

DELAYED EXPLOSION OF A PART OF THE FRIDMAN UNIVERSE, AND QUASARS

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A hypothesis interpreting quasistellar radio sources as parts of the Fridman universe delayed in expansion is considered. The matter comprising these delay cores expands beyond its Schwarzschild sphere at different times for different cores. Collision with matter falling in from outside is held responsible for the quasar phenomenon. The exact solution of the gravitational equations describing a cosmological model with delayed cores is derived. The expansion of each core prior to its interaction with the external matter repeats the expansion of the entire Fridman model. The possibility of a situation in which the epoch of expansion of the universe and of the delayed cores is preceded by the epoch of their contraction is also considered.

The prodigious quantity of energy stored in powerful Cygnus A type sources of radio-frequency emission has been interpreted in recent contributions to the literature [1-9] as the result of release of that energy in the gravitational collapse of quasars, or quasistellar radio sources. The discovery of 3C-273 type objects which appear to be at comparatively early stages in the development of the process of releasing that energy renders possible at least a partial verification of the hypotheses on the nature of the mechanism underlying the conversion of gravitational energy into energy associated with radio-frequency emission and cosmic rays. No satisfactory mechanism has been proposed to date to account for this phenomenon. Several authors have suggested abandonment of the generally accepted theory of gravity (Hoyle et al., [2]), but their arguments are inconclusive.

An attempt is made in the present article to approach the solution of the problem of quasars and radio galaxies from another point of view, still within the framework of generally accepted physical concepts (i.e., within the framework of the standard relativity theory and physics of elementary particles).

V. A. Ambartsumyan long ago expressed the notion that significant masses of matter are capable of remaining for a protracted time in a state different from the stellar state and different from

the state of a rarefied gas. These masses are known as D-bodies. Explosions on the part of such D-bodies are responsible, in Ambartsumyan's view, for the existence of stellar associations. Recently, V. A. Ambartsumyan [10] emphasized that considerable masses of matter and of relativistic particles could be ejected from the centers of galaxies, and that the tempestuous phenomena in giant galaxies and radio galaxies are due to the activity of the galactic nuclei.

The hypothesis advanced below is based on the usually accepted physical laws, as mentioned earlier, and does not presuppose any violation or suspension of those laws; in a certain sense this hypothesis is an extension of the idea expressed by V. A. Ambartsumyan on the conjectured D-bodies.

The possibility of a situation directly opposite to that of a gravitational collapse has been considered by several authors [11-13]; this situation is the expansion of a gravitating sphere beyond its Schwarzschild sphere. The concrete materialization of this possibility is examined in the present paper. We consider here a homogeneous isotropic Fridman cosmological model. We assume that at the initial time (the instant of infinite density), not all the matter included in the model began to undergo explosion. Certain regions, which will be assumed spherically symmetrical for simplicity in

our treatment, were delayed and failed to expand for some period with respect to the time in the co-moving system (for this possibility and the underlying reasons, see below).

The expansion of these parts of the model takes place only later. Their matter emerges beyond the gravitational radius, and the energy stored in that matter is converted to the energy of cosmic rays either directly, or by way of interaction with the matter falling into the gravitational field of a delayed core* of this type. Expansion not of the entire delayed matter all at once, but of successive discrete shells is also possible, i.e., repeated explosions and ensuing effluxes of matter are a distinct possibility.†

In the case of gravitational collapse, as we know, the radius of the star tends to the gravitational radius in an infinite time as seen by an external observer, while the radiation decays exponentially, and the rays emitted by the star after cutting through the surface of the Schwarzschild sphere never reach the external observer. This observer will never find out what happened after the contraction to the critical sphere. In the case of "anticollapse," or expansion beyond the Schwarzschild sphere, the kinematics of the matter will be directly opposite to that in the collapse case, but this will not mean a simple time reversal of the collapse pattern to the external observer. That observer will see the anticollapse proceed in a fundamentally different way. Actually, the rays reaching the observer emerge from the surface of the star as the star contracts. The change in the sign of the time converts collapse into anticollapse, and emerging rays into reentrant rays. But in constructing the picture as seen by the observer in the case of anticollapse, we have to take the rays emerging from the stellar surface again, rather than the entering rays. This is a time reversal of rays which were penetrating the surface of the star from outside to inside in the case of collapse. These rays pierce the Schwarzschild sphere, as we know. Consequently, following the change in the sign for the time, these rays will emerge beyond the Schwarzschild sphere and will reach the external observer. He witnesses the entire expansion process, beginning with point dimensions [13]. This state of affairs comes about because the physical extension of the Schwarzschild solution to the space-time region "inside" the Schwarzschild sphere (the T-region [11]) is double-valued. In one extension, the coordinate spheres of any reference system in the T-region necessarily contract, whereas in the other extension these coordinate spheres will be

obliged to expand, no matter how we constrain them to move.

As shown in reference [12], the choice between these two extensions of the Schwarzschild solution to the T-region is not arbitrary, but is governed by the conditions under which this region took shape. If the region formed in the compression of the sphere to a radius inferior to the gravitational radius R_g , then all the coordinate spheres in that region will contract. If the velocities of the matter with outwardly directed dimensions inferior to R_g are assigned from the very beginning, then all coordinate spheres will expand.‡

Consider a gravitating sphere expanding beyond its Schwarzschild sphere. Let the total rest mass of the matter comprising a sphere M_0 be

$$M_0 = \int_V \underline{n} m_0 dV,$$

where \underline{n} is the particle density, m_0 the particle mass, and dV an element of the proper volume of the sphere. Note that the fact of the co-moving space being non-Euclidean implies $dV \neq 4\pi r^2 dr$. Depending on the energy which this matter possesses, this space may: 1) either fly apart to infinity, 2) expand to some finite dimensions greater than the gravitational radius, after which it will begin to contract again, or 3) it may expand to dimensions less than the gravitational radius, and then again contract.

The gravitational field set up by the body is determined not only by the quantity of matter and its distribution, but also by its kinetic energy (since the energy corresponds to the mass $m = E/c^2$). The gravitational field is thus characterized by the total energy stored in the matter, and conversely the gravitational mass M of the body, determining the

*It is generally acknowledged that the gravitational field will not vary in time when spherically symmetrical motion is involved, so that the gravitational field of the core remains consistently invariant.

†We stress here that we have no knowledge whatever to date of any object about which it could be said with certainty that it is in a stage of relativistic collapse. But we can state with some certainty that the metagalaxy is currently in a relativistic "anticollapse" phase. If the metagalaxy is finite and spherically shaped, then that part of it which has been studied furnishes evidence that the boundary of the metagalaxy clearly lies under its Schwarzschild sphere [11].

‡Another alternative is that the initial velocities be assigned such that the energies stored in the matter will be inadequate to support the expansion of the sphere to dimensions greater than the gravitational radius (see below), whereupon the "expanded" T-region will be replaced by a "contracting" T-region [12, 16], but the reverse change is still out of the question.

external field, characterizes the total energy of the matter. The gravitational mass M is defined as follows:

$$M = 4\pi \int_0^r \frac{\rho + pv^2/c^4}{1 - v^2/c^2} r^2 dr, \quad (1)$$

where r is a radial coordinate so chosen that the surface of the sphere will be $4\pi r^2$. Depending on the relationship between M and M_0 , one of the cases of motion of the matter enumerated above will take place. When the energy is sufficiently large (when M is sufficiently large compared to M_0), the matter will fly apart to infinity. We shall assume a homogeneous sphere. When $M \geq M_0$, we have case (1) (flying apart to infinity), when $M_0 > M > 4/3\pi M_0$, we have case (2), and when $4/3\pi M_0 > M$, we have case (3).**

Note that in case (3) the co-moving space is a semiclosed space [14,15]. The external observer sees only the early phases of the expansion of this semiclosed universe. The further evolution of the matter remains beyond reach of his observation [16].

Below, we derive an exact solution of the Einstein equations describing the Fridman universe with cores delayed in expansion. Here we focus on the causes of the delay in expansion of a part of the matter.

There are two possibilities open. First, there is the plausible point of view that no metric space-time continuum exists at all prior to the instant when $\rho = \infty$. Then other essentially different space-time forms of motion of matter unknown to us may have existed. From that standpoint, the question of the delayed explosion of a part of the Fridman universe, just as the question of the cause of the explosion itself, lie outside the purview of contemporary physics. And the delay may be formulated simply as an initial condition, similarly to the way the expansion law itself is formulated in the Fridman model [23].

Another point of view places the contraction prior to the epoch of expansion. It is asserted by Lifshits, Sudakov, and Khalatnikov [17] that no singularity ($\rho = \infty$) will be attained at any arbitrary distribution and motion of matter. This means that the inhomogeneity in the distribution of matter and the asymmetry of the motion, developing in the course of the contraction, either cause a transition to the expansion phase or else bring about an asymptotic contraction. But if the densities attain very high values (on the order of 10^{93} g/cm³), quantum fluctuations of the metric will come to the fore [18]

and relativity theory will be invalidated. Elucidation of the problem of just how contraction could become transformed to expansion (if possible at all) in large volumes at a stage when the energy of the gravitational interaction exceeds the proper energy stored in the matter (on the order: $GM^2/R > Mc^2$) will be the object of a separate investigation. We shall not probe into that area here. In analyzing the approach placing contraction ahead of expansion, we shall assume that at the instant when $\rho = \infty$ is attained formally in the solution of the equations, a transition is possible from the solution describing contraction to the solution describing expansion [19]. Clearly, the validity of this joining of the solutions depends on the validity of the underlying assumption above.

From this vantage point the delay is seen to be due to those processes at work in the contraction phase. A concrete example of a model involving contraction followed by expansion and the delay in the expansion of a part of the matter is presented at the end of the article.

We stress the fact that this problem of joining the solutions may be omitted entirely in the hypothesis under consideration, i.e., we need rely only on the first viewpoint expounded above, wherein the processes occurring prior to the instant when $\rho = \infty$ are disregarded, just as in the Fridman cosmology.

Note one other essential circumstance. In non-relativistic physics the spherically symmetrical disintegration of matter proceeds from the state $\rho = \infty$ in such a way that all the particles will fly out from the center. For all particles then the state $\rho = \infty$ is achieved at a single point in space--at the center†† (but at different times, generally speaking). If we were to imagine a part of the matter at first starting out by expanding, and another part held back in its expansion, then, since at $\rho = \infty$ all the particles were at the same place, the delay in the expansion of that part of the matter might be due, say, to an exchange of energy at that critical state. In the general theory of relativity the situation is seen in a fundamentally different way. There always exists, in the case of spherical symmetry, a reference system in which collapse of the entire matter will take place simultaneously, but particles of different values of the radial coordinate will col-

**For detailed calculations in case (3), see references [14,15].

††It is conceivable, of course, that the infinite density may be assigned to some sphere, rather than to the center (the artificiality of such an assignment is obvious). But it is important then that the particles lying earlier on different spheres be collected on a single space sphere.

lapse not only at the one spatial point, at the "center" that is, but also at an infinite distance from each other [20].^{††} As a consequence, any exchange of energy at $\rho = \infty$ is entirely out of the question.

Now we shall assume that there exist in the Fridman universe cores delayed in their expansion. We infer from the arguments adduced above that the cores do not exchange energy with the remaining matter at the time $\rho = \infty$, and that this matter will consequently expand in exactly the same way as in the fully homogeneous model^{***} (the vacuole model [21, 22], see Fig. 1).

The vacuole model is an exact solution of the Einstein equations. The sole requirement imposed on the cores of the vacuoles in this model is that their gravitational masses be equal to the mass which the matter filling the vacuole uniformly with a density equal to the total density of the model would have. This means that the radius r_0 of the vacuole in the Schwarzschild system of coordinates is related to the mass M of the core by the formula $r_0 = (3M/4\pi\rho)^{1/3}$, where ρ is the density of the matter expanding outside the vacuole. Galaxies and stars form subsequently from this expanding matter. It is not excluded that at least the cores of some galaxies will be precisely those parts of the Fridman universe delayed in their expansion. Only after a certain time span has elapsed will these delayed cores again resume their expansion (different cores doing so at different times). As mentioned earlier, the entire matter comprising the core need not necessarily expand all at once. It may be ejected in discrete shells and may flow out continuously.

Consider one such core. We place the origin of the coordinate frame in the core. The customary matter comprising the metagalaxy may be repre-

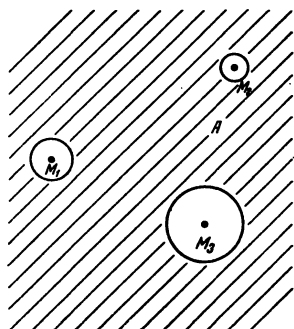


Fig. 1. Vacuole model with delayed cores M_1, M_2, M_3 . A) expanding matter of metagalaxy.

sented as a continuous homogeneous expanding sphere surrounding the vacuole. This matter does not set up any gravitational field in the vacuole, as we know. The evolution of the core matter will therefore proceed independently of the metagalactic matter.

As an example, let us construct the simplest model of this evolutionary process. Our assumption will be that the expansion of the core repeats to exactitude the expansion of the metagalaxy. This is a natural assumption, once we acknowledge that there is no energy exchange between the core and the remaining matter in the initial state (see above). Consider an explosion occurring as a single event, i.e., an expansion of the entire core all at once. The delay in time may be arbitrary.

We now formulate a solution describing this model. Observational data show the density of matter in the metagalaxy to be close to the critical density $\rho = 3H^2/8\pi G$, where H is Hubble's constant, and G is Newton's gravitational constant. This means that the matter comprising the cores will fly apart at a close to parabolic velocity. It is precisely this case which we shall treat here.

When $M \gg M_\odot$ it is no longer important to take the pressure into account so soon after the instant when $\rho = \infty$ (see the preceding footnote). Let $p = 0$. The metric appears in the form

$$ds^2 = c^2 dt^2 - g_{rr}(t,r) dr^2 - R^2(t,r) (d\theta^2 + \sin^2 \theta d\varphi^2).$$

In the case of parabolic motion, Tolman's familiar solution [23] yields

$$g_{rr} = \left(\frac{\partial R}{\partial r} \right)^2,$$

$$R = \left(\frac{3c}{2} \right)^{2/3} F^{1/3}(r) (t - t_0(r))^{2/3},$$

$$\frac{8\pi G}{c^2} \rho = \frac{\partial F}{\partial r} \left| \frac{\partial R}{\partial r} \right|^2 R^2,$$

where $F(r)$ and $t_0(r)$ are arbitrary functions satisfying only the most general requirements imposed on the tensor $g_{\mu\nu}$ in the general relativity theory. We determine F and t_0 as follows:

^{††}One exception is the case of a strictly homogeneous and isotropic collapse, but even in that case an energy exchange would be impossible.

^{***}This does not refer to matter preceded by a discharge wave from the edges of the vacuole. Since the role of pressure falls off rapidly with the expansion, this portion of the matter will be insignificant.

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$$F = \begin{cases} ar^3 & \text{for } 0 \leq r \leq r_1; \text{ core matter;} \\ ar_1^3 & \text{for } r_1 < r < r_2; \text{ vacuum;} \\ a \left(\frac{r_1}{r_2} r \right)^3 & \text{for } r_2 \leq r; \text{ metagalactic matter;} \\ a = \text{const} \end{cases}$$

$$t_0 = \begin{cases} c(r_2 - r_1) & \text{for } 0 \leq r \leq r_1; \text{ core matter;} \\ c(r_2 - r) & \text{for } r_1 < r < r_2; \text{ vacuum;} \\ 0 & \text{for } r_2 \leq r; \text{ metagalactic matter.} \end{cases}$$

Clearly, the matter expands when $r \geq r_2$, as in the homogeneous Fridman model with expansion beginning at $t = 0$. When $0 \leq r \leq r_1$, we also have a Fridman solution, but the onset of expansion will not correspond to the time $t = c(r_2 - r_1)$, i.e., expansion will set in later. The reference frame used in the vacuum consists of free test particles whose velocities tend to zero as $R \rightarrow \infty$.

Figure 2a shows the space-time of this model. Regions containing matter are shaded. The density

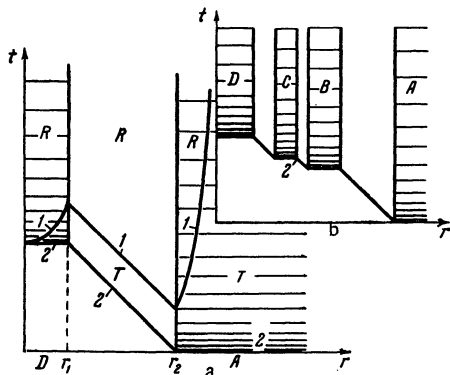


Fig. 2. Space-time of model with vacuole core delayed in expansion. 1) $t = s(r)$, 2) $R = 0$. A) Matter of metagalaxy; B) 1st shell of core; C) 2nd shell; D) core.

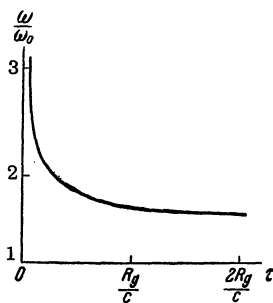


Fig. 3. Observed frequency of light emitted by surface of star expanding beyond Schwarzschild sphere.

of the shading corresponds to the density of the matter present. The $R(t, r) = 0$ line is a singularity in the solutions of the Einstein equations. R denotes regions of space-time in which a reference frame can be introduced where $R = r$, as in ordinary plane space. T denotes regions where this cannot be done, but where a reference frame such that $R = t$ does exist (see [11]).††† In a vacuum the latter regions correspond to the "interior" of the Schwarzschild sphere. The variable $t = s(r)$ is the boundary between R-regions and T-regions. The Schwarzschild sphere serves as the boundary in a vacuum. In the case of successive ejection of discrete shells, the space-time of the model will appear as shown in Fig. 2b. The pattern of expansion of the gravitating sphere beyond the Schwarzschild sphere, as it unfolds to the eyes of the external observer, is described in detail in [13]. Figure 3 plots the time variation of the frequency of light in the case of a ray arriving at the observer from the center of an apparent disk. In a time $\tau = 0.28 R_g/c$ after the arrival of the first rays emerging at the instant expansion of the surface begins from a point, the observer sees rays leaving the surface at the instant the Schwarzschild sphere intersects the surface, and sees the rays at the center of the apparent disk.‡‡‡ The apparent frequency of these rays will be double the frequency at which they were emitted.

As mentioned earlier, the parts delayed in expansion may be accretion cores of the surrounding space. Then, after emerging from under the Schwarzschild sphere, the expanding matter will collide with in-falling matter, and the observer will see processes resulting from those collisions. The matter falling in may have been core matter ejected earlier. Objects which are termed quasars are,

††† In fully homogeneous matter, the division into R-regions and T-regions has meaning only when the origin of coordinates is indicated. In the given case there is no homogeneity throughout all space.

‡‡‡ The grossly erroneous value $1.054 R_g/c$ for τ is given in [13].

from that point of view, the result of a similar process. Clearly, when $M \approx 10^5$ to $10^8 M_\odot$, the energy released will be sufficient to account for the processes at work in powerful radio galaxies.

If the proposed hypothesis corresponds to reality, then the observation of quasars might in principle yield an answer to the question of the state of matter at earlier stages in the expansion of the metagalaxy, since the matter in quasars is repeating that process. In considering the instants close to $\rho = \infty$, when $\rho > \rho_{\text{nuc}}$, the pressure is, of course, not to be neglected. For masses $M \gg M_\odot$ the effect of the boundary of a star can be completely neglected. In fact, the duration of the expansion to a state where nuclear reactions come to an almost complete halt is specified by the formula

$$t = \sqrt{\frac{4.5 \cdot 10^5}{\rho_{\text{nuc}}}},$$

where ρ_{nuc} is a density of the order of nuclear density. Consequently, $t \approx 10^{-5}$ sec. During this time, the discharge from the boundary has time to traverse a distance comprising the following fraction α of the radius of a sphere of mass M_\odot :

$$\alpha = (M / \odot)^{-1/2}.$$

This is proof that the discharge factor may be safely neglected when $M \gg M_\odot$.¹

Let us now examine the possibility of combining the picture of expansion with a delayed central core and the picture of preliminary contraction. This may seem contradictory at first glance. Actually, the combining of contraction and expansion envisages a commencement of expansion immediately following the instant when $R = 0$, $\rho = \infty$ is attained. But the expansion of the core began later than the expansion of the entire metagalaxy. Accordingly, contraction of the core to a point must take place later as well. But the core cannot contract any later than the surrounding space. Figure 4 shows this contradiction expressed by an explosion between $R_1 = 0$ and $R_2 = 0$, by lines of collapse for compression and expansion when $r < r_2$. However, the last assertion is valid only in plane space. In a curved universe contraction of the surface to a point would be possible without the volume bounding the surface tending to zero. Remember that there always exists some reference system in this case in which collapse will take place at the same instant but at different places (in contrast to the classical result!). Clearly, in that system contraction and expansion join along the line $R = 0$, corresponding to $t = \text{const}$, in precisely the same way

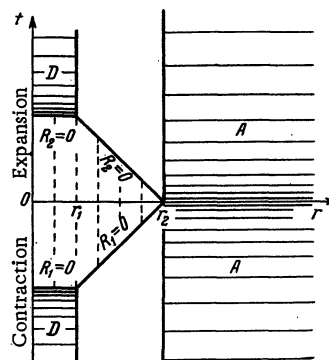


Fig 4. Space-time of vacuole model. Contraction of core and external matter leads to expansion.

as in the Fridman solution. This process corresponds to the identification of the points of identical r on $R_1 = 0$ and $R_2 = 0$ in Fig. 4 (as shown by broken lines), i.e., to the ejection of an entire piece of the set (r, t) intervening between $R_1 = 0$ and $R_2 = 0$.²

Thus, contraction of the core will occur earlier than contraction of the remaining matter, while expansion will occur later, and no contradiction of any kind results.

From this standpoint the reason for the delay in core expansion is the earlier contraction of the cores in the collapse phase, and this in turn is a result of the development of inhomogeneities in the contraction phase. It is possible then that not only quasistellar sources but other inhomogeneities as well which lead to the observed distribution of matter throughout the metagalaxy will appear in the contraction phase. Let us recall once more that no questions involved in the joining of contraction and expansion are obligatory in the proposed hypothesis.

In summary, the author would like to stress two crucial points. First, the arguments advanced share nothing in common with Hoyle's concept [1, 26] of the continuous creation of matter (increase in the baryonic charge in expansion). In this hypothesis the world lines of the baryons do not terminate anywhere. The gravitational action of the cores delayed in expansion remains consistently unaffected.

On the other hand, we note that the proposed hypothesis is viewed as only one of several possible explanations of large-scale catastrophic phenomena

¹Of course, this refers only to the dynamics and composition of the matter; it is precisely the surface layer which is important for observations.

²Note that in this construct we do not encounter any of the paradoxes, as they are termed in [24, 25], in the motion of rays and particles through T-regions.

meriting close scrutiny, along with attempts to account for these phenomena as the result of gravitational contraction of large masses. This hypothesis could also be discussed from a cosmological vantage point as a plausible picture of the evolution of systems like the metagalaxy.

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All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. *Some or all of this periodical literature may well be available in English translation.* A complete list of the cover-to-cover English translations appears at the back of this issue.