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THE QUASI-STELLAR RADIO SOURCES 3C 48 AND 3C 273

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ABSTRACT

The spectra of two quasi-stellar radio sources, 3C 48 and 3C 273, have been studied in detail. We present as full conclusions as we can derive from the redshift, luminosity, emission-line, and continuous spectra. Together with the radio-frequency data and the light variability, these indicate the presence of very large total energies in a relatively small volume of space. We deliberately have not attempted to discuss the origin of these large energies, nor do we discuss the numerous other physical problems concerned with suggested mechanisms in the quasi-stellar objects.

We first consider other explanations for the large redshifts, in particular the possibility of gravitational redshift. The presence of relatively narrow emission lines excludes objects near 1 $M_\odot$ which are stable because of the small emitting volume. The presence of forbidden lines sets an upper limit to the gas density. Together with a limit to gravitational perturbations on our Galaxy, this leads to a lower limit of 10^{11} $M_\odot$, condensed to a 10^{17}-cm radius. Whether such large masses can be even quasi-stable has not yet been demonstrated.

We then adopt the interpretation that the redshifts are cosmological in origin. The absolute visual magnitudes are about -26 for 3C 273 and -25 for 3C 48. The forbidden lines of high ionization potential are quite strong in 3C 48 relative to hydrogen. By analogy with planetary nebulae and assuming normal abundances, with astrophysical details given in the appendices, we derive the electron density, N_e, probably near to or less than 3×10^4 cm^{-3}; the electron temperature is not very high, and the mass is about 5×10^6 $M_\odot$ within a radius of 10 pc or more. The emitting volumes are obtained from N_e and the observed luminosity in Hβ and Mg II. The forbidden lines and the Balmer lines are optically thin, but Mg II is optically thick, leading to discussions in the appendices. For 3C 273, in which the forbidden lines are weaker, the surprising weakness of [O II] permits a closer estimate of N_e near 3×10^6 cm^{-3} and a mass of 6×10^5 $M_\odot$ within a radius of about 1 pc.

The light variations observed in both, with cycles of 10 years or less, suggest the presence of a source of optical continuum with a diameter of 1 pc, possibly much less. We urgently need continued observations of the absolute intensities of lines and continuum and their variations. The thermal energy supply in the H II region is small. The ionized gas must be of low density in the region in which the radio frequencies are generated, because of free-free absorption and Faraday rotation, i.e., $N_e < 10$ cm^{-3} if $R = 500$ pc for the radio source.

We explore models for synchrotron generation of radio and optical frequencies. If $R = 500$ pc, total energies required for radio emission are relatively low, about 10^{57} erg at equipartition. The lifetimes for exhausting the total energy supply are about 10^6 years. If we wish to obtain optical synchrotron from the same volume as produces radio frequencies, the equipartition energy reaches 10^{58} erg. If optical synchrotron radiation is to arise within a volume 1 pc^3, however, the total energy is small, 10^{54} erg, the life about a year, and serious problems arise, such as cosmic-ray proton collisional loss, and inverse-Compton effect electron loss. Models for the jet radio source 3C 273A offer no particular difficulty.

We review conditions under two possible age estimates of the inner components of the quasi-stellar objects. At 10^3 years, the object can be in expansion, with a velocity compatible with the emission-line width, about 1000 km/sec. The energy supply is sufficient for the radio spectrum, and the kinetic energy of the H II region is nearly enough to maintain the optical emission. On this hypothesis, the jet in 3C 273 and the nebular wisps in 3C 48, which are 150000 light-years in size, must have originated in a separate event.

If the age is 10^6 years, the H II region energy is much too small; in addition, its small radius and large internal motion would need to be stabilized by a gravitational mass near $10^9\ M_\odot$, inside the H II region. The radiated energy in 10^6 years is near 10^{60} erg, so that nuclear-energy sources of $10^9\ M_\odot$ are required. The simplest model of the quasi-stellar sources is one in which a small mass of $10^9\ M_\odot$ is surrounded by shells of increasing radius in which the optical continuum, the emission lines, and the radio continuum, respectively, originate. The relation of these objects to the most intense radio galaxies is unclear. The quasi-stellar sources have small optical size, a high ratio of optical to radio emission, and an optical luminosity so high that, if their age exceeds 1000 years, continued input of energy is required from some not directly observable source. Table 11 gives a brief résumé of numerical results.

I. INTRODUCTION

The present paper deals with optical objects of *stellar* appearance that have been associated with radio sources. The first radio source so identified was 3C 48, for which Matthews, Bolton, Greenstein, Münch, and Sandage (1960) announced the stellar appearance of the associated optical object. Subsequently, the radio sources 3C 196 and 3C 286 were identified with similar optical objects (Matthews and Sandage 1963), as was the source 3C 147 (Schmidt and Matthews 1964). The optical spectra of these four quasi-stellar objects appeared quite dissimilar, and no satisfactory identifications of the emission features could be obtained. The identification of the radio source 3C 273 with a bright object of stellar appearance provided a clue when it was found that its spectrum could be understood on the basis on an unexpectedly large redshift (Schmidt 1963). The spectrum of 3C 48, although of a rather different nature, could be explained by an even larger redshift (Greenstein and Matthews 1963).

The present discussion is limited to the *quasi-stellar radio sources* 3C 48 and 3C 273; the observational data are given in Sections II and III. The possibility of interpreting the redshift as the gravitational effect of either very dense or very massive objects is discussed in Section IV. The finally adopted interpretation of these quasi-stellar radio sources as distant, superluminous objects in galaxies, or intergalactic objects, is discussed in the remaining sections.

II. THE SOURCE 3C 273

A remarkably detailed study of the radio source 3C 273 has been made by Hazard, Mackey, and Shimmins (1963) using lunar occultations. They found that the source is double, with a separation of 19.5″ between the components. Component A has a diameter of 4″ at 400 Mc/s, and a spectral index of 0.9. Component B has a diameter of 3″ at 400 Mc/s, with a spectral index near zero. This component has a core with a diameter of about 0.5″ which contributes about half the flux at higher frequencies (Hazard, private communication). The position of each of the components was determined with an accuracy of around 1″.

The two components coincide almost precisely with a thirteenth-magnitude star and the end of a faint jet. Figure 1 shows an enlarged portion of a 200-inch photograph taken by Sandage. The star is 1″ east of Component B and the end of the jet 1″ east of Component A. The jet is between 1″ and 2″ in width, begins 11″ and ends 20″ from the star. This is one of the few radio sources where one of the two components coincides with the optical object. Another case may be M87 where an asymmetric jet is also present.

Spectra of the star were taken in December, 1962, and January, 1963, with the prime-focus spectrograph at dispersions of 190 and 400 Å/mm. Figure 2 shows a spectrum with original dispersion of 400 Å/mm. The broad emission lines were found in a pattern resembling the Balmer series with an unexpectedly large redshift, $z = \Delta\lambda/\lambda_0 = 0.158$ (Schmidt 1963). The identification with the Balmer series was confirmed by Oke's (1963) observation of $H\alpha$ redshifted to λ 7590 $\pm$ 10 Å. The wavelength of 7590 Å is strongly affected by its chance superposition on the head of the atmospheric A-band of O_2. An infrared spectrum taken by Schmidt shows $H\alpha$ as an asymmetrical double wide line, the faint red component at 7617 Å corresponding with the first minimum in the absorption of the O_2-band. Thus, the wavelength of $H\alpha$ cannot be obtained with high accuracy.

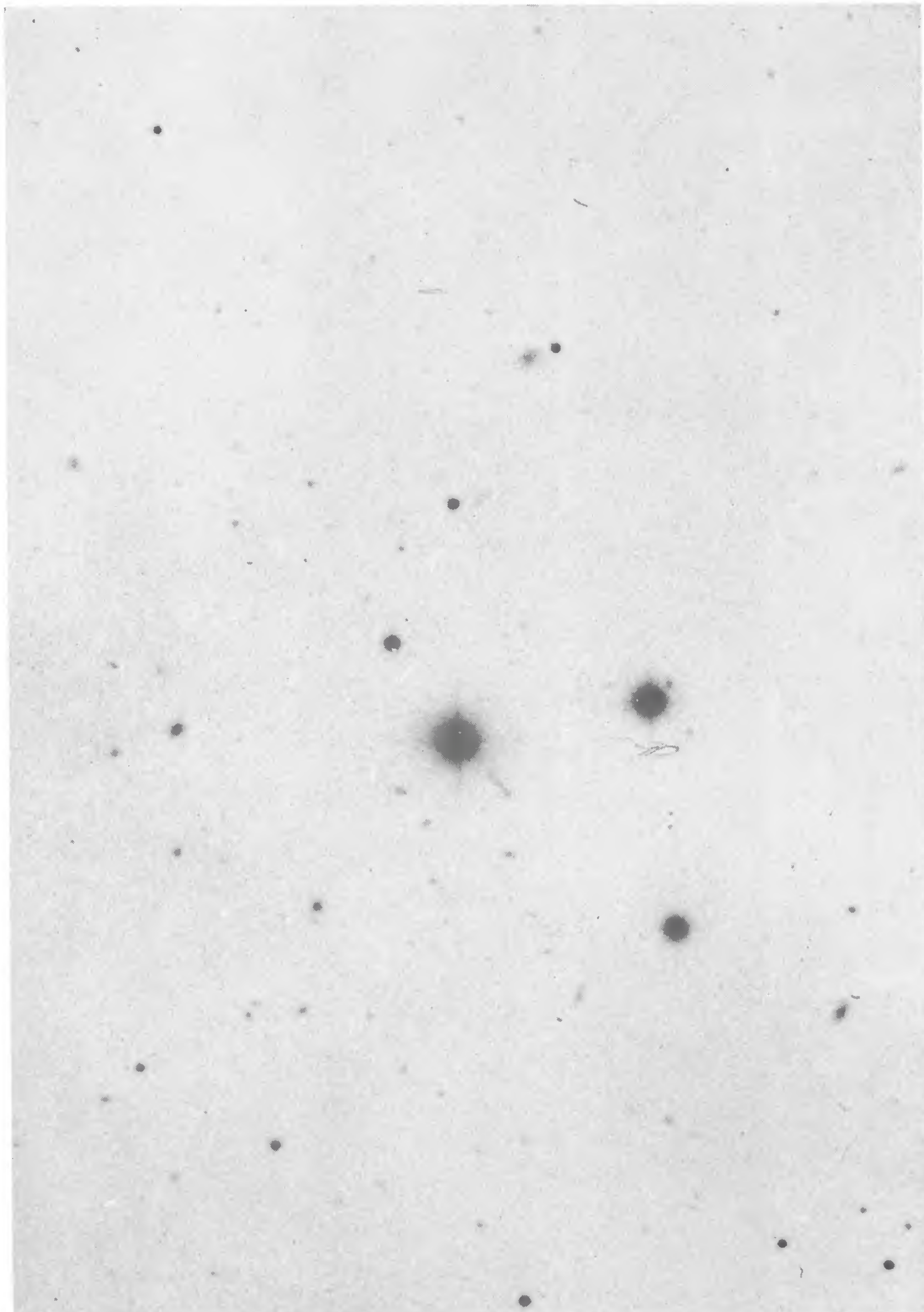

FIG. 1.—Enlarged portion of 200-inch photograph of 3C 273 (from a 103a-D plate taken by A. R. Sandage); north is up, east to left. The weak narrow jet visible in position angle 223° reaches to about 20″ from the quasi-stellar object.

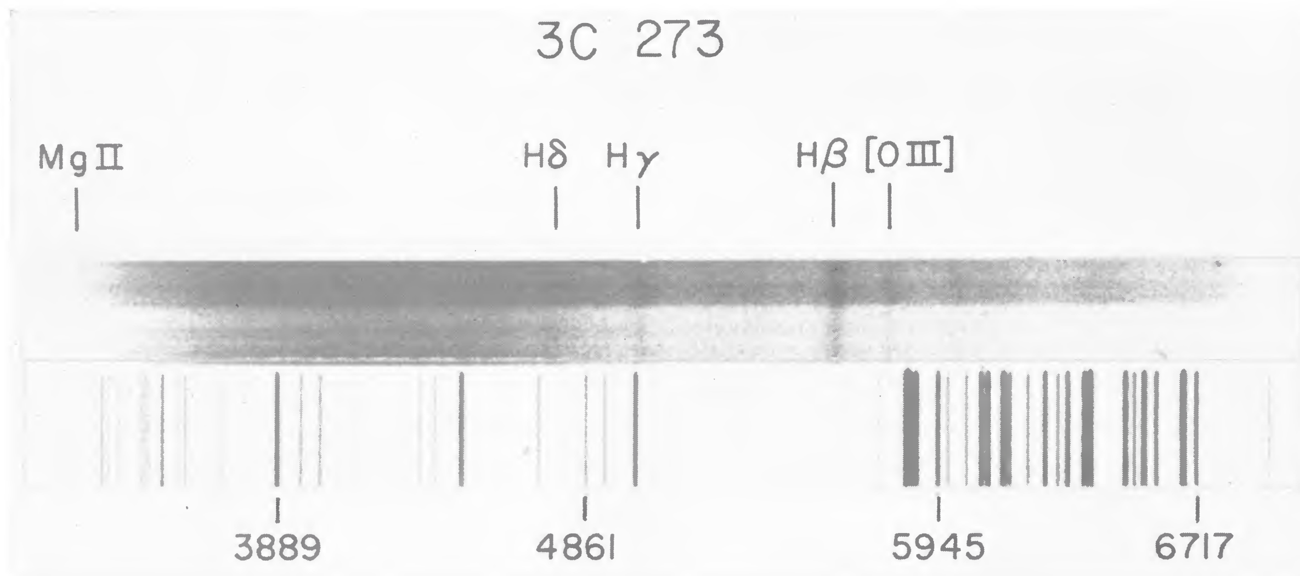

FIG. 2.—Spectrum of the quasi-stellar object 3C 273B, 400 Å/mm original, 103a-F, January 23, 1963. The comparison spectrum is H + He + Ne. Exposure over the upper half of slit was three times that over the lower half. Redshifted emission lines of H and [O III] are indicated; also the barely visible line of Mg II, confirmed on denser exposures.

Table 1 contains the observed wavelengths and identifications. Besides the Balmer lines, the identifications involve the Mg II doublet at about 2800 Å, and the strongest forbidden line of [O III]. The fit is quite satisfactory for the identifiable lines. The mean redshift is 0.1581 $\pm$ 0.0004; this internal mean error corresponds to about ± 4 Å per line and is quite reasonable for the dispersion and line quality. The identified lines are about 50 Å wide. No detailed explanation of the broad lines can be given, although the $\lambda\lambda$ 4490–4675 group (possibly also present in 3C 48) could be the blend of He II, C III, C IV, and N III found in hot emission-line stars or in Type II supernovae. Interstellar Ca II absorption lines have been found by Preston (private communication), and subsequently also on a Palomar spectrum with a dispersion of 85 Å/mm. This plate also shows a weak, sharp emission line at 4318 Å, corresponding to λ 3727 of [O II]. It has the correct shift, but cannot come from the same physical volume or have been formed under the same excitation conditions as the other lines.

TABLE 1

EMISSION LINES IN 3C 273

Wavelength*		Identification	λ/λ_0
λ	λ_0		
3239	2798	Mg II	1.1576
4595	3970	Hϵ	1.1574
4753	4101	Hδ	1.1590
5032	4340	Hγ	1.1594
5200–5415	(4490–4675)	...	...
5632	4861	Hβ	1.1586
5792	5007	[O III]	1.1568
6005–6190	(5186–5345)	...	...
6400–6510	(5527–5622)	...	...
		Weighted mean	1.1581 $\pm$ 0.0004

* A sharp, weak line at 4318 Å is possibly λ 3727 [O II] emission.

The continuum is blue with a nearly flat energy distribution, bluer than that of the quasi-stellar objects discussed by Matthews and Sandage (1963). The apparent brightness of the star is $m_V = +12.6$, or a flux of 3.5×10^{-25} erg cm^{-2} sec^{-1} (c/s)$^{-1}$ at λ 5600, according to Oke (1963). According to new measures by Oke, the Balmer gradient is not steep. His further measurements provide information on the excitation mechanism, and on the relative importance of Paschen, Balmer, and synchrotron continua.

Smith and Hoffleit (1963) have inspected the Harvard Patrol plates and found that the brightness of the star is nearly the same as it was 80 years ago. They report fluctuations up to 0.4 mag. with a rough period of about 10 years and scattered brightenings of a month's duration. Sharov and Efremov (1963) reported similar variations.

The proper motion is less than 0.01″/yr according to Luyten (1963), and less than 0.001″/yr according to Jefferys (reported at the Dallas Conference on Gravitational Collapse and the December 1963 AAS meeting).

III. THE SOURCE 3C 48

The radio source is smaller than 1″ and apparently single. The photographic image of 3C 48 is stellar except for faint reddish wisps about 12″ N.-S. by 5″ E.-W. The star is 3″ north of the center of these weak features, whose distribution of intensity is irregular (Matthews and Sandage 1963).

The first spectra of the stellar object, obtained in October, 1960, by Sandage, were sufficiently abnormal to show that this object was not an ordinary star or an extragalactic nebula of moderate redshift. An excellent spectrum at dispersion 190 Å/mm by Münch and several by Greenstein at dispersions of 190 and 400 Å/mm in 1961 established the existence of a nearly featureless continuum extending far into the ultraviolet and of weak, broad, emission lines not observed in normal or peculiar stars. Some details were reported by Greenstein and Münch (1961).

Figure 3 shows two spectra (6-hour exposures) of the blue region, original dispersion 190 Å/mm; the faint emission features are real, broad, and of low contrast. The colors ($B - V = +0^{m}.41$, $U - B = -0^{m}.59$) are not far from those of a cool white dwarf and indicate the extension of the spectrum into the ultraviolet. A trace of weak, sharp, Ca II absorption lines is seen, and also, on one plate, a line near λ 3888. The stationary H- and K-lines are interstellar and are somewhat strong for the high latitude and low dispersion used.

TABLE 2

PROBABLE REAL LINES WITH IDENTIFICATIONS IN 3C 48*

Int.	λ	λ_0	Quality	Identification	λ/λ_0
10 E.....	3832.3	{2796, 2803}	Broader than average	Mg II	1.3697:
1 A.....	3892	....	Sharp	?	....
1 A.....	3934.7	3933	Sharp	Ca II, interstellar?	....
1 A.....	3969	3968	Sharp	Ca II, interstellar?	....
2 E.....	4048.2	4047	Broad	Hg I, night sky	....
1 E.....	4065.7	2975:	Broad	[Ne V]	1.3667:
0 A.....	4139	....	Broad	....	....
0 A.....	4151	....	Broad	....	....
1 E.....	4166	....	Broad	Probably night sky	....
0 E.....	4205.3	....	Sharp	....	....
3 E.....	4356	4358	Sharp	Hg I, night sky	....
0 A.....	4553.7	....	Broad	....	....
3 E.....	4575	3346	Broad	[Ne V]	1.3673
5 E.....	4685.0	3426	Broad	[Ne V]	1.3676
5 E.....	5097	{3726, 3729}	Sharp	[O II]	1.3676
1 E.....	5136	....	Sharp	....	....
3 E.....	5288	3869	Sharp	[Ne III]	1.3668
10 E.....	5935	4340	Broad	Hγ	1.3675
2 E.....	6349:	4640–50	Broad	....	....
20 E.....	6646	4861	Very broad	Hβ	1.3672
				Weighted mean (λ/λ_0) ...	1.3675±0.0002

* It is possible that [O III] is present and strong at the extreme red end of the plate sensitivity.

Table 2 contains a list of lines, measured at least twice, from which all obvious night-sky lines and dubious stellar lines have been omitted. Most of the features of Table 2 are probably real. Figure 4 is a sketch of the most significant featues, in the blue, derived from the mean of direct-intensity tracings of two plates at 190 Å/mm, uncorrected for plate sensitivity. The exposures, 5–6 hours in length, have airglow features, the Herzberg bands of O_2, some city mercury lines, and slight traces of N_2 and N_2^+ (low-latitude aurorae). Broad minima are suspected in the blue, at observed λλ 4230, 4310. The λ 3832 feature has a half-width of 35 Å, as compared to the other weaker emissions, which average 23 Å.

The coincidence in wavelength of the strong emission feature in 3C 48 (at 4685 Å) with

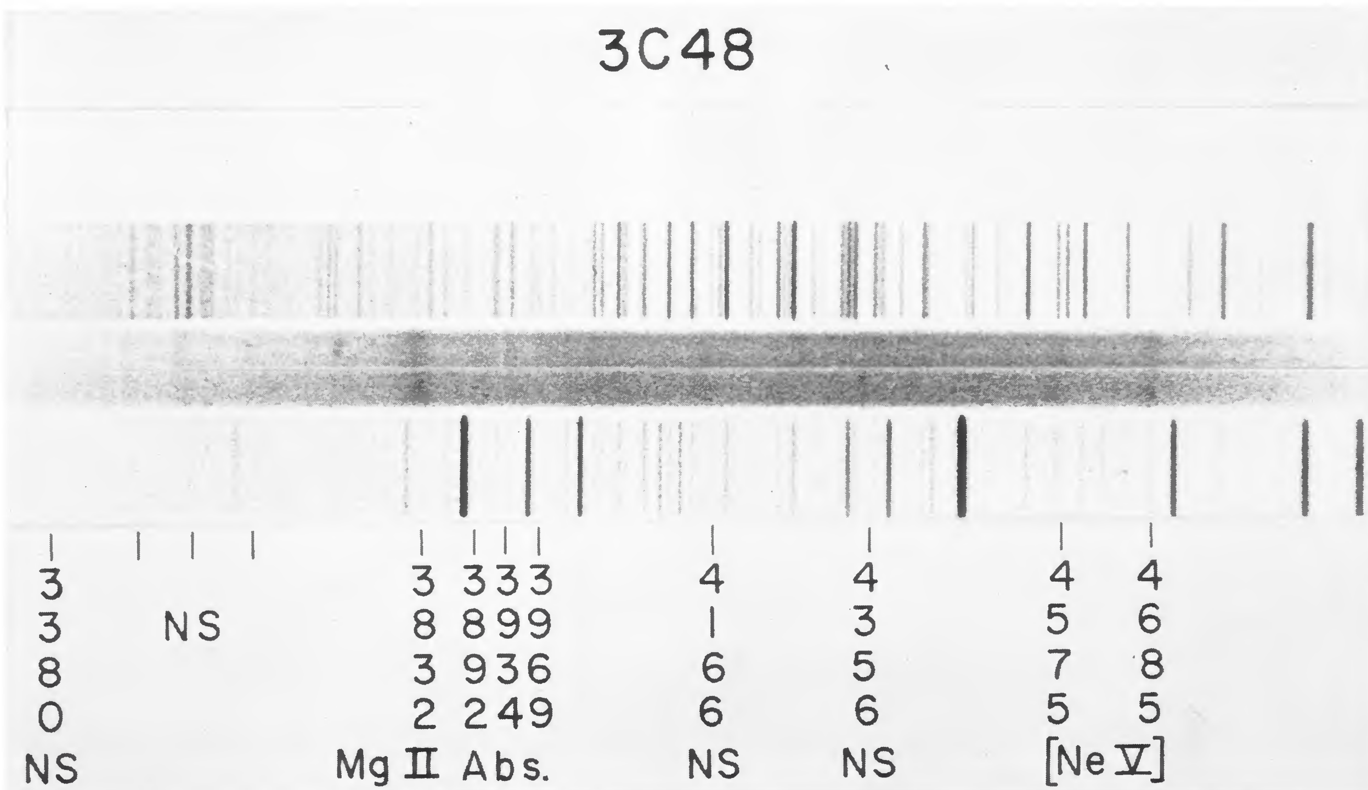

FIG. 3.—Two prime-focus spectra of the quasi-stellar object 3C 48, 190 Å/mm, IIa-O baked; upper November 12, lower December 20, 1960. The symbol *NS* indicates night-sky emission; *Abs.* is absorption. Upper comparison A + Ne, lower H + He + A. Redshifted lines of Mg II and [Ne V] are indicated.

the λ 4686 line of He II figured importantly in earlier attempts to explain the spectrum as that of a star of zero redshift. After the discovery by Schmidt of a large redshift in the Balmer spectrum of 3C 273 it was found that the strongest lines in 3C 48 can be identified (see Table 2) using a redshift $\Delta\lambda/\lambda_0 = 0.3675 \pm 0.0003$ (Greenstein and Matthews 1963). The wavelength of the blended Mg II resonance doublet is somewhat discrepant, possibly caused by self-absorption in an expanding envelope. A recently obtained low-dispersion spectrum at 760 Å/mm in the red shows the redshifted hydrogen lines Hγ and Hβ. A direct-intensity tracing is shown in Figure 5. The rapid drop in the 103a-F plate sensitivity at longer wavelengths and the presence of the atmospheric B-band make identifications from λ 6700 to λ 6850 difficult. But there is some evidence for a broad sensitivity maximum here, not measured, but corresponding to the redshifted wave-

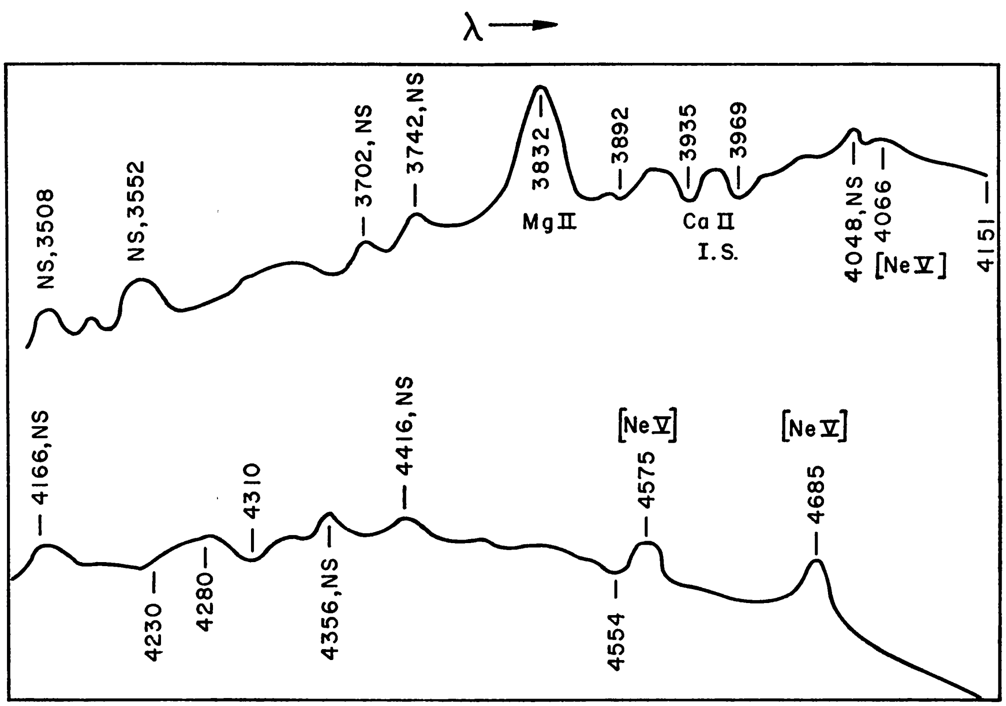

FIG. 4.—Mean of two microphotometer tracings of 3C 48 (not on an intensity scale). There is a possible slight contamination of λλ 3935, 3969 by moonlight. *NS* means night-sky emissions. Redshifted lines of Mg II and [Ne V] are indicated.

lengths of the N1 and N2 [O III] lines. These have been confirmed by spectral scans by Oke. The hydrogen lines are 80 Å wide near the continuum level, suggesting a Doppler half-half-width of 1100 km/sec. There is a possible broad feature whose corrected wavelength is near λ 4643.

IV. THE POSSIBILITY OF GRAVITATIONAL REDSHIFTS

Redshifts as large as those found for 3C 48 and 3C 273 have thus far only been encountered in distant galaxies. Both sources have a dominant optical component with an angular diameter of less than 1″, simulating the appearance of a star. The observed redshifts cannot be explained as velocity shifts of ordinary stars; the low upper limit to the proper motion of 3C 273 in conjunction with a transverse velocity of the same order as the observed redshift would lead to a minimum distance of 10 Mpc, and an absolute magnitude higher than −16. We shall discuss in the present section a possible interpretation in terms of gravitational redshifts. We divide the considerations into those relevant to collapsed "neutron" stars and to very massive objects.

Let us investigate the consequences of assuming that the redshift is of gravitational origin. The gravitational redshift is

$$\frac{\Delta\lambda}{\lambda_0}=\frac{GM}{Rc^2} = 1.47\times 10^5 \frac{M/M_\odot}{R}, \quad (1)$$

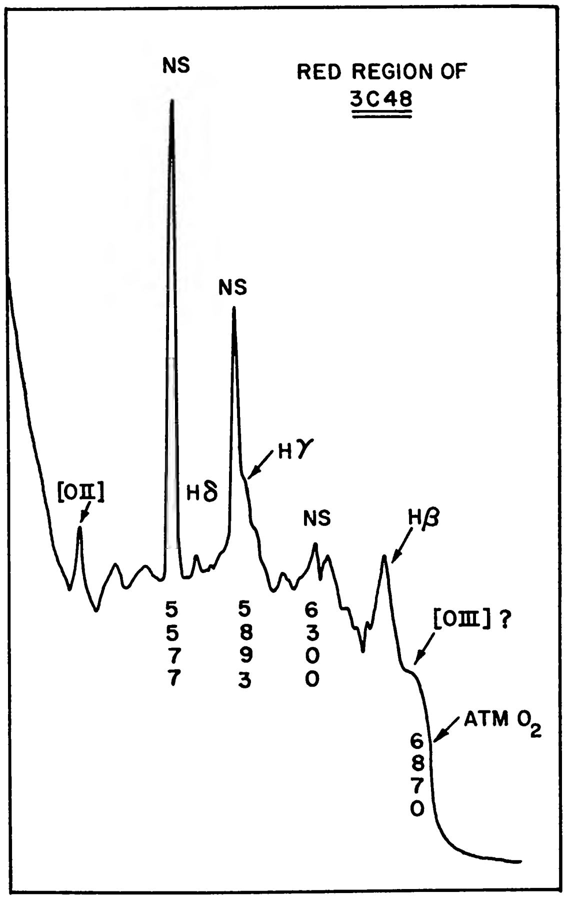

FIG. 5.—Microphotometer tracing of a low-dispersion spectrum of 3C 48 in the red, November 19, 1963. The source was near $V = 16^{m}.5$ on all dates for which spectra of 3C 48 are reproduced.

where R is the mean radius in centimeters. The width w of the emission lines, though considerable, is but a small part of the redshift $\Delta\lambda$. If the line width is due solely to the variation of gravitational potential over the region containing ionized gases, they must be confined within a shell with thickness ΔR, such that

$$\Delta R/R = w/\Delta\lambda . \quad (2)$$

Table 3 shows some relevant data for the two sources.

Stars of very high density (approaching that of compressed nuclear matter) may result from the collapse of a massive star or of a supernova of Type II. A stable stellar nucleus could remain surrounded by a gaseous envelope. The gravitational potential energy of such dense objects approaches the rest-mass energy. The star will contain neutrons or, at higher densities, protons and hyperons. Work by Wheeler (1958), Cameron (1959), and Salpeter (1961) gives stable models in the range of central densities, ρ_c, 10^{15} to 10^{19} gm/cm^3. The gravitational redshift at their surface gives z in the range 0.1 up to about 0.6 (with considerable uncertainty at the upper bound). Unfortunately, the theory of collapsed stellar configurations is still in doubt and incomplete. Oppenheimer and Volkoff (1939) give a maximum redshift near $z = 0.11$ for a neutron gas with degenerate pressure. Cameron's models, with a specific nuclear repulsion law, apparently permit a larger mass and redshift. Cameron found for $\rho_c = 10^{13}$ gm/cm^2, $z = 0.002$; for $\rho_c = 10^{15}$, $z = 0.14$ (near 3C 273); and for $\rho_c = 2 \times 10^{15}$, $z = 0.35$ (close to 3C 48). Ambartsumian and Saakyan (1961) have also studied the properties of the neutron-baryon stars and found an error in Cameron's application of his equation of state. Essentially, Ambartsumian and Saakyan (1961) first found that the maximum mass lay near 1.03 $M_\odot$, at a density where hyperons were the most abundant constituent. Saakyan (1963) found that Cameron's masses, even with the latter's equation of state, were too large by a factor of approximately $\frac{3}{2}$. Ambartsumian and Saakyan find a maximum value of $M_\odot/R_{\rm km} = 0.28$ for an incompressible baryon gas, which gives a maximum $z = 0.42$. This is just above the observed value (0.37) for 3C 48. A general discussion by Bondi apparently will make possible evaluation of the maximum possible z. His present limit, $z < 1.5$, involves negative density gradients. A more useful limit, as low as $z < 0.6$, seems within the range of plausibility.

TABLE 3

INTERPRETATION INVOLVING GRAVITATIONAL REDSHIFT

	3C 48	3C 273
$\Delta\lambda/\lambda_0$	0.367	0.158
w (Ångstroms)	$\sim$30	$\sim$50
R (cm)	$4.0 \times 10^5\ M/M_\odot$	$9.3 \times 10^5\ M/M_\odot$
$\Delta R/R$	0.016	0.07

A much more positive argument arises from evaluation of the emission from ionized hydrogen. At an electron temperature T_e and electron density N_e the emissivity in Hβ is (Aller 1956)

$$E(\mathrm{H}\beta) = 2.28 \times 10^{-19}\ N_e^2 T_e^{-3/2} b_4 \exp\ (0.98 \times 10^4/T_e)\ , \tag{3a}$$

where b_4 measures the deviation from equilibrium population of the upper level of Hβ. We assume $T_e = 10^4$ ° K, and a typical average nebular value of $b_4 = 0.16$. Then we compute

$$E(\mathrm{H}\beta) = 1.0 \times 10^{-25}\ N_e^2 \text{ erg sec}^{-1} \text{ cm}^{-3}\ . \tag{3b}$$

The observed brightness of Hβ in 3C 273 is 3.4×10^{-12} erg cm^{-2} sec^{-1}. Equating this to the emission from a volume, V, of ionized hydrogen at a distance, r, yields the relation

$$10^{-25}\ V\ N_e^2 = 3.4 \times 10^{-12}\ 4\pi r^2\ . \tag{4a}$$

Here $V = 4\pi R^2 \Delta R$, and $\Delta R = 0.07\ R$ is obtained in Table 3 from the width of the emission lines. Finally we have

$$N_e^2 R^3 r^{-2} = 4 \times 10^{14} \text{ cm}^{-5}\ . \tag{4b}$$

Let us first consider the possibility of a collapsed body of nearly normal stellar mass. The absence of observable proper motion makes a distance less than 100 pc very unlikely. Introducing this minimum distance into equation (4b) leads to the inequality

$$N_e^2 R^3 \geq 4 \times 10^{55}\ \mathrm{cm}^{-3}\,. \tag{4c}$$

If the mass of the object is 1 $M_\odot$, then $R \approx 10^6$ cm, from Table 3, and we need $N_e \geq 6 \times 10^{18}$ cm^{-3}. This density is very much larger than the maximum electron density of about 10^8 cm^{-3} which is imposed by the appearance of a forbidden line in the spectrum. This enormous discrepancy cannot be removed by any other choice of T_e, or of b_4, in equation (3a), and we are forced to the conclusion that the spectrum of 3C 273 cannot be explained by gravitational redshift near a star of about 1 $M_\odot$. Similar conclusions can be reached with regard to 3C 48, in which the forbidden lines are very strong, making the upper limit to N_e even lower.

The total luminosity in Hβ at $N_e = 6 \times 10^8$ cm^{-3} exceeds 10^{30} erg sec^{-1}. A black body of $R = 10^6$ cm and $T = 10^4$ ° K emits 7×10^{24} erg sec^{-1}; therefore the emission-line flux exceeds that from a black body, which is impossible. In fact, so high an electron density would result in a largely neutral gas opaque to its own radiation. The monochromatic flux at Earth from an object of radius R cm at a distance of r_{pc}, using the infrared approximation to the black body, is

$$F_\nu d\nu = 10^{-73}\left(\frac{R}{r_{\mathrm{pc}}}\right)^2 T\nu^2 d\nu\ \mathrm{erg\ sec^{-1}\ cm^{-2}\,(c/s)^{-1}}\,. \tag{5}$$

With $R = 10^6$ cm, $r_{\mathrm{pc}} = 10^2$, $\nu = 6 \times 10^{14}$, we have $F_\nu \approx 4 \times 10^{-36}\ T$ erg sec^{-1} cm^{-2} (c/s)$^{-1}$ at 5000 Å. Since the observed $F_\nu \approx 4 \times 10^{-25}$ for 3C 273, we require that $T = 10^{11}$ ° K if the black-body radiation of a 10-km star is to produce the continuum.

The discovery of hyperdense stars would be of great interest in theoretical physics; our conclusions suggest that optical detection is hopelessly difficult. From the theoretical physics side, in addition, if only neutron degeneracy determines the pressure, the redshifts already observed are too large to be explained in this manner.

Another possible interpretation of a gravitational redshift, from equation (1), is that it arises in an object of very great mass and moderate radius, e.g., $10^8\ M_\odot$ and $R = 10^{14}$ cm. We disregard for the moment questions regarding the stability of such a configuration. If such an object is located in or near our Galaxy, an additional limiting condition on its distance can be derived from its gravitational perturbations on the local dynamics of the Galaxy. The acceleration for stars near the Sun due to a single radio source certainly should be less than 10 per cent of that of the whole Galaxy, which gives

$$M/M_\odot \leq 10^{-35}\, r^2\,. \tag{6}$$

The emission in Hβ, from equation (3b), yields a flux at the Earth of

$$F(\mathrm{H}\beta) = 10^{-25}\, N_e^2 R^2\, \Delta R\, r^{-2}\,. \tag{7}$$

Combining equations (1), (6), and (7),

$$r^4 \geq N_e^{-2}(\Delta R/R)^{-1}\,(\Delta\lambda/\lambda_0)^3\, 10^{114.5} F(\mathrm{H}\beta)\,, \tag{8a}$$

$$M/M_\odot \geq N_e^{-1}\,(\Delta R/R)^{-1/2}(\Delta\lambda/\lambda_0)^{3/2}\, 10^{22.2}\,[F(\mathrm{H}\beta)]^{1/2}\,. \tag{8b}$$

The Hβ fluxes are 3.4×10^{-12} and 4.4×10^{-14} erg cm^{-2} sec^{-1} for 3C 273 and 3C 48, respectively; the redshifts and $\Delta R/R$ from Table 3 give approximately the same results for the two sources:

$$r_{\mathrm{pc}} > 8 \times 10^{6} N_e^{-1/2}\,, \qquad M/M_{\odot} > 7 \times 10^{15} N_e^{-1}\,. \tag{9}$$

We will see that the spectra, containing forbidden lines, are not consistent with an N_e very different from 10^7 cm^{-3} for 3C 273, and a somewhat lower value, 10^5 cm^{-3} for 3C 48. The resulting values are for 3C 273, $r_{\mathrm{pc}} \geq 2500$, $M/M_{\odot} \geq 7 \times 10^8$ and for 3C 48 $r_{\mathrm{pc}} \geq 25000$ and $M/M_{\odot} \geq 7 \times 10^{10}$. These minimum values are derived for the unlikely case of a gravitational perturbation 10 per cent that of the attraction of the Galaxy. From the structure of the equations, when we lower this upper limit to the perturbation, we increase the mass and radius in equation (9). The high latitude of 3C 273 ($b = 60°$) and the fact that the above mass and distance are minimum values make it quite safe to conclude that the interpretation of the redshift as a gravitational effect also requires an extragalactic nature for these quasi-stellar objects. We are reminded here of the gravitationally collapsing masses investigated by Hoyle and Fowler (1963; see also Fowler and Hoyle 1964) with a view to gaining some understanding of the origin of the very large energies required in the strong extragalactic radio sources. At present, there is no assurance from the theory, and it may even be viewed as unlikely, that $10^{11}\, M_{\odot}$ within a radius of 0.01 pc are stable. If gravitational implosions should occur, it is difficult to see what train of theoretical arguments would lead to a gaseous shell of fixed radius and 10^{-4} pc thickness. Under large gravitational attraction, only a very narrow range of temperature (or explosion velocities) could produce such a scale height. It is difficult also to understand why the density distribution would be such as to make the emission lines quite symmetrical. On this hypothesis the red wing would originate in the inner part and the blue wing in the outer part of a thin shell.

We believe that the postulation of stable, or nearly stable, objects of size less than 1 pc, with masses near those of galaxies, in nearby intergalactic space is not really justified at present. Consequently, we reject gravitational redshift as a basis for explanation of the large redshifts observed in 3C 48 and 3C 273. If stable, massive configurations exist, we must re-examine this possibility. The mass-radius relation would have to be such as to give larger gravitational redshifts for fainter objects. Although the range of shifts is small (a factor of 3 with recent discoveries), if we interpret the shifts as cosmological we deduce consistent optical and radio absolute fluxes for sources at various redshifts. The unprecedented combination of redshift, apparent luminosity, and appearance of 3C 48 and 3C 273 will require an unorthodox explanation.

V. EMISSION LINES IN A SUPERLUMINOUS GALAXY

Let us assume as the surviving hypothesis that the redshifts for 3C 48 and 3C 273 are cosmological redshifts. With a Hubble constant of 100 km sec^{-1} Mpc^{-1} the distances are 1100 and 474 Mpc, respectively. The absolute visual magnitudes become about -25 and -26, respectively, making these objects the brightest yet known in the universe.

The upper limits for the angular diameters of the optical starlike components, from visual inspection, are 1″ for 3C 48, and $\frac{1}{2}$″ for 3C 273, corresponding to 5 kpc, and 1 kpc, respectively. The southern end of the optical wisps of 3C 48 is at a distance of about 50 kpc in projection. The end of the jet in 3C 273 is also at a projected distance of about 50 kpc from the quasi-stellar object. If these features have been ejected from the central objects, this event took place at least 2×10^5 years ago.

At these distances, corrections to luminosities depend on the specific world model. For an approximate determination of the fluxes in the emission lines we use the formulae given by Sandage (1961) based on Mattig (1958). For the energy contained within an emission line, in which we integrate over the entire profile, the correction to the inverse-square law is of the form $(1 + z/2)^2$ if the cosmological constant and deceleration parameter are zero. It is $(1 + z)^2$ for the steady-state model. We use the $(1 + z/2)^2$ correction in Table 4 to obtain the luminosity, L, in each line from the measured equivalent widths, W. Note that the emission-equivalent widths, in units of the neighboring continuum, are only poorly determined, because of the low contrast and width of the lines. In addition we used interpolated estimates of the flux in the continuum, as measured by Matthews and Sandage (1963) for 3C 38 and Oke (1963) for 3C 273.

The spectra observed are not too different from those of planetary or diffuse nebulae, except for the presence of Mg II, hitherto undetected because of terrestrial ozone absorption, but probably present in other nebulae. In the study of spectra of emission regions it is the distribution over various levels of ionization that introduces the greatest uncertainty in the determination of element abundances. This problem exists even more strongly for the spectra at hand, since the ultraviolet radiation field is unknown.[1] We have

TABLE 4

EMISSION-LINE LUMINOSITIES

		3C 48	3C 273
r Mpc		1100	474
$(1+z/2)^2$		1.40	1.16
$4\pi r^2(1+z/2)^2$ cm^2		2.0×10^{56}	3.1×10^{55}
W(Ångstroms):	Hβ	28	86
	Mg II	10*	11*
	[Ne v]	6*	Absent
	[O II]	12*	Absent?
	[O III]	Present	24*
L (erg sec^{-1}):	Hβ	6.4×10^{42}	8.8×10^{43}
	Mg II	3.1×10^{42}*	4.0×10^{43}*
	[Ne v]	1.7×10^{42}*	...
	[O II]	3.3×10^{42}*	?
	[O III]	Present	3.1×10^{43}*

* The sum of W of the two lines of each doublet is given, except for W of [O III], which is for λ 5007 only. L is the sum over each doublet.

[1] *Note added in proof.*—The levels of ionization of common elements have been computed approximately by House (*Ap. J. Suppl.*, **8**, 307, 1964) for coronal conditions, far from Boltzmann equilibrium in the absence of ultraviolet quanta. Collisional ionization is balanced by collisional and radiative recombinations. House's computations were made specifically for laboratory or solar plasma, i.e., very high electron density, 10^{15} cm^{-3}. At lower electron density, they are incorrect if radiative ionization rates are in any way appreciable. For us, the most critical question is whether the weakness of [O II] in 3C 273 could arise from such anomalous collisional ionization; we give below his values of the logarithm of the fractional concentration of an ion, in units of the total number of all ions of that element present, as a function of T_e in eV (1 eV = 11600° K).

EQUILIBRIUM	LOG N(ION)/N(ELEMENT)				
	T=1 eV	1.8 eV	3 eV	6 eV	10 eV
O^+/O	−1.5	0.0	−0.2	−1.9	−4.9
O^{++}/O	...	−4.5	−0.8	−0.3	−1.7
Ne^{++}/Ne	...	−5.0	−1.4	−0.1	−1.5
Ne^{++++}/Ne	...	...	...	−5.0	−0.8
Mg^+/Mg	0.0	−0.7	−2.6	−3.8	−4.9

Inspection shows that there is no T_e which makes O^+ weak and leaves O^{++} present without highly ionizing Mg^+. Consequently, the weakness of [O II] in 3C 273 is confirmed as a density effect even if the ionization is collisional. The presence of [Ne III] and [Ne v] in 3C 48 requires $T_e > 2$ eV, i.e., higher than we used, and their weakness in 3C 273 suggests $T_e < 3$ eV. The probable values of electron temperature in the collisional case do not differ much from those obtained by our use of an analogy with planetary nebulae.

tried to circumvent the problem as well as possible by comparing the spectra with those of planetary nebulae and normalizing our computations to give intensities observed in planetaries of similar excitation. We assume that the chemical composition of the emitting regions is "normal," and that the electron temperature is $\theta_e = 0.3$ (slightly higher than normal). (See Appendix I for a detailed discussion of θ_e.)

We have revised and improved the expressions given by Aller (1954) for η_2, the deviation from thermodynamic equilibrium in the levels producing the nebular lines; we use, in particular for [O II], the work of Seaton and Osterbrock (1957) and compute the populations of the $^2D_{5/2}$ and $^2D_{3/2}$ levels separately before adding the intensities. The predicted emission per cubic parsec is given in Table 5 on a scale where O^+ and O^{++} have the same abundance, taken as 10^{-4} that of hydrogen, and where Ne^{++} and Ne^{++++} are taken as 10^{-5} that of hydrogen; we assume $N(H^+) = N_e$. Where more than one nebular line is present, the sum of the intensities is given. Elements like Ne and O have a concentration near this value, and the distribution over the various ionization levels could result in Table 5 being approximately correct. The table also contains the predicted intensity of the hydrogen line, Hβ, per pc^3, derived from equation (3a). We use higher temperature $\theta_e = 0.3$, $b_4 = 0.16$, and normal composition instead of equation (3b) to give results tabulated in Table 5.

TABLE 5

PREDICTED LUMINOSITIES IN FORBIDDEN LINES, Hβ AND Mg II

($\theta_e = 0.3$; log $E(X)$ in erg sec^{-1} pc^{-3})

LOG N_e CM^{-3}	LOG ρ $M_\odot$ PC^{-3}	LOG $E(X)$					
		[O II]*	[O III]*	[Ne III]†	[Ne V]†	Hβ	Mg II‡
2	0.4	35.1	35.7	34.8	34.7	34.0	33.4
3	1.4	37.0	37.7	36.8	36.7	36.0	35.4
4	2.4	38.7	39.7	38.8	38.7	38.0	37.4
5	3.4	40.0	41.7	40.8	40.7	40.0	39.4
6	4.4	41.0	43.5	42.7	42.7	42.0	41.4
7	5.4	42.0	44.8	44.6	44.6	44.0	43.4
8	6.4	43.0	45.8	46.1	46.2	46.0	45.4

* The ions of oxygen, O^+/O^{++}, vary greatly from one planetary nebula to another; the ratio is often less than 0.1. We adopt $O/H = 10^{-3}$, $O^+/H = 3 \times 10^{-5}$, and $O^{++}/H = 5 \times 10^{-4}$.

† The mean abundance used is Ne/O = 0.2 and the ions are spread evenly over Ne^{++} to Ne^{++++}, i.e., $Ne^{++}/H = 7 \times 10^{-5}$. Lower excitation planetaries have Ne^{++}/Ne^{++++} greater than unity.

‡ The abundance Mg/H is taken as 3×10^{-5}. The very rough ionization equilibrium in planetaries varies enormously at the low ionization-potential end, but $(1 - x_{Mg^+})$ is taken as 10^{-3}, with Mg^{++} assumed dominant.

The Appendix contains a detailed study of the [Ne V] and Mg II lines. A very serious problem, comparable to that of Lyman-α, is raised by the high opacity in the Mg II lines. The strength of Mg II results from its high abundance, the low energy required, 4.42 eV, and the large collisional cross-section, $\Omega \approx 25$. The latter has been estimated by van Regemorter (1960*a*, *b*) for collisions of moderate energy. From his work, the Ca II resonance doublet has $\Omega \approx 6$, and since the normal abundance ratio Ca/Mg is near 0.06, the product of abundance times cross-section is near 0.015. The ionization of Ca^+ would also be greater than that of Mg^+, so that the great strength of Mg II is not completely unexpected; Daub (1963) had derived its intensity in discussing planetary nebulae. In the rocket solar spectra, the Mg II resonance doublet is outstandingly strong (Friedman 1963). The collisional de-excitation of the upper level of the Mg II line is negligible for $N_e < 10^{15}$ cm^{-3}. The emission of Mg II summed over the two lines is

$$E(\text{Mg II}) = N_2 h\nu A_{21} = N_i(\text{Mg}^+) N_e \frac{\Omega_{12}}{g_1} C h\nu 10^{-4.42\theta_e}, \quad (10)$$

where

$$C = 8.54 \times 10^{-6}\, T_e^{-1/2}\,.$$

Let $N_i(\text{Mg}^+)$ be $Z_x N_e$, where $Z_x N_e$ represents the Mg^+/H ratio. Then if Mg^+ were the dominant stage of ionization of Mg, $\log Z_x = -4.5$ using its normal solar abundance. Retaining x_{Mg^+}, its level of second ionization as a parameter, gives the effective number $\log N_i(\text{Mg}^+) = -4.5 + \log(1 - x_{\text{Mg}^+}) + \log N_e$. We neglect recombination as a source of population of the upper level of the λ 2800 doublet. Finally, the emission per cubic parsec in the Mg II doublet is

$$E(\text{Mg II}) = 2.6 \times 10^{32}\, N_e^2 (1 - x_{\text{Mg}^+})\,. \tag{11}$$

It is very difficult to estimate the level of ionization of Mg^+ as compared to O^{++} or Ne^{++} or Ne^{++++}. The latter are usually the dominant states of ionization in high-excitation planetary nebulae. The results given by Aller and Menzel (1945) and Aller (1954) indicate a quite flat distribution of concentration over the various ions with O^+/O^{++} near 0.1. The data on NGC 7009 and 7027 obtained by Aller suggest, after considerable extrapolation, that Mg^+, with an ionization potential of 15 eV, is 10^{-3} of total Mg, the balance being Mg^{++}, with an 80-eV ionization potential. Consequently, in Table 5 we have assumed $1 - x_{\text{Mg}^+} = 10^{-3}$.

TABLE 6

COMPARISON OF OBSERVED AND PREDICTED LINE INTENSITIES (NORMALIZED TO Hβ)

	LOG $E(X)/(\text{H}\beta)$			
	[O II]	[O III]	[Ne III]	[Ne V]
3C 48	−0.3	Present	(−0.6):	−0.6
NGC 7027	−0.8	+1.2	−0.1	−0.1
Table 5, $N_e = 10^5$	0.0	+1.7	+0.8	+0.7
Correction based on NGC 7027	−0.8	−0.5	−0.9	−0.8
IC418	+0.3	+0.2	−1.5	Abs.
Table 5, $N_e = 10^3$	+1.0	+1.7	+0.8	+0.7
Correction based on IC418	−0.7	−1.5	−2.3	−2.3

The energies are large enough so that only a few cubic parsecs are needed, if $N_e > 10^5$, to match the observed emission-line strengths. We cannot view the predicted relative intensities as final, because of the uncertainties of abundances and concentration of ions. In Table 6 we give observed line intensities (O'Dell 1963) in the high-excitation planetary nebula, NGC 7027 (which has $\log N_e = 3.7$, $\theta_e = 0.3$) and repeat the row, $\log N_e = 5$ from Table 5. Note that 3C 48 runs parallel to NGC 7027, except that it has stronger [O II], weaker [Ne V], and possibly weaker [Ne III], for which only a rough estimate can be given. The corrections to Table 5 suggested by NGC 7027, on the average, are a factor of −0.8 in the logarithm, but the variations are smaller than a factor of 10. Just as in planetary nebulae, we have a considerable range of ionization level, with 3C 48 and 3C 273 differing very appreciably. The presence of strong [O III] in 3C 48 may be suggested by the results shown in Figure 5 where there seems to be a strong, excess emission longward of Hβ. Oke kindly informs us that photoelectric scans have, in fact, shown that [O III] is present. The rough intensities in 3C 48 give corrections which are not very different from those suggested by NGC 7027 and suggest a slightly lower level of ionization, since [O II] is stronger and [Ne V] weaker. We adopt a set of corrections shown in the fourth line of Table 6 and recompute part of Table 5, shown in Table 7A. No correction to the predicted Mg II intensities is made, since observations in planetary nebulae are not available. The correction to the O and Ne ionic concentrations may reflect peculiarities of ionization, a slightly lower than normal abundance, or most probably a wide variation in density and size of the emitting region for different ions.

The spectrum of 3C 273 is similar to that of a low-excitation planetary nebula in that it shows no [Ne V] or [Ne III]. However, no low-excitation planetaries are known in which the λ 3727 line of [O II] is weak or absent, as is the case in 3C 273. The most likely explanation is that the electron density is considerably higher than the values of 10^3 to 10^4 cm^{-3} usually encountered in planetary nebulae.

In Table 6 we attempt to obtain corrections for 3C 273 also, with less certain results. The [O III] line λ 5007 is present, but is not strong. No clear evidence for the presence of [O II] λ 3727 is found, although on one higher-dispersion plate (85 Å/mm) a trace of a

TABLE 7A

CORRECTED PREDICTED EMISSIVITIES FOR 3C 48; RADIUS AND HYDROGEN MASS DERIVED FROM OBSERVED Hβ LUMINOSITY

LOG N_e CM^{-3}	LOG $E(X)$, ERG SEC^{-1} PC^{-3}						LOG R_{pc}	LOG M(H)/$M_\odot$
	[O II]	[O III]	[Ne III]	[Ne V]	Mg II	Hβ		
2.	34.3	35.2	33.9	33.9	33.4	34.0	+2.73	9.2
3.	36.2	37.2	35.9	35.9	35.4	36.0	+2.06	8.2
4.	37.9	39.2	37.9	37.9	37.4	38.0	+1.39	7.2
5.	39.2	41.2	39.9	39.9	39.4	40.0	+0.73	6.2
6.	40.2	43.0	41.8	41.9	41.4	42.0	+0.06	5.2
7.	41.2	44.3	43.7	43.8	43.4	44.0	−0.61	4.2
log $L(X)$ observed.	42.5	(Strong)	(Present)	42.2	42.5	42.8	. . . .	. . . .

sharp line is seen at the correct wavelength, allowing for redshift. We compare the spectrum of 3C 273 with that of the low-excitation planetary nebula IC418. The planetary shows weak [Ne III] (3C 273 shows none), and the [O III] line λ 5007 is about 1.5 times Hβ in strength. In 3C 273 this line is about $\frac{1}{3}$ Hβ in strength, suggesting either lower ionization of O or a very high density. Table 5 shows that the low ratio of [O II]/[O III] requires very high density. It seems best to use the corrections to Table 5 derived from IC418 in Table 6; Table 7B gives the predicted emissions. The [O II] line seems to be present, but very faint and with anomalous structure; the predicted [O II]/[O III] ratio changes by 60 from $N_e = 10^4$ to $N_e = 10^7$ cm^{-3}. While our observations do not require so large a ratio they are consistent with the high-density case.

Comparing relative line intensities in C3 48 with a planetary nebula suggests that it has a high ionization level. A high density, less than that at which forbidden lines weaken with respect to permitted lines, is suggested by data in Table 7A. An upper limit, $N_e \leq 3 \times 10^5$ cm^{-3}, is suggested, with $N_e < 10^5$ cm^{-3} a more probable value. For 3C 273, in Table 7B, with fewer lines observed, a lower limit, $N_e > 3 \times 10^5$ cm^{-3}, is suggested by the weakness of [O II], and $N_e \leq 3 \times 10^7$ cm^{-3} is suggested by the presence of [O III]. The upper limit of N_e in 3C 273 can also be estimated[2] from the observed wavelength of Hγ, which for high density would be affected by the λ 4363 line of [O III]. For $\theta_e = 0.3$

[2] We owe this remark to Dr. N. Woolf.

and $N_e = 3 \times 10^7$ we derive from formulae given by Aller (1956) that the λ 4363 line would have a strength about 60 per cent of that of Hγ, leading to a shift of +10 Å relative to Hγ in the observed, redshifted wavelength of the blend. As may be seen from Table 1, the value of z for Hγ is larger than the average redshift by an amount corresponding to about 6 Å in observed wavelength. Since the individual wavelengths are not better than about 4 Å, an electron density of 3×10^7 cm^{-3} is just possible for the assumed T_e of 16800° K.

We adopt an electron density $N_e = 3 \times 10^4$ cm^{-3} in 3C 48, and $N_e = 3 \times 10^6$ cm^{-3} in 3C 273. The uncertainty in each value of N_e is a factor of 10 on either side. The volume emissivity in Hβ is $10^{30} N_e^2$ erg sec^{-1} pc^{-3}. We obtain the form of the relationships between R, N_e, and the observed L(Hβ) as

$$\begin{aligned} \log R_{\text{pc}} &= \tfrac{1}{3} \log L(\text{H}\beta) - \tfrac{2}{3} \log N_e - 10.21 \,, \\ \log M(\text{H})/M_\odot &= \log L(\text{H}\beta) - \log N_e - 31.61 \,. \end{aligned} \tag{12}$$

Radii and hydrogen masses computed from the observed Hβ luminosities of 3C 48 and 3C 273 are listed in the last columns of Tables 7A and 7B. For 3C 48, with $N_e = 3 \times$

TABLE 7B

CORRECTED PREDICTED EMISSIVITIES FOR 3C 273; RADIUS AND HYDROGEN MASS DERIVED FROM OBSERVED Hβ LUMINOSITY

LOG N_e CM^{-3}	LOG $E(X)$, ERG SEC^{-1} PC^{-3}						LOG R_{pc}	LOG M(H)/$M_\odot$
	[O II]	[O III]	[Ne III]	[Ne V]	Mg II	Hβ		
4	38.0	38.2	36.5	36.4	37.4	38.0	+1.75	8.3
5	39.3	40.2	38.5	38.4	39.4	40.0	+1.08	7.3
6	40.3	42.0	40.4	40.4	41.4	42.0	+0.42	6.3
7	41.3	43.3	42.3	42.3	43.4	44.0	−0.25	5.3
8	42.3	44.3	43.8	43.9	45.4	46.0	−0.91	4.3
log $L(X)$ observed	(Weak)	43.5	....	....	43.6:	43.9	....	....

10^4 cm^{-3} we find $R = 11$ pc, $M = 5 \times 10^6 M_\odot$, while for 3C 273, $N_e = 3 \times 10^6$ cm^{-3} leads to $R = 1.2$ pc, $M = 6 \times 10^5 M_\odot$. The effect of the uncertainty in N_e, which we estimate to be ± 1 in the logarithm, on the final R and M of the gas cloud is easily estimated from equation (12).

VI. AGES, LIGHT VARIATIONS, AND ENERGY SUPPLY

The southern end of the optical wisps of 3C 48 is at a distance of about 50 kpc, in projection, from the quasi-stellar object. The end of the jet of 3C 273 is also at a projected distance of about 50 kpc from the quasi-stellar object. If these features have been ejected from the central object, this must have taken place at least 2×10^5 years ago.

The upper limits to the angular diameters of the optical, starlike components from visual inspection are 1″ for 3C 48 and 0.5″ for 3C 273, corresponding to diameters of 5 kpc and 1 kpc. The widths of the emission lines correspond to 2000 and 3000 km/sec, respectively. These speeds are much larger than the escape velocities of about 100 km/sec, determined from the mass of hydrogen and the radius of the emission-line region. If material is indeed escaping from a very small inner region with the observed velocities, and if we may assume that these velocities represent the mean expansion velocity in the

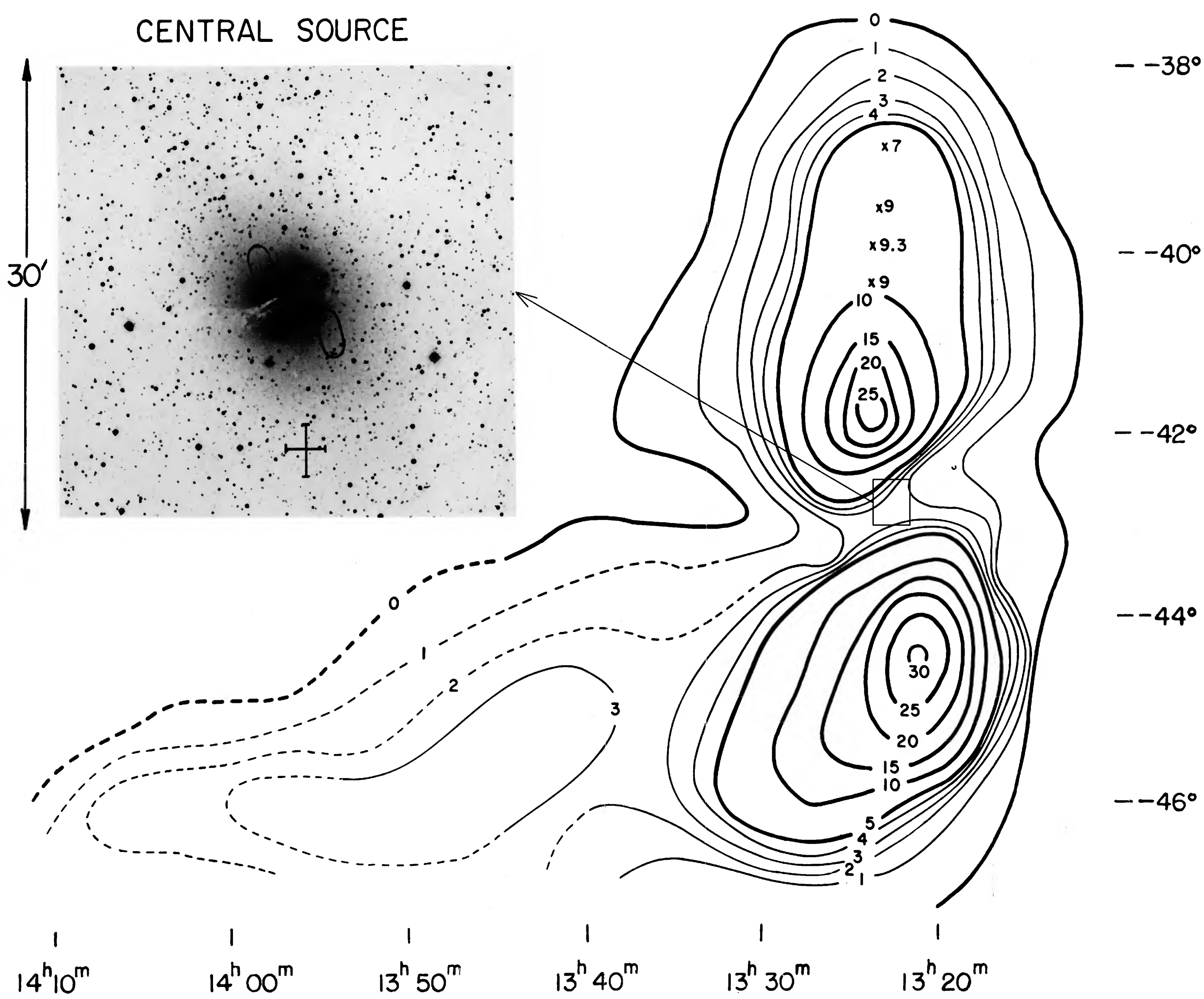

FIG. 2.—The radio source Centaurus A. The outer radio contours are taken from Bolton and Clark (*P.A.S.P.*, **72**, 29, 1960). The model of the inner double source was determined by Maltby (*Nature*, **191**, 793, 1961).

past, we obtain from the upper limit of size a maximum age of 2×10^6 years for 3C 48 and of 3×10^5 years for 3C 273. It would be more realistic to use here the diameters of the gaseous emission regions as determined in the preceding section. Together with the above velocities, these lead to ages of 10^4 and 10^3 years for 3C 48 and 3C 273, respectively.

These ages are considerably smaller than the minimum ages mentioned above. One possibility is that several events have occurred, one 2×10^5 years earlier than the other. Alternatively, the ages derived from the size of emission regions and the line widths would be meaningless if the emission regions are *not* expanding.[3] In that case there must be a mass of about $10^9\ M_\odot$ at the center of the gas cloud. The presence of a mass of this order is also attractive in view of the energy requirements to be discussed presently.

The discovery of light variations in 3C 48 by Matthews and Sandage (1963) and in 3C 273 by Smith and Hoffleit (1963) imposes quite stringent conditions on the light-travel time across these objects. The brighter object, 3C 273, has been studied for a long period. Smith and Hoffleit (1963) and Smith (1964) give results which may be summarized as follows: (1) there are small cyclic variations with a period of about 13 years; (2) flashes lasting of the order of a week or a month may occur, during which the object is up to 1 mag. brighter; and (3) in 1929 a sharp drop in brightness of $0^m.4$ occurred, after which normal brightness was restored by 1940. The cyclic variations with a period around 13 years present no problems as far as the light-travel time is concerned; the diameter of the optical model of the gas cloud for 3C 273 found in Section V is 7 light-years.

Were the 13-year period a rotation at 1500 km/sec, which is compatible with the line width, the object would be only 10^{16} cm in radius. If there is an "oscillation," it does not follow the $P\rho^{1/2}$ law for the mass and radius deduced. If there were an oscillation, it would require information transfer at nearly the velocity of light.

The flashes, which may last for a week or a month, do not necessarily put a restriction on the size of the object as a whole. Consider a region one light-month, i.e., 7×10^{16} cm, in radius. An event affecting this entire volume would be blurred by light-travel time over about one month. The matter is quite transparent when highly ionized. But when compressed, when recombination occurs, its hydrogen will be opaque in the ultraviolet. At $N_e = 10^7$ cm^{-3} the recombination time is about 3 days; there are 5×10^{23} atoms per cm^2, and if neutral, the hydrogen will have an ultraviolet opacity of 5×10^6. But an opaque sphere of this size radiates about 10^{46} erg sec^{-1} at $T = 10^4$ ° K, comparable to the visual luminosity of the flashes in 3C 273. (We have obviously neglected the problem of transfer and of the conversion of ultraviolet quanta into visual radiation, but the order of magnitude of the energy radiated is satisfactory.) A sudden cooling and density condensation thus results in a large and rapid increase of the thermal continuum. There would seem to be no severe problem in understanding the short duration of light flashes if there is such a filamentary or condensation structure, but there is an energy-content problem. Let the gas have m electron volts per proton-electron pair. A sphere one light-month in radius at $N_e = 10^7$ cm^{-3} contains $3 \times 10^{46}\ m$ ergs, so that the energy supply is sufficient for $3m$ seconds. Thermal energies give $m \approx 3$, ionization energies $m \approx 13$, and kinetic energies (at 1000 km/sec) $m \approx 6000$. Thus a maximum flash duration of only a few hours is possible for a region a light-month in radius. Larger total

[3] Electron scattering broadens emission lines when the density is high. The scattering optical depth, t, would be near 10 for 3C 273 and 1 for 3C 48. At a fixed hydrogen-line strength, t varies as $R^{-1/2}$; thus the effectiveness of scattering arises from the small size. Thermal electron velocities are $600(T_e/10^4)^{1/2}$ km/sec, so that an initially sharp emission line would be broadened to a width, at half-intensity, of 1000 km/sec at 10000° K. Multiple electron scattering slows the diffusion of quanta out of the H II region and, therefore, further slows the light variations. In both sources the emission lines have greater velocity widths, and in addition the line profiles seem to vary from line to line. For example, [O III] is sharper than Hβ in 3C 273, and [O II] is sharper than Hγ or [Ne V] in 3C 48. Only if T_e were greater than 10^5 ° K and the sizes less than a parsec would electron and Compton scattering dominate the line widths.

available energies and lifetimes occur only when velocities of 10^4 km/sec or larger are present.

Brief consideration of known means of supplying energy to produce light variations in the gas cloud yields largely negative results. Secondary nuclear-energy sources could be found in shock waves colliding at high velocities, heating to an equivalent temperature of 10^8 ° K at 1000 km/sec. The reaction rates (Milford 1959; Greenstein 1957) are such as to yield about 10^{34} $M/M_{\odot}$ erg sec^{-1}, where M is the mass of gas involved in the collision, from such nuclei as deuterium (lower temperatures) or C^{12} colliding with protons. Obviously only a small fraction of Mc^2 is so released. Supernova explosions usually produce about 10^{50} ergs of visible radiation in the 10^6 sec of their maxima; this is a barely detectable addition to the light of even 3C 48, for example. Supernovae may be sources of radioactive elements produced in the so-called r-process, sources too small, however, to maintain the enormous observed luminosities of the gas cloud for the total life span, 10^5 years. Short-lived decays yield about 2×10^{40} $M/M_{\odot}$ erg sec^{-1}, for a mean lifetime near 30 days, but this is small compared to the energy required for the light flashes, unless multiple supernova detonations occur.

Colgate and Cameron (1963) have computed the efficiency and rate of conversion into visible light of the kinetic energy of a supernova shock wave colliding with an interstellar gas cloud of 10^{-16} gm cm^{-3} density. They assume that supernovae of 100 $M_{\odot}$ are formed and release 3×10^{54} ergs in their explosion, i.e., about 2 $M_{\odot}$ c^2, mostly in fast particles. This is very large compared to the observable radiative output of a supernova. They can obtain, by an ingenious mechanism, up to 5×10^{46} erg sec^{-1} total light, of which about one-tenth might be in the visible. It should be remembered that the large energy output of 2 $M_{\odot}$ c^2 is not justified by any observed feature of a supernova light-curve or spectrum. The total input from the explosion of many such objects could explain the total light and its variability. Whether their model of multiple Type II supernova explosions really applies to these objects, for which nothing is known thus far about stellar content (if any), remains an open question. Their suggestion for rapid conversion of shock-wave energy into light may be an important one when the physical conditions within the gas cloud are better known. Similarly, shock waves around the outer fringe of the gas cloud may be the site of high-energy particle production for synchrotron emission.

While short-lived flashes can be thought of as localized phenomena, this does not apply to long-term decreases below the normal level of brightness. Such a steep drop occurred in 1929, with a decrease of 0.4 mag. for 3C 273 within a period of 7 months. This requires a light-travel time of 7 months across a volume containing at least 40 per cent of the source's radiation, implying a radius of not more than 0.3 pc. The gas cloud would have just this radius for the upper limit of the electron density, 3×10^7 cm^{-3}, derived in Section V.

For 3C 48 less detailed information is available over long periods (Smith and Hoffleit 1961). No secular variation and no fluctuations exceeding 0.3 mag. were found. However, photoelectric observations (Matthews and Sandage 1963; Sandage 1964) have shown it to be variable. The optical flux has changed by a factor of 1.4, apparently independent of wavelength, over a period of 600 days. This variation is too large to be caused by the emission lines only. The radius of the object emitting the continuum cannot be more than 1 pc. If the emission lines originate in this body, the electron density from Table 7A would have to be 10^6 cm^{-3}. This density is somewhat higher than the upper limit of 3×10^5 cm^{-3} derived from the emission-line spectrum in the previous section.

The above discussion suggests that the continuum radiation in 3C 48, and probably also in 3C 273, originates from a smaller volume than the emission-line spectrum. Perhaps the continuum is connected with the body of about 10^8 or 10^9 $M_{\odot}$, which we suspect to be present in these sources (see below, and Sec. IX). The optical continuum in 3C 48 may be synchrotron radiation according to Matthews and Sandage (1963). This would shift the burden of explaining the light variations to models concerned with the non-thermal radiation. Obviously, continuing accurate spectrophotometry in absolute units of the emission lines and their intensity relative to the continuum in both objects would be of great value in further studies of the light variations. Increases in electron temperature or in synchrotron background decrease the apparent emission-line strength; the added synchrotron optical continuum could disappear in a few days. Increase in the thermal electron density causes a quadratic increase in brightness of hydrogen lines and continua but no relative change. The decay time is inversely proportional to the density, with a time constant of $\alpha_H N_e$. According to Aller (1956), $\alpha_H \approx 2 \times 10^{-8}$ days^{-1} cm^3. Thus if $N_e \geq 10^7$ cm^3, hydrogen light variations would occur in 5 days or less, after a sudden brightening caused by an increase of N_e. The time dependence of relative concentrations of ions of oxygen is shown by Aller (1956, Fig. IV:6). He finds recombination rates near one day for $N_e = 10^7$ cm^{-3}, permitting rapid variations in 3C 273. At the lower density of 3C 48, variations with a time scale of a year are plausible. The low intensity of [O II] in 3C 273 cannot be explained as a non-equilibrium ionization level, frozen in for the full 10^5-year time scale, without an unacceptably low N_e.

We have noted several times the fundamental question of the total energy supply. The total radiation from 3C 273 amounts to about 10^{54} ergs per year. If the sources are at least 2×10^5 years old, at constant luminosity, we are faced with the inescapable fact that the energies in the observable gas are insufficient. Thermal and ionization energies are trivial, only a few electron volts per proton. For a source of mass $M/M_\odot$ and luminosity $L/L_\odot$, the total energy released in t years needs to be $10^{-16}\, t(L/L_\odot)/(M/M_\odot)$ ergs per proton. Since, for 3C 273, the gas cloud has an L/M or more than 10^7 and $t > 10^5$, each proton must supply 10^{-3} ergs, of about 10^9 eV. From nuclear physics, only complete annihilation of matter is adequate. If the ordinary nuclear yield of a few MeV per nucleon is the energy source, there must be a nuclear-energy reservoir of several hundred times greater mass than the gas cloud we observe. If gravitational collapse were capable of yielding radiant energy near Mc^2, only $10^5\, M_\odot$ is required; but if the more plausible value of the efficiency is only a few per cent (Fowler and Hoyle 1964) we require the energy from gravitational collapse of more than $10^7\, M_\odot$. Energy production through nuclear processes is less efficient by another factor of 10. If all the energy has come from nuclear fusion, we require a mass of at least $10^8\, M_\odot$. We saw above that a mass of about $10^9\, M_\odot$ would stabilize the fast-moving gas; this object may well be the source of the total energy emitted by the quasi-stellar components of 3C 48 and 3C 273. Continued flow of energy from this massive component into the gaseous nebula is required, and thence into the large radio-emitting region.

VII. FREE-FREE EMISSION, ABSORPTION, AND FARADAY ROTATION

The enormous strength of the hydrogen emission lines in 3C 273 and 3C 48 suggests that free-free emission may be large in either optical or radio wavelengths. An expression for the absorption, per cm, including stimulated emission was derived by Elwert (1948); it does not differ substantially from those more recently derived, e.g., by Oster (1961).

$$k_\nu = \frac{8\pi e^6 N_e^2}{3(6\pi)^{1/2} c (m k T_e)^{3/2} \nu^2} \left[\frac{3^{1/2}}{\pi} \ln \frac{(2kT_e)^{3/2}}{4.22\, m^{1/2} e^2 \nu}\right]. \tag{13}$$

If we denote the bracketed term as $g_{\rm III}$, we get in cgs units

$$\tau_{ff} = 0.017\, g_{\rm III} \frac{N_e^2 R}{\nu^2 T_e^{3/2}}, \tag{14a}$$

$$g_{\rm III} = 10.6 + 1.9 \log T_e - 1.3 \log \nu\,. \tag{14b}$$

From radio to optical frequencies with $\theta_e = 0.3$, $g_{\rm III}$ drops from about 8, at 100 Mc/s, to unity in the optical range. There, a quantum-mechanical expression for the Gaunt factor replaces equation (14b) for $g_{\rm III}$.

Since the emissivity in a Balmer emission line (eq. [3a]) and τ_{ff} (eq. [14]) at radio frequencies both depend primarily on $N_e^2\, T_e^{-3/2}$, we can determine their ratio practically independently of N_e and T_e. For $\theta_e = 0.3$, $b_4 = 0.16$, $g_{\rm III} = 8$, the ratio is

$$\begin{aligned} \tau_{ff}/E(\mathrm{H}\beta) &= 1.3 \times 10^{18}\, \nu^{-2}\, R\,, \\ \tau_{ff} &= 1.3 \times 10^{18}\, \nu^{-2}\, R\, L(\mathrm{H}\beta)/V\,, \\ \tau_{ff} &= 3 \times 10^{-20}\, L(\mathrm{H}\beta)(\nu\, R_{\rm pc})^{-2}\,. \end{aligned} \tag{15}$$

We found in the preceding section that the radius of the emission region of 3C 273 may be about 1 pc. The luminosity in Hβ is 9×10^{43} erg/sec. The opacity will be large for frequencies less than 10^{12} c/s, i.e., over the whole radio spectrum down to wavelengths of 0.3 mm. If we were to give the radius the maximum value, 500 pc, permitted by the stellar appearance and radio diameter, the opacity would still be large for frequencies less than 3×10^9 c/s. This would require that the flux density below 3000 Mc/s be proportional to ν^2. However, the observations show Component B of 3C 273 to have a flux density almost independent of frequency (Hazard, Mackey, and Shimmins 1963).

The probable radius of 3C 48, if $N_e = 3 \times 10^4$, is about 10 pc. In that case the gaseous emission region would be optically thick in free-free for frequencies below 10^4 Mc/s. At the upper limit to its size, near 1500 pc, it would become optically thick and the spectrum would turn downward below 300 Mc/s, well within the usual radio-frequency range. This is also not observed. Thus, that part of the volume of these sources containing the gases emitting the optical line spectrum (as well as the continuous spectrum of 3C 273) cannot also produce the radio-frequency spectrum. Any radio-frequency emission produced in the dense gas cloud is therefore limited to the black-body radiation of an optically thick cloud, i.e., to a brightness temperature equal to the relatively low electron temperature.

No substantial optical emission can occur from the volume containing the radio-frequency synchrotron electrons. The radio-frequency optical depth $\tau_{ff} \geq 1$ if $N_e R_{\rm pc}{}^{1/2} \geq 2\nu$ Mc/s. Thus, if a volume of radius 500 pc produces non-thermal radio noise at $\nu = 10^2$ Mc/s, then $N_e < 10$ cm^{-3}. Thus our dense, massive, expanding or turbulent gaseous nebula must be surrounded by a relatively low density region containing high-energy electrons. A model with shock fronts expanding into a near vacuum and maintaining this very large density gradient could provide the necessary geometry for particle acceleration, with the massive gas clouds providing some of the driving force required.

There exists a large effect of the thermal electron cloud and the magnetic field required by synchrotron theory on the original polarization of the synchrotron radiation. The Faraday rotation is, in radians,

$$\Delta\theta = 81\lambda^2 \int N_e B_{||}\, dR_{\rm pc}\,, \tag{16}$$

where λ is in cm, $B_{||}$ is the parallel component of the field, and the integral is taken over dR in parsecs. Thus in our typical model for the gas, with $R_{\rm pc} \approx 1$ and $N_e \approx 3 \times 10^6$ in 3C 273, and $R_{\rm pc} \approx 10$, $N_e \approx 3 \times 10^4$ in 3C 48, the rotations are $\Delta\theta \approx 10^9\, \lambda^2 B$ and $10^7\, \lambda^2 B$, respectively, when $B_{||} = B$. So large a rotation could depolarize the radiation rapidly within the normally used decimeter band widths, if B is greater than 10^{-5} gauss.

Thus on two grounds, the absence of free-free absorption and the existence of intrinsic source polarization and only moderate Faraday rotation, as is observed in these sources (see Sec. VIII), we can exclude models in which the radio-frequency synchrotron radiation comes from the same volume of space as the emission lines. The optical synchrotron radiation could do so, but if it does, its good fit in 3C 48, after reasonable extrapolation onto the radio-frequency curve, would be accidental. Since according to Oke the optical continuum in 3C 273 is largely free-free, the radio and optical continua are quite unrelated in 3C 273. We will see that a satisfactory synchrotron model can be derived for the energy and magnetic field. For 3C 48, at least, the optical and radio continuum could be produced in the same volume, possibly just outside the dense gas cloud. With our upper limit of $N_e = 10$ cm^{-3} in this outer region, collisional losses by higher-energy particles need not be large.

The existence of spatial inhomogeneities of density and temperature under these circumstances is quite plausible. Threadlike, filamentary structures are common in supernova shells, radio sources, and solar prominences. The emission lines could arise in cool condensations in a much hotter and more massive envelope. At 10^8 ° K and the same $N_e^2 R$, the opacity from equation (14a) is reduced by a million. The hot gas is optically thin except for $\nu < 500$ Mc/s, and has a nearly flat spectrum. For 3C 273, with $N_e^2 R_{\rm pc}^3 = 2 \times 10^{13}$,

$$L_\nu(ff)\,d\nu = 2 \times 10^{32} e^{-h\nu/kT_e} T_e^{-1/2} d\nu \,, \qquad L(ff) = 3 \times 10^{42} T_e^{1/2} \text{ erg sec}^{-1} \,. \tag{17}$$

At $T_e = 10^4$ ° K most of the energy is in the visible, but at $T_e = 10^8$ ° K, it lies in the far ultraviolet. If the quantity $N_e^2 R^3$ were the same for the hot and cold gas, the hot gas would have one-tenth the visual luminosity, but one hundred times the total radiated energy. A lower T_e for the hot gas gives too low a brightness temperature and a wrong energy distribution for free-free emission to be the source of the observed radiation. The reason for so great a temperature differential, from 10^8 to 10^4 ° K, is hard to find, as is the energy source to balance the high value of $L_{ff} = 3 \times 10^{46}$ erg sec^{-1} from the hot gas. The large ultraviolet flux would be a source of excessive ionization, since 10^8 ° K is equivalent to 10 keV, rather than the observed ionization level, which is below 0.1 keV. Therefore, the hypothesis of large temperature fluctuation leads to no useful model. Density fluctuations are very plausible, and require that proper averages be taken over the quantities $N_e^2 R^3$ in emission, $N_e^2 R$ or $N_e R$ in absorption.

VIII. MODELS FOR FIELD AND PARTICLE ENERGIES FROM SYNCHROTRON EMISSION

Our goal is to determine the order of magnitude of the energies in magnetic field and particles required to explain the radio continuum, and perhaps the optical continuum, as synchrotron radiation. Matthews and Sandage (1963) have shown that the energy distribution of 3C 48 at radio wavelengths can be extrapolated smoothly to also account for the optical continuum. This allows interpretation of the whole continuum radiation as synchrotron radiation with critical frequency of about 6×10^{14} sec^{-1}. The optical continuum of 3C 273 cannot be smoothly joined with the radio continuum of Source B. The source has a flux density at radio wavelengths that is several hundred times larger than the flux density at optical wavelengths. Yet the latter shows a slight increase with increasing wavelength, according to Oke (1963). This does not rule out the possibility that part of the optical continuum is synchrotron. Conversely, the smooth fit of radio and optical continuum in 3C 48 does not guarantee the synchrotron origin for the latter. We will consider models of magnetic field and particle energies with and without optical synchrotron radiation.

An investigation of the bolometric correction for optical synchrotron emitters has been made by Greenstein (1964), who has also studied the far-ultraviolet synchrotron radiation as a possible source of ionization in radio galaxies. The results are that there is a minimal value to the bolometric correction, near 1.3 mag., when the cutoff frequency lies near the optical frequency, i.e., $\nu_c \approx 10^{15}$ sec^{-1}. If a high-energy tail is added to the distribution of energies of the relativistic electrons, sufficient ultraviolet synchrotron radiation can be emitted to account for ionization up to 100 eV without greatly increasing the bolometric correction. The absolute visual magnitude of 3C 48 is about -25; for 3C 273, M_v is of the order of -26, and since 3C 273 is quite blue it is bolometrically the brighter of the two by far. The minimum bolometric correction for synchrotron radiation raises M_b to -27.4. Consequently, we must consider the problem of producing over 10^{46} erg sec^{-1} of optical synchrotron radiation in a small volume. The energy distributions of 3C 48 and 3C 273B place quite different requirements on the cutoff frequency; $\nu_c \approx 6 \times 10^{14}$ sec^{-1} for 3C 48 and either a larger value for 3C 273B, if its blue continuum is synchrotron, or a much lower value if it is free-free. We will study different values of ν_c, from 6.3×10^{14} sec^{-1} to 6.3×10^{10} sec^{-1}, and obtain models for the luminosities corresponding to the cutoff energies. In addition, we cannot be certain that the optical and radio continua arise from the same volume. The light variability, probably largely in the continuum, suggests a much smaller volume if the optical radiation is synchrotron in origin.

Assume that the relativistic electrons giving radiated energy of power-law index $n = (m - 1)/2$ have a number density given by a power law:

$$N(E)\,dE = \frac{N_0 V\,dE}{E^m}. \tag{18}$$

The total particle energy content in electrons and protons is

$$U_p = p \int_{E_1}^{E_2} N(E) E dE = \frac{p N_0 V}{2 - m} E^{2-m} \Big|_{E_1}^{E_2}, \tag{19}$$

where we have assumed that heavy particles carry $(p - 1)$ times the energy of the electrons. The total synchrotron radiation over all frequencies is approximately

$$L = 2.4 \times 10^{-3} B^2 \frac{N_0 V}{3 - m} E^{3-m} \Big|_{E_1}^{E_2}. \tag{20}$$

The cutoff frequency ν_c is defined by a dimensionless parameter β,

$$\beta \equiv \frac{\nu_c}{6.3 \times 10^{18}} = B E_2^2. \tag{21}$$

The lifetime, in days, of an electron of energy E, in ergs, is

$$t_{1/2}(E) = (206 B^2 E)^{-1}. \tag{22}$$

The energy content in the magnetic field is

$$U_M = \frac{B^2 V}{8\pi}, \tag{23}$$

and for convenience let the total particle energy $U_p = \alpha U_M$.

For values of m well below 2, only E_2 is needed in equations (19) and (20), since $E_2 \gg E_1$. It is of interest to investigate briefly the case $m = 1$, corresponding to a flat spectral energy distribution ($n = 0$), such as that exhibited by the bluer quasi-stellar objects. The luminosity is then

$$L = 0.03\, U_M U_p \frac{E_2}{V p}, \tag{24}$$

and we obtain

$$L = \frac{0.03\, \alpha U^2_M \beta^{1/2}}{p V B^{1/2}}. \tag{25}$$

For a quick reconnaisance, equation (25) is useful. We will assume that the optical and radio emission arises in the same region, of radius R_{pc} (in parsecs). The magnetic field B (in gauss) is given as a function of L (in erg sec^{-1}) and the parameters α, β, and p as

$$\log B = \tfrac{2}{7} \log \frac{pL}{\alpha} - \tfrac{6}{7} \log R_{pc} - \tfrac{1}{7} \log \beta - 14.79 \,. \tag{26}$$

We have an upper limit to R_{pc} from the angular diameter, and L is known. The total energy, $(1 + \alpha)U_M$, is obtainable from

$$\log U = \log(1 + \alpha) + \tfrac{4}{7} \log \frac{pL}{\alpha} + \tfrac{9}{7} \log R_{pc} - \tfrac{2}{7} \log \beta + 25.11 \,. \tag{27}$$

For $p = 1$ (no protons) and $\alpha = 1$, the usual equipartition assumption, we obtain values of B which may at first seem reasonable. Using $L = 2.5 \times 10^{46}$ erg/sec for 3C 273, $\beta = 10^{-4}$ (i.e., $\nu_c = 6.3 \times 10^{14}$ sec^{-1}) and adopting $R_{pc} = 500$ (see below), we find the value of $B = 5.4 \times 10^{-4}$ gauss. The most energetic "optical" electrons present have $E_2 = 0.4$ erg or 250 BeV and a lifetime, $t_{1/2}$, of 100 years. But the derived U (the sum of U_p and U_M) is too small—only 3×10^{56} erg, an energy supply sufficient to maintain the present total optical energy output for only 400 years. (This maximum time scale is evaluated from $t_0 = U/L$, i.e., assuming constant luminosity and complete conversion of both U_p and U_M into radiation.) If the total particle energy is 100 times that of the electrons, i.e., $p = 100$, then for $\alpha = 1$ the field is $B = 2 \times 10^{-3}$ gauss, and the total energy content 4×10^{57} ergs, sufficient for at most 5000 years at the present rate of radiation if the entire proton energy can eventually be converted into relativistic electrons. The total energy can be increased if we admit non-equipartition of energies of field and particles, but the gain is by a factor of less than 10 for values of α between 0.01 and 100. In non-equipartition cases, there is possibly some preference for values of α less than unity, if we think that the magnetic fields cause particle acceleration, i.e., B greater than in the equipartition case.

In further considerations of the properties of 3C 273, and in finding the energies involved in a model for 3C 48, we will account for the properties of each source specifically. We have adopted $p = 100$, i.e., the relativistic proton (or heavy particle) total energy store 99 times that of the electrons. Burbidge (1956) evaluated the direct production of electrons or positrons from meson decay in energetic stationary proton–cosmic-ray collisions, giving $p \approx 30$. At the very best, neutrino- and gamma-ray losses insure that $p > 10$. Only a direct electron injection could give small p. We give the parameters of interest in Table 8. For given values of n, ν_c, L, and E_1 the field strength B determines E_2, N_0V, and U_p through equations (19)–(21). For 3C 48 we use $n = 0.67$ and $\nu = 6.3 \times 10^{14}$ sec^{-1} ($\beta = 10^{-4}$) from a discussion by Matthews and Sandage (1963) to derive data given in Table 8 for Case A. Case B in the table is based on $\nu_c = 6.3 \times 10^{10}$ sec^{-1} ($\beta = 10^{-8}$), i.e., the optical continuum has a non-synchrotron origin or arises in a smaller volume with larger B. Because n is larger than 0.5, the total particle energy depends strongly on the lower-energy cutoff of the relativistic electron spectrum, E_1. We will take this as 100 MeV for electrons, assuming that at lower energy they are lost by inelastic collisions so that $E_1 \approx 1.6 \times 10^{-4}$ erg. Since the volume occupied by the radio emission is not directly measured, in order to make the problem concrete, for the energy-supply time scale, t_0, we have assumed an undetectably small synchrotron process volume of 10^9 pc^3 for Cases A and B, the same as the observed radio volume of the core of 3C 273B (see below). Lovell (private communication) informs us that 3C 48 is still unresolved at 180000 λ.

From our analysis of the emission lines, and from the light variations, we believe that the optical radiation of 3C 48 comes largely from a small volume that is less than about 10 pc in radius. In Case C we assume that the optical continuum is of synchrotron origin, has $n = 0.67$ and $\nu_c = 6.3 \times 10^{14}$ sec^{-1}, but arises from a volume $V = 4 \times 10^3$ pc^3. The radio-frequency spectrum has low total luminosity in 3C 48, compared to that in optical frequencies, so we can still use $L = 2 \times 10^{46}$ erg sec^{-1}.

The source 3C 273 has a more complicated structure at radio wavelengths. We shall not here consider Source A that is associated with the optical jet. Source B, associated with the quasi-stellar object, has an extended part with a diameter of about 4″ with a central component with a diameter of about 0.5″ (Hazard 1964), or about 1.1 kpc. We shall only be concerned with this core of 3C 273B. Accordingly, we have adopted for the source of continuum radiation in Cases A and B a volume of 10^9 pc^3, corresponding to a radius of about 500 pc. Observations at Michigan (Dent and Haddock 1964) suggest that 3C 273B may have a flat radio-frequency spectrum at very high frequencies. We

TABLE 8

PARTICLE AND MAGNETIC ENERGIES IN ERGS FOR SYNCHROTRON MODELS OF 3C 48

($n = 0.67$, $m = 2.34$)

CASE A

LUMINOSITY ALL SYNCHROTRON; $\nu_c = 6.3 \times 10^{14}$ SEC^{-1}, $L = 2 \times 10^{46}$ ERG SEC^{-1}; RADIO AND OPTICAL CONTINUUM FROM 10^9 PC^3 VOLUME

LOG B	LOG E_2 (erg)	LOG N_0V	LOG U_p	LOG U_M	TIME SCALES	
					log $t_{1/2}(E_2)$ (days)	log t_0 (yrs)
0	−2.0	50.1	53.8	63.1	−0.3	9.3
−1	−1.5	51.7	55.5	61.1	+1.2	7.3
−2	−1.0	53.4	57.2	59.1	+2.7	5.3
−3	−0.5	55.1	58.8	57.1	+4.2	5.0
−4	0.0	56.7	60.5	55.1	+5.7	6.7
−5	+0.5	58.4	62.2	53.1	+7.2	8.4

CASE B

LUMINOSITY OF SYNCHROTRON ORIGIN ONLY AT RADIO FREQUENCIES; $\nu_c = 6.3 \times 10^{10}$, VOLUME $V = 10^9$ PC^3; $L = 10^{45}$ ERG SEC^{-1}

LOG B	LOG E_2 (erg)	LOG N_0V	LOG U_p	LOG U_M	log $t_{1/2}(E_2)$ (days)	log t_0 (yrs)
0	−4.0	48.7	52.5	63.1	1.7	10.6
−1	−3.5	50.4	54.2	61.1	3.2	8.6
−2	−3.0	52.1	55.8	59.1	4.7	6.6
−3	−2.5	53.8	57.5	57.1	6.2	5.1
−4	−2.0	55.4	59.2	55.1	7.7	6.7
−5	−1.5	57.1	60.8	53.1	9.2	8.3

CASE C

LUMINOSITY OF SYNCHROTRON ORIGIN AT OPTICAL FREQUENCIES; $\nu_c = 6.3 \times 10^{14}$ FROM 4×10^3 PC^3 INNER CORE; $L = 2 \times 10^{46}$ ERG SEC^{-1}

LOG B	LOG E_2 (erg)	LOG N_0V	LOG U_p	LOG U_M	log $t_{1/2}(E_2)$ (days)	log t_0 (yrs)
+1	−2.5	48.4	52.2	59.7	−1.8	5.9
0	−2.0	50.1	53.8	57.7	−0.3	3.9
−1	−1.5	51.7	55.5	55.7	+1.2	2.1
−2	−1.0	53.4	57.2	53.7	+2.7	3.4

have assumed for 3C 273B (core) a flux density of about 10^{-25} W m^{-2} (c/s)$^{-1}$, independent of frequency (i.e., $n = 0$, $m = 1$). At the distance of 3C 273 this corresponds to a flux of about 3×10^{33} erg sec^{-1} (c/s)$^{-1}$. We have actually used the total luminosities given in Table 9, Case A for $\nu_c = 6.3 \times 10^{13}$ sec^{-1}, and Case B with $\nu_c = 6.3 \times 10^{10}$ sec^{-1}. In Case A the cutoff was arbitrarily placed in the infrared, in such a way that synchrotron radiation is negligible in the visual range. Since m is rather smaller than 1, the values of U_p and L depend mostly on E_2, very little on E_1. We have again used $p = 100$, in deriving data given in Table 9.

TABLE 9

PARTICLE AND MAGNETIC ENERGIES, IN ERGS, FOR SYNCHROTRON MODELS OF 3C 273B

($n = 0$, $m = 1$)

CASE A

LUMINOSITY ALL SYNCHROTRON;* $\nu_c = 6.3 \times 10^{13}$ SEC^{-1}, $L = 2 \times 10^{47}$ ERG SEC^{-1}; RADIO AND OPTICAL CONTINUUM FROM 10^9 PC^3 VOLUME

LOG B	LOG E_2 (erg)	LOG N_0V	LOG U_p	LOG U_M	TIME SCALES: log $t_{1/2}(E_2)$ (days)	TIME SCALES: log t_0 (yrs)
0	−2.5	55.2	54.7	63.1	0.2	8.3
−1	−2.0	56.2	56.2	61.1	1.7	6.3
−2	−1.5	57.2	57.7	59.1	3.2	4.3
−3	−1.0	58.2	59.2	57.1	4.7	4.4
−4	−0.5	59.2	60.7	55.1	6.2	5.9
−5	0.0	60.2	62.2	53.1	7.7	7.4

CASE B

LUMINOSITY OF SYNCHROTRON ORIGIN ONLY AT RADIO FREQUENCIES; $\nu_c = 6.3 \times 10^{10}$ SEC^{-1}, $L = 2 \times 10^{44}$ ERG SEC^{-1}; VOLUME $= 10^9$ PC^3

0	−4.0	55.2	53.2	63.1	1.7	11.3
−1	−3.5	56.2	54.7	61.1	3.2	9.3
−2	−3.0	57.2	56.2	59.1	4.7	7.3
−3	−2.5	58.2	57.7	57.1	6.2	6.0
−4	−2.0	59.2	59.2	55.1	7.7	7.4
−5	−1.5	60.2	60.7	53.1	9.2	8.9

CASE C

LUMINOSITY OF OPTICAL CORE IS OF SYNCHROTRON ORIGIN FROM SMALL VOLUME = 1 PC^3, $\nu_c = 6.3 \times 10^{15}$ SEC^{-1}; $L = 5 \times 10^{46}$ ERG SEC^{-1}

+1	−2.0	51.6	51.6	56.1	−2.3	+1.8
0	−1.5	52.6	53.1	54.1	−0.8	−0.2
−1	−1.0	53.6	54.6	52.1	+0.7	+0.3
−2	−0.5	54.6	56.1	50.1	+2.2	+1.8
−3	+0.5	55.6	57.6	48.1	+3.2	+3.3

* The very high luminosity in Case A arises as follows: There is considerable difficulty in assuming the optical radiation to be of synchrotron origin, according to Oke. In this model we extrapolate the flat radio spectrum to the infrared and set a cutoff energy such that it does not contribute to the visual and blue continuum. If ν_c is reduced, L is reduced in proportion.

Inspection of Tables 8 and 9, Cases A and B, shows that in all cases equipartition between field and total particle energies (with the assumed $p = 100$) is reached at a magnetic-field strength of between 10^{-3} and 3×10^{-3} gauss. The corresponding upper limits to the lifetime $t_0 = (U_M + U_p)/L$ at present luminosity for optical synchrotron radiation are almost 10^5 years and 10^4 years for 3C 48 and 3C 273, respectively. The maximum lifetime t_0 for radio radiation in 3C 48 is barely larger than that for the optical synchrotron radiation, i.e., about 10^5 years. The maximum lifetime t_0 for radio radiation in 3C 273 is 10^6 years. The decay times for the "optical" electrons in 3C 48 are 10 years, for $B = 3 \times 10^{-3}$ gauss, and about 100 times longer for the highest-energy electrons producing radio frequencies. Thus variability of the continua is plausible on optical but not on radio frequencies.

There is no particular reason why in these sources there should be equipartition between field and particle energies. The range of magnetic field which is plausible for these non-equipartition cases is limited by the following considerations. If there is a magnetic field in the gaseous emission-line region, we might interpret the line broadening as an Alfvén velocity. Since $v_A \approx 2.2 \times 10^{11} BN_e^{-1/2}$, from the observed $v = 10^8$ cm sec^{-1}, we obtain $B = 0.1$ gauss for the inner region of 3C 48. It is improbable that B in the radio-emitting region is larger than in the gas clouds, so we should use $B < 10^{-1}$ gauss. We have already shown from Faraday rotation and from the lack of free-free self-absorption at radio frequencies that N_e is small in the outer region; an Alfvén velocity q times that of light requires that $B = 0.14\, qN_e^{1/2}$, again setting the upper limit to the field at $B < 0.5\, q$ gauss.

The observed Faraday rotation also carried information about B (cf. eq. [16]). Gardner and Whiteoak (1963) give for 3C 273 a Faraday rotation of -8.2 radians per m^2. The polarizations observed, ranging from $2\frac{1}{2}$ per cent at 21-cm wavelength to 4 per cent at 10-cm wavelength, must be due to Component B, which has a much flatter spectrum than Component A. Seielstad (1963) derived a rotation of $+11 \pm 6$ radians per m^2 for 3C 273, and $+48 \pm 9$ radians per m^2 for 3C 48. We find from these small rotations and equation (16) that N_eB is about 10^{-8} to 10^{-7} for both sources, if $R \approx 500$ pc. The requirement that no free-free absorption in the radio-emitting region occurs sets an upper limit to N_e of about 10 cm^{-3} (cf. Sec. VII). The corresponding lower limit of 10^{-8} in B is quite low. Since the field must be larger, the small rotation observed requires very low N_e.

Arguments leading to the value of the field strength B in these sources will involve the age of the objects and the possibility of a steady renewal of the high-energy electrons from some unknown central source, as in the Crab Nebula. This very interesting question of evolution of radio sources and the interchange between energy modes, U_p and U_M, is outside the scope of the present paper. If there is a steady renewal from an unknown energy source of the high-energy electrons, the values of t_0 listed in Tables 8 and 9 are lower limits. It should be noted that, if there is no such steady injection of high-energy particles and magnetic energy from the unknown source, t_0 must be a quite generous upper limit to the past lifetime, because it assumes complete conversion of the total energy content of the source into synchrotron radiation from electrons, neglecting collisional losses in U_p and the dissipation of the field, U_M, by loss of flux lines, when, in fact, the efficiency is probably quite low.

One most interesting case for which we may evaluate the energy density is given in Tables 8 and 9 and labeled Case C. Here the optical continuum is interpreted as synchrotron radiation from the same inner volume containing a gas nebula which produces the emission lines. In 3C 48 the radius was 10 pc or more; in 3C 273, less than 1 pc. The differences in the derived properties arise largely from the difference in volumes. The equipartition values of B are near 0.1 gauss, and the total energies are 10^{56} and 10^{53} erg, respectively. The lifetime of the high-energy electrons is a few days, but t_0 is 100 years and 1 year for 3C 48 and 3C 273. These total lifetimes, based on exhaustion of the energy supply, are near the radius, measured in light years, i.e., relativistic particles, even traveling in straight lines, would barely traverse the volume before their energy must be exhausted.

Even more startling is the energy density of relativistic particles. At equipartition in 3C 273B (Case C, $\alpha = 1$, $p = 100$), we find $B \approx 0.5$ gauss, $U_p = 3 \times 10^{53}$, i.e., 0.01 erg cm^{-3}, 6×10^9 eV cm^{-3}. If we evaluate an equivalent temperature from U_p, we find $T = 1000°$ K. The gas has $N_e kT \approx 10^{-5}$ erg cm^{-3}, but if we include translational energy $\frac{1}{2}\rho v^2$, it has 0.05 erg cm^{-3}. Thus, the cosmic rays have nearly the same energy as the observable gas. The energy density of radiation near the surface is $\approx 0.05\ L/R^2c \simeq 0.02$ erg cm^{-3} also. The loss rate to the high-energy protons can be evaluated from the nuclear collisions with stationary protons, which have a cross-section of 4×10^{-26} cm^2. The lifetime of the protons against collision is 10 years. The synchrotron lifetime for high-energy electrons is near 10^5 sec. The inverse Compton effect is large and should be evaluated. The energy loss is measured by $c\sigma_0\gamma^2 u$, where σ_0 is the Compton cross-section, 6.6×10^{-25} cm^2, $\gamma = E/m_0c^2$ is the electron energy and u is the photon energy density. With values used of L and R in Case C, an electron loses energy at the rate of 4×10^{-8} erg sec^{-1}, or has a lifetime of 2×10^5 sec. If we slightly increase u or γ the inverse Compton effect can be very important compared to synchrotron losses. It is suggestive that the balance is so close; if inverse Compton effect dominates, the photons are "heated" by the cosmic rays, and therefore the thermal gas will also be heated. The proton nuclear collisions produce an energy loss of U_p/τ where τ is their nuclear-collision lifetime; this is about 10^{45} erg sec^{-1}, less than the luminosity by 1–2 orders of magnitude. Thus the small radius model for optical synchrotron radiation is nearing the boundaries of the plausible. At much higher energy concentrations (i.e., higher luminosities from so small a volume), other physical effects may provide a limit.

IX. THE JET, 3C 273A

The stronger radio source in 3C 273 is associated with a visible jet about $1.5'' \times 10''$. We have no conclusive spectroscopic or colorimetric data as to the nature of its continuum, nor even an accurate apparent magnitude. The radio-frequency observations suggest an index $n = 0.77$ and a flux of 4.2×10^{-25} W m^{-2} (c/s)$^{-1}$ at 400 Mc/s. Let us assume that the entire radiation is synchrotron and has $\nu_c = 6.3 \times 10^{14}$ sec^{-1}, as does 3C 48. We then predict the optical apparent magnitude of the jet and find it to be $16\overset{m}{.}6$, brighter than seems plausible for an object not much brighter than the sky background. A spectral index of $n = 0.9$ and the same ν_c predicts an apparent magnitude near 19^m, which is probably more nearly correct. The total luminosity of the jet is not very sensitive to the frequency range adopted, varying as $\nu_c^{0.1}$, and is found to be 2×10^{44} erg sec^{-1}. The volume producing optical radiation is observed to be about 3×10^{11} pc^3. If we use our equations for the case $\alpha = 1$, we find $B \approx 6 \times 10^{-4}$ gauss, $E_2 \approx 200$ BeV, the total energy $U = 2 \times 10^{60}$ erg. For other values of B, with $p = 100$, the results are given in Table 10. Note that especially if $U_M > U_p$ the lifetime of the highest-energy electrons becomes short, suggesting that the electron acceleration processes must continue, even in this source of very low energy density, 10^{-7} erg cm^{-3}. The total lifetime, t_0, however, is very long, more than 10^8 years, so that a single event storing 10^{60} erg and occurring 10^5 years ago could easily provide the required energy. At constant luminosity, the total radiation in 10^5 years is 10^{57} erg, a small fraction of the minimum U of 10^{60} erg. We cannot yet make a full study of 3C 273A. The details of radio structure of the jet are not as simple as the simple cylindrical model we adopt. There is a brightening at the far end, and there consequently may be density and field fluctuations.

If the optical luminosity is not of synchrotron origin, our analysis would be modified only slightly. Cutting ν_c to 6×10^{10} sec^{-1}, and using the radio power-law index of 0.77 gives $L = 10^{44}$ erg sec^{-1}. If we examine analogous solutions for 3C 48 and 3C 273B in Tables 8 and 9 (Case B) and change the volume to 3×10^{11} pc^3, the minimum energy is then 4×10^{58} erg, $B = 3 \times 10^{-4}$ gauss and t_0 is 10^7 years.

It is of course possible that the jet has hydrogen emission rather than synchrotron continuum. From equation (3b) the Hβ luminosity from 3×10^{11} pc^3 is $10^{42} N_e^2$ erg sec^{-1}, the total hydrogen emission is about $3 \times 10^{43} N_e^2$ erg sec^{-1}. From $L = 10^{44}$ erg sec^{-1} we obtain $N_e = 1.7$ cm^{-3}, or a mass of $10^{10} M_\odot$, a rather high value; such an N_e is not excluded by considerations of radio-frequency opacity (eq. [15]), but could give large Faraday rotation (eq. [16]). So far, one spectrum of the far end of the jet has been obtained. It shows only a weak, bluish continuum.

X. DISCUSSION

We have considered the explanation of the observed redshifts in terms of (1) Doppler effect from a high-velocity star, (2) gravitational redshift, and (3) cosmological redshift.

The first two alternatives have been shown to lead to an extra-galactic nature of the object. In case 1 the small proper motion results in a distance measured in megaparsecs and a luminosity closer to that of galaxies than of stars. The four quasi-stellar objects with known velocity are all receding from us (Schmidt and Matthews 1964). These facts,

TABLE 10

SYNCHROTRON RADIATION FROM THE JET, 3C 273A
($L = 2 \times 10^{44}$ erg sec^{-1}; $\nu_c = 6.3 \times 10^{14}$ sec^{-1};
$n = 0.9$; volume $= 3 \times 10^{11}$ pc^3)

LOG B	LOG E_2 (erg)	LOG N_0V	LOG U_p	LOG U_M	TIME SCALES	
					log $t_{1/2}(E_2)$ (days)	log t_0 (yrs)
−2	−1.0	52.4	57.4	62.2	2.7	10.4
−3	−0.5	54.3	59.3	60.2	4.2	8.5
−4	0.0	56.2	61.2	58.2	5.7	9.4

when combined with the problem of how to accelerate an apparently very large star (or stellar system) to velocities that are an appreciable fraction of that of light, make case 1 an exceedingly unlikely interpretation.

We have shown that alternative 2 also leads to an extra-galactic nature of these objects. In fact, the mass would have to be of the order of that of a galaxy or more. It seems rather likely on the basis of current theoretical work that objects of such a mass, condensed to a diameter of less than 1 pc, cannot be stable and thus cannot exist for any length of time. The thinness of the surrounding emission-line shell would also be a problem. Altogether, we believe that it is quite unlikely (but not definitely disproven) that gravitational redshifts explain the spectrum of the quasi-stellar objects.

Accordingly, we have adopted the interpretation of the redshifts as cosmological redshifts. The ensuing lengthy astrophysical discussion of the emission spectra gave radii for the gaseous nebulae of about 1 pc for 3C 273, and 10 pc or more for 3C 48. The light variations seem to require even smaller sizes for the source of optical continuum radiation, especially for 3C 48. The non-thermal character of the radio spectrum shows that it must originate outside the gas cloud which emits the emission lines. We find it attractive to think of a model in which a small inner core produces most of the optical continuum, surrounded by a gas cloud producing the emission lines and thermal continuum. This would itself be surrounded by the radio-emitting regions.

The models for optical continuum synchrotron radiation from an inner volume small enough to admit of the light variations encounter serious difficulties. The high energy-density in a region containing a gaseous nebula produces rapid loss of cosmic-ray protons; the electrons are lost rapidly by either inverse Compton-effect or synchrotron radiation, so that it is nearly impossible to maintain the high-energy particle supply.

An important parameter in further considerations regarding the quasi-stellar objects is their age, i.e., their lifetime as objects producing large optical luminosity from an intrinsically small volume. Let us consider the consequences of two quite different estimates of their age, namely, 10^3 years and 10^6 years.

Age 10^3 years.—This is the age, to an order of magnitude, that follows from the size of the gaseous nebula and the interpretation of the widths of the emission lines as caused by expansion. It also seems a lower limit to the possible age of 3C 273, because the secular decrease in optical light amounts to less than 0.1 mag. per century (Smith 1964). The total energy output would amount to 10^{57} ergs, or the rest-mass energy of about 500 suns. This amount can be supplied by nuclear fusion of 10^5 $M_\odot$ of hydrogen to helium. Such a mass is less than that of the gases producing the emission lines; the expansion of the fast-moving gas would be unimpeded. Any of the synchrotron emission models in Tables 8 and 9 (Cases A and B) would have sufficient energy content to maintain present radiation over 10^3 years. *Not* explained by so short an age would be the radio halo of 3C 273B, the radio source 3C 273A and the jet, or the optical wisps near 3C 48. We assume here that all these features originated in what is now the quasi-stellar object. Their existence requires either a number of separate events, or a much larger age.

Age 10^6 years.—The above objections are met if we assume an age of about 10^6 years. Total energy output would amount to 10^{60} ergs, requiring gravitational collapse of a large mass or the nuclear energy for 10^8 $M_\odot$ of hydrogen. This would probably involve a total mass of some 10^9 $M_\odot$. Such a mass could stabilize the large internal motions of the observed H II region at the radius of a few parsecs, derived from the electron density and the intensity of the emission lines. Synchrotron emission models (Tables 8 and 9) with $B \approx 10^{-3}$ gauss (close to equipartition of energies) do not have sufficient energy content to last for 10^6 years at present luminosity. Either there is a steady injection of high-energy particles or a non-equipartition of energy. In the latter case we would consider field of 0.01–0.1 gauss most likely.

We discussed above a model of the quasi-stellar objects consisting of an optical-continuum source (radius < 1 pc) surrounded by an emission region (radius ≈ 1 pc [3C 273], ≈ 10 pc [3C 48]) and by a radio-emitting region. If there is indeed an object with a mass of about 10^9 $M_\odot$ present in these objects, then, presumably, this would be inside the small optical-continuum source. The radius of the 10^9 $M_\odot$ object could have any value below 1 pc. Its gravitational redshift, if any exists, cannot easily be observed because the observed redshift of emission lines refers to a distance from this mass of about 1 pc, where the gravitational effect is negligible. The radius of a Schwarzschild sphere is $2GM/c^2$, about 10^{-4} pc for such a mass. It would be important to know whether continued energy and mass input from such a "collapsed" region are possible.

It is not yet possible to establish the role of these quasi-stellar radio sources in the evolution of galaxies or radio galaxies. No trace of a galaxy around these objects has been found as yet. It is not certain at present that this really excludes the possibility of a galaxy being present; seeing and scattering in the photographic emulsion make detection of a low surface-brightness galaxy, containing a stellar object 100 times brighter, very difficult.

The quasi-stellar sources might have been thought to be a precursor stage of the radio galaxies. Their radio luminosities are about equal to that of the most intense radio galaxies. However, the linear sizes of the radio-emitting regions in the quasi-stellar sources show a range quite similar to that seen in the radio galaxies (Schmidt and Matthews 1964). This does not support the idea that radio galaxies start their radio life as a quasi-stellar source. Either the quasi-stellar stage can occur at any time in the life of a radio galaxy, or the two phenomena may be completely unrelated, with the quasi-stellar objects primary intergalactic condensations.

In Table 11 we give a final résumé of the properties of possible models. The "distance" r is obtained simply from czH^{-1}, with $H = 100$ km sec^{-1} Mpc^{-1}. The Hβ luminosity is observed; the N_e refers to the H II regions producing permitted and forbidden lines, the internal velocity, v, is deduced from the line widths. The N_e in 3C 273 is determined from the weakness of [O II]; in 3C 48 a wide range is possible, and we give nearly the largest acceptable. The angular diameter gives the sizes, but R(H II) is derived from L(Hβ) and N_e, so the masses depend on $N_e R^3$. The total luminosity is well determined for the visual region, but bolometric corrections may be large. If the radiation is of synchrotron origin, for various ν_c, the L becomes fixed, and the total L and U are given in the last columns.

TABLE 11

RÉSUMÉ OF DATA ON QUASI-STELLAR SOURCES

	3C 48	3C 273
z	0.367	0.158
r (Mpc)	1100	474
Luminosities (erg sec^{-1}):		
Hβ	6×10^{42}	9×10^{43}
Visual	10^{45}	4×10^{45}
Radii (R_{pc}):		
Optical	<2500	<500
Radio	<2500	500
H II region	≥10	1
H II region:		
N_e cm^{-3}	$\leq3\times10^4$	3×10^6
$M/M_\odot$	$\geq5\times10^6$	6×10^5
v (km/sec)	1000	1500
Equipartition synchrotron models		
L (erg sec^{-1}):		
Case A	2×10^{46}	2×10^{47}
Case B	10^{45}	2×10^{44}
Case C	2×10^{46}	5×10^{46}
U (erg):		
Case A	10^{58}	2×10^{58}
Case B	2×10^{57}	3×10^{57}
Case C	4×10^{55}	4×10^{53}

We are much indebted to J. B. Oke for as yet unpublished information on the photoelectronic scans of the continuum of 3C 273 and of the red region of the spectrum of 3C 48; we are grateful to many colleagues for very stimulating discussions and for much observational data before publication.

APPENDIX I

ELECTRON TEMPERATURES

Although the present material is not sufficient for a quantitative analysis, the appearance on three plates of what seems to be the auroral line, λ 2973 of [Ne V], gives the possibility of estimating the electron temperature in 3C 48 by a new method. The situation is the same as for λ 4363 of [O III] except that λ 2973 will be important at higher electron temperatures. It may be observable in planetary nebulae from rockets or satellites. Consequently, an approximate derivation of the ratio of the auroral to the nebular lines of [Ne V] was made. The various terms in the general expression for b_3/b_2, the populations of states 3 and 2, were evaluated approximately to determine which could be neglected. The collisional cross-sections are only poorly known. The results are

$$\frac{b_3}{b_2}=\frac{C(\Omega_{12}+\Omega_{23})+A_{21}g_2}{C(\Omega_{13}+\Omega_{23})+A_{32}g_3}, \tag{AI.1}$$

$$\frac{E_{32}}{E_{21}}=1.13\times 10^{-4.17\theta_e}\left(\frac{1+4.5\times 10^{-6}N_e/T_e^{1/2}}{1+2.2\times 10^{-6}N_e/T_e^{1/2}}\right). \tag{AI.2}$$

Equation (AI.2) gives the ratio of the intensity of the auroral line E_{32} to E_{21}, the sum of the intensities of the two nebular lines. Note that if the term involving $N_e/T_e^{1/2}$ is less than unity, i.e., low N_e, one asymptotic limit to the intensity ratio for one of the nebular lines, λ 3426, is

$$\frac{E(\lambda 2973)}{E(\lambda 3426)}=1.55\times 10^{-4.17\theta_e}, \tag{AI.3}$$

and if N_e is very high, the other limit is

$$\frac{E(\lambda 2973)}{E(\lambda 3426)}=3.2\times 10^{-4.17\theta_e}. \tag{AI.4}$$

As a function of N_e the coefficient of $10^{-21000/T_e}$ changes only by a factor of 2. The crossover to equation (AI.4) occurs for $T_e = 20000°$ at $N_e > 6 \times 10^7$ cm^{-3}, a very high electron density, so that equation (AI.3) is the expression for most nebulae and for 3C 48. Consequently, the uncertainty of the cross-sections proves not to be a serious factor. If λ 2973 appears at all (i.e., with an intensity ratio to λ 3426 of even 0.1), we know that T_e is above 15000° K. It can become quite strong if $T_e > 20000°$ K; since λ 2973 is weak in 3C 48, it is improbable that $T_e > 40000°$ K.

From observation of planetary nebulae, we find associated with the small changes of T_e large changes in the relative intensity of forbidden lines of different ions; the direct effects of change of T_e are relatively small in the excitation of the nebular lines. In the absence of knowledge of the source of ionization (radiation from stars or synchrotron electrons or by collision), it seems impossible to do more than set a lower limit to T_e from the relative intensity of the forbidden lines. Inspection of spectra of planetary nebulae shows large variations in the [O II]/[O III] and [O II]/[Ne V] ratios, although no low-excitation objects resemble 3C 273 in the small [O II]/[O III] ratio, which is here a density effect. But if the auroral line of [Ne V], and other nebular and permitted lines of 3–4.4 eV excitation are to be present, very low T_e is excluded.

The hydrogen-line spectrum of 3C 273 excludes very high T_e, because of the reduced visibility of a hydrogen line superposed on the Paschen and free-free continua. The addition of synchrotron continuum will reduce the intensities predicted here. We can derive a rough theoretical expression for the ratio of Hβ to the continua on which it is superposed, and for the discontinuity in emission at the Balmer limit. The bound-free plus free-free, neglecting the quantum-mechanical g-factors, is

$$E_{ff+bf}d\nu \propto \frac{N_e^2 h\, d\nu}{T_e^{3/2}}\, e^{-h\nu/kT_e}\left(\frac{kT}{2Rh}+\sum\frac{10^{13.54\theta_e/n'^2}}{n'^3}\right), \tag{AI.5}$$

with R the Rydberg. The summation is to be taken over those series whose n'th limits are longward of the observed frequency, ν. The bound-bound is

$$E_{bb}\propto \frac{N_e^2 b_n 2hR}{T_e^{3/2}n'^3 n^3}\, 10^{13.54\theta_e/n^2}, \tag{AI.6}$$

where n' is the lower level and n the upper level. Let us write $E_{bb} = E_{ff+bf}\Delta\nu$, so that $\Delta\nu$ is the equivalent width, in frequency units, of a line, $n \to n'$, in terms of the neighboring hydrogen continua (neglecting two-photon emission). Then we obtain for Hβ, with $n = 4$, $n' = 2$, and $b_4 = 0.16$ (assumed constant) an expression for $\Delta\nu$ independent of N_e,

$$\Delta\nu = 2.2 \times 10^{12} \times 10^{3.39\theta_e} \frac{62.4\theta_e}{1+62.4\theta_e \Sigma(1/n'^3) 10^{13.54\theta_e/n'^2}}. \tag{AI.7}$$

The exponential dominates for large θ_e, and the equivalent width of Hβ is large; if $\theta_e < 0.1$ the free-free emission (represented by unity in the denominator) becomes large and the lines are weak. The values of $\Delta\nu$ are 4×10^{14} sec^{-1} at $\theta_e = 0.5$, 10^{14} sec^{-1} at $\theta_e = 0.3$, and 2×10^{13} sec^{-1} at $\theta_e = 0.1$. The summation permits evaluation of the Balmer emission discontinuity, which is a factor of over 20 at $\theta_e = 0.5$; no such jump is observed. The predicted and observed equivalent widths do not agree until $\theta_e = 0.05$, where $\Delta\nu = 8 \times 10^{12}$ sec^{-1}, as compared to 86 Å in 3C 273, which gives the same $\Delta\nu$. This value of θ_e is a minimum value because any non-thermal or additional thermal component in the continuum over that predicted from equation (AI.5) would reduce the observed strength of Hβ in units of the total continuum. The use of $b_4 = 0.16$ is also questionable; this is an average "observed" value in typical planetaries. Perhaps the range $0.1 \leq \theta_e \leq 0.3$ is a reasonably correct first approximation. Oke is discussing his photoelectric scans of the 3C 273 continuum and hydrogen lines in detail, including the two-photon emission, which is very blue.

The effect of increasing the electron temperature above the value of 17000° ($\theta_e = 0.3$) adopted in this paper is to reduce the predicted Hβ fluxes by $T_e^{-3/2}$. The behavior of the forbidden lines is quite complicated. If we could continue to use the standard expressions for b_2/b_1 (the deviations from Boltzmann populations), then the important terms are of the form

$$\frac{b_2}{b_1} \approx \frac{1}{1+\text{const. } T_e^{1/2}/N_e}. \tag{AI.8}$$

Note that the "high-density" limit ($b_2/b_1 \rightarrow 1$) is approached when $N_e/T_e^{1/2}$ becomes large. Therefore, an increase from T_e to T_e' causes the high-density limit to be reached at a new value, $N_e'^2 = N_e^2(T_e/17000)$. The weakness of [O II], λ 3727, was critical in our discussion of 3C 273. Raising T_e to 50000° results in an increase of N_e' by $3^{1/2}$; the Hβ emission-line luminosity is given by $N_e'^2/T_e'^{3/2}$, which is now $3^{-1/2}$ of the originally adopted values. Not all forbidden lines would change in the same way, since more complicated formulae than equation (AI.8) are required as the higher states become well populated. The very high electron temperatures might more grossly change the various ionization equilibrium. Another effect is strengthening of the auroral line of [O III] at 4363 Å. We derived in Section V an upper limit to the electron density of 3×10^7 cm^{-3} from the observed wavelength of the blend of Hγ and the λ 4363 line. If T_e were high, this upper limit to the electron density would be reduced to a value between 10^7 and 3×10^6 cm^{-3}.

A very high electron temperature, say $\theta_e < 0.1$, has still another consequence if the helium/hydrogen ratio is normal. Lines of a hydrogenic element like He II arise from levels of binding energy Z^2R/n^2. The line of He II corresponding to Hβ has $n' = 4$, $n = 8$, $Z = 2$, and an emission proportional to

$$\frac{N(\text{He II}) N_e Z^6 b_8}{T_e^{3/2} n'^3 n^3} 10^{13.54 Z^2 \theta_e/n^2}. \tag{AI.9}$$

If b_8(He II) $= b_4$(H), the Pickering series line equals Hβ only if N(He II) $= N_e$; however, for λ 4686, $n' = 3$, $n = 4$, the binding energy is larger, and λ 4686 = Hβ at N(He II) $= 0.03\, N_e$, if $\theta_e = 0.1$. Since neither He I or He II is observed so far, the electron temperature may not be very high.

APPENDIX II

SELF-ABSORPTION

A few additional considerations of the possible self-absorption of the emission lines, forbidden and permitted, are necessary. The large broadening suggests that the problem is less serious than in planetary nebulae; the velocity width of $\pm 10^8$ cm/sec in 3C 48 and 3C 273 spreads the absorption coefficient over a frequency width $\Delta\nu_D = 4 \times 10^{12}$ sec^{-1}. If there are $N_i(X)$ ions of element X per cm^3, the mean optical depth across a sphere of radius R, in a forbidden line arising from the ground state, is

$$\langle \tau \rangle_\nu = \frac{\pi e^2}{m c} \frac{2RN_i(X)}{\Delta\nu_D} \frac{g_2 A_{21}}{g_1} \frac{m c^3}{8\pi^2 e^2 \nu^2} . \tag{AII.1}$$

Converting to R_{pc}, $g_2/g_1 = 1$, $N_i(X) \approx 10^{-4} N_e$ for strong forbidden lines, and evaluating $\langle\tau\rangle_\nu$ at the undisplaced frequency $\nu = 7 \times 10^{14}$ sec^{-1}, we find

$$\langle \tau \rangle_\nu \approx 10^{-8} A_{21} R_{pc} N_e . \tag{AII.2}$$

The values of A_{21} range from about 10^{-1} to 10^{-4} sec^{-1}, so that $\langle\tau\rangle_\nu$ approaches unity for such strong lines as [Ne V] and [O III] only if $R_{pc}N_e > 10^9$ and for [O II] if $R_{pc}N_e > 10^{12}$. Thus, in general, the forbidden lines have low mean optical depth of 10^{-2} to 10^{-5}.

Are the emitting volumes also transparent in the Balmer lines? This may depend on the method of excitation, radiative or collisional. Consider first recombination in a nebula optically thick in Lyman-α. Then the population of the level $n = 2$ is given by

$$N_2 = \frac{g_2 b_2}{2} N_e^2 \left(\frac{h^2}{2\pi m k}\right)^{3/2} 10^{3.39\theta_e} T_e^{-3/2} . \tag{AII.3}$$

Since Lyman-α is optically thick, $b_2 \approx 1$. We obtain the mean absorption in Hβ proportional to N_2 from equations (AII.1) and (AII.3) as

$$\langle \tau \rangle_\nu (\mathrm{H}\beta) \approx 7.8 \times 10^{-12} b_2 10^{3.39\theta_e} T_e^{-3/2} R_{pc} N_e^2 . \tag{AII.4}$$

Note that if $b_2 \approx 1$, $\theta_e \approx 0.3$, $\langle\tau\rangle_\nu(\mathrm{H}\beta) \approx 4 \times 10^{-17} R_{pc}N_e^2$. From Table 7B, or equation (12), note that $N_e^2 R_{pc} \approx 2 \times 10^4 N_e^{4/3}$, which is about 5×10^{13} for $N_e = 10^7$ cm^{-3}. Therefore, $\langle\tau\rangle_\nu$ (Hβ) is at most 0.002, at the high-density limit of N_e suggested by the weakness of [O II]. Since lower values of density are probable for 3C 48, it appears that Hβ, and also Hγ and the Balmer continuum are optically thin.

A separate consideration is needed for the case where the radiation field is neglected and the ionization as well as the excitation is by collision. For collision, $b_2 \approx 1$. Chamberlain (1953) computes the collisional ionization and excitation of hydrogen over suitable ranges of θ_e. His values of b_n are compared with those in the recombination case in his Tables 2 and 3. For low n, collision and recombination differ by about a factor of only 2 if the nebula is optically thick in the Lyman lines. The Balmer decrements also do not differ greatly, especially at high T_e. No modification in the conclusions drawn from equation (AII.4) are necessary.

APPENDIX III

THE INTENSITY OF Mg II EMISSION IN AN OPTICALLY THICK NEBULA

We find from equation (AII.2) that the optical thickness of the Mg II resonance line is large, since $A_{21} \approx 10^8$ sec^{-1}. The concentration of Mg II is $Z(\mathrm{Mg})(1 - x_{\mathrm{Mg}^+})N_e$, which we will denote by αN_e; the number of atoms per cm^3 in the ground level is then $N_1 = \alpha N_e$. Let us first consider the standard collisional treatment, which we will find leads to a significant contradiction. We resolve this by study of the effect of the nebular radiation itself on the population of the upper level. Consider a two-level atom with collisional excitation and de-excitation rates F_{12} and F_{21}, respectively, and a spontaneous decay probability of A_{21}. The population of the upper level N_2 is obtained from a collisional value, b_2(coll.), given by the steady-state condition

$$N_2 A_{21} + F_{21} = F_{12} \tag{AIII.1}$$

and the collisional rates

$$F_{12} = \frac{8.54 \times 10^{-6}}{T_e^{1/2}} \frac{N_1 N_e \Omega 10^{-\theta\chi}}{g_1} , \qquad F_{21} = \frac{8.54 \times 10^{-6}}{T_e^{1/2}} \frac{N_2 N_e \Omega}{g_2} . \tag{AIII.2}$$

The value of b_2(coll.) so obtained is small, and therefore collisional de-excitation can be neglected, so that b_2(coll.) simplifies to

$$b_2(\text{coll.}) = \frac{8.54 \times 10^{-6}}{T_e^{1/2}} \frac{N_e \Omega}{g_2 A_{21}} \approx 10^{-15} N_e . \tag{AIII.3}$$

The rate of emission for the optically thin case is obtained from either the total number of collisions within the volume V, or from the number of atoms in the upper state times A_{21}. The lumnosity in Mg^+ is then, as used before in equation (10),

$$E(\text{Mg}^+) = \frac{8.54 \times 10^{-6} a N_e^2 \Omega h\nu 10^{-4.42\theta_e}}{T_e^{1/2} g_1} \frac{4\pi R^3}{3} \; (\tau_\nu < 1) . \tag{AIII.4}$$

But the optical depth is, in fact, very large; through an object of radius R, with a total Doppler width of $\Delta\nu_D$, the mean line absorption averaged over the line is $N_1 B_{12}\, h\nu\, 2R/\Delta\nu_D \approx 10^{-2} N_e R_{\text{pc}}$. We can show that unless the synchrotron radiation is much larger than the black-body radiation, we can neglect both the normal black-body-stimulated emission and that stimulated by the synchrotron radiation.

The atomistic equation of transfer valid over the frequency range $\Delta\nu_D$ within the line can be taken as

$$dI_\nu = -N_1 B_{12} h\nu I_\nu ds + \frac{N_2 A_{21} h\nu ds}{4\pi} , \tag{AIII.5}$$

which leads to

$$-\frac{dI_\nu}{h\nu N_1 B_{12} ds} = I_\nu - \frac{g_2 b_2(\text{coll.}) A_{21}}{g_1 4\pi B_{12}} e^{-h\nu/kT_e} . \tag{AIII.6}$$

With the optical depth defined as $dt_\nu = -h\nu\, N_1\, B_{12} ds$ the transfer equation becomes

$$\frac{dI_\nu}{dt_\nu} - I_\nu = -\frac{b_2(\text{coll.})}{4\pi} \frac{2h\nu^3}{c^2} e^{-h\nu/kT_e} . \tag{AIII.7}$$

If we take the simple geometry of a plane-parallel slab, the emergent intensity is $I_\nu(0)$, constant over $\Delta\nu_D$,

$$I_\nu(0) = \frac{2h\nu^3}{c^2} e^{-h\nu/kT_e} \frac{b_2(\text{coll.})}{4\pi} [1 - e^{-t_\nu(2R)}] , \tag{AIII.8}$$

where $t_\nu(2R)$ is the total optical thickness. The energy emitted, as evaluated from equation (AIII.8), must be multiplied by the band width $\Delta\nu_D$ and proves to be the thermal emission times b_2, if $t_\nu(2R)$ is very large. For a plane the flux is $\pi I_\nu(0)$. The total luminosity is then, for $t_\nu = \infty$,

$$E(\text{Mg}^+) = \frac{2h\nu^3}{c^2} 10^{-4.42\theta_e} \pi R^2 b_2(\text{coll.}) \Delta\nu_D . \tag{AIII.9}$$

We have here integrated over the entire line width $\Delta\nu_D$, and evaluated a surface emission, in contrast to the volume emission given by equation (AIII.4). It can be shown that if $t_\nu(2R)$ is small in equation (AIII.8), the total luminosity approaches that given by equation (AIII.4) within a trivial factor of $2\pi/(4\pi/3)$. If a spherical geometry had been used this factor would be unity.

Returning to the optically thick case of equation (AIII.9), we compute the surface emissivity, which is now linearly dependent on N_e but independent of the abundance of Mg, as essentially that of a black body reduced by the factor b_2(coll.), i.e.,

$$\begin{aligned} E(\text{Mg}^+) &= \frac{8.54 \times 10^{-6} N_e \Omega}{T_e^{1/2} g_2 A_{21}} \frac{2h\nu^3}{c^2} 10^{-4.42\theta_e} \pi R^2 \Delta\nu_D \\ &\approx 8 \times 10^{19} \Delta\nu_D N_e R_{\text{pc}}^2 \approx 3 \times 10^{32} N_e R_{\text{pc}}^2 \text{ erg cm}^{-3} \text{ sec}^{-1} . \end{aligned} \tag{AIII.10}$$

If we apply this result to 3C 273, with $E = 4 \times 10^{43}$ and $N_e = 10^7$, we find the large radius of 140 pc, where all other lines gave $R \approx 1$ pc. We believe that the radius is 1 pc and that a new approach to the excitation mechanism is required by the intense radiation field of the line itself. The actual flux at the surface in the Mg II line reaches the enormous value of 10^6 erg cm^{-2} sec^{-1}, approximately equal to daylight.

The absorbed radiation within the Mg^+ resonance line is multiply scattered, as long as ionizations or absorptions from its upper level can be neglected. The problem of the emergent intensity of the diffused light is equivalent to the largely unsolved problem of Lyman-α in a planetary nebula with internal motions. The diffusion of the λ 2800 radiation differs from that of Ly-α in that its upper level is populated easily by collision (only 4.4 eV excitation) as against recombination for Ly-α (10.2 eV). The upper level of the resonance lines of Mg^+, 3^2S–3^2P^0 ($\lambda\lambda$ 2795.5, 2802.7) communicates by the absorptions of 3^2P^0–3^2D ($\lambda\lambda$ 2798.0, 2790.8) to levels which can reradiate these strong, nearly coincident ultraviolet lines or, by forbidden transitions 3^2D–4^2S or 3^2D–3^2S, return to the ground state. The population of 3^2P^0, however, seems to be too small at reasonable N_e to complicate the situation in any significant way beyond the two-level pure scattering problem. At $N_e = 10^7$ cm^{-3} and $b_2 \approx 10^{-9}$ if the optical depth is of the order of 10^5, only one scattering in 10^4 leads to an "absorption" process, i.e., the scattering albedo is very near unity.

We cannot neglect, therefore, the effect of radiative excitation on the population of the upper level, and proceed to re-evaluate b_2, including both radiative and collisional effects. Equation (AIII.1) should be rewritten as

$$N_2(A_{21} + B_{21} 4\pi J_\nu) + F_{21} = N_1 B_{12} 4\pi J_\nu + F_{12} , \tag{AIII.11}$$

where J_ν is the mean intensity in the line over all directions. The terms involving the stimulated emission ($N_2 B_{21} 4\pi J_\nu$) and collisional de-excitation F_{21} are, in fact, small. We follow an analysis by Münch (1962) to whom we are indebted for several interesting discussions. We obtain the result that b_2(rad. + coll.) $\approx b_2$(rad.) is

$$b_2(\text{rad.} + \text{coll.}) = \frac{[(8.5 \times 10^{-6} N_e \Omega)/(T_e^{1/2} g_1 A_{21})] + \mathfrak{J} e^{h\nu/kT_e}}{[(8.5 \times 10^{-6} N_e \Omega)/(T_e^{1/2} g_1 A_{21})] + \mathfrak{J} + 1} . \tag{AIII.12}$$

Here the effective mean intensity is

$$\mathfrak{J} = \frac{c^2 4\pi}{2 h\nu^3} \frac{J_\nu}{\Delta\nu_D} \tag{AIII.13}$$

and must be derived from the radiation field in the line itself. In equation (AIII.12) in the numerator, only the second term proves to be appreciable and in the denominator only unity. Therefore, the best estimate of b_2(rad. + coll.) is

$$b_2(\text{rad.} + \text{coll.}) \approx \frac{c^2}{2 h\nu^3} 4\pi J_\nu e^{h\nu/kT_e} . \tag{AIII.14}$$

As before, the line width is $\Delta\nu_D$, so that the luminosity for an optically thick nebula from equation (AIII.8) becomes

$$I_\nu(Mg^+) = J_\nu . \tag{AIII.15}$$

Equation (AIII.15) represents merely the conservation of energy, i.e., that every resonance quantum absorbed within the volume ultimately is emitted at the surface, except for the small neglected factors in equation (AIII.12). The b_2 derived in equation (AIII.14) is linear in J_ν; if we take the observed flux from 3C 273 at the surface and compute J_ν from $J_\nu \approx \frac{1}{2} F_\nu$, we find b_2(rad. + coll.) $\approx 2 \times 10^{-5}$, or b_2(rad.) $\approx 2 \times 10^{10}$ b_2(coll.)/N_e. Thus we have an increase in the predicted radiation by about 2000, or a required radius of $R \approx 1$ pc, as computed earlier from the total volume emissivity. A more detailed study of this problem would involve us in a very difficult problem. A major simplification in this case is that the Mg^+ analog of Balmer-α is here coincident with the Ly-α transition of Mg^+, because of the wide spacing of the 3^2S, 3^2P^0, and 3^2D levels. We have neglected recombination in the population of 3^2P^0 throughout this paper. No computations have been made of the value of b_2 for recombination; the energy levels are widely split and the degeneracy which makes hydrogen relatively simple is absent. If we treat the Mg^+ ion as if it were hydrogenic, the approximate value of the recombination emission is $10^{32}\, b_2 N_e^2 R_{pc}^3$ or about $b_2\, 10^{45}$ erg sec^{-1} for 3C 273. Thus unless b_2(recombination) exceeds 10^{-2} there is little likelihood of pure recombination being important. If recombination computations can be carried through for this complex case and b_2 is large, it is possible that an additional emission varying as $N_e^2R^3$ would then be found. In that case the $N_e^2R^3$ required would be less than that predicted in this paper.

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