} times the maximum line intensity. With the use of cooling of the photomultiplier, even lower intensities can be measured, but the cooling makes it difficult to maintain exact alignment and maximum resolving power of the spectrometer. Therefore in the following no cooling of the photomultiplier is employed.

b) *Determination of Plasma Parameters*

The electron temperature t_e is determined from the slope of the Balmer continuum and, with known t_e, the electron density N_e from the absolute intensity of the continuum. Also the relative and absolute intensities of high Balmer lines are used for temperature and density determinations (Schlüter 1963), yielding results consistent with that of the continuum measurements. For the conditions considered in the following $t_e = 1.89 \times 10^3$ ° K, $N_e = 1.3 \times 10^{13}$ cm^{-3}. The spectroscopic determination of N_e, the parameter most important for the line-profile studies, is supported by an independent determination from the transmission of microwaves (4 or 9 mm wavelength) through the plasma, as previously described in detail by Lisitano and Tutter (1961). The error for N_e is estimated to be less than 10 per cent due to the combination of different methods.

The gas temperature t_g can be determined from the broadening of Hβ slightly outside the plasma ellipsoid considered where the electron density is drastically reduced and therefore Stark broadening negligible (as assured from observations of profiles of high Balmer lines and from microwave transmission measurements). Taking into account small broadening contributions due to Zeeman effects, fine structure, and apparatus broadening, $t_g = 1.85 \times 10^3$ ° K is obtained. However in the following—though for Hδ—the observed interplay of the different non–Stark-broadening contributions shall be confirmed by analysis. Therefore the knowledge of the Doppler temperature from another independent determination is desirable. Such a determination is realized by adding traces of helium and observing the Doppler broadening of isolated Zeeman components of the first members of the 2^1P–n^1S series, eliminating π-components with a polarization filter (Vidal 1964). The slope of the log of the line intensity plotted versus the square of the distance from the line center yields the Doppler temperature t_g virtually identical with the value above. The result $t_g \approx t_e$ appears appropriate for the specific type of recombination controlled plasma considered. The magnetic field strength is determined with Hall probes.

More details about the determination of the plasma parameters and the apparatus have been given previously (Schlüter 1961; Schlüter and Ransom 1965).

IV. ANALYSIS

For a comparison to theory clearly in the regime of collisional electron broadening, a low principal quantum number n must be selected according to equations (4) and (8). On the other hand, for reasons of improved accuracy it is desirable to perform the comparison for high enough n so that Stark broadening (half-width approximately $\sim\lambda^2 n^2$ (Bergstedt, Ferguson, Schlüter, and Wulff 1961)) is still stronger than Doppler broadening and other broadening contributions. Under the conditions considered, the Hδ-profile satisfactorily meets both demands and is therefore studied in the following. Moreover, in the case of Hδ the interesting transition region to quasi-static electron broadening in the very far parts of the line wings is still within the range of accurate detection achieved here, although the line core and large parts of the line wings are in the region of collisional electron broadening as desired. The conclusions obtained in the following for Hδ under conditions favorable for experimental accuracy apply at least qualitatively also to neighboring lines, as the smooth changes in the profiles from Hδ to higher or lower lines indicate (Ferguson and Schlüter 1962; Vidal 1964).

a) Determination of Non–Stark Broadening

The broadening contribution to the profile of Hδ from effects other than Stark effect are determined experimentally. The Hδ-profile is observed outside the plasma ellipsoid in a region where, as mentioned, the electron density and thus Stark broadening are drastically reduced, but where gas temperature and magnetic field strength and thus Doppler broadening, Zeeman effect, and fine-structure influence and apparatus broadening are unchanged. The accuracy of quantitative statements about the actual Stark broadening depends to a large degree on the certainty with which the contributions of non-Stark effects are known. Therefore, a confirmation of the measurement of these contributions by analysis is desirable in order to assure the absence of any unnoticed source of essential experimental error.

First the apparatus broadening contribution shall be discussed though this contribution is relatively small for the core of the profile considered in the following, becoming more important in the line wings (because of a dependence of the broadening function on $\Delta\lambda$ with approximately the $-\frac{4}{2}$ power). The apparatus contribution $f_A(\Delta\lambda)$ is observed by using narrow argon, cadmium, and iron lines, the optical arrangements and spectrometer settings being the same as for the other experiments. The observations are well accounted for if the theoretical broadening function (Unsöld 1955)

$$f_1(\Delta\lambda) = \frac{\Delta\lambda_0}{\pi^2} \frac{\sin^2(\pi\Delta\lambda/\Delta\lambda_0)}{\Delta\lambda^2} \tag{9}$$

is folded with the broadening function f_2 describing the influence of the finite width of the spectrometer slits according to

$$f_A(\Delta\lambda) = \int_{-\infty}^{+\infty} f_1(\Delta\lambda - \Delta\lambda') f_2(\Delta\lambda') d(\Delta\lambda'). \tag{10}$$

Very accurate agreement with observation is obtained if the actual resolving power $\lambda/\Delta\lambda_0$ of the grating is taken as 155000, the value for a perfect arrangement being 180000. The slit function f_2 corresponds in the case of an entrance slit only to a square distribution with the width of the slit (10 μ) multiplied with the dispersion (about 2.6 Å/mm). Here, however, the exit slit (10 μ) in front of the photomultiplier has to be taken into account by another folding procedure analogous to expression (10). Thus the experimentally determined apparatus broadening is backed up by analysis, except of course, for a weak ghost in each side of a line. These ghosts are found, however, to constitute rather local perturbations, particularly after folding of the apparatus profile with all other broadening functions.

The dominant non–Stark-broadening contribution is given by the Doppler broadening. It is easily calculated since the Doppler temperature is known from the slope of the log of the line intensity I plotted versus $\Delta\lambda^2$, obtained for isolated Zeeman components of helium lines, as described before. Actually the accurate Doppler temperature has to be determined by trial and error by folding Doppler profiles with the apparatus profile determined for the correct wavelength as discussed above, and then comparing the slope of the resulting log $I(\Delta\lambda^2)$ in the core of the line with the observed slope. As mentioned, practically the same result (1.85×10^3 ° K) for t_g is obtained from the Hβ-profile outside the plasma ellipsoid, taking into account in this case also the Zeeman pattern. (The Hα-line is not used, because here optical thickness may start affecting the profile.) Since the Doppler temperature inside the plasma ellipsoid cannot be appreciably different from the one outside due to the relatively large mean free path of the gas particles, the Doppler broadening can be considered to be well known.

The influence of fine structure and Zeeman effect is relatively small. Nevertheless the splitting pattern and the profile resulting from superposition of the different Doppler broadening components is calculated. In the case of anomalous Zeeman effect of Hδ there are (end-on) thirty possible transitions with various relative intensities (Back and Landé 1925; White 1934; Moore 1949; Unsöld 1955). For the field strength considered (about 10^3 gauss) corrections have to be made, since transitions to Paschen-Back effect sets in. Inaccuracies in the corrections actually performed here are of minor importance, particularly since even for the line core the contribution of Zeeman effect to the total non–Stark-broadening profile is small to begin with.

By folding the apparatus profile (determined experimentally and found consistent with analysis) with the Doppler profile for known t_g, including the small contributions of Zeeman effect and fine structure, a profile is found satisfactorily backing up the total non–Stark-broadening profile measured; except for the limited region of a small line ghost, there is agreement within about 5 per cent. The folding procedures and other lengthy calculations are performed with the aid of a digital computer (CDC 1604) of the University of Texas Computation Center.

b) Comparison to Stark-broadening Theory

The non–Stark-broadening profile being known with high accuracy, the Stark broadening according to different theoretical approaches is now easily discussed by folding, analogous to expression (10), the non–Stark- and Stark-broadening function together and comparing the resulting total theoretical profile with the observed total profile. As mentioned, for Hδ this procedure is still rather sensitive since all theoretical broadening contributions considered in the following discussions are larger than the non-Stark contributions.

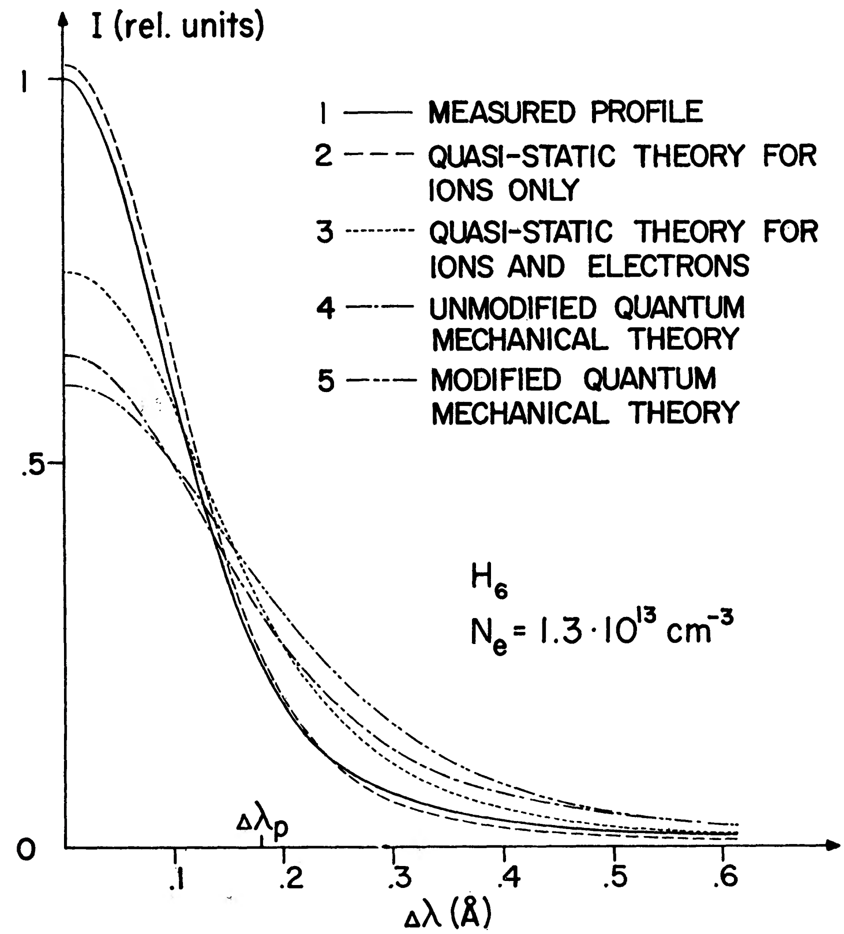

FIG. 1.—Comparison of measured profile of Hδ with theoretical profiles according to quasi-static theory of ions only and ions plus electrons (low-frequency component of Mozer and Baranger), unmodified (Griem 1960) and modified (Griem 1962) generalized impact theory.

Curve *2* of Figure 1 shows the main body of the total theoretical profile thus obtained if the Stark broadening were to be given merely by the ion contribution. According to equations (4) and (8), in this case the quasi-static approximation for ion broadening is doubtlessly appropriate virtually for the whole profile. For curve *2* a quasi-static profile is used which takes into account shielding and correlation effects according to the low-frequency field distribution function of Mozer and Baranger (1960). The comparison to the measured total profile, curve *1* of Figure 1, clearly shows that the ion contribution alone is almost sufficient to account for the observation, i.e., for the main body of the line. Figure 1 shows the "red" side of Hδ; but practically no differences between the two sides of the profile are observed. Possible asymmetries stay within the limits of experimental errors (within 10 per cent). Quadrupole and quadratic Stark effect contributions (Nguyen-Hoe, Drawin, and Herman 1964) which may occur appear to be of limited importance in this case.

The profile given by curve *2* of Figure 1 has virtually the same half-width as the observed profile of curve *1*. However, toward the wings the observed profile exhibits intensities becoming substantially larger than those of curve *2* (ion Stark broadening only); this is discussed further below.

The modified impact theory, in the approximation previously outlined, takes into account collisional electron broadening in addition to quasi-static ion broadening. Curve *4*

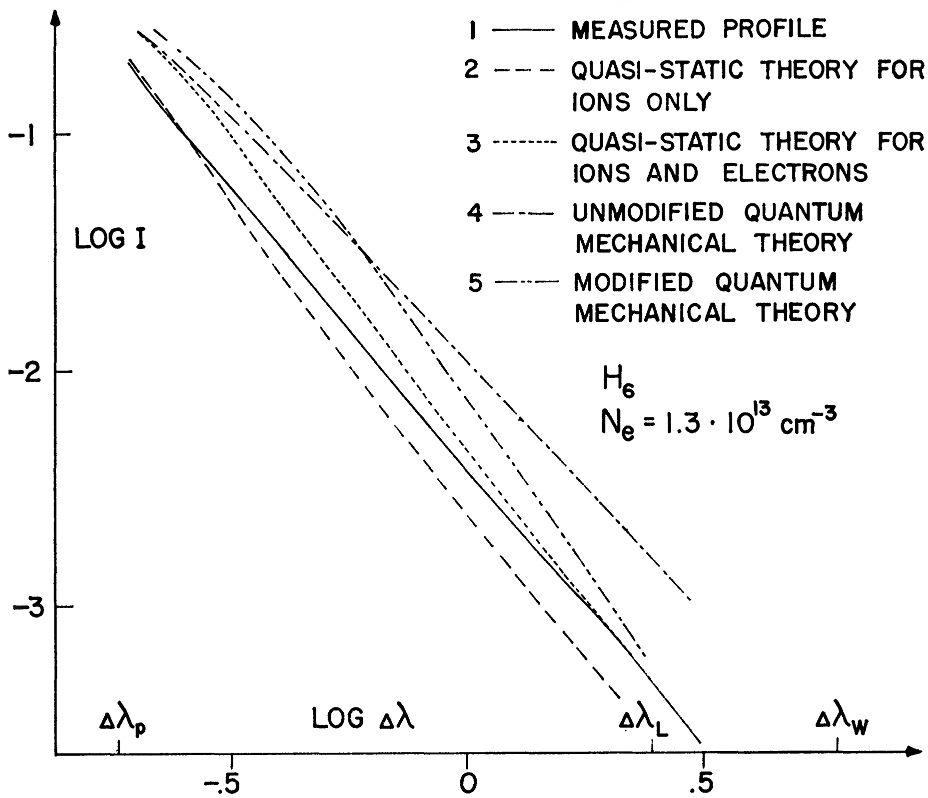

FIG. 2.—Extension of Fig. 1 into the line wings

of Figure 1 results from the unmodified theory (Griem 1960), curve *5* from the modified form of the theory (Griem 1962). Here in both cases the contribution due to the ions is taken as a quasi-static one including shielding and correlation effects according to the low-frequency component of Mozer and Baranger (1960). The increased broadening of the modified version in the main body of the profile stems from the inclusion of the strong collision term in the theory; the reduction of the electron damping constant for $\Delta\lambda > \Delta\lambda_p$ becomes noticeable further in the wings. $\Delta\lambda_p$ according to equation (5) is indicated in Figure 1. Comparison with the measured profile (curve *1*) shows that the generalized impact theory in both versions considerably overestimates the Stark broadening in the main body of the profile and, as Figure 2 demonstrates, also in the line wings, though the modified version (curve *5*) approaches the measured profile (curve *1*) far in the line wings when $\Delta\lambda$ approaches $\Delta\lambda_w$ of equation (4), due to the interpolation to quasi-static electron broadening incorporated into the impact theory in its modified version.

Quasi-static broadening for $2 \times N_e$, including again shielding and correlation effects, is also too strong for the main body of the profile (curve *3* of Fig. 1), which is clearly in the collisional regime of electron broadening. However, the more the domain of quasi-static electron broadening is approached according to $\Delta\lambda_L$ and $\Delta\lambda_w$ of equations (4) and (8), the more this approximation (curve *3* of Fig. 2) approaches the measured profile. (Actually a tiny bump appears in the curves of Fig. 2 at about $\Delta\lambda = 1$ Å due to a grating ghost. But this small bump is very narrow and therefore omitted; the curves are interpolated through this narrow ghost region.) At about $\Delta\lambda = \Delta\lambda_L$ the observed profile appears to go over into the profile for quasi-static broadening of ions as well as electrons (curve *3*). Figure 2 seems to favor the estimate $\Delta\lambda_L$ of equation (8) compared to the estimate $\Delta\lambda_w$ of equation (4). But there is some ambiguity as to where curve *1* actually

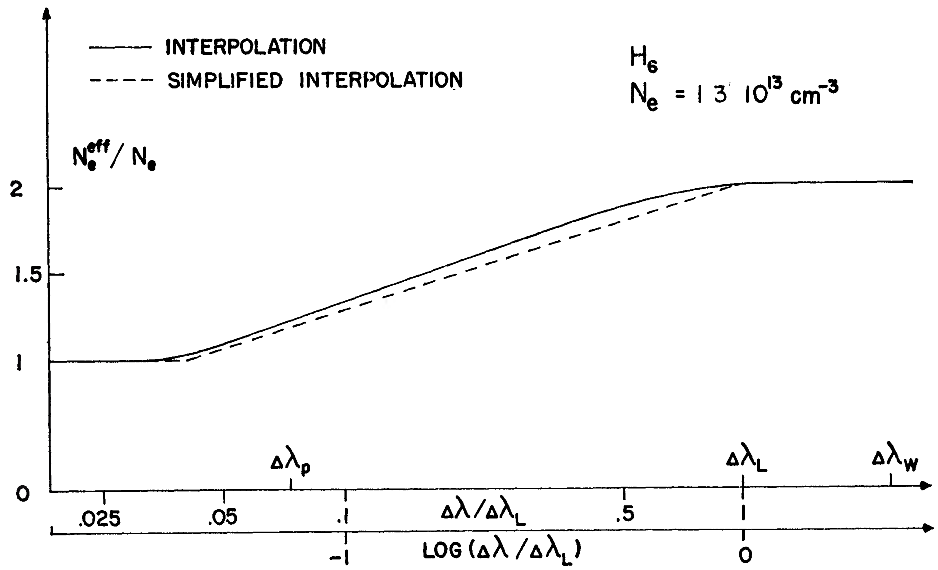

FIG. 3.—Effective perturber density N_e^{eff}, to be used in the quasi-static approximation (low-frequency component of Mozer and Baranger) at different distances from the line center $\Delta\lambda$ in order to describe the observed profile. $\Delta\lambda_L$ is the estimate of eq. (8) according to Unsöld for the transition to the quasi-static regime of the electron broadening.

reaches curve *3*, caused by possible experimental errors and depending, of course, on the accuracy with which the transition is defined. Possible experimental errors affecting the comparisons in Figure 2 are estimated not to exceed 15 per cent for the part of the curves actually shown. Curve *1* is still observable for larger $\Delta\lambda$ than shown and seems to follow curve *3* even further out into the wing. However, this part of the wing is not included since there the apparatus function is not as accurately known as before, thus affecting the accuracy of the theoretical curves containing the apparatus contribution for larger $\Delta\lambda$ than actually shown in Figure 2. It should be mentioned that there is consistency (within the given limits of experimental uncertainties) with previous measurements (Ferguson and Schlüter 1963) which did not eliminate non–Stark-broadening contributions.

The above comparisons between experiment and the different theoretical approaches are not expected to be essentially affected by possible experimental errors. Inaccuracies in the measurements of the profiles and the plasma parameters are expected to introduce uncertainties in the comparisons of about 10 per cent for the main body of the profile, growing possibly to about 15 per cent for the far wings, as mentioned.

The observed profile can be empirically described by a profile containing only quasi-static broadening (taking into account shielding and correlation effects) in a smooth transition from broadening for $1 \times N_e$ (ions only) to $2 \times N_e$ (ions and electrons). Figure 3 shows how this transition has to be made in detail in order to achieve a good description of the observation in the main body and the wings of the line. The effective density to be used in the quasi-static approximation is plotted versus the log of $\Delta\lambda$ in units of the transition value $\Delta\lambda_L$. A profile thus obtained is, of course, to be renormalized; but here the renormalization is close to 1 since the transition from $1 \times N_e$ to $2 \times N_e$ starts at relatively low intensities of the profile. Therefore the good approximation of the observations in the main body of the profile shown by curve *2* is essentially retained; the center intensity, e.g., drops to only a few per cent (~5 per cent) below the observed center intensity. Practically the same agreement with the observations in the main body as well as the wings of the profile is obtained if, instead of the smooth solid curve of Figure 3, the dotted curve is used: $1 \times N_e$ for $\Delta\lambda/\Delta\lambda_L$ smaller than about $\frac{1}{25}$, $2 \times N_e$ for $\Delta\lambda/\Delta\lambda_L > 1$, a linear transition with log $(\Delta\lambda/\Delta\lambda_L)$ for intermittent values. Even further changes of the curve given are permissible within the experimental uncertainties cited above, for instance, a transition to a perturber density of $2 \times N_e$ at $(\Delta\lambda_L + \Delta\lambda_w)/2$. However a drastic qualitative deviation from Figure 3 would not be consistent with the observations, and the (solid) curve of Figure 3 is quite representative in reflecting the transition to quasi-static broadening of ions plus electrons in the range of $\Delta\lambda_L$ and $\Delta\lambda_w$. This agrees well with the findings reported for higher lines (Schlüter *et al.* 1965) which left little doubt about the applicability of the quasi-static approximation with $2 \times N_e$ for regions beyond[6] $\Delta\lambda_L$ and $\Delta\lambda_w$.

Finally it might be pointed out that the experiments of Boldt and Cooper (1964) lead to an effective quasi-static perturber density as a function of log $(\Delta\lambda/\Delta\lambda_L)$—for a somewhat smaller range of $\Delta\lambda/\Delta\lambda_L$—quite similar to that of Figure 3, although in that case a much higher density (by about a factor 10^4), a higher temperature (by about a factor 6), and a quite different line (Lyman-α) are considered.

V. SUMMARY AND DISCUSSION

After non–Stark-effect contributions are taken into account, the Stark broadening of Hδ under the conditions considered is well approximated by the quasi-static ion contribution (including shielding and correlation effects according to Baranger and Mozer [1960]) in the main body of the profile. For the half-width of the profile the influence of the ions is predominant, even if possible experimental errors are taken into account. Considerably larger half-widths result from the application of the generalized impact theory in the approximations presently available. These differences between experiment and theory exceed the uncertainties estimated (Griem 1960, 1962) for the theoretical approximations. (The uncertainties are due to approximations used in calculating electron impact broadening for a high quantum number n and due to the modifications recently introduced, which have to some extent the nature of interpolations; the omission of shielding and correlation effects of the ions is avoided here by the use of the low-frequency component of Mozer and Baranger [1962] in all comparisons to experiment.) However, the inaccuracies encountered in the approximate calculations are estimated on the basis of comparisons (Griem 1960) to more accurate calculations (Griem *et al.* 1960) under somewhat different conditions than considered here, and they may therefore be larger here. Considerations concerning the independence of ion and electron contributions, such as given by Traving (1960, pp. 75, 76) may be of some importance. A possible source of theoretical inaccuracies recently discussed by Van Regemorter (1964) may also have to be considered, though one should expect its influence to be minor and not very specific for the case of Hδ considered here. It must be pointed out that other hydrogen

[6] Details on the construction of broadening functions with variable density of perturbers, in particular of automatically normalized ones, are contained in an article by F. N. Edmonds, H. Schlüter, and C. Wells (to be published).

investigations, most of which are discussed in the Introduction, state better agreement though in somewhat varying degree. In any case, the over-all situation seems to be far from satisfactory and further experimental and theoretical investigations for various conditions seem to be desirable.

Though the electron broadening contribution for the main body of the profile is observed to be limited to a minor role compared to that of the ions, the electron contributions become comparable toward the line wing and finally equal to the ion contribution, as expected theoretically. For low-density and low-temperature observations such as investigated here, the electron contributions can be essentially taken into account according to a relatively simple empirical description.

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