TERMINATION OF THE PROTON-PROTON CHAIN IN STELLAR INTERIORS*

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ABSTRACT

The roles of the various H^2-burning and He^3-burning reactions in the termination of the proton-proton chain in stellar interiors have been re-examined using (1) recent experimental determinations of the cross-sections for the $H^2(p, \gamma)He^3$ and $He^3(\alpha, \gamma)Be^7$ reactions, (2) theoretical studies of the rates of the relevant beta-decay reactions, and (3) a re-examination of the previous cross-section factor determinations for the other possible reactions in the proton-proton chain.

I. INTRODUCTION

A number of review articles have discussed the termination of the proton-proton chain, for example, Bethe (1939), Salpeter (1952), Burbidge, Burbidge, Fowler, and Hoyle (1957), and Fowler (1960). However, recent experimental studies of the $H^2(p, \gamma)He^3$ reaction by Griffiths, Lal, and Scarfe (1963) and recent theoretical and experimental studies of the termination of the chain via the $He^3(\alpha, \gamma)Be^7$ reaction (Kavanagh 1960; Bahcall 1962*b;* Parker and Kavanagh 1963) make it desirable to re-examine the roles of various reactions in the termination of the proton-proton chain. In the course of this re-examination it has been found that, contrary to statements in some previous review articles, further experimental investigation of the Li^4 system is necessary before one can definitely rule out the possibility that Li^4 plays an important role in the termination of the proton-proton chain.

In the calculations described below we have adopted many of the definitions and notations of Burbidge *et al.* (1957). The cross-section factor, $S(E)$, is defined by the following equation:

$$S(E) = \sigma(E)E \exp\,(31.28\, Z_1Z_0A^{1/2}/E^{1/2})\ \text{keV-barns}\,, \tag{1}$$

where σ is the reaction cross-section in barns, E is the center-of-mass energy in keV, Z_1 and Z_0 are the charges of the interacting nuclei in units of the proton charge, and A is the reduced mass, $A_1A_0/(A_1 + A_0)$, in atomic mass units. The mean reaction rate, P, of a non-resonant nuclear reaction can be written

$$P = 7.20 \times 10^{-19} n(1)n(0) f S\tau^2 e^{-\tau}\,(1+\delta_{10})^{-1}(AZ_1Z_0)^{-1}\ \text{reactions cm}^{-3}\,\text{sec}^{-1}\,, \tag{2a}$$

$$P = 2.62 \times 10^{29} \rho^2 \frac{X_1X_0}{A_1A_0} f S \tau^2 e^{-\tau}(1+\delta_{10})^{-1}(AZ_1Z_0)^{-1}\ \text{reactions cm}^{-3}\,\text{sec}^{-1}\,, \tag{2b}$$

where

$$\tau = 42.48\left(Z_1^2Z_0^2\frac{A}{T_6}\right)^{1/3}. \tag{2c}$$

In equations (2*a*), (2*b*), and (2*c*) T_6 is the temperature in millions of degrees; δ_{ij} is the Kronecker delta,

$$\delta_{ij} = 1 \quad \text{if} \quad i = j\,, \qquad \delta_{ij} = 0 \quad \text{if} \quad i \neq j\,;$$

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ρ is the density in gm cm^{-3}; $n(i)$ is the number density in particles per cm^3 of nuclei of type (i); X_i is the fraction by weight of nuclei of type (i) in the medium,

$$n(i) = 6.023 \times 10^{23} \rho \frac{X_i}{A_i}; \tag{3}$$

S is the cross-section factor in keV-barns defined in equation (1); and f is the electron-screening factor discussed by Salpeter (1954). In the weak-screening approximation appropriate for the sun, Salpeter has shown that

$$f = \exp\,(0.188\, Z_1 Z_0 \zeta \rho^{1/2} T_6^{-3/2})\,, \tag{4a}$$

where

$$\zeta = \left[\sum_i \left(X_i \frac{Z_i^2}{A_i} + X_i \frac{Z_i}{A_i}\right)\right]^{1/2}. \tag{4b}$$

It is convenient to introduce a quantity, $\langle i, j\rangle$, defined by the relations

$$\begin{aligned} \langle i, j\rangle &\equiv \frac{P(1+\delta_{i,j})}{n(i)n(j)}\ \mathrm{cm^3\,sec^{-1}}, \\ \langle i, j\rangle &= \langle \sigma v\rangle_{ij}\,. \\ \langle i, j\rangle &= \langle j, i\rangle, \end{aligned} \tag{5}$$

where $\langle i, j\rangle$ is the thermal average of the cross-section times the relative velocity of particles i and j. Using equation (5) we can write the mean lifetime of a nucleus (i) for interaction with nuclei of type (j) as follows:

$$\tau_j(i) = [\langle i, j\rangle n(j)]^{-1}\ \mathrm{sec}\,. \tag{6}$$

The rate, $R_j(i)$, at which nuclei of type (i) are consumed by interaction with nuclei of type (j) can be written

$$\begin{aligned} R_j(i) &= P(1+\delta_{i,j}) \\ &= \langle i, j\rangle n(i) n(j)\ \text{nuclei cm}^{-3}\,\text{sec}^{-1}\,. \end{aligned} \tag{7}$$

The fraction of the nuclei (i) destroyed that are consumed by interaction with nuclei (j) is thus

$$F_j(i) = \frac{R_j(i)}{\sum_k R_k(i)}, \tag{8}$$

and the fraction of the nuclei (i) produced that are consumed by interaction with nuclei (j) is

$$\Phi_j(i) = \frac{R_j(i)}{\text{Rate of production of } (i)}\,. \tag{9}$$

Under equilibrium conditions the rates of production and consumption of nuclei (i) are equal, and therefore

$$F_j(i) = \Phi_j(i)\,. \tag{10}$$

The fraction of the He^4 that is produced by the interaction of nuclei (i) and (j) is defined as

$$C_{\mathrm{He}^4}(i, j) = \frac{1}{(1+\delta_{ij})} \frac{\langle i, j\rangle n(i) n(j)}{\text{Rate of production of He}^4}\,.$$

The cross-sections for the nuclear reactions that occur in the proton-proton chain cannot in general be measured at the energies of interest to astrophysicists (of the order of 20 keV). Hence, for the cross-section factor, extrapolations from higher-energy measurements must be used to obtain a zero-energy intercept, S_0, and an average value for the derivative at low energies, $(dS/dE)_0$. Note that $S_0 = S(E = 0)$ is not identical with the $S_o = S(E = E_o)$ of Burbidge, Burbidge, Fowler, and Hoyle (1957). Christy and Duck (1961) and Tombrello and Parker (1963) have discussed the use of direct-capture calculations to obtain reliable extrapolations from high energy data. The Q-value and the values of S_0 and $(dS/dE)_0$ for the various reactions in the proton-proton chain are listed in Table 1. Estimates of the absolute errors involved in the determination of S_0 are included for those cases where it was felt that such estimates were meaningful. In the calculations described below, we have followed Caughlan and Fowler (1962) and have included the temperature dependence of S by writing $S(T_6)$ as

$$S(T_6) = S_0\left\{1 + \frac{5}{12\beta} T_6^{1/3} + \frac{(dS/dE)_0}{S_0}\left[E_0(T_6) + \tfrac{35}{36} kT_6\right] + \ldots\right\}, \tag{11a}$$

where

$$\beta = 42.48\ (Z_1^2 Z_0^2 A)^{1/3} \tag{11b}$$

and

$$E_0(T_6) = 1.220\ (Z_1^2 Z_0^2 A T_6^2)^{1/3}\ \text{keV}\,. \tag{11c}$$

II. NUCLEAR ABUNDANCES

In the proton-proton chain, unlike the CNO bi-cycle, many nuclear reactions occur in which neither of the interacting nuclei is a proton. Furthermore, the interacting nuclei have abundances which are not independent of each other but are instead determined by the physical conditions in the star in which the reactions are occurring. Because many reactions interrelate the various nuclear species, there is no simple way of expressing the abundance of each of these species in terms of the proton abundance, $X_{\rm H}$. Hence, it is not possible to express our results for the pertinent reactions in the form of a simple table of $[\log(\rho\tau X_{\rm H}/100)]$ versus T_6, as Caughlan and Fowler (1962) have done for the CNO bi-cycle. Instead, in the discussion that follows, we present general expressions which can be used to calculate the quantities of interest and actually carry out such calculations (except where noted) only for the case of $\rho = 100$ gm cm^{-3} and $X_{\rm H} = X_{\rm He} = 0.50$.

The proton and alpha-particle abundances are described by the parameters $X_{\rm H}$ and $X_{\rm He}$ defined above. The electron density is given by the usual expression,

$$n(e) \cong 6.023 \times 10^{23} \rho \frac{(1 + X_{\rm H})}{2}\ \text{cm}^{-3}. \tag{12}$$

In order to calculate the abundances of deuterium and He^3, it is necessary to consider the various reactions involved in the proton-proton chain. The discussions in Sections III and IV will show that the following reactions are the important ones for producing and destroying deuterium and He^3 under normal stellar conditions:

$$\begin{array}{ll} (a)\ \mathrm{H}^1(p, \beta^+ \nu)\mathrm{H}^2\,, & (d)\ \mathrm{He}^3(d, p)\mathrm{He}^4\,, \\ (b)\ \mathrm{H}^2(p, \gamma)\mathrm{He}^3\,, & (e)\ \mathrm{He}^3(\tau, 2p)\mathrm{He}^4\,, \\ (c)\ \mathrm{H}^2(\tau, p)\mathrm{He}^4\,, & (f)\ \mathrm{He}^3(\alpha, \gamma)\mathrm{Be}^7\,, \end{array} \tag{13}$$

where we have followed the convention that $\tau \equiv \mathrm{He}^3$ inside the parentheses in the same way that $p \equiv \mathrm{H}^1$, $d \equiv \mathrm{H}^2$, $t \equiv \mathrm{H}^3$, and $\alpha \equiv \mathrm{He}^4$. Thus, it should be noted that reactions (13c) and (13d) are identical except that (13c) is written so as to emphasize its role as

TABLE 1

REACTION PARAMETERS (PROTON-PROTON CHAIN)

Reaction	Q-Value* (MeV)	S_0 (keV-barns)	$(dS/dE)_0$ (barns)	$\tau_j(i)$† (yr.)	Reference
$H^1(p, \beta^+\nu)H^2$	$+1.442$	$(3.36\pm0.4)\times10^{-22}$	$+2.7\times10^{-24}$	8.85×10^{9}	Fowler (1960)
$H^2(p, \gamma)He^3$	$+5\ 493$	$(2.5\ \pm0.4)\times10^{-4}$	$+7.9\times10^{-6}$	4.43×10^{-8}	Griffiths *et al.* (1963)
$H^2(d, p)H^3$	$+4.032$	100	$+0.525$	5.32×10^{5}	Arnold *et al.* (1954)
$H^2(d, n)He^3$	$+3.268$				
$H^2(\tau, p)He^4$	$+18.352$	$6.7\ \times10^{3}$	$+27$	1.15×10^{-4}	Arnold *et al.* (1954)
$He^3(d, p)He^4$	$+18.352$	$6.7\ \times10^{3}$	$+27$	8.86×10^{8}	Arnold *et al.* (1954)
$He^3(\tau, 2p)He^4$	$+12.859$	$1.1\ \times10^{3}$		5.28×10^{5}	Good *et al.* (1954)
$He^3(\alpha, \gamma)Be^7$	$+1.586$	(0.47 ± 0.05)	-2.8×10^{-4}	$9\ 70\times10^{5}$	Parker *et al.* (1963)
$Be^7(e^-, \nu)Li^7$	$+0.861$ (89 7%)			3.86×10^{-1}	Bahcall (1962*b*)
$Be^7(e^-, \nu)Li^{7*}$	$+0.383$ (10.3%)				Taylor *et al.* (1962)
$Li^7(p, \alpha)He^4$	$+17.347$	120	0.0	1.80×10^{-5}	Sawyer and Philips (1953)
$Li^7(p, \gamma)Be^8(\alpha)He^4$	$+17.347$	<1		$>2\times10^{-3}$	Sawyer and Philips (1953)
$Be^7(p, \gamma)B^8$	$+0.135$	(0.030 ± 0.01)	0	8.81×10^{1}	Kavanagh (1960)
$B^8(\beta^+\nu)Be^{8*}(\alpha)He^4$	$+18.074$			3×10^{-8}	Bahcall (1962*a*)
$He^3(e^-, \nu)H^3$	-0.018			2×10^{10}	Bahcall and Wolf (1963)
$H^3(p, \gamma)He^4$	$+19.813$	1.5×10^{-3}	$+2.7\times10^{-5}$	1.68×10^{-8}	Perry and Bame (1955)
$He^3(p, \beta^+\nu)He^4$	$+19\ 795$				

* König, Mattauch and Wapstra (1962).

† τ calculated for $X_H = X_{He} = 0.5$, $\rho = 100$ gm cm^{-3} and $T_6 = 15$.

a deuterium-burning reaction and (13*d*) is written so as to emphasize its role as a He^3-burning reaction.

Assuming that reactions (13*a*)–(13*f*) are the only important ones, we can write

$$\dot{n}(H^2) = \tfrac{1}{2}\langle H^1, H^1\rangle n(H^1)n(H^1) - \langle H^2, H^1\rangle n(H^2)n(H^1) - \langle H^2, He^3\rangle n(H^2)n(He^3) \quad (14a)$$

and

$$\begin{aligned}\dot{n}(He^3) = {} & \langle H^2, H^1\rangle n(H^2)n(H^1) - \langle He^3, H^2\rangle n(He^3)n(H^2) \\ & - \langle He^3, He^3\rangle n(He^3)n(He^3) - \langle He^3, He^4\rangle n(He^3)n(He^4)\,.\end{aligned} \quad (14b)$$

Over a wide range of stellar temperatures the rates of production and consumption of H^2 and He^3 are equal (see Secs. III*a* and IV*a*, respectively) so that

$$\dot{n}(H^2) = 0\,, \quad (15a)$$

and

$$\dot{n}(He^3) = 0\,. \quad (15b)$$

Let II $= n(H^2)/n(H^1)$, III $= n(He^3)/n(H^1)$, and IV $= n(He^4)/n(H^1)$. Then under the equilibrium conditions defined by equations (15*a*) and (15*b*) equations (14*a*) and (14*b*) can be rewritten as

$$\tfrac{1}{2}\langle H^1, H^1\rangle - \langle H^2, H^1\rangle \mathrm{II} - \langle H^2, He^3\rangle \mathrm{II\ III} = 0\,, \quad (16a)$$

and

$$\langle H^2, H^1\rangle \mathrm{II} - \langle He^3, H^2\rangle \mathrm{III\ II} - \langle He^3, He^3\rangle \mathrm{III\ III} - \langle He^3, He^4\rangle \mathrm{III\ IV} = 0\,. \quad (16b)$$

Combining equations (16*a*) and (16*b*), we obtain

$$\mathrm{II} = \tfrac{1}{2}\langle H^1, H^1\rangle/(\langle H^2, H^1\rangle + \langle H^2, He^3\rangle \mathrm{III})\,, \quad (17a)$$

and

$$\begin{aligned}&\langle He^3, H^2\rangle\langle He^3, He^3\rangle \mathrm{III}^3 + [\langle He^3, He^3\rangle\langle H^2, H^1\rangle + \langle He^3, H^2\rangle\langle He^3, He^4\rangle \mathrm{IV}]\,\mathrm{III}^2 \\ &+ [\tfrac{1}{2}\langle He^3, H^2\rangle\langle H^1, H^1\rangle + \langle H^2, H^1\rangle\langle He^3, He^4\rangle \mathrm{IV}]\,\mathrm{III} - \tfrac{1}{2}\langle H^2, H^1\rangle\langle H^1, H^1\rangle = 0\,.\end{aligned} \quad (17b)$$

The coupled equations (17*a*) and (17*b*) were solved as a function of temperature for the case $X_H = X_{He} = 0.50$, that is, IV $= 0.25$, and the resulting $n(H^2)/n(H^1)$ and $n(He^3)/n(H^1)$ ratios are plotted in Figures 1 and 2. It should be noted that except for screening corrections and except under the special conditions described in Section IV*f* $n(H^2)/n(H^1)$ and $n(He^3)/n(H^1)$ are independent of density. Using the equilibrium abundances shown in Figures 1 and 2 for H^2 and He^3, for the case $X_H = X_{He} = 0.50$ and $\rho = 100$ gm cm^{-3}, we have calculated the mean lifetimes (defined by eq. [5]) associated with each of the reactions in the proton-proton chain; the results are plotted in Figures 3, *A*, and *B*.

III. DEUTERIUM-BURNING REACTIONS

The $H^2(p, \gamma)He^3$ reaction is the only reaction usually discussed in connection with deuterium burning in the interiors of stars operating on the proton-chain. The discussion in this section shows that, in fact, the $H^2(p, \gamma)He^3$ reaction is the most important deuterium-burning reaction in stellar interiors under normal conditions. There are, however, other deuterium-burning reactions, and we discuss them in order to determine under what, if any, conditions they contribute significantly to the consumption of deuterium in the proton-proton chain.

a) $H^2(p, \gamma)He^3$

Griffiths *et al.* (1963) have measured the cross-section for the $H^2(p, \gamma)He^3$ reaction in the energy region from 16 to 35 keV and have obtained the following intercept parameters for $S(E)$:

$$S_0 = (0.25 \pm 0.04) \times 10^{-3} \text{ keV-barns},$$

and

$$(dS/dE)_0 = +0.79 \times 10^{-5} \text{ barns}.$$

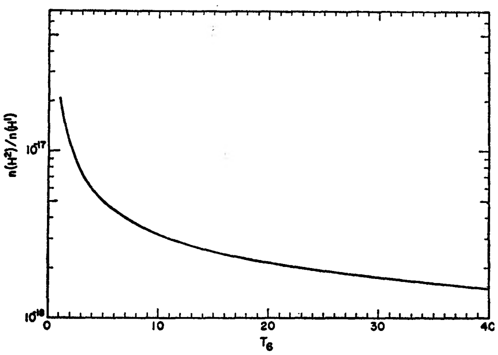

FIG. 1.—The ratio, $n(H^2)/n(H^1)$, of the deuterium equilibrium abundance to the proton abundance is shown for $X_H = X_{He} = 0.5$ as a function of T_6, the temperature in millions of degrees.

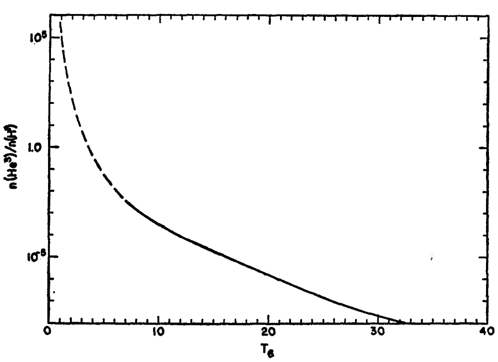

FIG. 2.—The ratio, $n(He^3)/n(H^1)$, of the He^3 equilibrium abundance to the proton abundance is shown for $X_H = X_{He} = 0.5$ as a function of T_6. The dashed curve below $T_6 = 6$ indicates that in this temperature region the actual He^3 abundance will probably be less than the equilibrium value plotted here.

A comparison of the mean lives for the $H^2(p, \gamma)He^3$ reaction and the $H^1(p, \beta^+ \nu)H^2$ reaction (see Fig. 3, *A*) shows that the $H^2(p, \gamma)He^3$ reaction is sufficiently rapid at temperatures above a few hundred thousand degrees to guarantee that the rate of consumption of H^2 via the $H^2(p, \gamma)He^3$ reaction will be equal to the rate of production of H^2 via the $H^1(p, \beta^+ \nu)H^2$ reaction. Hence, the assumption that $\dot{n}(H^2) = 0$ is justified at such temperatures, and the abundance of deuterium equals its equilibrium value.

b) $H^2(d, p)H^3$ and $H^2(d, n)He^3$

The cross-sections for these reactions have been measured by Arnold, Phillips, Sawyer, Stovall, and Tuck (1954) over a deuteron-energy range from 13 to 113 keV. Summing their cross-sections for these two reactions, one gets the following intercept parameters for the total $S(E)$:

$$S_0 = 100 \text{ keV-barns},$$

and

$$(dS/dE)_0 = +0.525 \text{ barns} .$$

A comparison of the mean lives for the $H^2 + H^2$ reactions and the $H^1(p, \beta^+ \nu)H^2$ reaction (see Fig. 3, *A*) shows that the $H^2 + H^2$ reactions are also sufficiently fast to guarantee an equilibrium abundance for deuterium. Moreover, a comparison of the cross-section factors for the $H^2 + H^2$ reactions and the $H^2(p, \gamma)He^3$ reaction shows that the burning of deuterium by deuterium has a considerable advantage over the $H^2(p, \gamma)He^3$ reaction because of the much larger cross-section factor for the $H^2 + H^2$ reaction. However, the relative deuterium-to-proton abundance, which (Fig. 1) is $\lesssim 10^{-17}$ for $T_6 \geq 1$, completely

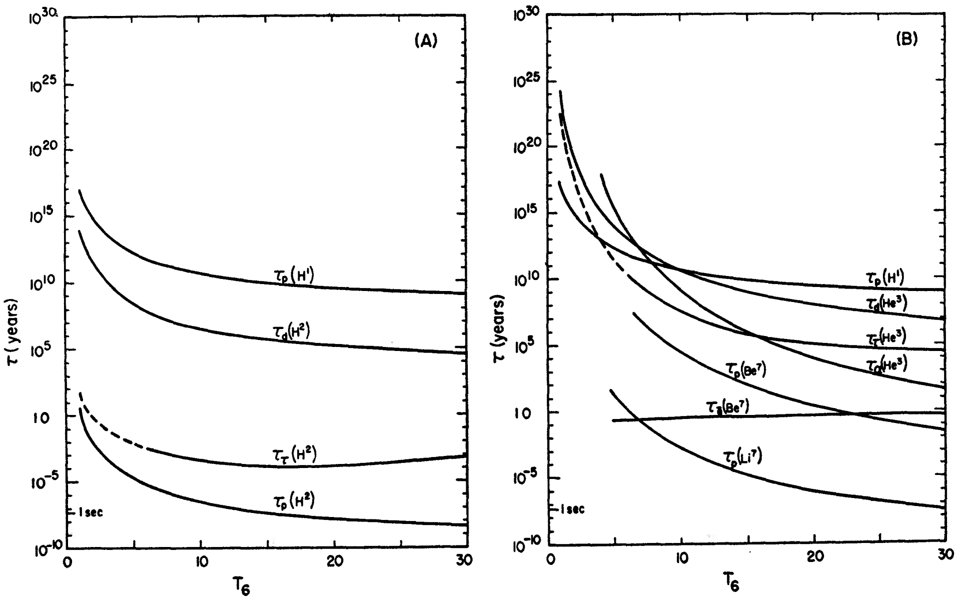

FIG. 3.—*A*, The mean lives of deuterium for the various H^2-burning reactions are compared to the mean life of the $H^1(p, \beta^+\nu)H^2$ reaction; all mean lives are plotted as a function of T_6 for $X_H = X_{He} = 0.5$ and $\rho = 100$ gm cm^{-3}. The dashed curve has the same significance as in Fig. 2. *B*, The mean lives of He^3 for the various He^3-burning reactions and the mean lives associated with the various $A = 7$ products of the $He^3(\alpha, \gamma)Be^7$ termination are compared to the mean life of the $H^1(p, \beta^+\nu)H^2$ reaction. All mean lives are plotted as functions of T_6 for $X_H = X_{He} = 0.5$ and $\rho = 100$ gm cm^{-3}. The dashed curve has the same significance as in Fig. 2.

overcomes this advantage. A calculation of the fraction of deuterium burned by both the $H^2 + H^2$ reactions,

$$F_d(H^2) = \frac{\langle H^2, H^2 \rangle \text{II}}{\langle H^2, H^1 \rangle + \langle H^2, H^2 \rangle \text{II} + \langle H^2, He^3 \rangle \text{III}}, \tag{18}$$

assuming an equilibrium abundance of He^3 and $X_H = X_{He} = 0.5$, shows that this fraction is less than 8.41×10^{-14} at all temperatures. Thus, the $H^2 + H^2$ reactions can be ignored in stellar-interior calculations.

c) $H^2(\tau, p)He^4$

Arnold *et al.* (1954) have measured the cross-sections for the $H^2(\tau, p)He^4$ reaction over a deuteron-energy range from 36 to 93 keV. From their measurements one can obtain the following intercept parameters:

$$S_0 = 6700 \text{ keV-barns} ,$$

and

$$(dS/dE)_0 = +27 \text{ barns} .$$

The fraction of deuterium burned by this reaction,

$$F_\tau(\mathrm{H}^2) = \frac{\langle \mathrm{H}^2, \mathrm{He}^3 \rangle \mathrm{III}}{\langle \mathrm{H}^2, \mathrm{H}^1 \rangle + \langle \mathrm{H}^2, \mathrm{H}^2 \rangle \mathrm{II} + \langle \mathrm{H}^2, \mathrm{He}^3 \rangle \mathrm{III}}, \tag{19}$$

depends critically on the relative abundance of He^3 to protons. A plot of $F_\tau(\mathrm{H}^2)$ is shown in Figure 4 for the case where III is assumed equal to its equilibrium value. The range of validity of the assumption of He^3 equilibrium is discussed in Section IV*a*. From equations (14*a*) and (14*b*) it is clear that, in the normal situation where the He^3 abundance is increasing, the maximum value of the He^3-to-proton ratio is obtained from the equilibrium condition. Hence, Figure 4 represents an upper limit on the fraction of deuterium burned by the $\mathrm{H}^2(\tau, p)\mathrm{He}^4$ reaction in the interiors of stars.

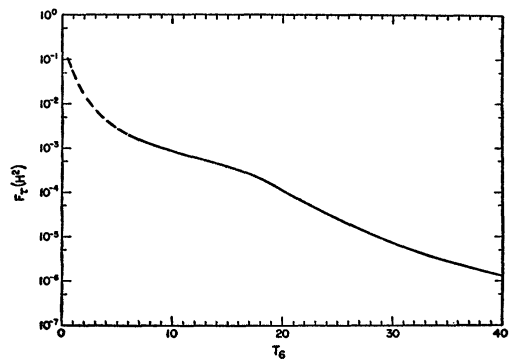

FIG. 4.—The fraction of deuterium consumed by the $\mathrm{H}^2(\tau, p)\mathrm{He}^4$ reaction is shown as a function of T_6 for $X_{\mathrm{H}} = X_{\mathrm{He}} = 0.5$. The dashed curve has the same significance as in Fig. 2.

IV. He^3-BURNING REACTIONS

A large number of reactions play important roles in the conversion of He^3 to He^4. In the discussion below, we examine the conditions under which each He^3-burning reaction plays a significant role.

a) The Abundance of He^3

In all the calculations described in Sections III, IV, and V we have assumed that the abundance of He^3 is equal to its equilibrium value (Fig. 2). However, at temperatures below 6×10^6 ° K, the mean lifetime of a He^3 nucleus is sufficiently long ($\geq 10^{10}$ years) that it is unlikely that He^3 equilibrium can be established during the time that a star of one solar mass spends on the main sequence ($\sim 10^{10}$ years). Hence at temperatures below 6×10^6 ° K the He^3 abundance will almost certainly be less than its equilibrium value, perhaps several orders of magnitude less.

For temperatures below $T_6 = 6$, the actual He^3 abundance depends on the mass, age, and chemical composition of the star as well as the local temperature. Thus, the He^3 abundance cannot be determined in general but must be calculated separately for each case. Lack of He^3 equilibrium seriously affects many of the quantities of interest in the proton-proton chain. In this paper, the curves which depend sensitively on the He^3 abundance have been calculated for an equilibrium He^3 abundance but have been dashed in the temperature region below $T_6 = 6$. We have assumed an equilibrium abundance of He^3 in order to call attention to the fact that at very low temperatures ($< 4 \times 10^6$ ° K) it is possible to build up large relative abundances of He^3 and also because of the large range of He^3 abundances that are possible.

A lack of He^3 equilibrium will have only a small effect on our conclusions regarding the burning of deuterium. The importance of the $H^2(\tau, p)He^4$ reaction will be drastically reduced at low temperatures so that $F_\tau(H^2)$ will probably reach a maximum value of about 10^{-3} in the neighborhood of $T_6 = 5$ and then fall rapidly at still lower temperatures. One further result of this decrease in the already minor role of the $H^2(\tau, p)He^4$ reaction as a H^2-burning reaction will be a slight increase in the abundance of deuterium at low temperatures. This latter effect, however, is small, amounting to only 5 per cent at $T_6 = 1$ and falling to less than $\frac{1}{3}$ per cent by $T_6 = 5$.

In our discussions of He^3 burning, we note the effect of a non-equilibrium abundance of He^3 on the various quantities derived. The principal result is a reduction in the importance of the $He^3(\tau, 2p)He^4$ termination at temperatures below $T_6 = 6$.

b) $He^3(p, \gamma)Li^4$

The importance of the stability, or instability, of Li^4 to proton emission has long been recognized in relation to such questions as the speed and efficiency of the termination of the proton-proton chain, the ages of astronomical systems, and the production of high-energy solar neutrinos (cf. Bethe 1939; Reeves 1959; and Fowler 1960).

Because of the great astrophysical importance of knowing whether Li^4 is particle stable, Bashkin, Kavanagh, and Parker (1959) investigated the particle stability of this isotope by searching for high-energy positrons from the beta-decay of Li^4. Bashkin *et al.* established an experimental upper limit on the cross-section for the formation of Li^4 by the reaction $He^3(p, \gamma)Li^4$ that is 25 times smaller than the minimum theoretical cross-section predicted by Christy (1959) if Li^4 were particle stable and had a 1^- or 2^- ground state. On this basis Bashkin *et al.* were able to conclude that Li^4 is probably unstable to proton emission.

Contrary to what has been assumed in previous review articles, we assert that the instability of Li^4 to proton emission does not completely rule out its possible importance as a termination for the proton-proton chain via the reactions

$$He^3(p, \gamma)Li^4(\beta^+\nu)He^4 .$$

If the proton-unstable ground state of Li^4 corresponds to a narrow low-lying resonance in the $He^3 + p$ system, then one can show, either by assuming the Breit-Wigner form for the cross-section or by statistical-mechanics arguments, that in an environment of He^3's and protons the equilibrium abundance of Li^4 is given by

$$n(Li^4) = n(He^3)\,n(H^1)\,\omega\left(\frac{8\pi\hbar^2}{3m_p kT}\right)^{3/2} e^{-E_r/kT}\ \text{cm}^{-3}, \tag{20}$$

where E_r is the amount by which the Li^4 ground state is unbound, ω is the statistical weight, $(2J_{Li^4} + 1)/[(2J_{He^3} + 1)(2J_p + 1)] = \frac{1}{4}(2J_{Li^4} + 1)$, and all the other symbols have their usual meaning.

The mean life of He^3 for conversion into He^4 via the Li^4 termination is

$$\tau_p(He^3) = \frac{n(He^3)}{n(Li^4)}\,\tau_{\beta^+}(Li^4), \tag{21}$$

or, from equation (20),

$$\tau_p(He^3) = 0.8205 \times 10^6\left[\frac{\tau_{\beta^+}(Li^4)}{\rho X_H(2J_{Li^4}+1)}\right]T_6^{3/2}\exp\frac{E_r(\text{keV})}{0.0862\,T_6}, \tag{22}$$

where $\tau_{\beta^+}(Li^4)$ is the mean Li^4 positron lifetime. On the basis of theoretical discussions by Breit and McIntosh (1951) and by Werntz and Brennan (1963), we conclude that the mean beta-decay lifetime of Li^4 lies within the limits

$$10^{-3}\ \text{sec} \le \tau_{\beta^+}(Li^4) \le 4 \times 10^3\ \text{sec} .$$

This large range in the possible lifetimes is due to the unknown spin and parity of the Li^4 ground state. Shell-model considerations suggest that the ground state may be 1^- or 2^-, and more complicated arguments indicate it may even be 0^+ (Werntz and Brennan 1963).

Since the properties of the ground state of Li^4 are at present completely unknown, we have calculated $\tau_p(He^3)$ for a number of values of E_r as a function of T_6 using equation (22) and assuming $J^\pi(Li^4) = 1^-$ (our results depend only slightly on this assumption) and $\tau_{\beta^+}(Li^4) = 10^{-3}$, 1.0, and 10^{+3} sec. The results of these calculations are plotted in Figure 5 for $\tau_{\beta^+}(Li^4) = 1.0$ sec, and $\tau_p(He^3)$ is compared to $\tau_p(H^1)$ and $\tau(He^3)$, the mean

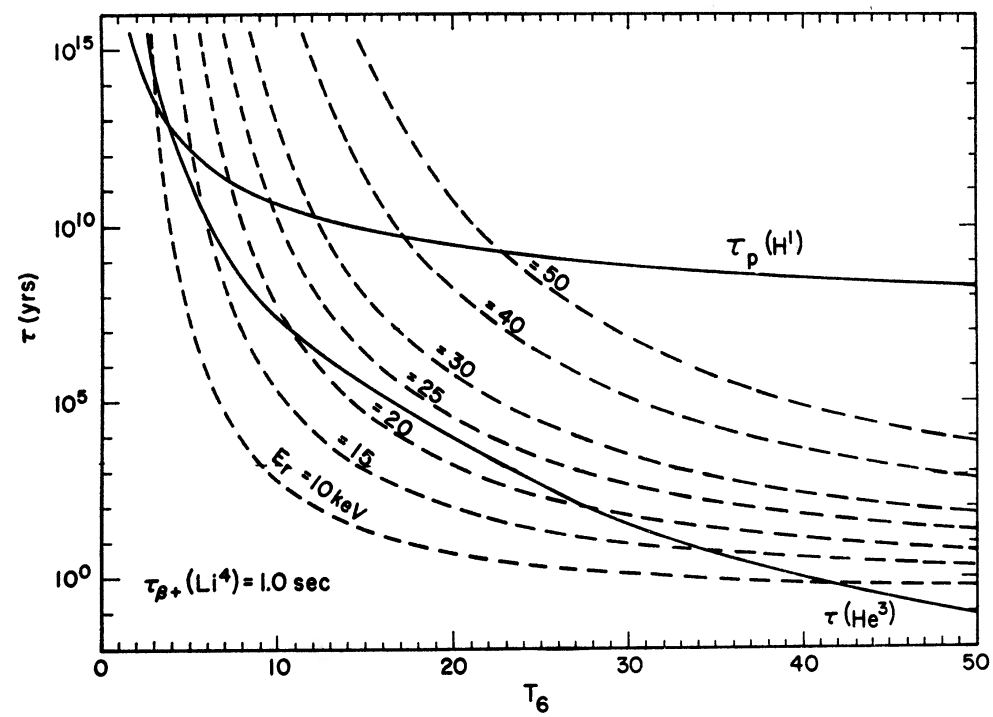

FIG. 5 —The mean life of He^3 to the Li^4 termination is shown as a function of T_6 for $X_H = 0.5$, $\rho = 100$ gm cm^{-3}, $J^\pi(Li^4) = 1^-$, $\tau_{\beta^+}(Li^4) = 1\ 0$ sec and various values of E_r. For comparison, the mean life of the $H^1(p, \beta^+\nu)H^2$ reaction and $\tau(He^3)$, the mean life of He^3 to burning by H^2, He^3, and He^4, are also shown

life of He^3 to the He^3-burning reactions (13*d*), (13*e*), and (13*f*). The quantity $\tau(He^3)$ is defined by

$$\tau(He^3)^{-1} = \tau_d(He^3)^{-1} + \tau_\tau(He^3)^{-1} + \tau_\alpha(He^3)^{-1}\,. \tag{23}$$

Since $\tau_p(He^3)$ is proportional to $\tau_{\beta+}(Li^4)$, the curves in Figure 5 can easily be scaled to values of $\tau_{\beta^+}(Li^4)$ other than 1.0 sec.

The fraction of the He^3 consumed that is converted to He^4 via Li^4 is

$$F_p(He^3) = \frac{\tau(He^3)}{\tau(He^3) + \tau_p(He^3)}\,. \tag{24}$$

The quantity $F_p(He^3)$ is plotted as a function of E_r in Figure 6 for $T_6 = 15$ and for the cases of $\tau_{\beta^+}(Li^4) = 10^{-3}$, 1.0, and 10^{+3} sec. A comparison with results at other temperatures indicates that, in the temperature region around $15 \times 10^6\,°$ K, $F_p(He^3)$ is fairly insensitive to temperature A temperature change of $5 \times 10^6\,°$ K shifts the curves in Figure 6 by only 1 or 2 keV.

From Figures 5 and 6 we conclude that E_r must be less than 40 keV in order for the

Li^4 termination to be of importance in the astrophysically interesting region, $5 \leq T_6 \leq 30$.

Since the completion of the proton-proton chain by the $He^3(p, \gamma)Li^4(\beta^+\nu)He^4$ termination would produce a flux of high-energy solar neutrinos, we can use the measurements of Davis (1964) of the solar-neutrino flux to establish a lower limit on the possible value of E_r. Our argument is an elaboration of one described briefly by Bahcall (1964).

Davis (1964) has established an upper limit on the solar-neutrino flux at the earth's orbit of

$$[\varphi_\nu \bar{\sigma}_\nu (Cl^{37})] \leq 3 \times 10^{-34} \text{ sec}^{-1} \text{ Cl}^{37} - \text{atom}^{-1} . \tag{25a}$$

Bahcall (1964) has shown that the average cross-section for absorption of Li^4 neutrinos by Cl^{37} (including transitions to excited states) is

$$\bar{\sigma}_{\nu(Li^4)}(Cl^{37}) \cong 2 \times 10^{-42} \text{ cm}^2 . \tag{25b}$$

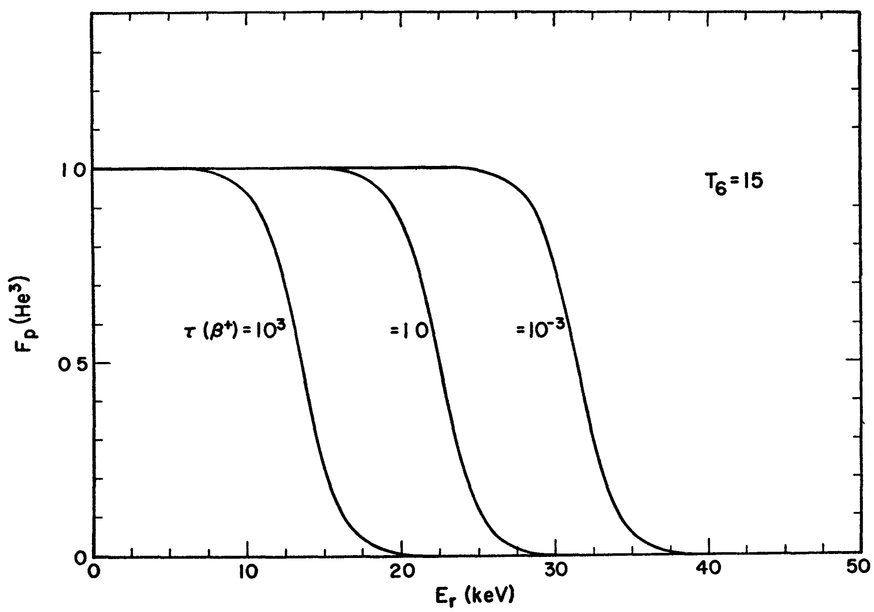

FIG. 6.—The fraction of He^3 consumed by the Li^4 termination is shown as a function of E_r for $X_H = 0.5$, $\rho = 100$ gm cm^{-3}, $T_6 = 15$, $J^\pi(Li^4) = 1^-$ and $\tau_{\beta^+}(Li^4) = 10^{-3}$, 1.0, 10^3 sec.

This cross-section, combined with the experimental upper limit of Davis, implies the following upper limit on the flux of Li^4 neutrinos at the earth's orbit:

$$\varphi_\nu(Li^4) \leq 1.5 \times 10^{+8} \text{ cm}^{-2} \text{ sec}^{-1} . \tag{25c}$$

Thus the energy flux, $\epsilon(Li^4)$, at the earth's orbit from proton-proton reactions terminated by Li^4 in the Sun is less than $2.5 \times 10^{+9}$ MeV cm^{-2} sec^{-1}. Comparing this energy flux with the solar constant, $\epsilon(\odot) = 8.7 \times 10^{+11}$ MeV cm^{-2} sec^{-1}, we find that

$$\frac{\epsilon(Li^4)}{\epsilon(\odot)} \leq 0.29 \times 10^{-2} . \tag{26a}$$

The fractional energy flux due to Li^4 terminations can also be expressed as

$$\frac{\epsilon(Li^4)}{\epsilon(\odot)} = \frac{(16.8)F_p(He^3)}{(16.8)F_p(He^3) + (13.1)F_{He^3}(He^3) + (25.7)F_{He^4}(He^3)} . \tag{26b}$$

It follows from the work of Sears (1964) or Bahcall *et al.* (1963) that $F_{He^3}(He^3) \cong 4F_{He^4}(He^3)$. Hence, if we neglect the small contribution due to Li^4, $F_{He^3}(He^3) = 0.8$ and $F_{He^4}(He^3) = 0.2$. Inserting these numbers in equations (26*a*) and (26*b*), we find

$$F_p(He^3)|_\odot \leq 0.27 \times 10^{-2}. \tag{27}$$

Since

$$C_{He^4}(He^3, p) = [1 \times C_{He^4}(He^3, He^4) + 2 \times C_{He^4}(He^3, He^3)] \times F_p(He^3),$$

we find

$$C_{He^4}(He^3, He^4) \cong \tfrac{1}{3},\ C_{He^4}(He^3, He^3) \cong \tfrac{2}{3},\ \text{and } C_{He^4}(He^3, p)|_\odot \leq 0.45 \times 10^{-2}. \tag{28}$$

Finally, comparing this limit on $F_p(He^3)|_\odot$ with Figure 6, we see that the Bahcall (1964), Davis (1964), and Sears (1964) results imply that

$$E_r \geq 20 \text{ keV}, \tag{29}$$

unless the Li^4 mean life to beta decay is considerably longer than 4000 sec.

It should be emphasized that at present nothing is known experimentally about the ground state of Li^4 except its probable instability to proton emission. Hence, although it is a priori unlikely that the Li^4 ground state lies in the narrow region from 20 to 40 keV above the $He^3 + p$ threshold, the only experimental limit that can be placed at present on the astrophysical importance of the Li^4 termination derives from the solar neutrino measurements of Davis.

b′) $He^3(p, \beta^+\nu)He^4$

Even if the ground state of Li^4 proves to be unbound by several MeV, He^3 can be converted into He^4 by the direct beta-decay reaction $He^3(p, \beta^+\nu)He^4$.

Salpeter (1952) has shown that the $He^3(p, \beta^+\nu)He^4$ termination is unimportant for temperatures between $T_6 = 5$ and $T_6 = 30$. An extrapolation of Salpeter's calculations suggests, however, that at temperatures for which $T_6 \leq 1$ the $He^3(p, \beta^+\nu)He^4$ reaction may be the fastest He^3-burning reaction except for the special conditions discussed in Section IV*f*. This extrapolation provides, however, only a rough estimate, and a further investigation of the $He^3(p, \beta^+\nu)He^4$ reaction is necessary for a full understanding of the operation of the proton-proton chain at temperatures below $T_6 = 1$. At a temperature of 1×10^6 ° K, however, the mean life of He^3 to the $He^3(p, \beta^+\nu)He^4$ reaction is probably $\sim 10^{22}$ years, and this termination is therefore incapable of producing a He^3 equilibrium abundance or of generating any significant amount of energy.

c) $He^3(d, p)He^4$

This reaction was discussed in Section III*c* in connection with H^2 burning; we now examine its role as a He^3-burning reaction. The fraction of the He^3 destroyed that is consumed by the $He^3(d, p)He^4$ reaction is

$$F_d(He^3) = \frac{\langle He^3, H^2\rangle \text{II}}{\langle He^3, H^2\rangle \text{II} + \langle He^3, He^3\rangle \text{III} + \langle He^3, He^4\rangle \text{IV}}, \tag{30}$$

and the fraction of the He^3 produced that is consumed by the $He^3(d, p)He^4$ reaction is

$$\Phi_d(He^3) = \frac{\langle He^3, H^2\rangle \text{III}}{\langle H^2, H^1\rangle}. \tag{31}$$

The fraction of He^4 that is produced by the $He^3(d, p)He^4$ termination is

$$C_{He^4}(He^3, H^2) = \frac{\langle He^3, H^2\rangle \text{II}}{\langle He^3, H^2\rangle \text{II} + \tfrac{1}{2}\langle He^3, He^3\rangle \text{III} + \langle He^3, He^4\rangle \text{IV}}. \tag{32}$$

The quantities $F_d(\mathrm{He}^3)$ and $C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{H}^2)$ have been calculated as a function of temperature, assuming an equilibrium abundance of He^3; the results are shown in Figures 7 and 8. (The assumption of an equilibrium He^3 abundance implies that $\Phi_d[\mathrm{He}^3] = F_d[\mathrm{He}^3]$.)

Figures 7 and 8 show that the $\mathrm{He}^3(d, p)\mathrm{He}^4$ reaction is important as a He^3-burning reaction only for temperatures below $T_6 = 6$, where we have shown that the He^3 abundance is usually less than its equilibrium value. In such a non-equilibrium situation, equations (30), (31), and (32) show that, although the fraction $\Phi_d(\mathrm{He}^3)$ is reduced (not all the He^3 produced will be destroyed), both $F_d(\mathrm{He}^3)$ and $C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{H}^2)$ will be enhanced. Hence at low temperatures ($T_6 \lesssim 3$), it is likely that whatever He^3 is converted into He^4 is converted via the $\mathrm{He}^3(d, p)\mathrm{He}^4$ reaction, except under the special conditions discussed in Sections IVf and b'.

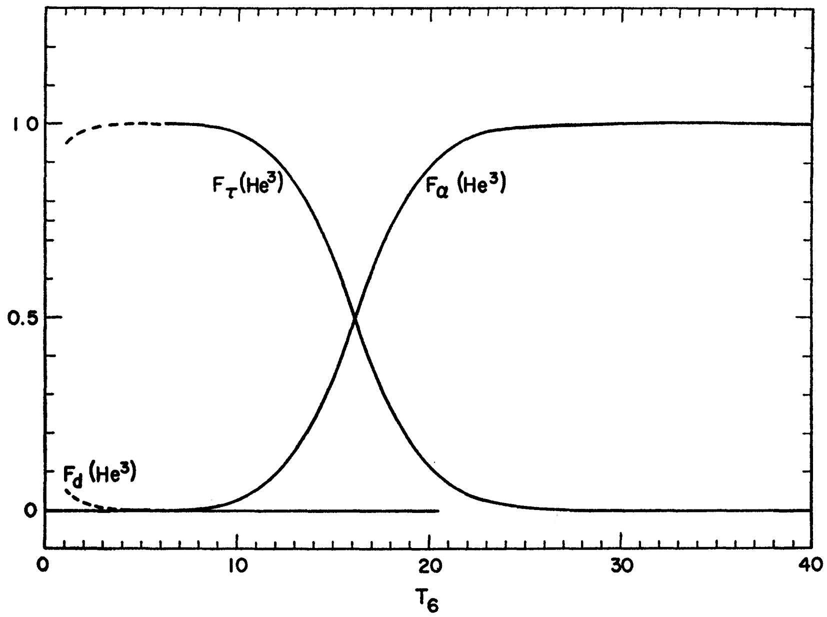

FIG. 7.—The fractions of He^3 consumed by the various He^3-burning reactions are shown as a function of T_6 for $X_{\mathrm{H}} = X_{\mathrm{He}} = 0.5$ and $\rho = 100$ gm cm^{-3}. The He^3-consumption fractions were calculated assuming an equilibrium abundance of He^3 and neglecting possible Li^4 terminations. The dashed curves have the same significance as in Fig. 2.

Despite the "relative" importance of the $\mathrm{He}^3(d, p)\mathrm{He}^4$ reaction at low temperatures, it should be noted that for $T_6 \lesssim 3$, $\tau_d(\mathrm{He}^3) \gtrsim 10^{16}$ years and hence this reaction never contributes significantly to the energy generation.

d) $He^3(\tau, 2p)He^4$

The cross-section for the $\mathrm{He}^3(\tau, 2p)\mathrm{He}^4$ reaction has been measured by Good, Kunz, and Moak (1954) over a range of He^3 energies from 100 to 800 keV. Their low-energy results are not inconsistent with the following intercept parameters for $S(E)$:

$$S_0 \sim 1100 \text{ keV-barns},$$

$$(dS/dE)_0 \sim 0 \text{ barns}.$$

However, a plot of S versus E_{CM} for the $\mathrm{He}^3(\tau, 2p)\mathrm{He}^4$ reaction using the data of Good *et al.* suggests that the value of S_0 quoted in the literature for this reaction [$S_0 \approx 1100$ keV-barns] may have been determined by the limitations of their experiment, for example, by the uncertainty in the average beam energy and by the straggle of the beam through their target. The correct value of S_0 may be as much as a factor of 5 or even 10 different from the value quoted above. Further experimental clarification of this problem is urgently needed.

We have calculated the relevant fractions, $F_\tau(\mathrm{He}^3)$, $\Phi_\tau(\mathrm{He}^3)$ and $C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{He}^3)$, and these are plotted as functions of the temperature in Figures 7 and 8 for the case $X_{\mathrm{H}} = X_{\mathrm{He}} = 0.50$ and $\dot{n}(\mathrm{He}^3) = 0$. The quantity $C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{He}^3)$ is defined by analogy with equation (32)

$$C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{He}^3) = \frac{\frac{1}{2}\langle \mathrm{He}^3, \mathrm{He}^3\rangle \mathrm{III}}{\langle \mathrm{He}^3, \mathrm{H}^2\rangle \mathrm{II} + \frac{1}{2}\langle \mathrm{He}^3, \mathrm{He}^3\rangle \mathrm{III} + \langle \mathrm{He}^3, \mathrm{He}^4\rangle \mathrm{IV}}. \tag{33}$$

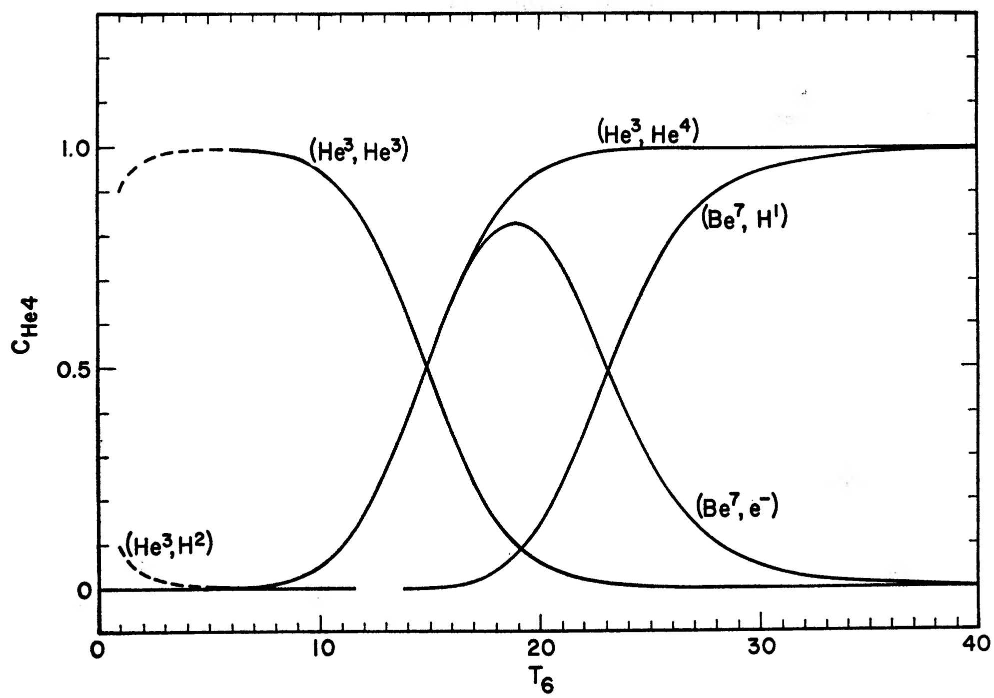

FIG. 8.—The fractions of He^4 produced by various terminations are shown as a function of T_6 for $X_{\mathrm{H}} = X_{\mathrm{He}} = 0.5$ and $\rho = 100$ gm cm^{-3}. The He^4-production fractions were calculated assuming an equilibrium abundance for He^3 and neglecting possible Li^4 terminations. The dashed curves have the same significance as in Fig. 2.

Since a non-equilibrium abundance of He^3 will reduce all three of the fractions, $F_\tau(\mathrm{He}^3)$, $\Phi_\tau(\mathrm{He}^3)$, and $C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{He}^3)$, at temperatures below $T_6 = 6$ the curves in Figures 7 and 8 represent upper limits.

e) $\mathit{He}^3(\alpha, \gamma)\mathit{Be}^7$

Experimental measurements on the $\mathrm{He}^3(\alpha, \gamma)\mathrm{Be}^7$ reaction by Holmgren and Johnston (1959) and by Parker and Kavanagh (1963) have shown that the cross-section for this reaction is sufficiently large so that at high temperatures the burning of He^3 by alpha-particles competes successfully with the $\mathrm{He}^3(\tau, 2p)\mathrm{He}^4$ reaction. From their measurements, Parker and Kavanagh have obtained the following intercept parameters for $S(E)$:

$$S_0 = (0.47 \pm 0.05) \text{ keV-barns},$$

$$(dS/dE)_0 = -2.8 \times 10^{-4} \text{ barns}.$$

The fractions, $F_\alpha(\mathrm{He}^3)$, $\Phi_\alpha(\mathrm{He}^3)$, and $C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{He}^4)$ have been calculated for the $\mathrm{He}^3(\alpha, \gamma)\mathrm{Be}^7$ reaction and are plotted in Figures 7 and 8 for an equilibrium abundance of He^3. The quantity $C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{He}^4)$ is defined by analogy with equation (32) as

$$C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{He}^4) = \frac{\langle \mathrm{He}^3\ \mathrm{He}^4 \rangle \mathrm{IV}}{\langle \mathrm{He}^3, \mathrm{H}^2 \rangle \mathrm{II} + \frac{1}{2} \langle \mathrm{He}^3, \mathrm{He}^3 \rangle \mathrm{III} + \langle \mathrm{He}^3, \mathrm{He}^4 \rangle \mathrm{IV}}. \tag{34}$$

If the He^3 abundance is less than its equilibrium value, then $\Phi_\alpha(\mathrm{He}^3)$ is decreased, and both $F_\alpha(\mathrm{He}^3)$ and $C_{\mathrm{He}^4}(\mathrm{He}^3, \mathrm{He}^4)$ are increased relative to the values plotted in Figures 7 and 8. However, at the low temperatures where the non-equilibrium abundance of He^3 is important, the $\mathrm{He}^3(\alpha, \gamma)\mathrm{Be}^7$ reaction is so infrequent [$F_\alpha(\mathrm{He}^3) \lesssim 10^{-4}$ at $T_6 = 6$; $\lesssim 10^{-9}$ at $T_6 = 2$], that we can safely ignore non-equilibrium considerations for this reaction.

The following reactions are possible ways to convert Be^7 into two He^4's and terminate the proton-proton chain:

$$\begin{aligned}
&(a)\ \ \mathrm{Be}^7(e^-, \nu)\mathrm{Li}^7(p, \alpha)\mathrm{He}^4\,; \\
&(b)\ \ \mathrm{Be}^7(p, \gamma)\mathrm{B}^8(\beta^+\nu)\mathrm{Be}^{8*}(\alpha)\mathrm{He}^4\,; \\
&(c)\ \ \mathrm{Be}^7(d, p)\mathrm{Be}^8(\alpha)\mathrm{He}^4\,; \\
&(d)\ \ \mathrm{Be}^7(\tau, p)\mathrm{B}^9(p)\mathrm{Be}^8(\alpha)\mathrm{He}^4\,, \\
&\qquad \mathrm{Be}^7(\tau, \alpha)\mathrm{Be}^6(p)\mathrm{Li}^5(p)\mathrm{He}^4; \text{ and} \\
&(e)\ \ \mathrm{Be}^7(\alpha, \gamma)\mathrm{C}^{11}(\beta^+\nu)\mathrm{B}^{11}(p, \alpha)\mathrm{Be}^8(\alpha)\mathrm{He}^4\,.
\end{aligned} \tag{35}$$

Of these reactions, only (*a*) and (*b*) are sufficiently fast to guarantee equilibrium; (*c*) is too slow because of the small relative abundance of deuterium and (*d*) and (*e*) are too slow because of their larger coulomb barriers.

From Figure 3, *B*, we can see that the $\mathrm{Li}^7(p, \alpha)\mathrm{He}^4$ reaction is sufficiently fast at all temperatures of interest so that the rate of equation (35*a*) is determined by the $\mathrm{Be}^7(e^-, \nu)\mathrm{Li}^7$ reaction. The positron mean life of B^8 in stellar interiors does not differ significantly (Bahcall 1962*a*) from its terrestrial value of about 1 sec, and so from Figure 3, *B*, we can see that the rate of equation (35*b*) is determined by the $\mathrm{Be}^7(p, \gamma)\mathrm{B}^8$ reaction.

Bahcall (1962*b*) and Kavanagh (1960) have studied terminations (35*a*) and (35*b*). Bahcall has shown that for the $\mathrm{Be}^7(e^-, \nu)\mathrm{Li}^7$ reaction the mean lifetime of Be^7 for continuum electron capture in stars like the Sun is

$$\tau_{e^-}(\mathrm{Be}^7) = \frac{10^9 T_6^{1/2}}{2.12\rho(1 + X_\mathrm{H})}\ \mathrm{sec}. \tag{36}$$

From Kavanagh's measurements of the cross-section for the $\mathrm{Be}^7(p, \gamma)\mathrm{B}^8$ reaction at proton energies of 800 and 1400 keV, Christy and Duck (1961) have obtained the following intercept parameters:

$$S_0 = (0.030 \pm 0.010)\ \mathrm{keV\text{-}barns}, \qquad (dS/dE)_0 = 0\ \mathrm{barns}\,.$$

The fraction of Be^7 consumed via reaction (35*a*) is

$$F_{e^-}(\mathrm{Be}^7) = \frac{\tau_p(\mathrm{Be}^7)}{\tau_{e^-}(\mathrm{Be}^7) + \tau_p(\mathrm{Be}^7)}, \tag{37a}$$

and the fraction consumed via reaction (35*b*) is

$$F_p(\mathrm{Be}^7) = \frac{\tau_{e^-}(\mathrm{Be}^7)}{\tau_{e^-}(\mathrm{Be}^7) + \tau_p(\mathrm{Be}^7)}. \tag{37b}$$

From equations (37*a*) and (37*b*) we find that the fraction of He^4 produced through each of these terminations is

$$C_{He^4}(Be^7, e^-) = C_{He^4}(He^3, He^4)F_{e^-}(Be^7)\,, \tag{38a}$$

$$C_{He^4}(Be^7, H^1) = C_{He^4}(He^3, He^4)F_p(Be^7)\,. \tag{38b}$$

The two fractions, $C_{He^4}(Be^7, e^-)$ and $C_{He^4}(Be^7, H^1)$, are plotted in Figure 8 as functions of temperature and are compared with the C_{He^4}'s for the terminations discussed in Sections IV*c* and IV*d*.

f) $He^4(e^-, \nu)T^3$

At low temperatures the He^3-burning reactions (13*d*), (13*e*), and (13*f*) occur more slowly than the He^3-producing reactions (13*a*) and (13*b*). Hence at such temperatures the proton-proton chain would effectively end at He^3 if the only reactions possible were those listed in equation (13). Schatzman (1958) has, however, discussed an alternative way of closing the proton-proton chain via the reactions

$$He^3(e^-, \nu)H^3\,, \tag{39a}$$

$$H^3(p, \gamma)He^4\,. \tag{39b}$$

Bahcall and Wolf (1964) have recently reinvestigated terminations (39*a*) and (39*b*) using the modern theory of nuclear beta-decay and have also considered some alternative ways of burning the tritium produced by reaction (39*a*). The reader is referred to their accompanying paper for further details. In our discussion we summarize some of the conclusions of Bahcall and Wolf. Iben (1963) is currently studying models for stars of less than 1 solar mass in order to determine under what conditions reactions (39*a*) and (39*b*) actually terminate the proton-proton chain.

The reaction $He^3(e^-, \nu)H^3$ is an endothermic electron-capture reaction with a threshold energy of about 18 keV (Langer and Moffat 1952). Bahcall and Wolf have shown that the rate of the $He^3(e^-, \nu)H^3$ reaction is negligibly small for densities less than a critical density,

$$\rho_{cr} = 1.87\mu_e \times 10^{+4}\ \text{gm cm}^{-3}\,,$$

where μ_e is the mean molecular weight per electron. This critical density is determined by the requirement that the Fermi energy for a completely degenerate gas of electrons be equal to the threshold energy of 18 keV.

The electron-capture lifetime for a He^3 nucleus is

$$\tau_{1/2}(He^3) = (ft_{1/2})_{lab}K^{-1}\,, \tag{40}$$

where $(ft_{1/2})_{lab}$ is the terrestrial ft-value for the exothermic tritium beta-decay and is equal to 1070 ± 70 sec (Porter 1959) and K is the generalized phase-space factor. Accurate analytic and numerical expressions for K have been given by Bahcall and Wolf for both non-degenerate and degenerate situations. For degenerate situations with densities such that $\rho_{cr} < \rho < 10^6$ gm gm^{-3}, they find

$$K = 8.22 \times 10^{-10}(E_F - 18)^3[1 + 0.019(E_F - 18)]\,. \tag{41}$$

In equation (41) E_F is the Fermi energy in keV, that is,

$$E_F = 511\left\{\left[1.018\left(\frac{\rho}{\mu_e}\right)^{2/3} \times 10^{-4} + 1\right]^{1/2} - 1\right\}\text{keV}. \tag{42}$$

Perry and Bame (1955) studied the $H^3(p, \gamma)He^4$ reaction at center-of-mass energies from 75 keV to 4.7 MeV, and from their measurements one can obtain the following intercept parameters for the cross-section factor

$$S_0 = 1.5 \times 10^{-3} \text{ keV-barns},$$
$$(dS/dE)_0 = 2.7 \times 10^{-5} \text{ barns}.$$

Two other tritium-burning reactions of interest are $H^3(t, 2n)He^4$ and $H^3(\tau, np)He^4$. We estimate on the basis of work by Jarmie and Allen (1958) and Youn, Osetinskii, Sodnom, Govorov, Sizov, and Salatskii (1961) that these reactions have S_0's of 300 keV-barns and 1000 keV-barns, respectively. Using these intercept parameters Bahcall and Wolf have shown that the mean lifetime of tritium to these three nuclear reactions is less than the electron-capture lifetime of He^3 at all relevant stellar temperatures and densities. Hence, the lifetimes for the terminations via $He^3(e^-, \nu)H^3$ are determined entirely by the electron-capture lifetime given by equation (40).

The electron-capture lifetime of He^3 for the $He^3(e^-, \nu)H^3$ reaction is only 5000 years for a density of $2.2 \times 10^4 \mu_e$ gm cm^{-3}($E_F = 20$ keV). Thus at densities only slightly greater than ρ_{cr} the $He^3(e^-, \nu)H^3$ reaction is considerably faster than the $H^1(p, \beta^+\nu)H^2$ reaction at all temperatures of interest for main-sequence stars. Hence, at sufficiently high densities reactions (39*a*) and (39*b*) can establish an equilibrium abundance of He^3.

g) Solar He^3 Burning

In conclusion, we note that stellar-model calculations by Iben and Sears (1962) suggest that over the Sun as a whole about two-thirds of the proton-proton chain terminations occur via the $He^3(\tau, 2p)He^4$ reaction, one-third via the $He^3(\alpha, \gamma)Be^7(e^-, \nu)Li^7$ reactions, and 1/1000 via the $He^3(\alpha, \gamma)Be^7(p, \gamma)B^8$ reactions. The ratio of $He^3(\tau, 2p)He^4$ completions to $He^3(\alpha, \gamma)Be^7$ completions is approximately reversed at the center of the Sun compared to the average values quoted above.

V. He^4 PRODUCTION AND ENERGY GENERATION

Over most of the temperature range in which the proton-proton chain operates the $H^1(p, \beta^+\nu)H^2$ reaction is the slowest link in the chain. However, the rate of He^4 production, R_α, is not proportional to the rate of the $H^1(p, \beta^+\nu)H^2$ reaction, $\frac{1}{2}\langle H^1, H^1\rangle n(H^1)n(H^1)$. Instead we must write

$$R_\alpha = \chi_\alpha[\tfrac{1}{2}\langle H^1, H^1\rangle n(H^1)n(H^1)]\text{cm}^{-3}\text{ sec}^{-1}, \tag{43}$$

where χ_α is a dimensionless function of temperature. The temperature variation of χ_α arises because the terminations $He^3(d, p)He^4$ and $He^3(\tau, 2p)He^4$ both require two $H^1(p, \beta^+\nu)H^2$ reactions for each He^4 produced ($\chi_\alpha = \frac{1}{2}$) while the $He^3(\alpha, \gamma)Be^7$ terminations produce one He^4 for every $H^1(p, \beta^+\nu)H^2$ reaction ($\chi_\alpha = 1$). Writing R_α explicitly,

$$\begin{aligned} R_\alpha = \langle He^3, H^2\rangle n(He^3)n(H^2) &+ \tfrac{1}{2}\langle He^3, He^3\rangle n(He^3)n(He^3) \\ &+ \langle He^3, He^4\rangle n(He^3)n(He^4), \end{aligned} \tag{44}$$

we see that

$$\chi_\alpha = \frac{\langle He^3, H^2\rangle \text{III II} + \frac{1}{2}\langle He^3, He^3\rangle \text{III}^2 + \langle He^3, He^4\rangle \text{III IV}}{\frac{1}{2}\langle H^1, H^1\rangle}. \tag{45}$$

The quantity χ_α is plotted as a function of temperature in Figure 9 assuming that $X_H = X_{He} = 0.5$ and that the He^3 abundance is equal to its equilibrium value.

At low temperatures, the He^3 abundance is less than its equilibrium value and the proton-proton chain tends to terminate with the production of He^3. Thus at low temperatures χ_α is much less than the equilibrium values shown in Figure 9 and, in fact, approaches zero as the temperature goes to zero. For the conditions in which the chain is terminated by the $He^3(e^-, \nu)H^3(p, \gamma)He^4$ reactions, χ_α is equal to unity. Note that χ_α is one-half the quantity φ_α defined by Fowler (1958, 1960); the latter quantity was defined in terms of the $He^3(\tau, 2p)He^4$ reaction instead of the $H^1(p, \beta^+\nu)H^2$ reaction.

The rate of useful energy generation (i.e., energy not carried off by neutrinos) is not proportional to the rate of the $H^1(p, \beta^+\nu)H^2$ reaction or even to R_α. If we write the rate of useful energy generation, R_ϵ, as

$$R_\epsilon = \chi_\epsilon[\tfrac{1}{2}26.73\langle H^1, H^1\rangle n(H^1)n(H^1)]\text{MeV cm}^{-3}\text{ sec}^{-1}, \tag{46}$$

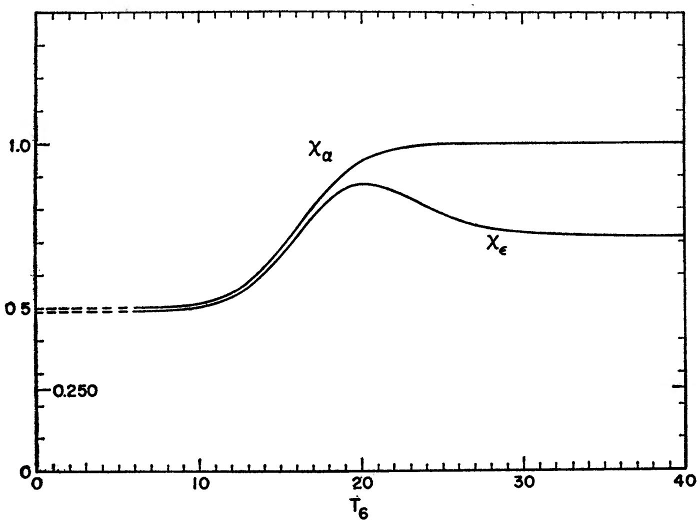

FIG. 9.—The He^4-production and energy-generation factors, χ_α and χ_ϵ, are shown as functions of T_6 for $X_H = X_{He} = 0.5$ and $\rho = 100$ gm cm^{-3}. An equilibrium abundance is assumed for He^3 and any Li^4 terminations are neglected. The dashed curves have the same significance as in Fig. 2.

where 26.73 MeV is the Q-value for converting $4H^1$ into He^4, then the additional variation of χ_ϵ is caused by the differences in the neutrino-energy losses of the various terminations. The "useful" Q-values for the various terminations of the proton-proton chain are (König, Mattauch, and Wapstra 1962):

$He^3(p, \gamma)Li^4(\beta^+\nu)He^4$	16.8 MeV
$He^3(p, \beta^+\nu)He^4$	16.8 MeV
$He^3(d, p)He^4$	26.22 MeV
$He^3(\tau, 2p)He^4$	26.22 MeV
$He^3(\alpha, \gamma)Be^7(e^-, \nu)Li^7(p, \alpha)He^4$	25.67 MeV
$He^3(\alpha, \gamma)Be^7(p, \gamma)B^8(\beta^+\nu)Be^{8*}(\alpha)He^4$	19.1 MeV
$He^3(e^-, \nu)H^3(p, \gamma)He^4$	26.47 MeV

Considering only the burning of He^3 by H^2, He^3, and He^4, we can write R_ϵ explicitly as

$$\begin{aligned} R_\epsilon = {} & 26.22\langle He^3, H^2\rangle n(He^3)n(H^2) + \tfrac{1}{2}26.22\langle He^3, He^3\rangle n(He^3)n(He^3) \\ & + 25.67\langle He^3, He^4\rangle F_e(Be^7)n(He^3)n(He^4) \\ & + 19.1\langle He^3, He^4\rangle F_p(Be^7)n(He^3)n(He^4). \end{aligned} \tag{47}$$

It should be noted that with a few refinements χ_ϵ is essentially one-half the quantity $\psi(\alpha)$ defined by Fowler (1958, 1960), since the latter quantity was defined in terms of one

$He^3(\tau, 2p)He^4$ reaction instead of one $H^1(p, \beta^+\nu)H^2$ reaction. Thus we can express χ_ϵ as

$$\tfrac{1}{2}\chi_e = 0.981 \frac{\langle He^3, He^2\rangle}{\langle H^1, H^1\rangle} \text{ III II} + \tfrac{1}{2}0.981 \frac{\langle He^3, He^3\rangle}{\langle H^1, H^1\rangle} \text{ III III}$$
$$+ 0.960 \frac{\langle He^3, He^4\rangle}{\langle H^1, H^1\rangle} F_{e^-}(Be^7)\text{ III IV} + 0.715 \frac{\langle He^3, He^4\rangle}{\langle H^1, H^1\rangle} F_p(Be^7)\text{ III IV}, \tag{48}$$

or

$$\chi_\epsilon = \chi_\alpha[0.981 C_{He^4}(He^3, H^2) + 0.981 C_{He^4}(He^3, He^3) + 0.960 C_{He^4}(Be^7, e^-) + 0.715 C_{He^4}(Be^7, H^1)]\,. \tag{49}$$

The factor χ_ϵ is plotted as a function of temperature in Figure 9 assuming an equilibrium He^3 abundance and $X_H = X_{He} = 0.5$. When the proton-proton chain tends to stop at

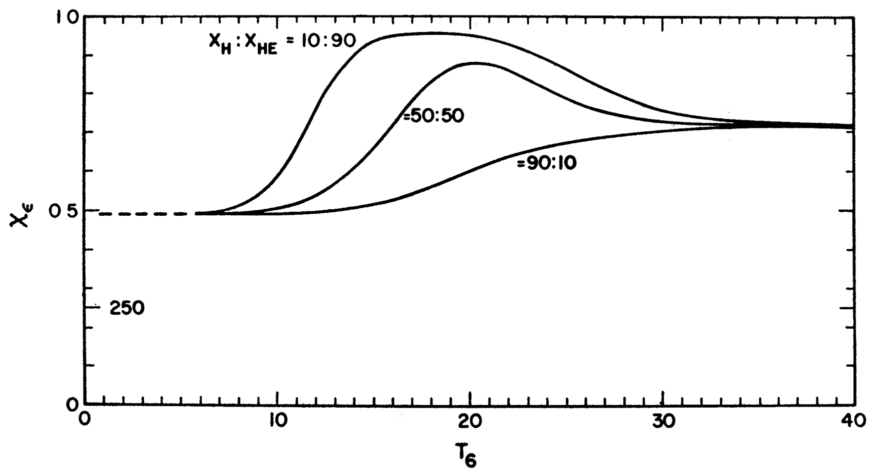

FIG. 10.—The energy-generation factor, χ_ϵ, is shown as a function of T_6 for $\rho = 100$ gm cm^{-3} and for various values of (X_H, X_{He}). An equilibrium He^3 abundance is assumed and Li^4 terminations are neglected. The dashed curve has the same significance as in Fig. 2.

the production of He^3, only 6.68 MeV of useful energy are produced for each $H^1(p, \beta^+\nu)H^2$ reaction. In such a situation, the He^3 abundance is less than its equilibrium value, and χ_ϵ approaches a value of 0.250.

The energy generation in the pp-chain can be calculated explicitly from the following expression based on $S_0(pp) = (3.36 \pm 0.4) \times 10^{-22}$ keV-barns:

$$E_{pp} = (4.1 \pm 0.5)\rho X_H^2 \chi_\epsilon T_6^{-2/3} \exp(-33.804 T_6^{-3/2}) \times (1 + 0.012 T_6^{1/3} + 0.008 T_6^{2/3} + 0.00065 T_6 + 0.188 \zeta \rho^{1/2} T_6^{-3/2})\, 10^6 \text{ ergs gm}^{-1}\text{ sec}^{-1}\,. \tag{50}$$

All calculations discussed thus far in this paper were made assuming that $X_H = X_{He} = 0.5$. Many of the quantities we have calculated depend strongly on X_H and X_{He}. In order to illustrate this dependence we have plotted in Figure 10 χ_ϵ as a function of temperature for hydrogen and helium abundances of ($X_H = 0.9$; $X_{He} = 0.1$), ($X_H = 0.5$; $X_{He} = 0.5$), and ($X_H = 0.1$; $X_{He} = 0.9$).

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