ON THE POSSIBILITY OF DETERMINING HUBBLE'S PARAMETER AND THE MASSES OF GALAXIES FROM THE GRAVITATIONAL LENS EFFECT*

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Summary

The gravitational lens effect is applied to a supernova lying far behind and close to the line of sight through a distant galaxy. The light from the supernova may follow two different paths to the observer, and the difference Δt in the time of light travel for these two paths can amount to a couple of months or more, and may be measurable. It is shown that Hubble's parameter and the mass of the galaxy can be expressed by Δt, the red-shifts of the supernova and the galaxy, the luminosities of the supernova "images" and the angle between them. The possibility of observing the phenomenon is discussed.

1. *Introduction.*—In 1937 Zwicky suggested that a galaxy, due to the gravitational deflection of light, may act as a gravitational lens. He considered the case of a galaxy A lying far behind and close to the line of sight through a distant galaxy B. If the line of sight through the centre of B goes through A, the "image" of A will be a ring around B, otherwise two separated "images" appear, on opposite sides of B. The phenomenon has later been discussed by Zwicky (1957) and Klimov (1963), and they both conclude that the possibility of observing the phenomenon should be good. In the present paper the case of a supernova lying behind a galaxy is considered. Two "images" of the supernova may then be seen, and we will show that from one such "double image" observation, Hubble's parameter and the mass of the deflecting galaxy can be determined. The possiblity of observing such a "double image" will be discussed.

2. *Determination of Hubble's parameter and the masses of galaxies.*—We consider a supernova S lying far behind and close to the line of sight through a distant galaxy, B, which will then act as a gravitational lens. For simplicity, we assume

1. The deflecting galaxy is spherically symmetric.
2. The red-shifts of S and B are small.

We can then apply the results previously obtained in the case of a star acting as a gravitational lens (Refsdal 1964). Using the same notation, we have

$$\alpha = \sqrt{\alpha_0^2+\beta^2} \approx \alpha_0\left(1+\tfrac{1}{2}\frac{\beta^2}{\alpha_0^2}\right) \approx \alpha_0 \tag{1}$$

$$\alpha_0 = \frac{4}{C}\frac{\sqrt{G\mathcal{M}}}{\sqrt{na_B}} \tag{2}$$

$$\alpha_1 - \alpha_2 = \beta \tag{3}$$

$$\frac{L_1}{L_2} = \frac{\alpha_1^2}{\alpha_2^2} \tag{4}$$

$$\Delta t \approx na_B\alpha\beta C^{-1}\left(1-\tfrac{1}{8}\frac{\beta^2}{\alpha^2}\right) \approx na_B\alpha\beta C^{-1} \approx na_B\alpha_0\beta C^{-1}. \tag{5}$$

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From (2) and (5) we obtain

$$\Delta t = \frac{16G}{C^3}\frac{\beta}{\alpha_0}\mathscr{M}. \tag{6}$$

With $\beta/\alpha_0 = 0{\cdot}2$ and $\mathscr{M} = 3\times 10^{11}\mathscr{M}_\odot$ we obtain $\Delta t = 55$ days. Due to the rapid change in the magnitude of S, it should then be possible to determine Δt. To be precise, (4) has to be changed to

$$\frac{L_1(t)}{L_2(t+\Delta t)} = \frac{\alpha_1^2}{\alpha_2^2}. \tag{7}$$

Our second assumption involves that the linear distance–red-shift relation is valid.

$$a_B = Z_B C H^{-1}, \quad a_S = Z_S C H^{-1} \tag{8}$$

where Z_B and Z_S are the red-shifts of S and B, respectively, and H is Hubble's, parameter. Hence,

$$n = Z_S/(Z_S - Z_B). \tag{9}$$

From (3), (5), (8) and (9) we obtain

$$H = \frac{Z_S Z_B \alpha(\alpha_1 - \alpha_2)}{\Delta t (Z_S - Z_B)}. \tag{10}$$

From (3) and (6) we get

$$\mathscr{M} = \frac{\Delta t \alpha C^3}{16G(\alpha_1 - \alpha_2)}. \tag{11}$$

We note that H and $\mathscr{M}$ depend only on observable quantities. The quantity most difficult to determine experimentally seems to be $\alpha_1 - \alpha_2$, because α_1 and α_2 are nearly equal. Using (7), and noting that $\alpha_1 + \alpha_2 = \alpha$, we get

$$\beta = \alpha_1 - \alpha_2 = \alpha\,\frac{\sqrt{L_1/L_2} - 1}{\sqrt{L_1/L_2} + 1}. \tag{12}$$

By L_1/L_2 we understand $L_1(t)/L_2(t+\Delta t)$. Equations (10) and (11) may now be written as

$$H = \frac{Z_S Z_B \alpha^2}{\Delta t (Z_S - Z_B)} \cdot \frac{\sqrt{L_1/L_2} - 1}{\sqrt{L_1/L_2} + 1}. \tag{10 a}$$

$$\mathscr{M} = \frac{\Delta t C^3}{16G}\,\frac{\sqrt{L_1/L_2} + 1}{\sqrt{L_1/L_2} - 1}. \tag{11 a}$$

Most galaxies are far from spherically symmetric, and corrections for this may be necessary. To do this, the angular mass distribution in B must be known. We suppose these corrections may be easily carried out in the case of elliptical galaxies because of their symmetry. As the average mass of elliptical galaxies is believed to be greater than for other types of galaxies, the elliptical galaxies are best suited for our purpose. Another possible error will be the scattering or absorption of the light from S while passing B. It is reasonable to believe that the fractional reduction of L_2 will be greater than for L_1, because ray 2 passes nearer to the centre of B than ray 1, giving a greater value of L_1/L_2. Due to the selective character of this effect, L_1/L_2 will depend on the frequency. Corrections could be estimated if observations at different frequencies can be carried out.

3. *Possibility of observing the effect.*—We shall consider only supernovae of type 1, which have an absolute magnitude $\mathcal{M} = -16$. They are the brightest type of supernovae, occurring on the average about once a year per 500 galaxies. The average number of galaxies per unit volume is about 0·03 Mpc^{-3} (Allen 1963). Masses and diameters of galaxies show a great spread, but for simplicity we assume all galaxies to have a mass $\mathcal{M} = 3 \times 10^{11} \mathcal{M}_\odot$, and a diameter $D = 6000$ pc. In Table I the number of galaxies $N(a)$ within a distance a from us is given for different values of a. Choosing $a_S = a$ and $a_B = 0{\cdot}5a$ we get $\alpha_0 = \alpha_0(a)$ as given in Table I. $\alpha_0(a)$ is roughly equal to the mean value of α_0 for supernovae within the distance a. $u(a)$ is the angular radius of a galaxy at the distance $0{\cdot}5a$, and $m(a)$ is the apparent magnitude of a supernova of type 1 at the distance a. For small β $m(a)$ may, due to the lens effect, decrease by several magnitudes.

TABLE I

a (pc)	$m(a)$	α_0 (sec of arc)	$2u$ (sec of arc)	$N(a)$	$P(a)$
10^8	19	10	25	$1{\cdot}3 \times 10^5$	0·43
3×10^8	21·4	5·7	8·2	$3{\cdot}4 \times 10^6$	100
6×10^8	22·9	4·1	4·1	$2{\cdot}7 \times 10^7$	3400
10^9	24	3·1	2·5	$1{\cdot}3 \times 10^8$	43000
3×10^9	26·4	1·8	0·82	$3{\cdot}4 \times 10^9$	10^7

In the previous paper, (Refsdal 1964), it was shown that L_2 decreases rapidly as β increases. Thus, we will only be interested in cases for which $\beta < 0{\cdot}3\alpha_0$, giving $L_1 > 1{\cdot}44 L_N$ and $L_2 > {\cdot}044 L_N$. Among $N(a)$ galaxies there will exist a certain number of pairs whose angular separation is smaller than $0{\cdot}3\alpha_0(a)$. $P(a)$ is the expected number of such pairs, a random distribution of the galaxies being assumed. This assumption is reasonable because pairs for which both galaxies belong to the same cluster are of no interest in this case, due to their small mutual distance.

We have assumed the light rays from S to pass outside the deflecting galaxy, and thus we require $\alpha_2 > u$, which, by using (3) and noting that $\alpha_1 + \alpha_2 = \alpha$, gives

$$\alpha - \beta > 2u. \tag{13}$$

In the case of $\beta = 0$ we see from Table I that equation (13) requires

$$a_S > a_S(\text{min}) = 6 \times 10^8 \text{ pc}, \tag{14}$$

and consequently

$$\alpha_0(a) < \alpha_0(a_S(\text{min})) = \alpha_0(\text{max}) = 4.1''. \tag{15}$$

Masses and diameters of galaxies show a great spread, and it is therefore interesting to notice that $a_S(\text{min})$ and $\alpha_0(\text{max})$ are proportional to $\mathcal{M}^{-1}D^2$ and $\mathcal{M}D^{-1}$, respectively. The galaxy M87 NGC 4486, (Allen 1963), is believed to have a mass $4 \times 10^{12} \mathcal{M}_\odot$ and a diameter $1{\cdot}3 \times 10^4$ pc. For a galaxy of this type we get $a_S(\text{min}) = 2 \times 10^8$ pc. and $\alpha_0(\text{max}) = 25''$.

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For $a = 6 \times 10^8$ pc $P(a)$ is equal to 3400, giving about 7 supernova–galaxy pairs per year. Let us roughly estimate that (13) will be fulfilled for 5 per cent of these pairs, so that a "double image" of a supernova within a distance 6×10^8 pc should be possible to observe every third year. The apparent magnitude of the "images" will be 21–23 for S_1 and 21–24 for S_2. For greater values of a $P(a)$ increase rapidly, in fact it is proportional to a^5.

It is possible that our estimate of the mass of galaxies is too great. If we consider only elliptical galaxies as possible gravitational lenses, the estimate of $\mathscr{M}$ is perhaps more realistic. About 20 per cent of all galaxies are elliptical, and the expected number of observable "double images" will be reduced by 80 per cent.

Star-like objects with intense emission both in the radio range and the optical range have recently been discovered (Greenstein 1963). Their absolute visual luminosity are of order −24, and it is possible that flashes occur in the optical region, lasting about one month, and with an amplitude about $0{\cdot}5^{\mathrm{m}}$.

If so, observations at greater distances than with supernova will be possible. The distances may be so great that we can no longer assume Z to be small. The result of our calculations will then depend on the cosmological model we choose, giving a possibility of testing the different models. This will be discussed in a subsequent paper.

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