

BINARY STARS AMONG CATAclysmic VARIABLES. V. PHOTO-ELECTRIC AND SPECTROSCOPIC OBSERVATIONS OF THE ULTRA-SHORT-PERIOD BINARY NOVA WZ SAGITTAE

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ABSTRACT

A new model for recurrent Nova WZ Sagittae is advanced based on approximately 3000 ultraviolet photoelectric observations made at Lowell and at Lick, and several nights' spectroscopic runs with the prime-focus spectrograph of the Hale reflector. A simultaneous photoelectric and spectroscopic run over $2\frac{1}{2}$ periods ($P = 0.05668786$ days) on August 4, 1962 (U.T.) establishes that the W UMa-type eclipse curves and the radial velocities from the hydrogen emission lines are approximately 90° out of phase. Maximum velocity of recession occurs near principal minimum. The new model attributes this effect to a rapidly receding stream ejected from the dark star toward the primary component. The properties of the radiation field of the white-dwarf primary are derived from tracings of $H\gamma$ in absorption and are shown to be consistent with the observed geometrical extent of the line-emitting regions.

The best present estimates of the geometrical and dynamical parameters of the system are: $M_1 = 0.59 M_\odot$, $M_2 = 0.03 M_\odot$, $M_2/M_1 = 0.05$, $i \sim 82^\circ$, $R_1 = 0.87 \times 10^9$ cm, $R_2 \sim 7 \times 10^8$ cm, $a = 3.7 \times 10^{10}$ cm, where the subscript 1 refers to the white-dwarf primary. The numbers are rather uncertain, but there is little doubt that the mass ratio is unusually small, though some of the observational details remain poorly explained. However, the mass of the secondary is similar to that of L726-8B, in which case the value of the radius derived from the eclipse data agrees with the radius of L726-8B.

The proposed emission of gravitational waves is such that the gain in phase is 1 sec in 12 years. It is likely that mass transfer and mass loss during outbursts of WZ Sge severely mask the period changes resulting from the emission of gravitational waves.

I. INTRODUCTION

In Paper III of this series (Kraft 1964), it was suggested that all old novae are close binaries, consisting of a blue subdwarf and a red star which overflows its lobe of the inner zero-velocity surface defined by the restricted problem of three bodies. As a consequence of the overflow through the inner Lagrangian point, material would be expected to leave the red star and take up an orbit in a ring or disk surrounding the blue component, in the manner of Kuiper's (1941) ejection of "Type A." Under plausible assumptions, the model predicts that widened or doubled emission lines should be associated with systems seen edge-on, i.e., systems expected to eclipse.

Of the old novae, doubled emission lines have been found in the spectra of DQ Her and WZ Sge; both have been shown to eclipse (Walker 1954; Krzeminski 1962). Among U Gem variables, doubled lines are found in the spectra of U Gem and Z Cam (Kraft 1962, Paper I); the former has very pronounced eclipses (Mumford 1964, Paper IV), and the latter, shallow ones (Krzeminski and Mumford, private communication). Despite the gathering of considerable photoelectric material by Walker, Mumford, and Krzeminski, no other stars of these two groups have been found to be eclipsing binaries, except Nova T Aur (1891) (Walker 1963). On the rather underexposed spectrograms obtained recently by Kraft, the emission lines of H and He II, which are very wide, are so feeble that it is not possible to judge with certainty whether they are doubled.

Among stars spectroscopically similar to old novae, but having no known major outbursts, both EM Cyg and EX Hya are eclipsing binaries; their emission lines are conspicuously doubled. WZ Sge and EX Hya are the eclipsing binaries of shortest known periods; the former will be rediscussed in this paper. Simultaneous photoelectric and spectroscopic observations now permit the development of a model that avoids the objections set forth in Paper III. It seems likely on the basis of this model, that WZ Sge radiates gravitational waves too inefficiently to be of interest as a test for the physical reality of such waves (Kraft, Mathews, and Greenstein 1962, Paper II).

II. PHOTOELECTRIC OBSERVATIONS

The 42-inch Lowell reflector was used in all observations except those of September, 1961, which were obtained with the Lick Observatory 36-inch Crossley reflector. A 1P21 photomultiplier was used throughout behind 3 mm of Corning 9863 at Lowell and 2 mm Schott UG-2 + 2 mm Corning 7910 at Lick.

Unfortunately, WZ Sge has a red optical companion about 7" west which is visually 1 mag. brighter than the nova, and which must be scooped into the diaphragm. The nova, however, is 2.8 mag. brighter than the companion in the ultraviolet, and therefore observations were obtained almost exclusively in the ultraviolet. The companion has $V = 14.27$, $B - V = +1.49$, and $U - B = +1.45$, values very kindly obtained by

TABLE 1
MAGNITUDES AND COLORS OF COMPARISON
STARS OF WZ SAGITTAE

Star (BD)	V	$B - V$	$U - B$	n
A = +17°4228. . . .	9.72	+1.03	+0 76	2
B	11.84	+0.39	+ .22	2
C = +17°4225.	8.76	+0.16	+0 13	6

Dr. Olin J. Eggen on one night at the 200-inch telescope. The ultraviolet observations, about 3000 in number, corrected for the presence of the companion, will be published elsewhere (Krzeminski 1964). In some of the sample photometric runs reproduced here in the figures, the ultraviolet magnitudes are expressed in terms of the magnitude difference WZ Sge *minus* comparison star C = BD+17°4225, reduced to no atmosphere. The observations have been left on the instrumental system of the particular telescope and filters; allowance was made for the difference in color of the variable, comparison star, and optical companion in the reduction. In Figure 1 the field of the comparison stars is shown, and in Table 1 the colors and magnitudes of these stars are given. The constancy of star C was checked against stars A = BD+17°4228 and B, which served also as extinction stars. The probable errors of the values given in Table 1 for star C are $V = \pm 0.007$, $B - V = \pm 0.006$, and $U - B = \pm 0.006$ mag.

The magnitudes and colors of the optical companion were checked twice at Lowell, and satisfactory agreement with Eggen's values was obtained. However, nothing can be said about the night-to-night constancy of this star, and if it were variable, the ultraviolet magnitudes assigned to the nova could be slightly affected. Over the period of the observations, it was found that the nova gradually faded (Fig. 2), but the rather improbable possibility that the average brightness of the nova remained constant and the optical companion decreased in brightness cannot be ruled out. However, random changes in ultraviolet brightness of the companion by as much as 0.5 mag. will account for only 20 per cent of the observed scatter in Figure 2.

According to proper motions measured by Luyten (private communication), the companion is purely optical and not physical.

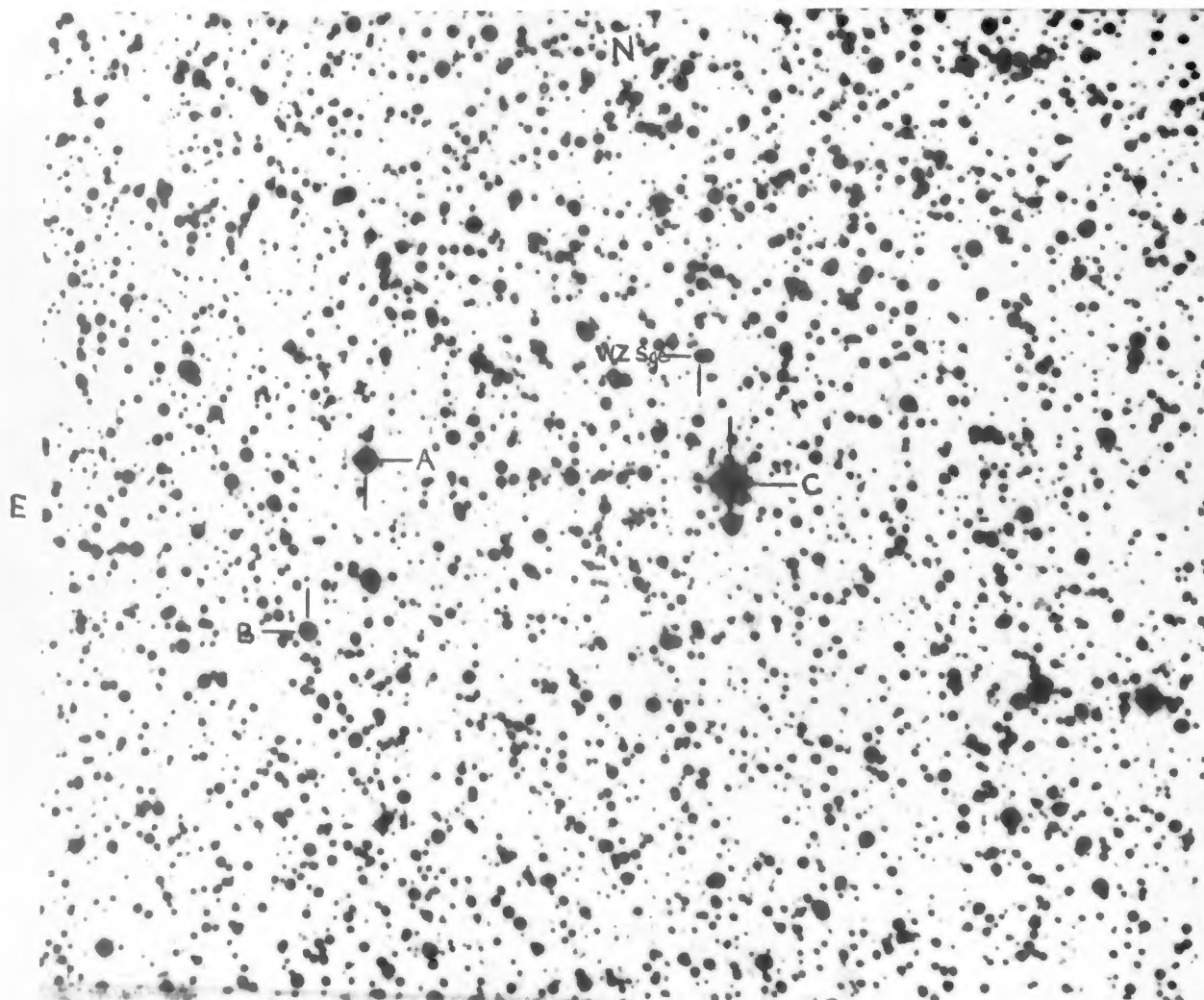


FIG. 1.—Field of Nova WZ Sge. The print is a copy of a portion of the Palomar Sky Survey (copyright National Geographic Society–Palomar Observatory Sky Survey).

The observations secured on two nights, August 4 and October 1, 1962 (U.T.), are shown in Figure 3 as an example. On August 4 a simultaneous spectroscopic run was also obtained (Sec. V). In general, the light-curves reproduce features previously described (Krzeminski 1962), namely, asymmetry of the primary minimum, dips around phase 0.25P (from primary minimum), shifted secondary minimum, and erratic intrinsic fluctuations ("flickering"). No extremely short periodic variations of the DQ Her type (Walker 1956, 1961) were found. The depth of primary minimum varies by nearly 0.5 mag., and it is of interest that the extreme range of depth was displayed within a few hours on the night when spectroscopic results were also secured.

III. THE PHOTOMETRIC PERIOD

The times of primary minimum are listed in Table 2 and were based on the ultraviolet observations, except the three obtained on JD 2437547 which were derived from observations in blue light. These were corrected for the presence of the optical companion.

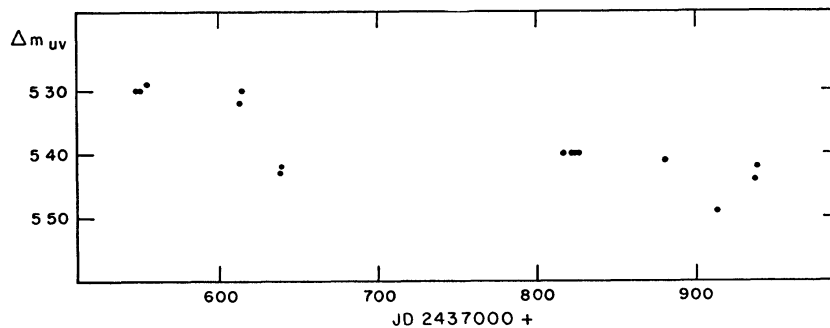


FIG. 2.—Change in average brightness of WZ Sge outside eclipse. The secular decrease shown here (ignoring the "dips") might result from a change in brightness of the optical companion, but more likely reflects a real change in the nova.

TABLE 2
OBSERVED TIMES OF PRIMARY MINIMA OF WZ SAGITTAE

JD $\odot$ 2437000+	O-C Day	JD $\odot$ 2437000+	O-C Day
547 7284*	0 0000	821 81325	-0 00094
547 78470	- 00038	821 87081	- 00007
547 8416*	- 0002	821 92774	+ 00017
548 7488*	0000	823 91182	+ 00018
549 7127*	+ 0002	825 89569	- 00003
549 76870	- 00045	880 71279	- 00009
549 82560	- 00024	880 76937	- 00020
549 8827*	+ 0002	880 82631	+ 00005
554 70090	- 00010	912 74124	- 00028
554 75789	+ 00021	912 79831	+ 00010
554 81447	+ 00010	937 62773	+ 00024
613 65631	- 00006	937 68399	- 00019
614 62077	+ 00071	937 74104	+ 00017
638 59885	- 00018	937 79766	+ 00011
639 61995	+ 00054	938 64795	+ 00008
816 82594	+ 00028	938 7039*	- 0007
816 88279	+ 00044	938 76104	-0 00021
816 93918	+0 00014		

* Lower weight; times of minima have not been used in deriving the period

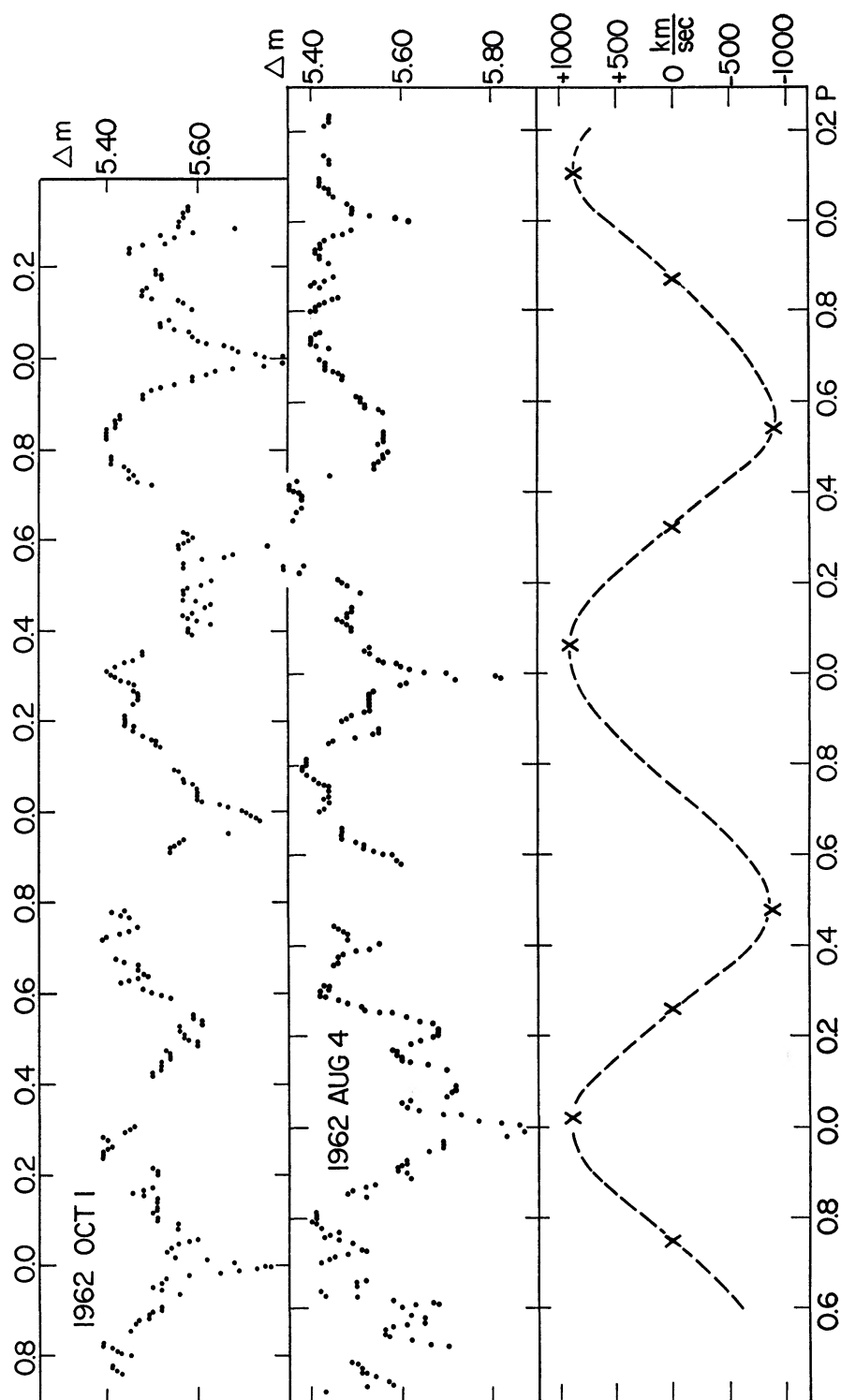


FIG. 3.—Photoelectric observations of October 1, 1962 (U.T.), and simultaneous photoelectric and spectroscopic observations of August 4, 1962 (U.T.). Heliocentric phases are computed from the equation given in Section III. The radial velocity-curve is a free-hand representation of the estimated times and velocities corresponding to maximum, minimum, and mean radial velocity.

In Table 2 we list also the residuals obtained from the elements: Primary Min. = $\text{JD}_{\odot} 2437547.72839 + 0.05668786 E$. The primary minima of WZ Sge are sharp and narrow. The relatively large scatter in the residuals of Table 2 results from distortion of the light-curve. As we will show in Section VI, the eclipse is, at least to some extent, that of a gaseous stream by a dark body, and the moment of minimum would therefore scarcely be expected to remain very stable.

IV. THE LIGHT-CURVE

Because the light-curve for each cycle is essentially unique, a composite light-curve was constructed (Fig. 4) to show better the general behavior of the star. Five nights were assembled, November 10 and 11, December 5 and 6, 1961, and June 6, 1962. Owing to the secular changes in brightness of WZ Sge (Fig. 2), the average level of brightness around phases 0.25P and 0.75P were determined each night, and all observations have been fitted to the level $\Delta m = +5.40$. The individual light-curves show clearly the effect of a "dip" in light centered around phase 0.25P, and the composite light curve has been fitted at phase 0.25P to the level $\Delta m = +5.40$, omitting the dip. These dips shift back and forth in phase, and are catalogued in Table 3. The composite light-curve shows the effects of the dips very clearly; they occur between primary and secondary minimum, but not at opposite quadrature.

TABLE 3
TIMES OF MINIMA OF DIP AND ITS DEPTH

$\text{JD}_{\odot}$ 2437000+	Phase	Depth (mag)	$\text{JD}_{\odot}$ 2437000+	Phase	Depth (mag)
548 7627.....	0 246	0 09	821 8848	0 246	0 16
549 7281. . . .	.276	.17	821 9405 .	.228	.19
549 7846	.272	.10	823 9268 . .	.267	.16
554 7212	.356	.15	825 9173 .	.381	.10
554 7776	.351	.11	880 7252	.217	.16
613 6740	.311	.21	880 7795	.175	.13
614 6337. . . .	.240	.18	937 6402 .	.224	.09
638 6116	.222	.16	937 7014 .	.304	.07
639 6284	.159	.15	937 7545 .	.240	.11
816 8444	.331	.21	938 6590 .	.196	.08
816 8986	.287	.17	938 7195	0 264	0 05
821 8295	0 270	0 22			

Figure 4 indicates that the light-curve is of the W UMa type, and is quite symmetrical. If normal points are formed (omitting the aforementioned dips), the resulting light-curve reveals three important features: (1) the W UMa-type character of the changes in brightness; (2) the secondary minimum is symmetrical and flat-bottomed but is not centered between successive primary minima, i.e., the phase of secondary minimum is $0.544P \pm 0.003$ (p.e.), as determined from twenty-five minima; (3) the branch of the light-curve between 0.04P and 0.54P may be reflected about the mid-point of the secondary eclipse and fitted perfectly into the branch beyond 0.54P up to phase 0.80P; the later descending branch of the light-curve is much steeper. It is interesting to note that, if the reflected branch were simply continued beyond the principal minimum, one would "manufacture" a synthetic classical W UMa-type light-curve, with a shallower principal minimum at phase 0.044P.

V. RADIAL VELOCITIES AND SPECTRUM

The basic character of the spectrum photographed on August 3 and August 4, 1962 (U.T.), remains qualitatively the same as that described in Papers II and III, including the stationary white-dwarf absorption lines of hydrogen first observed by Greenstein

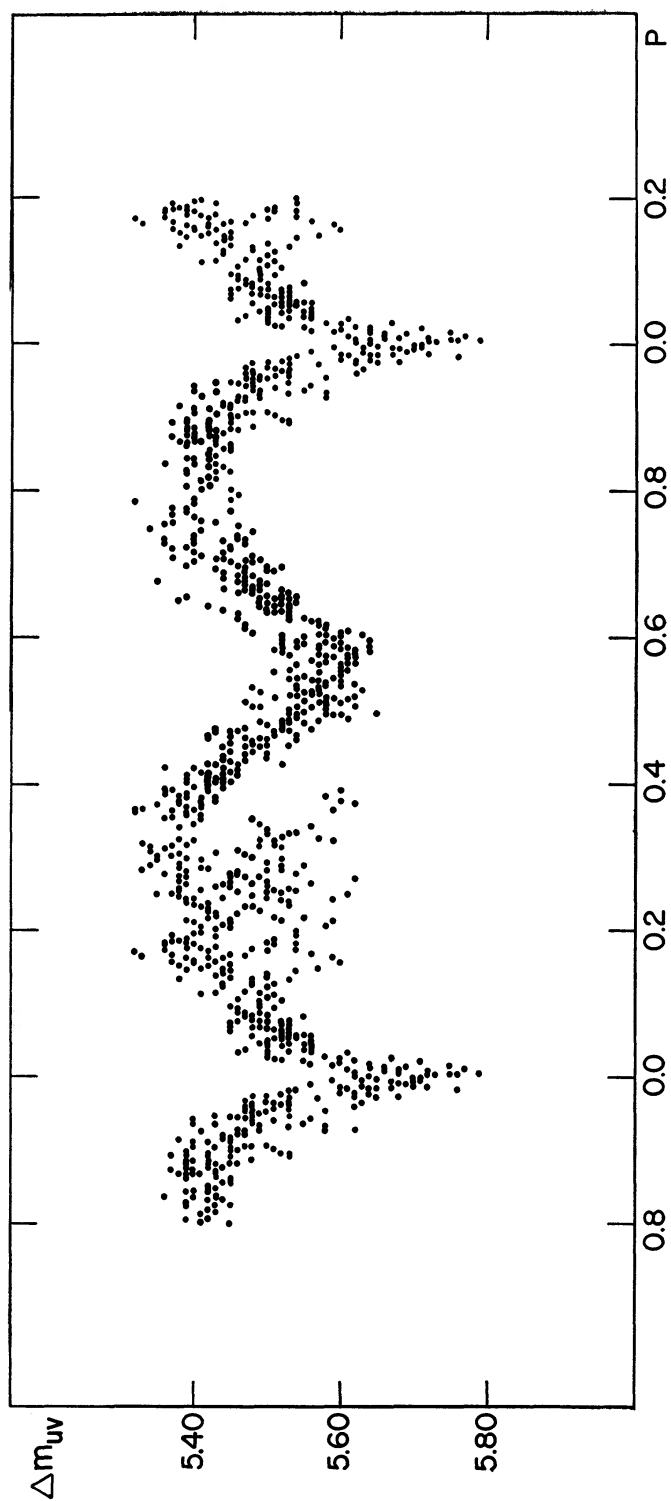


FIG. 4.—Composite light-curve of WZ Sge. Observations from five nights were assembled (see text). Note the W UMa-type light-curve, except for the dips centered around $\phi = 0.25P$.

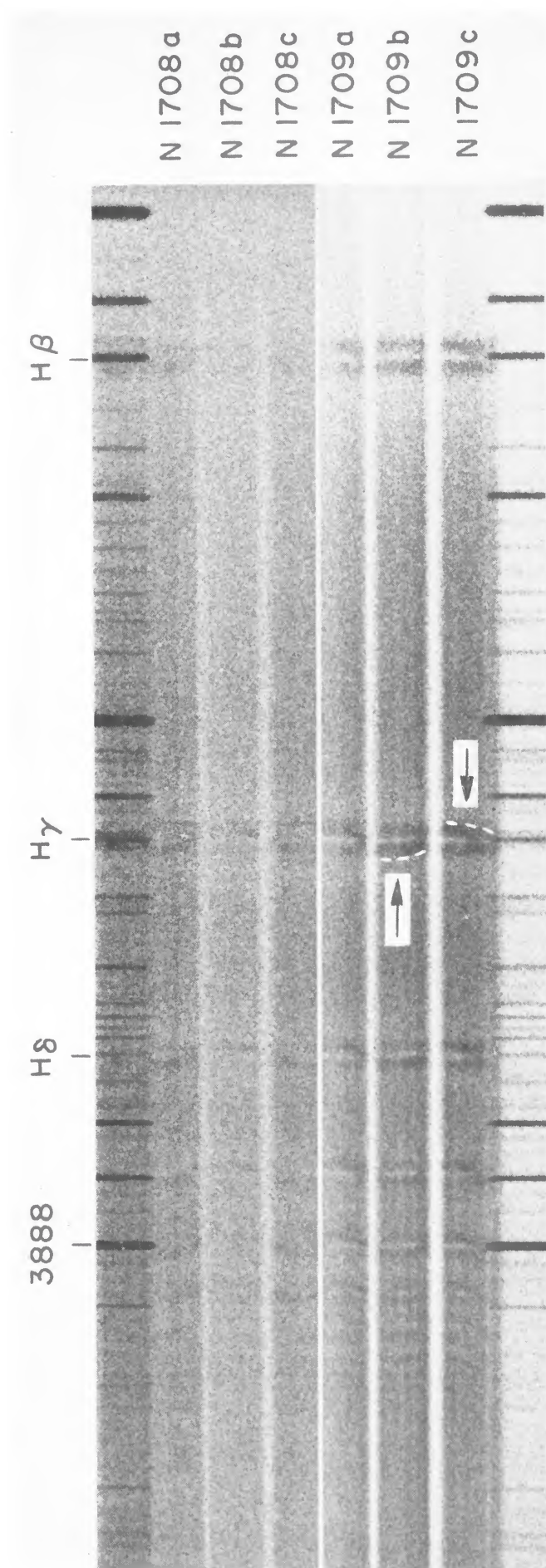


FIG. 5.—Spectrograms of WZ Sge, taken August 4, 1962 (U.T.). Time runs downward; the spectrograms are continuously trailed. Arrows show the excursion of the S-wave beyond the mean separation of the stationary emission components. This is less noticeable in the reproduction (though clearly visible on the original spectrograms) on the shortward (*left*) side (N1709b) than on the longward side (N1709c) because the S-wave emission region is almost totally eclipsed at secondary minimum. The faint narrow line longward of H γ in N1708 is due to mercury vapor in city street lights.

(1957), the stationary double hydrogen emission, and the *S*-wave emission first observed by Kraft (1961) when the star is trailed continuously over a long slit. The two spectrograms of August 4, obtained simultaneously with the photoelectric run, are reproduced in Figure 5. Because of the unusual method by which the spectrograms were obtained, we have measured only the radial velocity of the *S*-wave at its extremes, and estimated the times corresponding to maximum, minimum, and mean. We have not attempted to measure the velocities and times in (say) a two-coordinate measuring machine because the time coordinate depends entirely on the linearity of the telescope drift rate, a difficult matter to control at best. The critical moments are listed in Table 4 for August 4, and are plotted in Figure 3. This material, considered together with the body of spectroscopic data obtained earlier, leads to the following conclusions: (1) The mean spectroscopic period of the *S*-wave, averaged over all cycles, is 82 ± 3 (m.e.) min. This is not significantly different from the photometric period of 81.6 min. (2) The separation of the "stationary" emission components of $H\beta$ and $H\gamma$ measured on all adequately exposed plates, is, within the errors, constant and equal to 1440 ± 20 (m.e.) km/sec. The constant mean velocity is -25 ± 20 (m.e.) km/sec. It is assumed that this is the radial

TABLE 4
MAXIMUM, MINIMUM, AND MEAN RADIAL VELOCITY ON
AUGUST 4, 1962 (U.T.)

MAXIMA		MINIMA		MEAN	
U T.	$\phi(P)$	U T.	$\phi(P)$	U T	$\phi(P)$
5:02 . .	0.025	5:39 .	0 478	4:40	0 757
6:26	053	7:06 .	0 543	5:21	.258
7:52	0 108			6:48	323
				7:33 .	0 875

velocity of the star in space. (3) The amplitude of the *S*-wave is not constant, as earlier believed, but varies from about 650 km/sec on plate N1440 (July 14, 1961 U.T.) to about 850 km/sec on plate N1709 (August 4, 1962 U.T.) (see Fig. 5). The fact that this amplitude is variable and is *not* always equal to half the separation of the stationary emission components (viz., 720 km/sec) is the most important clue pointing to the need for revision of the model developed in Paper II. (4) Figure 3 shows that the *S*-wave is about 90° out of phase with respect to the eclipse data, in the sense that maximum and minimum velocities occur near principal and secondary minima, respectively, and systematic velocity is reached near photometric elongation. The descent from maximum to minimum velocity takes slightly less time than the ascent from minimum to maximum, indicating that the formal orbit is somewhat eccentric. On the average, the descent takes only 0.45P. In our opinion, the quotation of a formal eccentricity does not seem very meaningful in view of the variation in amplitude and the uncertainty of the time coordinate. There is a slight indication from the phasing of Figure 3 that maximum velocity is delayed slightly after principal minimum by 0.06P on the average. (5) The white-dwarf absorption spectrum is constant in velocity and line strength, so far as can be seen from inspection of the plates and measurement of microphotometer tracings at several critical phases on August 4, 1962 U.T.

In order to investigate changes in the emission-line intensities as a function of ultraviolet brightness, microphotometer tracings have been made during the first and third principal eclipses of the lower part of Figure 3, and the photometric elongation centered at 4:40 U.T., August 4. These are compared with Greenstein's (1957) estimates in Table 5, based on a plate guided in the conventional manner. We have also subtracted out the

underlying white-dwarf profile by selecting from Greenstein's (1960) computed hydrogen-line profiles ones which fitted best the wings of the hydrogen lines in WZ Sge. Inspection of Table 5 shows that the strengths of the hydrogen emission lines are fairly similar at elongation and during the shallow principal eclipse, though it must be admitted that the latter lasts for so short a time (~ 5 min.) that its time resolution is really quite doubtful. The numbers are somewhat larger than those given by Greenstein. However, there is no doubt that the emission lines decline in strength by about a factor of 2 relative to the white-dwarf continuum during the deep principal minimum. Indeed both the *S*-wave component and the double emission appear to be cut down, to judge from the relative intensities of the longward and shortward components of the latter.

Inspection of plates N1708 and N1709 (Fig. 5) also reveals three additional facts about the *S*-wave: (1) At secondary eclipse, only the *S*-wave is cut down, the double emission remaining apparently unaffected. This is at once obvious on plate N1709b, where the unusually large shortward excursion of the *S*-wave takes it beyond the shortward stationary component. (2) The *S*-wave achieves its maximum brightness near photometric elongations (i.e., apparent spectroscopic conjunctions). (3) At the shallow eclipse of 7:40 U.T., the *S*-wave is cut down much less than at the deep minimum of 5:00 U.T.

TABLE 5
STRENGTHS OF HYDROGEN-EMISSION LINES IN WZ SGE

EVENT	PLATE No	E W (Å)				AUG 4, 1962 U T
		H β	H γ	H δ	H ϵ	
Photometric elongation	N1708a	45:	21	12		$\sim$ 4:40
Principal eclipse (deep)	N1708b	23:	13	10:		$\sim$ 5:00
Principal eclipse (shallow)	N1709c	40	22	16		$\sim$ 7:40
Greenstein (1957)	N418	35	16	7:	5:	..

VI. INTERPRETATION AND DISCUSSION

a) Geometry of the Eclipse

WZ Sge is unlike any other eclipsing nova (Paper III) or dwarf nova (Kraft 1962) in that the velocity-curve is 90° out of phase with respect to the light-curve. The mass of gas responsible for the *S*-wave emission cannot share directly the dynamical motion of either star in the system. In an earlier treatment (Paper II), we had supposed that, in view of the absence of apparent velocity changes either for the double emission or for the white dwarf, the mass of the latter was large compared with the companion. The *S*-wave was assumed to arise in a "lump" rotating with the same velocity as the ring producing the double emission. The difficulty then arose that, if the mass of the lump were assumed to be infinitesimal with respect to the white dwarf, the *minimum* mass of the white dwarf turned out to be $1.62 M_\odot$, for $P = 81.6$ min. and $K \sim 650$ km/sec. This is already $\frac{1}{2} M_\odot$ larger than the theoretical upper mass limit for electron degenerate stellar configurations without interior hydrogen (Hamada and Salpeter 1961).

The new observational material indicates that this picture need not be correct. The fact that the amplitude of the *S*-wave is sometimes larger, sometimes smaller, than half the separation of the stationary emission components suggests that the *S*-wave is not a part of the rotating ring at all. However, the fact that (1) both are cut down during the deep principal minimum of August 4 and (2) the Balmer decrements of the stationary double and *S*-wave components of emission are the same within the errors implies that

the regions of formation are not widely separated. The rotation period of the ring could be much longer than 81.6 min., thus leading to a more comfortable mass for the white dwarf.

Now the light-curves refer to an ultraviolet region, one in which the Balmer lines and Balmer jump contribute a substantial fraction of the total light. We will show that the eclipses, as observed, result from the occultation not only of stellar surfaces but of line emitting gaseous regions as well. We postulate therefore the following model for WZ Sge:

1. The object is a binary star of rather large mass ratio, with the white dwarf the more massive component. The dark star fills its lobe of the inner zero-velocity surface defined by the restricted problem of three bodies. The dark star has a radius much larger than the radius of the white dwarf.

2. A strong stream, with variable ejection velocity (~ 600 to 900 km/sec) flows out of the red star at L_1 toward the white dwarf. This stream, the velocity of which exceeds greatly the relative velocity of the stars, gives rise to the S-wave. When the stars are approximately in conjunction, the stream has its maximum velocity of recession or approach.

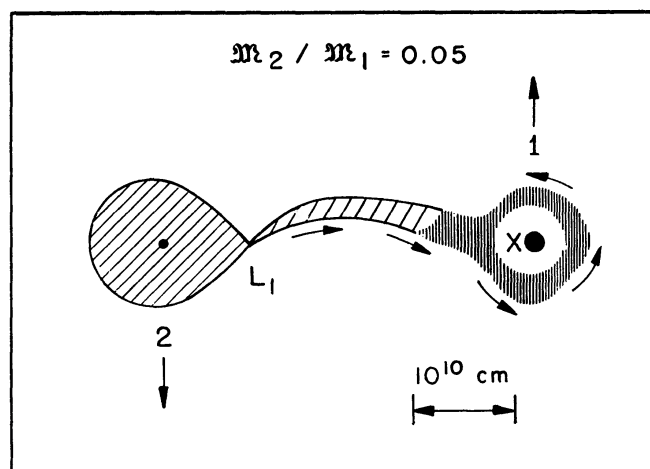


FIG. 6—Schematic model of WZ Sge (from above). The rotation is counterclockwise and "X" marks the center of mass. Gaseous material ionized by the white-dwarf radiation is marked with vertical hatching.

3. The white dwarf is surrounded by a rotating ring, the material of which is supplied from at least a portion of the stream ejected from the dark star.

4. Principal eclipse consists of the covering of the rotating ring and the ejected stream by the dark star. The white dwarf itself, as we shall demonstrate, is only partially eclipsed. At secondary eclipse the white dwarf passes in front of the ejected stream. The rotating ring has little or no continuous opacity, and has therefore no effect when it passes before the ejected stream. Thus, while the dark star is actually eclipsed at secondary minimum, it gives no light in any case and serves only as a source for the ejected stream projected before it. A purely schematic drawing of this geometrical picture is shown in Figure 6.

b) Properties of the White Dwarf

If the above rather complex picture, which seems to be required by the eclipse data, is correct, it must be internally consistent with the properties of the radiation field that can be produced by the white dwarf. If this were not so, we would be forced to invoke excessive ultraviolet radiation to support the ionization in the ring and stream, or perhaps collisional or other non-thermal processes.

Weidemann (1963) has shown that the profile of $H\gamma$, together with model atmosphere considerations, can be used to obtain the effective temperature T_e and surface gravity g of a white dwarf. Because of the mass ($\mathcal{M}$)-radius (R) relation for electron degenerate stellar configurations, the mass and luminosity can be determined as well.

In Table 6 we list the $U - B$, $B - V$ colors and hydrogen-line strengths for WZ Sge and several hydrogen (DA) white dwarfs as tabulated by Greenstein (1958, 1960); the colors of the white dwarfs were taken from the work of D. L. Harris. Unfortunately, we do not know the colors of WZ Sge at the epoch of our observations, for reasons already cited. Walker (1957) gives $U - B = -0.71$, $B - V = +0.09$. If this is typical we would have $B - V \sim 0.15$ if the contribution of the hydrogen emission lines were removed from the B filter. However, with this $B - V$, comparison with the other white dwarfs suggests $T_e \sim 10000$ – 15000° K range, and the electron temperature of the postulated rotating ring would probably be $\sim 10000^\circ$ K. Since the Balmer jump in emission appears to be present in the spectrum, a small contribution to the B and V filters is

TABLE 6
PROPERTIES OF DA WHITE DWARFS AND WZ SGE
(After Greenstein 1958, 1960; Weidemann 1963)

Name	$(B - V)$	$(U - B)$	$W(H\gamma)$	θ_e	$\log g$	$-\log R/R_\odot$	$\mathcal{M}/\mathcal{M}_\odot$
+82°3818	−0 02	−0 72	24 4	0 32	8 1	1 93	0 66
+73°8031	+ 01	− 66	37 3:	36	8 0	1 90	59
40 Eri B	+ 03	− 70	31 7	39	7 5	1 77	34
W485	+ 06	− 62	31 9	37	8 0	1 90	59
SA29-130	+ 07	− 54	39 4	43	8 0	1 90	59
L1512-34B	+ 17	− 58	39 6	38	8 35	2 00	82:
R808	+ 20	− 56	30 6:	52	8 5	2 04	90
L1491-27	+ 22	− 58	25 3:	0 52	8 25	1 97	0 76
WZ Sge (1957)	+0 09	−0 71	27 1:				

made by the Paschen continuum (cf. Kraft 1959), perhaps 5–10 per cent, and this in turn will make the Balmer lines in absorption appear slightly too weak. From our tracings outside eclipse, we estimate that the $H\gamma$ profiles, for $\Delta\lambda > 30 \text{ \AA}$ and $\Delta\lambda < -30 \text{ \AA}$ from the line center, are nearly the same as R808 or L711-10 (Table 5, and Greenstein 1960); the mean of $W(H\gamma)$ for these two stars is $28.4 \text{ \AA}$, thus confirming Greenstein's result for WZ Sge. However, with a small contribution of Paschen continuum, the true $H\gamma$ is stronger than this and we shall adopt for WZ Sge in what follows the parameters for W485. Thus we take $B - V = +0.06$, $\theta_e = 0.37$, $T_e = 13600^\circ$ K, $\log R/R_\odot = -1.90$, $\mathcal{M}/\mathcal{M}_\odot = 0.59$, $M_v = +11.2$, $M_{\text{bol}} = +10.4 \mu_e = 2$ (Weidemann 1963). The effective temperature seems compatible with the absence of bright He II, with the occasional presence of feeble bright He I ($\lambda 4471$) near photometric elongation, and the great strength of the hydrogen emission. The absolute magnitude is reasonable in view of the proper motion of $0''.079$ (Luyten 1960), from which one obtains a tangential velocity of 24 km/sec; at a distance of only 65 pc, small or zero reddening would be expected. The tangential velocity is thus of the same order as the radial velocity, and compares favorably with the known low-velocity character of most old novae and dwarf novae (Papers I and III).

c) The Line-emitting Regions

We assume, to begin with, that the stationary Balmer emission is a result of recombination in a rotating ring or disk surrounding the white dwarf. The total energy emitted in $H\gamma$ on August 4, 1962, is 0.8×10^{29} erg/sec. We find from the work of Cillié

(1932) that the rate of recombination per cm^3 onto the fifth level of hydrogen is $2.02 \times 10^{-14} N_e^2$ at an electron temperature $T_e = 10000^\circ \text{K}$. Since the relative probability of emitting $\text{H}\gamma$ is 0.22, the electron density can be obtained from

$$0.22 \times 2.02 \times 10^{-14} N_e^2 V = \frac{8\pi^2 c R^2}{(e^{hc/\lambda k T_e} - 1)} \cdot \frac{\Delta\lambda}{\lambda^4}, \quad (1)$$

where T_e is the effective temperature and R the radius of the white dwarf, λ is the wavelength of $\text{H}\gamma$, $\Delta\lambda$ the equivalent width of $\text{H}\gamma$, and V the volume of the emitting region. Thus we find

$$N_e^2 V = 2.27 \times 10^{54} \text{ cm}^{-3}. \quad (2)$$

This number may be a considerable overestimate for two reasons. First, we have neglected cascades from levels higher than the fifth. If we went to the opposite extreme and supposed that the fifth level was populated entirely by cascades from higher levels, $N_e^2 V$ would be reduced by about a factor of 10. Second, if T_e is lower, say 5000°K , $N_e^2 V$ will be reduced by about 50 per cent. As a compromise, we take $N_e^2 V = 5 \times 10^{53} \text{ cm}^{-3}$. As in earlier discussions (Papers I and III), we assume the mean ring (or disk) radius to be given $r = G\mathcal{M}_1/v^2$, where v is the mean rotational velocity of the ring, $\mathcal{M}_1$, the mass of the white dwarf, and the ring thickness is taken as $\frac{1}{10} r$. Thus $r = 4.34 \times 10^9 \text{ cm}$, and if $V \sim \pi r^2 (\frac{1}{10} r)$, we obtain $N_e = 4.5 \times 10^{12} \text{ cm}^{-3}$; if the ring has a thickness of (say) only $\frac{1}{10} r$ in the plane of the orbit, the electron density is 3 times larger.

For electron densities in the range 10^{12} – 10^{13} cm^{-3} , one can consider the correctness of the assumption that the Balmer emission results entirely from electron captures and cascades. For simplicity, we have considered a three-level hydrogen atom, corresponding to $n = 1$, $n = 2$, and the continuum. Ignoring electron collisions, but writing the equilibrium equations for the states for all radiative processes, we find that $N_1/N_e \sim 7 \times 10^{-5}$ and $N_2/N_e \sim 2 \times 10^{-5}$, where N_1 and N_2 are the numbers of neutral hydrogen atoms in the ground and first excited states, respectively. With $N_e = 4.5 \times 10^{12} \text{ cm}^{-3}$, we find the number of direct radiative excitations from the ground state into the upper level of $\text{H}\gamma$ is about the same as the number of captures from the continuum. (We take the mean ring radius r as a typical distance from the white dwarf; this corresponds to a geometrical dilution factor $W = 1.0 \times 10^{-2}$.) An exact solution for the population of all the levels of neutral hydrogen and for the value of N_e becomes therefore very difficult, and the various geometrical uncertainties of the eclipse geometry scarcely warrant a full-dress treatment. However, two points can be made. (1) The actual electron density is less than $4.5 \times 10^{12} \text{ cm}^{-3}$, since part of the emission measure results from direct excitation rather than electron capture; (2) the Lyman limit is optically thin to distances only slightly greater than the mean radius of the ring, but the Lyman lines are opaque. This latter point results from the geometry assumed in Section VIa, which remains to be justified. As we shall see later, however, the portion of the stream ejected from the dark star that is ionized is confined to a region near the center of mass of the system. The path length corresponding to $\tau \sim 1$ in the Lyman continuum probably does not exceed about 10^{10} cm . Thus $N_1 \times 10^{10} \times 10^{-17} \sim 1$ gives $N_1 \sim 10^7 \text{ cm}^{-3}$ as an estimate of the number of neutral hydrogen atoms per cm^3 in the ground state along a path through the ring and into the ejected stream. Our three-level atom would then require $N_e \sim 10^7 \times \frac{1}{7} \times 10^5 \approx 10^{11} \text{ cm}^{-3}$, a value about an order of magnitude smaller than that derived on the assumption of pure electron captures and subsequent cascades.

It appears, therefore, that the ultraviolet radiation of the white dwarf is sufficient to maintain the ionization of hydrogen in the ring and for a short distance into the stream ejected from the dark star.

From a comparison of the corrected colors of WZ Sge with those of other DA white dwarfs, we find for it an ultraviolet excess between 0.10 and 0.15 mag. This is no doubt due to the presence of the higher Balmer lines and jump in the U filter. However, the

colors were measured some time ago (Walker 1957), close to the time when Greenstein (1957) obtained his spectrograms. From Table 4, we see that the emission lines are presently about 40 per cent stronger relative to the white dwarf than they were in 1957. Unfortunately, we do not have new measures of $U - B$ and $B - V$, and we assume, therefore, that if these quantities were currently measured, the ultraviolet excess would be greater, perhaps 0.15–0.20 mag. Some small corroboration is provided by a calculation of the ratio of the energy in the Balmer jump to that in $H\beta$, which gives a contribution to the U filter of 35 per cent from the present strength of $H\beta$. The calculation assumes pure recombination, however, and the excess would be reduced because of direct radiative excitation. Thus, we find that perhaps 0.15–0.30 mag. decline in light could result from a total eclipse of the line-emission regions alone. However, because we expect the line-emitting regions a priori to be in the plane of the orbit, the white dwarf would be covered whenever the line-emitting regions are covered. Since the depth of principal eclipse on the average is only 0.30 mag., it is necessary to suppose that about 0.2 mag. of white-dwarf light is lost and 0.1 mag. of light from Balmer emission; in other words, one has a grazing eclipse of the white dwarf.

d) The Masses of the Components

A lower limit to certain detection of orbital velocity changes for the double emission feature is $2K = 75$ km/sec; if the mass of the white-dwarf primary is $0.59 M_{\odot}$ this sets an upper limit of $0.06 M_{\odot}$ ($M_2/M_1 = 0.10$) to the mass of the secondary, the velocity in the orbit for the two components then being 43 and 435 km/sec. The observational material at hand does not permit a sharp determination of the mass of the secondary, but it is of interest to explore the consequences of the limiting case.

Principal eclipse has an average duration of $0.10P$ at half-depth. If the secondary fills its lobe of the inner zero-velocity surface, and the eclipse is grazing, then the duration D can be computed for $M_2 = 0.06 M_{\odot}$. However, if the eclipse begins with a partial obscuration of the line-emitting region, the latter's extent in the plane of the orbit must be known in detail. If it is in the form of a very thin ring (thickness small compared with the size of the dark star), then the dark star cuts in at an apparent distance r from the white dwarf, and $D = 0.11P$. If the outer part of the ring, or a portion of the illuminated stream, extends to $2r$, $D = 0.22P$. These values suggest that the ring is spread out somewhat (perhaps more nearly in the form of a disk seen almost edge-on), with an outside radius between r and $2r$.

Again for $M_2 = 0.06 M_{\odot}$, we find that the mean radius of the rotating disk passes almost through the center of mass ($r = 4.3 \times 10^9$ cm; $a_1 = 3.4 \times 10^9$ cm). The ionized, and therefore illuminated, portion of the stream receding through L_1 (Fig. 6) cannot extend far beyond the center of mass toward L_1 , otherwise the transverse motion of the rotating frame of reference would distort the radial velocities; in other words, the observed maximum velocity of recession would tend to occur well *after* principal eclipse. The average delay in phase of the maximum velocity of recession after principal minimum is $0.06P$, and the reality of this is not established with much certainty.

Secondary eclipse can therefore be interpreted as the "annular" eclipse of this stream by the white dwarf. The diminution of light cannot exceed about 0.20 mag. if the eclipse is total. This must mean that, since the depth is ~ 0.15 mag. on the average, the stream cannot have a cross-sectional radius much in excess of 1×10^9 cm. Here we encounter the main objection to the present picture. If i is not 90° , as suggested by the depth of primary minimum, then it is difficult to obscure a sufficient amount for a drop of 0.15 mag. A possible way out of the difficulty would be to choose $M_2 < 0.06 M_{\odot}$, in which case the size of the secondary zero-velocity lobe would decrease, i would come closer to 90° , the difficulties at secondary minimum would tend to disappear, and the duration of principal eclipse would decline negligibly. For example, if $M_2 = 0.03 M_{\odot}$, the duration of principal eclipse is only 3 per cent shorter than for $M_2 = 0.06 M_{\odot}$ and the white

dwarf could easily cut off half the light of the stream. A set of physical parameters satisfying reasonably well all the observations is the following:

$$\mathcal{M}_1 = 0.59 \mathcal{M}_\odot, \mathcal{M}_2 = 0.03 \mathcal{M}_\odot, \mathcal{M}_2/\mathcal{M}_1 = 0.05, i \sim 82^\circ, R_1 = 0.87 \times 10^9 \text{ cm},$$

$$R_2 \sim 7 \times 10^9 \text{ cm}, a_1 = 1.7 \times 10^9 \text{ cm}, a_2 = 3.5 \times 10^{10} \text{ cm}.$$

The supposed mass of the dark star is similar to the secondary component of L726-8B (van de Kamp 1959) for which $M_v = 16$, $M_{\text{bol}} = 13$, and $T_e = 2700^\circ \text{ K}$ from its trigonometric parallax. Thus $R = 7 \times 10^9 \text{ cm}$, in exact agreement with the value estimated from the eclipse data.

e) Some Details of the Light-Curve

Three details of the light-curve require explanation.

1. *The delay of secondary minimum to phase 0.54P.*—One of the most unusual features of WZ Sge is that the velocity of the ejected stream ($\sim 750 \text{ km/sec}$) is considerably larger than the relative velocity of the two components in the system ($\sim 460 \text{ km/sec}$). There is no reason why the ejected stream should leave the dark star exactly in the direction of the line joining the two stars. If it were ejected, for example, in a retrograde direction through L_1 then the illuminated portion might lie in such a position (Fig. 6) that secondary eclipse would come too late, and primary eclipse perhaps somewhat too early. However, if the rotation of the line of apsides were very slow, the photometric and spectroscopic data do not rule out the possibility that the orbit is in reality slightly eccentric.

2. *The dips around phase 0.25P.*—It is possible that an irregular extension in the material streaming past the following hemisphere of the white dwarf might produce this effect. In a number of eclipsing binaries, viz., DQ Her (Kraft 1959) and VV Pup (Herbig 1960), with similar physical properties, there is evidence that the side of the ring following the star is brighter and more spread out than that preceding; this produces a delay in recovery from principal eclipse. In the present case, the grazing nature of the eclipse might contribute to a greater irregularity in recovery since turbulent motions perpendicular to the plane of the orbit could conceivably project bright elements of gas irregularly behind the limb of the dark star.

We could also speculate a little on an alternative possibility. The picture advanced above does not explain very well how the dips can be observed in every cycle. However, with a mass ratio of the order of 20 to 1, the possibility is opened that one or both of the *triangular* Lagrangian points is occupied by material, since if $\mu = \mathcal{M}_2/(\mathcal{M}_1 + \mathcal{M}_2) < 0.0385$, the triangular points are periodic stable solutions of the restricted three-body problem (Moulton 1935). Thus, if material should leave the system beyond the primary and tend to accumulate at or near the Lagrangian triangular point on the *following* side of the primary, this material would be eclipsed by the dark star at a phase centered on 0.17P. The absence of a corresponding eclipse at phase 0.33P by the white dwarf might be understandable if the extent of the material at the triangular point is large. Irregularity in the phasing of the dip might be expected by analogy with the changing location of the Trojan asteroids. Before such a picture could be accepted, however, we would need to resolve two difficulties: (i) There is no spectroscopic evidence for the ejection of a stream from the system beyond the primary. This stream should produce shortward displaced absorption cores near secondary minimum, which are not observed. Even so, the material occupying the triangular point might be primordial and perhaps needs not to be replenished. (ii) There are no emission lines in the system corresponding to the motion of the triangular point, which should have a range of velocity similar to that of the dark star ($2K \sim 900 \text{ km/sec}$). If the material forms simply a reflection nebulosity, which might be possible considering the local state of the physically and geometrically diluted radiation field of the white dwarf, then it must be in the form of dust or solid par-

ticles. No attempt is made here to study the possibility of forming such particles, or to study their rates of formation and destruction, etc.

3. *The change in the depth of principal eclipse on August 4, 1962 U.T.*—This is the most difficult detail of all. Spectroscopically, it appears that the S-wave component is less cut down when the eclipse is shallow than when it is deep. This suggests that the receding stream is less heavily obscured at $\text{JD}_{\odot} 2437880.827$ than at $\text{JD}_{\odot} 2437880.713$. Because the average level of brightness outside eclipse does not change more than a few hundredths of a magnitude, we are left with three possibilities: (i) The orbit is eccentric and the line of apsides is rotating very rapidly. This does not appear likely because it requires, among other things, that the mean eclipse depth vary periodically, which is not observed. (ii) The dark star changes in size irregularly. (iii) The stream moves out of the plane of the orbit irregularly, because of vertical turbulent motions and/or the action of a magnetic field. Both (ii) and (iii) are not impossible, but their *ad hoc* character does not enhance the attractiveness of the general picture advanced here.

f) *Period Changes; Gravitational Waves*

As already mentioned, there is no spectroscopic evidence for the secular ejection of mass from the system. The matter ejected by the secondary apparently flows into the ring around the white dwarf, and by mass-continuity arguments, presumably is accreted by the latter from the ring. The rate σ of transfer of mass can be estimated. If the stream has a cross-sectional area of $3 \times 10^{18} \text{ cm}^2$, and the flow velocity averages 750 km/sec, then for $N_e \sim 10^{11} \text{ cm}^{-3}$ we find $\rho = 10^{-13} \text{ gm cm}^{-3}$ and $\sigma \sim 10^{21} \text{ gm/yr}$. From Huang's (1963) discussion of the effect of mass transfer and loss on the orbital elements, we find that, if all the matter is accumulated by the primary, the rate of increase of period is 1 sec. in 10^6 years.

On the other hand, suppose the nova outburst ejects material to infinity. If the ejection occurs in a spherically symmetrical fashion, we can apply what Huang (1963) calls "Jean's mode of mass ejection." In the absence of spectroscopic data about the outbursts of WZ Sge (Payne-Gaposchkin 1957), we make the assumption, justified by experience with spectroscopically well-observed nova outbursts, that the kinetic energy of the ejected shell is of the same order of magnitude as the available outburst energy (cf. Aller 1954). In the case of WZ Sge, one has $E \sim 10^{41} \text{ erg}$, so that if the ejection velocities are typical, viz., 1000 km/sec, the mass carried off is $-\delta M \sim 10^{25} \text{ gm}$ per outburst. Thus from Huang's (1956, 1963) discussions one finds $dP/P \sim 10^{-8}$ per outburst, i.e., each 30 years. Thus the period would be increased by one second in almost 10^6 years, provided the outburst "period" remains 30 years. If, however, the ejection to infinity is unidirectional, the value of dP/P is of the same order of magnitude as that computed for the spherically symmetrical case.

These values may be compared with the rate of change of period resulting from the emission of gravitational waves (Kraft *et al.* 1963, Paper II). The rate of emission of energy $dE/dt = 0.5 \times 10^{30} \text{ erg/sec} \approx \frac{1}{30}$ the luminosity of the primary, the time for collapse of the system is 1×10^9 years and $dP/dt = -6 \times 10^{-14}$, so that the rate of decrease in period is about 3 sec. in 10^6 years. Since this is of the same order as the two rates computed above, we conclude that detection of gravitational waves from a period change would be very difficult, owing to the masking effect of mass transfer and mass loss.

VII. CONCLUDING REMARKS

The picture set forth here has several unique properties not commonly found in other close spectroscopic binaries. The period is the shortest known, the mass ratio ($\sim \frac{1}{20}$) is unusually small, and the ejection velocity from the secondary unusually large. Further photoelectric observations, however, could provide a valuable check on the validity of the model. For example, in visual light, secondary eclipse should virtually disappear and

primary eclipse would be less deep. Photoelectric observations in spectral regions outside the ultraviolet must unfortunately await the use of a telescope large enough to permit resolution of WZ Sge from its red optical companion.

The present picture provides a self-consistent model but does not explain how a system with an 81-min. period and with masses of $0.59 M_{\odot}$ and $0.03 M_{\odot}$ can have come into existence. It is puzzling that the primary should be a white dwarf, since in the age of the universe, a star of mass $0.6 M_{\odot}$ cannot have evolved to the white-dwarf stage. This objection can be overcome by assuming that the system has lost mass; however, Huang (1956, 1963) has shown that in this case, the period will increase. Thus if the white dwarf had formerly been a main-sequence star F5 or earlier, and had begun to evolve to the red-giant region of the H-R diagram, and had lost mass secularly either in the manner of α Her (Deutsch 1956), or by expanding through its lobe of the inner zero-velocity surface, the original period must have been still shorter than 81 min., an obvious *non sequitur*.

The only known processes tending to decrease the period are mass loss from the primary to the secondary, the emission of gravitational waves, and fast ejection of matter from the system by the *secondary*. In a later paper, the emission of gravitational waves and its possible effect on the evolution of close binaries will be considered in greater detail.

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