Statistics of Meteor Streams

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In the period 1954 to 1957 a random sample comprising 360 of the meteors photographed from two stations by the Baker Super-Schmidt meteor cameras were reduced under the "Short Trail" program (Hawkins and Southworth, 1958). The radiants and velocities of 359 meteors were sufficiently accurate to permit computation of dependable orbits (Hawkins and Southworth, 1961). Since this is the only random sample of photographic meteors with individually reliable orbits, it provides valuable material for a study of meteor distribution in space. The degree to which the meteors are grouped into streams is a basic feature of this distribution.

The major meteor streams are conspicuous in any random sample of photographic meteors and virtually all of their members are readily identifiable. Since in the sample there are many meteors that belong to each such stream, the characteristics of the stream are well defined, as is the range of each characteristic within the stream. Furthermore, these streams are well-known from other studies.

Members of a stream not previously known, or not well represented in the sample under study, are not as easy to identify. Early in the Short Trail program it became apparent that some of the meteors were associated into other groups or streams, apart from the previously known major streams. Numerous other meteors were doubtfully associated with streams. Several attempts, based upon a variety of principles, were made to classify the meteors into streams and a "sporadic" remainder. None of the classifications based on geocentric quantities—radiants, velocities, and dates of occurrence—was satisfactory. It became clear that a comprehensive quantitative criterion was required that would embrace all the elements of the orbit.

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A criterion for stream membership

We may regard a meteor stream as composed of those meteors originating from a single comet during a single ejection or over an interval of time. As yet, we cannot reliably identify stream membership by physical or chemical characteristics, although some physical differences have been observed between different streams (Jacchia, 1960). Meteors arising from a comet will move in orbits similar to the comet's orbit, at least for a while. If planetary perturbations or other forces disperse the meteors so that the similarity of their orbits can no longer be seen, the stream is effectively destroyed. Thus, in practice, a meteor stream is recognized and defined as a group of meteors in similar orbits.

Occasionally meteors from two or more comets may have similar orbits. If the different origins cannot be recognized from the orbits, then the meteors form a single stream in practice, although arising from more than one comet.

A quantitative criterion for stream membership therefore requires a quantitative measure of orbit similarity (or difference). We shall consider the difference between orbits to be the difference between meteors. Let $D(A, B)$ be such a quantitative measure of the difference between meteors A and B. Similarly, let $D(M,N)$ be of the same mathematical form, but let it measure the difference between a mean orbit M of a stream and an individual meteor N. The two measures $D(A, B)$ and $D(M, N)$ correspond to two possible ways of defining a stream.

(1) A stream may be defined, for example, as

all meteors N such that $D(M, N)$ does not exceed a given value D_M.

(2) We may define a stream by serial association between members. We may state that two meteors A and B are associated if $D(A, B)$ does not exceed a standard value D_S. A stream may then be defined as a group of meteors in which every member is associated with one or more other members, and all members are associated together directly or indirectly.

As a third possibility we may define a phase space in which each meteor is represented by a point and the distance between points is the difference between meteors. Then a stream can be defined as the meteors in a region of high density in the phase space. This form has theoretical advantages but requires far more data than we possess. We shall therefore use the first two alternatives.

No particular measure of orbit difference is specified a priori. Indeed, we have found no way of determining a "best" or "optimum" measure. The magnitude of the cause of orbit difference, when known, is often the best measure of the difference. We do not know the causes of the dispersion within meteor streams in any quantitative manner, however, and it would be quite wrong to assume the causes. Consequently, we have looked for a measure that is as simple as possible and is expressed in terms of orbital elements as far as possible, since these are the most familiar indices of orbit differences. Because differences within streams are small, our measure need not be accurate for large differences in order to be useful.

One orbit may differ from another in any of six independent ways. One of these, the position of the meteor in the orbit at a given time, is not significant for our purpose because the meteors of a stream rapidly spread out all along the orbit. Apart from the instantaneous position of the meteor, the orbit may be specified by five elements. These may be the usual elements: a, e, ω, Ω, i; or they may be any set, out of an infinite number of possible sets, of five independent functions of the usual elements: C_j, where $j=1, 2, 3, 4, 5$. Each of the five differences between two orbits A, B may be expressed as the difference between the values of an element: $C_j(A)-C_j(B)$.

The fact that all the observed meteors pass close to the orbit of the earth imposes one constraint on the observed five elements, and thus reduces the number of independent elements to four. The earth is not important in the evolution of meteor streams, however, so that this constraint is not relevant to the differences within streams. Accordingly, we disregard it in formulating D.

In order to construct a single measure comprising five independent differences, we draw an analogy with a five-dimensional orthogonal coordinate system and consider each element as a coordinate. Any orbit then corresponds to a point in this conceptual space, and the distance between two points is a natural measure of the difference between the two corresponding orbits. Although orthogonal, the coordinates need not be rectangular; usually these will be curvilinear coordinates. We may approximate them by rectangular coordinates in any sufficiently small region, however, and this approximation will prove to be adequate for our purpose. The difference between meteors A, B is then of the form

$$D(A, B)=\left\{\sum_{j=1}^{5} c_j^2\,[C_j(A)-C_j(B)]^2\right\}^{1/2}, \qquad (1)$$

where c_j are functions of the elements which are at our disposal and which determine the geometry of the space.

The value of D as a measure of difference depends on how well the c_j are chosen or can be chosen. The c_j, for our purpose, are scale factors determining the relative importance of the different C_j in D. Each c_j should be inversely proportional to the expected standard deviation of the corresponding element C_j in a stream.

An attempt to base the scale factors only on the streams would be an attempt to lift ourselves by our own bootstraps, because we are still trying to determine which meteors are stream members and which are not. We have checked the reasonability of the scale factors that were ultimately chosen by computing the mean standard deviation of each element on the assumption of an idealized dispersing mechanism. This is discussed below.

In accordance with all the foregoing, we have selected the elements and scale factors described below. Angles, and thus the differences of angular elements, are measured by their chords, i.e., by twice the sine of half the angle. This is convenient; for small angles it is a close approximation to radian measure, and for all angles from 0 to 2π it is a smooth continuous measure of the absolute value, without difficulties of sign or definition. We measure a difference in orbital plane by the angle I_{AB} between the orbital planes. I_{AB} comprises two elements and their scale factors; this appears explicitly in the identity

$$\left(2\sin\frac{I_{AB}}{2}\right)^2=\left(2\sin\frac{i_B-i_A}{2}\right)^2 +\sin i_A \sin i_B\left(2\sin\frac{\Omega_B-\Omega_A}{2}\right)^2. \quad (2)$$

Three elements describe an orbit within the orbital plane. We measure a difference of shape by the difference in eccentricity e, and of size by the difference in perihelion distance q. In contrast to the major semi-axis a, q is well determined by observation and has a small range of values. A difference in orientation of the orbit within the plane is measured by the angle Π_{AB} between the major axes, weighted by the scale factor e. Thus Π_{AB} is the difference between the longitudes of perihelion. When the orbits are in different planes, Π_{AB} is the difference between the longitudes of perihelion measured from the intersection of the orbits. In terms of familiar elements,

$$\Pi_{AB}=\omega_B-\omega_A+2\arcsin\left[\cos\frac{i_A+i_B}{2}\sin\frac{\Omega_B-\Omega_A}{2}\sec\frac{I_{AB}}{2}\right], \quad (3a)$$

or, to a sufficient approximation when i_A and i_B are small,

$$\Pi_{AB}=(\Omega_B+\omega_B)-(\Omega_A+\omega_A). \quad (3b)$$

By combining equations (1), (2), and (3a), we find

$$[D(A,B)]^2=[e_B-e_A]^2 +[q_B-q_A]^2+\left[2\sin\frac{i_B-i_A}{2}\right]^2 +\sin i_A\sin i_B\left[2\sin\frac{\Omega_B-\Omega_A}{2}\right]^2 +\left[\left(\frac{e_A+e_B}{2}\right)2\sin\frac{(\Omega_B+\omega_B)-(\Omega_A+\omega_A)}{2}\right]^2 \quad (4)$$

at low inclination, or, after substituting equation (3a) for (3b) in the last term, at any inclination. This expression is readily computed with a desk calculator and a special table of chords.

A model of stream dispersion

The scale factors c_j of the various elements in D, as shown in equation (4), were selected on a variety of grounds of simplicity, reasonability, and previous experience. By qualitative arguments we expected that, after scaling, the elements would make roughly equal contributions to D. The utility of D is empirically established below on known streams; nevertheless, a theoretical basis for D is clearly desirable if it can be found. Although we made this theoretical study after D had already been applied to the meteors, the validity of D is confirmed in the main.

An excellent measure of the difference between orbits, if it can be found, is the amount of perturbation that will transform one of the orbits into the other. A perturbation of an orbit arises from an accelerating force applied for a period of time; hence the velocity change so arising is the natural measure of the perturbation. In general, no single perturbation can suffice because the two orbits will not intersect; we may expect that an appreciable difference between a pair of orbits that were once the same will normally be the result of many perturbations. Since D comprehends all orbital elements, it appears possible that D is indeed a measure of total perturbation; we must determine, however, whether the different elements may be so combined.

We consider an idealized model for orbit change, postulating random vectorial increments $\delta\mathbf{V}$ to the vectorial orbital velocity $\mathbf{v}_H$ of a meteor. The natural unit of orbital speed at any particular distance r from the sun is the speed U of a body in a circular orbit of that radius; U varies as $r^{-1/2}$. We take δV to be proportional to U because, broadly speaking, we would expect most kinds of perturbation to be more intense nearer the sun. The test applied to D consisted in finding the sensitivities of the different elements to perturbations $\delta\mathbf{V}$; these sensitivities should be comparable. To measure the sensitivities, we computed the time averages, denoted "Grad," of the magnitudes of the gradients of the various elements in D with respect to perturbations $\delta\mathbf{V}$. They are shown in figure 1, plotted as functions of eccentricity. The computation of these gradients is described in the appendix; they are approximate, except for Grad I_{AB} which is exact. The value of Grad Π_{AB} on the graph has already been multiplied by the scale factor e. To obtain Grad q, the value in the figure must be multiplied by a. Broken lines in the figure indicate the extreme values of Grad q possible for a meteor observed on earth.

Grad e, e Grad Π_{AB}, and Grad I_{AB} are quite satisfactorily similar for values of $e \leq 0.85$. Grad q is a function of both a and e, but is comparable to the others for most observed values of a when $e \leq 0.85$. For eccentricities near unity the model implies that the different elements in D have very different sensitivities to perturbations of the kind postulated. At these high eccentricities, the time averages primarily reflect the conditions near aphelion; the effect of this is not clear. Possibly the apparent practical success of D for high eccentricity depends upon averaging overweighted elements with underweighted elements.

If this computation had been available earlier in the analysis, we could no doubt have modified the form of D at the cost of simplicity. Except when the eccentricity is nearly one, however, we may conclude that D in its present form is approximately proportional to the average velocity increment, in units of the circular speed, which will perturb one of the orbits into the other. Estimating the factor of proportionality from figure 1, we find that D is about 3/2 of the mean velocity increment.

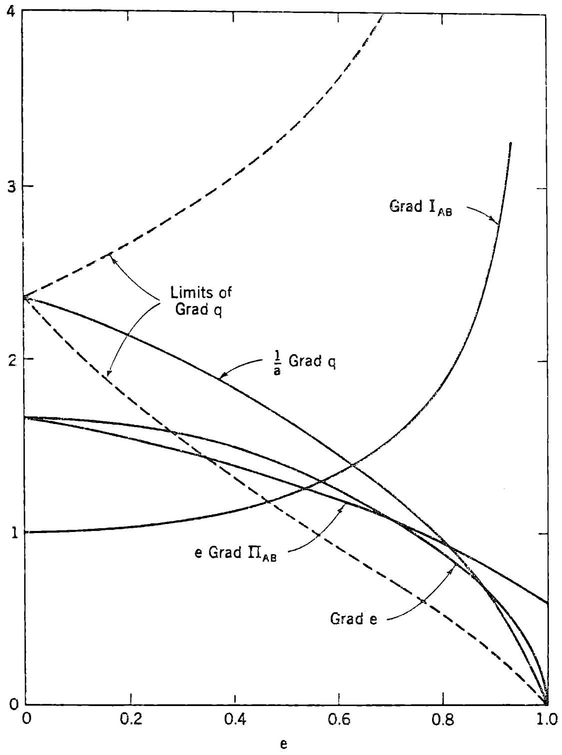

FIGURE 1.—Time average gradients of elements with respect to velocity perturbations which are proportional to the circular speed.

Application to known streams

We applied D first to the known streams that were sufficiently represented in the sample, using both forms of the stream criterion discussed above. A list of the members of each stream had already been prepared by use of conventional criteria. The difference $D(M,N)$ between the mean orbit M of the stream and the orbit of each member N was found, as were the differences $D(A,B)$ between pairs of meteors in each stream. The values of $D(A,B)$ were examined to find the minimum value $D_{S\ \min}$ of D_S such that all the members were associated. Table 1 summarizes the values of $D(M,N)$ and also includes $D_{S\ \min}$. There is obviously a close correlation between $D_{S\ \min}$ and the mean value of $D(M,N)$.

Both $D_{S\ \min}$ and the mean value of $D(M,N)$ are also correlated with the average spread of a stream radiant, as found by Wright, Jacchia, and Whipple (see Whipple, 1947; Wright and

TABLE 1.—*Distribution of values of $D(M,N)$ for previously identified members of known streams, and values of $D_{S\ \min}$ and O—C*

Streams	Range of $D(M,N)$								Total	Mean	$D_{S\ \min}$	O—C
	.00 to .04	.05 to .08	.09 to .12	.13 to .16	.17 to .20	.21 to .24	.25 to .28	.29 to .32				
Quadrantids	3	2							5	0.04	0.05	
Orionids	4	3	3	1	1				12	.08	.15	0.38
Perseids*	1	3	1	3	1				9	*.11	*.12	.68
Geminids	14	2							16	.02	.04	.22
S. ι Aquarids†			2						2	.10	.20	1.8
S. δ Aquarids	2	1							3	.03	.07	.80
N. δ Aquarids		1	1	1					3	.10	.15	1.2
α Capricornid		2							2	.08	.16	2.1
S. Taurid-Arietid		1	2	2	1	1	2	1	10	.18	.19	.83
N. Taurids		1	2	3		1			7	.13	.19	1.02
Andromedids		2							2	.08	.16	
Total	24	18	11	10	3	2	2	1	71	.09		

*Incomplete observations; shower peak missed.
†One meteor, characterized as a "very doubtful member" by Wright, Jacchia, and Whipple (1957), has been omitted.

Whipple, 1948, 1950, 1953; Wright, Jacchia, and Whipple, 1954, 1956, 1957). Their value of the average deviation $|\overline{O-C}|$ of an extended trail from the least-squares moving radiant appears in table 1 for nine streams. They have not determined a value of $|\overline{O-C}|$ for Quadrantids or Andromedids. Figure 2 shows the correlations with $D_{S\ \min}$ and the average $D(M,N)$. There is a well-known correlation

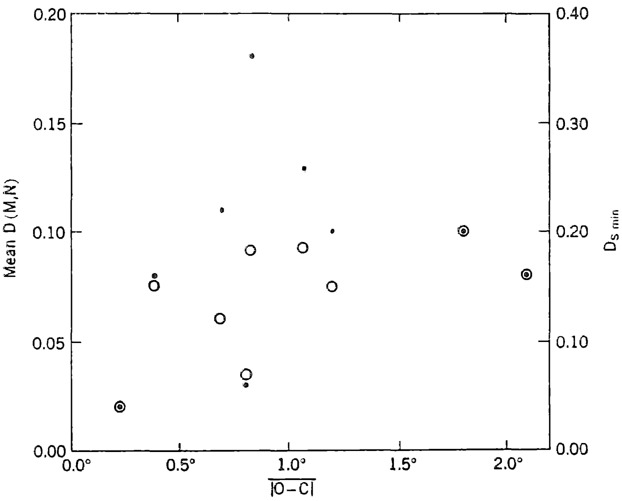

FIGURE 2.—Correlations of mean $D(M,N)$ (solid dots) and of $D_{S\ \min}$ (open circles) with Wright, Jacchia, and Whipple's average radiant deviations $|\overline{O-C}|$, for nine streams.

between radiant spread and shower duration (Wright, Jacchia, and Whipple, 1957), and a correlation of either $D_{S\ \min}$ or mean $D(M,N)$ with shower duration is easily shown. Thus all these measures of meteor orbit difference within a stream are intercorrelated.

It will be seen in table 1 that $D(M,N)$ exceeds 0.20 for only four meteors, one Northern Taurid and three Southern Taurid-Arietids. Thus we may use the condition $D(M,N) \leq D_M$, where $D_M = 0.20$, as an empirical test for membership in these known streams. Similarly, no value of $D_{S\ \min}$ exceeds 0.20, and we may use $D_S = 0.20$ as an empirical test. For all but four streams in table 1, either test gives the same results as conventional criteria, including all meteors previously selected as members and excluding all others. The Southern ι-Aquarid stream has presented difficulties to conventional classification (Wright, Jacchia, and Whipple, 1957); the D_S test includes two additional meteors, one of which had been considered a member but was subsequently rejected.

The Andromedid stream has recently been shown to be much more extensive than was believed when the members were identified for table 1 (Hawkins, Southworth, and Stienon, 1959) and both tests add several additional members. The D_S test adds one new Northern Taurid and one new Southern Taurid-Arietid, and excludes no previous members. The D_M test includes these two additional meteors and excludes one Northern Taurid and three Southern Taurid-Arietids. Almost all of the additional and rejected meteors are close to the dividing lines of the tests.

We conclude that D provides valid tests for membership in a stream known to exist. The essential value of these tests is that they have the same formulation for every stream and do not depend on prior study of it. Thus they may be used to look for new streams.

The numerical values $D_M = D_S = 0.20$, which were empirically determined from this sample of meteors, will require re-evaluation if applied to a different sample, particularly to a much larger sample. Certainly D_S should be decreased in a larger sample. If, as above, we use a four-dimensional point distribution as a model of the distribution of meteor orbits, D_S should vary inversely as the fourth root of the sample size in random samples.

Search for new streams

We next applied the D_S test to our entire sample by computing every value of $D(A,B) \leq 0.20$ for any pair of meteors in the sample, as follows. There already existed a card for each meteor, on which its orbital elements had been written. These cards were sorted and then compared in pairs. $D(A,B)$ was computed for every pair unless it could be seen by inspection that D exceeded 0.20. Because members of some known streams had already been treated they were not included in this process, but special pains were taken to see that any association existing between these stream members and other meteors was not overlooked.

The D_S test found 25 groups of meteors, which we may therefore consider as possible streams.

As will be discussed below, we define the "major" streams as those whose mean photographic rates exceed three per hour. Table 2 includes all others, termed "minor" streams, including the 25 newly-found possible streams. For the minor streams previously known it includes all members as defined by the D_S test. Three known streams, each having one member in our sample, are included for completeness; they are not considered further in this study.

One of the 25 possible streams has been called the "Cyclids" because of its nearly circular orbit. (This stream includes two meteors that were assigned the earth's orbit because their observed velocity falls short of the velocity of escape.) Each of the other streams is identified for convenience by a name formed in the traditional manner from a star near the radiant. Mean values of the orbital elements, the corrected radiant (equinox 1950.0), and the extra-atmospheric velocity are tabulated for each stream. Several of the streams near the ecliptic have members whose node differs by 180° from the rest. To form mean elements of streams, we have added 180° to the node of such meteors, and interpreted them as having negative inclinations. Individual values of the same quantities as for the streams, of the date of occurrence, and of $D(M,N)$ are tabulated for each member. One readily sees that the D_M test (i.e., $D \leq 0.20$) is also fulfilled for the great majority of the meteors in table 2.

Several of the possible streams in table 2 are discussed below, after a general discussion of the statistical significance of these streams.

Probability of chance association

To evaluate the importance of the "possible" streams listed in table 2, we clearly must know the probability that such groups would occur by chance. It is not proper to compare the observed groups with the groups that would be found in a random sample of orbits because, quite apart from any grouping, the observed distribution is far from random. Furthermore, we know the distribution of meteor orbits very imperfectly, with or without selection effects. In order to estimate the degree of chance grouping, therefore, we have taken the observed distribution and smoothed out its grouping as far as possible, and then looked for chance groups.

Each meteor is described by the elements q, e, ω, i, and Ω. The smoothing process consisted in removing the values of Ω from all the meteors not originally classified in known streams, mixing them up, and reassigning them at random; and in doing the same with the values of i. This substantially effaces the grouping. To offset any residual grouping, this process also eliminated the weak correlations between Ω or i and other elements. Any further randomizing is hardly feasible since there is an almost exact relation among q, e, and ω. The resulting artificial orbits in the range $0° \leq \Omega \leq 100°$ were then treated in the same way as the observed orbits. In this way, although the sample size was reduced, we would expect the same percentage of "stream" meteors if table 2 were a result of chance. Table 3 shows the groups found.

The artificial sample may be compared with the observed meteors not originally classified in known streams, as follows:

	Observed sample	Artificial sample
Meteors	290	103
New groups found	24	8
Known streams enlarged	4	
Meteors classified into groups:		
Number	119	22
Percent of sample	41	21

Since the random sample has produced 21 percent of "stream meteors," we conclude that approximately one half of the groups found by the D_S test are chance associations. Approximately 14 groups or 62 meteors in table 2 then are included by chance; the remaining streams and meteors are genuinely associated.

We do not know which groups in table 2 are spurious, but it is possible to develop criteria for the probability that a group is a genuine meteor stream.

It is clear that the chance associations in table 2 are principally the result of the non-random overall distribution of orbits. The chance associations occur almost entirely in the regions of the phase space most densely populated with orbits. We have stated earlier that the value of D_S should be adjusted to the total number in a sample. A more refined test should take account of crowding in part of a sample, but the present data are insufficient to warrant refinements. One result of the non-random distribution is obvious; the "Cyclid" stream, in all probability, does not exist as a physical entity but only represents the peak of the distribution of probability of meeting the earth. Another result of crowding will be the association into larger groups of streams which are genuinely distinct. The "σ Leonids" are certainly composite; possibly an improved test can separate this group into independent parts.

Hourly rates of streams; major and minor streams

For each stream, we have computed the average number of photographic meteors per hour that are members of that stream, as follows. The number of meteors photographed is taken as ten times the number in the sample, because the sample originally consisted of every tenth meteor photographed (Hawkins and Southworth, 1958). If there were two meteors from one stream on a single plate because additional meteors on the same plates as "tenth" meteors had been reduced, the additional meteor was not counted for this purpose. We define the period of visibility of each stream by the earliest and latest observed dates in the sample, and by the rising and setting of the mean radiant. The total number of hours of double-station observing time within these limits, and within the overall limits of the sample (26 February, 1952, to 3 July, 1954) was found from the records. The hourly rate is then the number of meteors photographed divided by the number of hours observed. Table 4 lists these hourly rates, and also the average $D(M,N)$ within each stream. A meaningful hourly rate for the "Cyclid" stream cannot be computed. The other columns of table 4 are discussed further on. Figure 3 is a plot of mean hourly rate versus mean $D(M,N)$.

Three streams—the Geminids, Orionids, and Quadrantids—are well separated from all others in figure 3 by their high hourly rate. Their internal dispersions are smaller than those of almost all the other streams. We shall call these the major streams, and treat them as a class apart.

The position of the Perseids in figure 3 is anomalous in view of the known importance of this shower. On closer examination, we find that the shower maximum in fact is hardly represented in our sample, observations having been prevented in 1952 by moonlight and almost entirely prevented in 1953 by clouds.

TABLE 2.—*Minor streams*

(Asterisk represents newly found members of known streams)

Stream	Greenwich date	a (a.u.)	e	i (deg)	☊ (deg)	π (deg)	Corr. R.A. (deg)	Rad. dec. (deg)	V_∞ (km/sec)	$D(M,N)$
KNOWN STREAMS										
South ι Aquarids	52 June 29.43208	1. 50	0. 852	7. 4	97. 6	51. 8	299. 3	−16. 1	32. 1	0. 38
	53 July 21.35918	1. 37	0. 921	2. 9	298. 2	87. 9	328. 7	−13. 9	35. 9	0. 24
	*53 Aug 6.17192	1. 71	0. 827	1. 9	313. 4	77. 8	331. 9	−13. 1	30. 9	0. 11
	53 Aug 8.19278	3. 32	0. 927	7. 5	315. 3	80. 8	335. 1	−15. 1	36. 1	0. 16
	Mean values	*1. 98*	*0. 882*	*1. 2*	*301. 1*	*74. 6*	*323. 8*	*−14. 6*	*33. 8*	*0. 22*
South δ Aquarids	52 July 25.44425	2. 85	0. 978	27. 3	302. 4	96. 2	335. 9	−17. 1	43. 0	0. 05
	52 July 28.31172	2. 36	0. 973	30. 0	305. 1	99. 3	339. 6	−16. 5	43. 7	0. 03
	52 July 29.40913	2. 40	0. 970	29. 4	306. 2	98. 8	340. 2	−16. 6	42. 2	0. 02
	Mean values	*2. 54*	*0. 974*	*28. 9*	*304. 6*	*98. 1*	*338. 6*	*−16. 7*	*42. 6*	*0. 03*
North δ Aquarids	52 Aug 25.35970	2. 38	0. 947	17. 0	152. 0	114. 9	354. 3	+4. 7	39. 0	0. 15
	53 Aug 10.43442	2. 41	0. 960	22. 5	137. 4	105. 2	342. 8	+0. 5	40. 5	0. 06
	53 Aug 13.19186	2. 50	0. 949	24. 1	140. 1	102. 8	343. 0	+2. 5	40. 0	0. 09
	Mean values	*2. 43*	*0. 952*	*21. 2*	*143. 2*	*107. 6*	*346. 7*	*+2. 6*	*39. 8*	*0. 10*
α Capricornids	52 Aug 22.20417	3. 19	0. 793	8. 3	149. 0	46. 6	321. 2	−2. 2	24. 3	0. 08
	53 Aug 8.36042	2. 65	0. 753	7. 7	135. 4	35. 5	309. 8	−6. 3	23. 5	0. 08
	Mean values	*2. 92*	*0. 773*	*8. 0*	*142. 2*	*41. 0*	*315. 6*	*−4. 2*	*23. 9*	*0. 08*
S. Taurid-Arietids	52 Sept 17.31692	2. 47	0. 894	4. 5	354. 3	118. 7	10. 6	+1. 4	33. 8	0. 28
	52 Sept 19.37056	1. 17	0. 736	6. 1	356. 3	125. 7	16. 8	+1. 2	26. 5	0. 17
	52 Sept 20.28367	1. 37	0. 741	3. 0	357. 2	118. 4	12. 5	+2. 2	29. 9	0. 26
	52 Sept 25.36076	1. 52	0. 846	7. 9	2. 2	134. 5	22. 8	+4. 0	31. 8	0. 10
	52 Oct 13.28086	1. 85	0. 840	5. 9	19. 8	142. 6	34. 8	+9. 2	31. 4	0. 10
	52 Oct 14.35000	1. 47	0. 844	6. 2	20. 9	154. 1	40. 6	+11. 5	31. 8	0. 27
	52 Oct 19.26487	1. 95	0. 806	4. 3	25. 8	138. 6	36. 0	+10. 1	29. 2	0. 08
	52 Oct 22.47095	2. 76	0. 829	5. 3	29. 0	128. 1	33. 6	+7. 4	27. 9	0. 12
	52 Oct 27.35625	1. 79	0. 869	9. 9	33. 8	163. 8	52. 5	+12. 5	33. 3	0. 42
	*53 Oct 9.24410	1. 73	0. 791	5. 4	195. 6	131. 9	24. 7	+15. 6	28. 8	0. 20
	53 Oct 9.24725	1. 45	0. 813	7. 5	15. 6	144. 1	34. 1	+7. 7	30. 4	0. 12
	Mean values	*1. 78*	*0. 819*	*5. 0*	*13. 7*	*136. 4*	*29. 0*	*+7. 5*	*30. 4*	*0. 19*
North Taurids	52 Oct 16.33975	2. 26	0. 912	3. 4	202. 9	155. 6	39. 7	+17. 4	35. 7	0. 12
	52 Oct 19.38343	1. 44	0. 782	5. 0	205. 9	149. 9	38. 8	+19. 6	28. 6	0. 12
	52 Oct 27.35799	2. 40	0. 856	3. 0	213. 8	148. 4	42. 6	+18. 9	31. 1	0. 13
	52 Nov 12.19596	2. 25	0. 839	3. 1	229. 7	162. 8	58. 4	+23. 0	30. 7	0. 10
	52 Nov 12.37045	2. 35	0. 836	2. 3	229. 9	159. 7	57. 1	+22. 1	29. 8	0. 10
	52 Nov 12.37333	1. 98	0. 855	5. 3	229. 9	172. 5	53. 0	+25. 1	32. 0	0. 23
	*53 Oct 10.21343	2. 18	0. 929	4. 7	196. 6	155. 5	36. 0	+16. 6	37. 4	0. 17
	53 Nov 3.29681	1. 97	0. 800	3. 6	220. 5	151. 4	47. 6	+21. 3	28. 8	0. 13
	Mean values	*2. 10*	*0. 851*	*3. 8*	*216. 1*	*157. 0*	*47. 9*	*+20. 5*	*31. 9*	*0. 14*

TABLE 2.—*Minor streams*—Continued

Stream	Greenwich date	a (a.u.)	e	i (deg)	☊ (deg)	π (deg)	Corr. R.A. (deg)	Rad. dec. (deg)	V_∞ (km/sec)	$D(M,N)$
	KNOWN STREAMS—Continued									
Andromedids (Bielids)	*52 Sept 20.28438	2. 80	0. 764	0. 8	357. 2	75. 0	351. 8	−4. 9	23. 0	0. 33
	*52 Sept 25.36001	2. 80	0. 775	4. 7	182. 2	83. 6	355. 2	+5. 5	23. 8	0. 25
	*52 Oct 9.19792	2. 50	0. 773	2. 1	195. 8	105. 2	13. 3	+8. 7	25. 2	0. 16
	*52 Oct 13.27778	2. 38	0. 787	8. 1	199. 8	116. 6	18. 5	+18. 1	26. 8	0. 32
	*52 Oct 21.32940	1. 75	0. 558	8. 0	27. 8	95. 4	22. 6	−10. 4	18. 1	0. 22
	*52 Oct 24.35000	2. 48	0. 707	4. 1	30. 8	100. 4	21. 4	+0. 6	20. 7	0. 07
	52 Nov 7.10870	2. 59	0. 708	8. 4	224. 6	108. 9	22. 9	+27. 6	20. 9	0. 20
	*52 Dec 9.26872	2. 68	0. 666	3. 5	77. 1	116. 8	44. 8	+6. 0	16. 5	0. 29
	*53 Oct 2.31090	2. 54	0. 773	3. 7	188. 8	97. 0	5. 4	+7. 7	24. 8	0. 16
	*53 Oct 6.28447	1. 81	0. 703	3. 6	12. 7	109. 6	16. 9	+2. 0	24. 1	0. 20
	*53 Oct 7.39307	1. 92	0. 649	1. 9	13. 8	93. 9	9. 3	+0. 3	20. 6	0. 07
	*53 Oct 10.31215	2. 39	0. 700	7. 5	16. 6	88. 3	12. 3	−9. 6	21. 2	0. 16
	53 Nov 7.46139	2. 29	0. 679	0. 0	224. 7	113. 5	31. 1	+12. 7	19. 8	0. 17
	*53 Nov 13.24583	2. 02	0. 577	3. 8	50. 5	102. 0	29. 5	+0. 6	16. 6	0. 25
	Mean values	*2. 35*	*0. 701*	*0. 4*	*25. 9*	*100. 4*	*18. 6*	*+4. 6*	*21. 6*	*0. 20*
	KNOWN STREAMS HAVING ONE MEMBER IN OUR SAMPLE									
North ι Aquarids	52 July 27.22705	3. 48	0. 947	15. 0	124. 1	76. 9	323. 8	−6. 8	38. 5	—
Draconids (Giacobinids)	53 Oct 9.19222	3. 37	0. 704	24. 6	195. 5	12. 5	270. 9	+47. 2	20. 2	—
Monocerotids	53 Dec 10.51604	20. 16	0. 992	39. 8	78. 1	209. 4	101. 6	+7. 7	43. 9	—
	NEWLY FOUND POSSIBLE STREAMS									
χ Geminids	53 Jan 13. 29493	2. 01	0. 739	1. 0	292. 8	208. 3	118. 4	+22. 1	25. 6	0. 11
	53 Jan 13. 45089	2. 58	0. 779	17. 2	292. 9	200. 6	123. 0	+41. 8	27. 0	0. 19
	53 Jan 13. 45411	2. 52	0. 798	11. 3	292. 9	208. 1	122. 4	+33. 2	27. 6	0. 13
	53 Jan 23. 38956	2. 41	0. 725	4. 2	123. 1	200. 0	117. 9	+14. 0	22. 7	0. 20
	Mean values	*2. 38.*	*0. 760*	*6. 3*	*295. 4*	*204. 2*	*120. 4*	*+27. 8*	*25. 7*	*0. 16*
ρ Geminids	53 Jan 15.40959	2. 41	0. 716	7. 4	294. 9	189. 0	111. 6	+34. 6	22. 3	0. 11
	53 Jan 19.19278	2. 55	0. 694	4. 4	298. 8	179. 5	105. 5	+31. 9	20. 1	0. 06
	53 Jan 21.37852	2. 81	0. 723	1. 1	301. 0	180. 6	105. 9	+24. 9	20. 0	0. 08
	54 Jan 28.17159	3. 34	0. 762	7. 9	307. 6	184. 1	114. 6	+37. 8	21. 0	0. 07
	Mean values	*2. 78*	*0. 724*	*5. 2*	*300. 6*	*183. 3*	*109. 4*	*+32. 3*	*20. 8*	*0. 08*

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TABLE 2.—*Minor streams*—Continued

Stream	Greenwich date	a (a.u.)	e	i (deg)	☊ (deg)	π (deg)	Corr. R.A. (deg)	Rad. dec. (deg)	V_∞ (km/sec)	$D(M,N)$
NEWLY FOUND POSSIBLE STREAMS—Continued										
σ Leonids	52 Feb 26.40208	1.65	0.621	8.9	336.7	244.6	164.6	+21.8	22.1	0.17
	52 Apr 26.28750	1.98	0.592	4.6	216.0	278.2	195.3	−17.6	18.5	0.35
	52 May 22.28093	2.04	0.581	2.2	61.1	295.9	219.6	− 9.7	17.3	0.51
	53 Feb 5.14883	2.71	0.858	0.8	136.0	244.8	147.2	+12.4	31.7	0.39
	53 Feb 17.20441	2.49	0.758	2.4	148.2	232.5	147.0	+ 9.7	25.0	0.28
	53 Feb 17.20626	1.89	0.720	14.5	328.2	244.6	162.9	+26.2	26.6	0.30
	53 Feb 18.35000	1.95	0.701	4.8	329.4	239.3	154.6	+17.7	24.1	0.20
	53 Mar 12.17613	1.53	0.456	9.6	171.3	232.3	150.4	−15.2	17.1	0.37
	53 Mar 12.27841	2.53	0.671	0.5	351.4	225.2	153.2	+12.2	18.7	0.32
	53 Mar 13.29792	1.55	0.414	5.6	172.4	217.9	143.0	− 6.6	14.6	0.47
	53 Mar 14.34103	1.68	0.596	4.4	173.4	255.1	167.6	− 3.1	20.6	0.15
	53 Mar 18.33892	1.35	0.492	2.3	177.4	264.3	174.7	− 2.8	18.6	0.24
	53 Mar 19.31853	2.91	0.731	11.5	358.4	238.8	174.8	+26.3	21.6	0.24
	53 Mar 19.39518	8.49	0.913	9.5	358.4	241.5	172.2	+20.0	24.7	0.30
	53 Mar 20.42303	2.14	0.625	7.5	179.5	239.8	159.8	− 9.0	19.0	0.23
	53 Mar 21.40331	2.71	0.710	4.3	0.4	240.8	169.6	+14.2	20.2	0.15
	53 Apr 9.29081	2.77	0.688	8.9	19.1	247.6	184.8	+20.9	19.0	0.20
	53 Apr 11.22535	2.58	0.682	4.9	21.0	257.4	186.3	+ 9.0	19.6	0.14
	53 Apr 11.34984	2.30	0.694	4.2	21.1	275.4	194.9	+ 1.4	21.9	0.29
	53 Apr 14.28612	2.01	0.605	2.6	24.0	267.3	191.0	+ 1.4	18.8	0.21
	53 Apr 15.24583	2.92	0.708	2.4	204.9	255.3	178.9	− 5.7	18.8	0.17
	53 May 6.28495	2.15	0.671	0.5	225.4	300.6	214.7	−14.8	21.4	0.56
	53 May 7.22324	2.37	0.674	1.0	46.3	291.7	211.0	−10.6	20.4	0.47
	54 Feb 6.38914	2.63	0.810	16.6	317.0	233.0	152.0	+30.5	29.0	0.43
	54 Feb 6.42292	2.45	0.807	2.6	317.0	236.4	145.4	+16.8	28.1	0.34
	54 Feb 26.30579	1.91	0.660	4.6	157.1	239.5	153.2	+ 3.2	22.3	0.19
	54 Mar 1.36968	2.01	0.639	7.3	340.2	231.9	159.3	+23.5	20.7	0.24
	Mean values	*2.43*	*0.670*	*2.2*	*359.0*	*250.8*	*171.4*	*6.4*	*21.5*	*0.29*
ξ Orionids	53 Mar 14.16204	1.94	0.488	4.2	173.2	169.1	80.6	+ 0.2	12.9	0.09
	54 Feb 11.42659	1.80	0.479	3.4	322.1	173.9	106.4	+36.5	13.9	0.09
	54 Mar 5.29565	1.84	0.472	1.0	164.1	185.6	113.6	+12.6	13.2	0.08
	Mean values	*1.86*	*0.480*	*0.9*	*159.8*	*176.2*	*100.2*	*+16.4*	*13.3*	*0.09*
ψ Ursa Majorids	53 Mar 18.33889	1.80	0.497	6.9	357.4	220.9	161.8	+31.8	15.6	0.08
	53 Apr 10.24583	1.63	0.398	12.0	20.0	222.5	185.4	+53.7	14.6	0.09
	Mean values	*1.72*	*0.448*	*9.4*	*8.7*	*221.7*	*173.6*	*+42.8*	*15.1*	*0.08*
α Bootids	53 Apr 9.35927	1.90	0.599	21.0	19.2	268.3	209.8	+27.6	22.4	0.10
	53 Apr 16.28881	2.13	0.674	16.0	26.0	282.3	210.4	+14.8	23.4	0.10
	Mean values	*2.02*	*0.636*	*18.5*	*22.6*	*275.3*	*210.1*	*+21.2*	*22.9*	*0.10*
γ Bootids	53 Apr 13.25625	3.76	0.786	26.6	23.0	259.5	213.0	+33.4	25.9	0.08
	53 Apr 21.36280	3.34	0.733	29.1	30.9	253.3	222.3	+42.7	24.5	0.08
	Mean values	*3.55*	*0.760*	*27.8*	*27.0*	*256.4*	*217.6*	*+38.0*	*25.2*	*0.08*
ξ Librids	53 Apr 13.25916	1.71	0.914	2.8	23.0	344.9	224.4	−15.6	37.0	0.09
	53 May 5.28417	2.95	0.925	3.2	224.4	353.3	239.1	−22.2	36.8	0.09
	Mean values	*2.33*	*0.920*	*0.2*	*213.7*	*349.0*	*231.8*	*−18.9*	*36.9*	*0.09*

TABLE 2.—*Minor streams*—Continued

Stream	Greenwich date	a (a.u.)	e	i (deg)	☊ (deg)	π (deg)	Corr. R.A. (deg)	Rad. dec. (deg)	V_∞ (km/sec)	$D(M,N)$
	NEWLY FOUND POSSIBLE STREAMS—Continued									
μ Draconids	53 Apr 16.42532	4. 32	0. 770	43. 6	26. 1	218. 3	256. 6	+59. 8	29. 6	0. 09
	53 Apr 21.36042	15. 72	0. 936	47. 1	30. 9	217. 9	263. 4	+61. 0	32. 4	0. 09
	Mean values	*10. 02*	*0. 853*	*45. 4*	*28. 5*	*218. 1*	*260. 0*	*+60. 4*	*31. 0*	*0. 09*
φ Bootids	53 May 7.33958	1. 50	0. 379	19. 9	46. 4	270. 3	231. 6	+40. 4	17. 6	0. 06
	53 May 8.38998	1. 39	0. 321	17. 9	47. 4	270. 1	231. 8	+41. 5	16. 2	0. 08
	54 June 4.24583	1. 70	0. 417	22. 7	73. 0	275. 0	245. 0	+50. 5	18. 2	0. 13
	Mean values	*1. 53*	*0. 372*	*20. 2*	*55. 6*	*271. 8*	*236. 1*	*+44. 1*	*17. 3*	*0. 09*
ξ Cepheids	53 May 9.42072	1. 00	0. 157	16. 4	48. 4	127. 4	341. 8	+63. 9	14. 7	0. 10
	54 Apr 28.30873	1. 15	0. 172	22. 1	37. 4	164. 7	313. 8	+68. 3	16. 6	0. 10
	Mean values	*1. 08*	*0. 164*	*19. 2*	*42. 9*	*146. 0*	*327. 8*	*+66. 1*	*15. 6*	*0. 10*
ω Ursa Majorids	53 May 13.16348	2. 04	0. 512	2. 8	52. 0	248. 3	178. 0	+15. 1	13. 6	0. 16
	53 June 5.17987	2. 48	0. 596	9. 4	74. 2	240. 5	161. 0	+49. 0	14. 8	0. 03
	53 June 6.19792	3. 55	0. 725	13. 3	75. 1	230. 6	143. 5	+57. 9	17. 2	0. 18
	Mean values	*2. 69*	*0. 611*	*8. 5*	*67. 1*	*239. 8*	*160. 8*	*+40. 7*	*15. 1*	*0. 12*
Cylids	52 May 19.21518	1. 03	0. 131	0. 8	58. 1	148. 7	57. 5	+27. 2	11. 5	0. 06
	52 Oct 19.44228	0. 95	0. 120	2. 7	26. 0	144. 7	50. 2	−7. 4	11. 4	0. 10
	53 Apr 11.14238	1. 00	0. 017	0. 0	—	101. 2	—	—	10. 6	0. 10
	53 Apr 15.24742	1. 00	0. 017	0. 0	—	101. 2	—	—	11. 0	0. 10
	54 June 3.25929	1. 08	0. 148	6. 2	72. 1	179. 9	76. 9	+62. 8	12. 2	0. 14
	Mean values	*1. 01*	*0. 087*	*1. 9*	*52. 1*	*135. 1*	*60. 5*	*+27. 5*	*11. 3*	*0. 10*
θ Ophiucids	52 May 21.36274	2. 62	0. 915	4. 5	240. 2	9. 4	256. 3	−25. 4	36. 0	0. 19
	53 June 4.20432	2. 34	0. 838	3. 0	253. 2	5. 2	262. 4	−25. 9	30. 8	0. 03
	53 June 8.37832	3. 42	0. 893	3. 1	257. 2	8. 2	266. 3	−25. 8	32. 4	0. 06
	53 June 16.30529	3. 11	0. 827	4. 5	264. 8	357. 0	265. 0	−28. 6	27. 4	0. 19
	53 June 16.31098	2. 86	0. 837	4. 9	264. 8	5. 7	269. 7	−28. 4	29. 0	0. 08
	Mean values	*2. 87*	*0. 862*	*4. 0*	*256. 0*	*5. 1*	*264. 0*	*−26. 8*	*31. 1*	*0. 11*
μ Draconids	52 June 17.36898	2. 83	0. 641	26. 7	86. 1	269. 4	238. 4	+60. 3	20. 6	0. 18
	53 June 8.28750	3. 28	0. 720	31. 8	77. 1	296. 2	255. 5	+37. 9	25. 0	0. 24
	53 June 13.26806	3. 70	0. 736	31. 3	81. 9	286. 4	253. 1	+46. 8	24. 1	0. 12
	54 June 9.37083	2. 81	0. 644	39. 6	77. 9	273. 5	267. 7	+56. 6	26. 7	0. 13
	54 June 11.38442	3. 27	0. 697	33. 6	79. 8	280. 1	255. 9	+51. 7	24. 4	0. 04
	54 June 23.21042	2. 74	0. 629	38. 2	91. 1	270. 1	267. 8	+66. 9	25. 9	0. 18
	Mean values	*3. 10*	*0. 678*	*33. 5*	*82. 3*	*279. 3*	*256. 4*	*+53. 4*	*24. 4*	*0. 15*
τ Herculids	53 June 20. 35000	2. 07	0. 521	28. 9	88. 7	291. 0	258. 6	+50. 4	21. 2	0. 10
	54 June 25. 24461	2. 81	0. 642	21. 3	93. 1	286. 2	237. 8	+46. 4	18. 8	0. 10
	Mean Values	*2. 44*	*0. 582*	*25. 1*	*90. 9*	*288. 6*	*248. 2*	*+48. 4*	*20. 0*	*0. 10*
μ Ophiucids	53 Aug 6. 16985	2. 53	0. 624	1. 2	313. 4	347. 0	272. 3	−27. 4	15. 5	0. 12
	53 Aug 7. 39845	2. 06	0. 515	7. 7	134. 5	330. 7	254. 0	+11. 6	13. 7	0. 13
	53 Aug 10. 22697	2. 36	0. 587	4. 3	137. 2	343. 1	266. 9	−6. 6	14. 5	0. 10
	Mean values	*2. 32*	*0. 575*	*3. 6*	*135. 0*	*340. 3*	*264. 4*	*−7. 5*	*14. 6*	*0. 10*

TABLE 2.—*Minor streams*—Continued

Stream	Greenwich date	a (a.u.)	e	i (deg)	☊ (deg)	π (deg)	Corr. R.A. (deg)	Rad dec. (deg)	V_∞ (km/sec)	$D(M,N)$
NEWLY FOUND POSSIBLE STREAMS—Continued										
ϑ Herculids	53 Aug 9.23255	2.59	0.611	18.6	136.3	327.5	257.1	+38.8	17.4	0.04
	53 Aug 13.42525	2.50	0.617	17.3	140.3	352.1	280.1	+26.6	17.7	0.13
	53 Aug 15.42275	2.06	0.518	18.7	142.2	341.2	270.2	+38.8	16.9	0.06
	Mean values	*2.38*	*0.582*	*18.2*	*139.6*	*340.3*	*269.1*	*+34.7*	*17.2*	*0.08*
β Cetids	52 Sept 14.37389	1.93	0.720	12.0	351.4	87.4	2.5	−15.7	25.0	0.09
	52 Sept 20.29047	1.40	0.592	15.5	357.2	96.7	14.0	−18.8	22.5	0.09
	Mean values	*1.66*	*0.656*	*13.8*	*354.3*	*92.0*	*8.2*	*−17.2*	*23.8*	*0.09*
Capri-cornids	52 Sept 16.33958	1.57	0.422	1.0	173.4	41.6	327.2	−9.0	13.9	0.13
	52 Oct 9.19742	1.46	0.314	8.9	195.8	17.6	278.6	+30.2	12.7	0.10
	53 Aug 13.35836	1.37	0.348	0.9	320.2	17.1	302.4	−23.7	13.6	0.14
	Mean values	*1.47*	*0.361*	*3.0*	*169.8*	*25.4*	*302.7*	*−0.8*	*13.5*	*0.12*
α Pegasids	52 Sept 19.23462	2.70	0.713	0.7	176.2	59.4	341.4	−6.3	20.0	0.20
	52 Sept 27.30972	1.79	0.599	10.9	184.1	79.8	349.8	+18.5	20.0	0.22
	52 Oct 16.21458	2.61	0.670	2.6	22.8	71.3	359.5	−7.7	17.5	0.13
	52 Nov 12.19033	3.41	0.717	7.6	229.7	70.1	342.2	+22.1	15.6	0.14
	52 Nov 12.19349	2.46	0.605	6.9	229.7	68.7	339.8	+22.4	14.4	0.12
	52 Nov 12.26391	3.42	0.719	7.4	229.8	70.6	343.0	+21.9	15.5	0.14
	53 Oct 3.25367	1.46	0.432	11.5	189.7	74.5	345.0	+28.5	16.5	0.24
	53 Oct 3.29383	1.73	0.501	5.4	189.7	64.0	343.1	+10.4	15.7	0.15
	53 Nov 6.33041	3.06	0.704	7.0	43.6	81.2	15.8	−15.5	16.8	0.25
	Mean values	*2.52*	*0.629*	*4.5*	*206.1*	*71.1*	*348.8*	*+10.5*	*16.9*	*0.18*
λ Draconids	52 Oct 12.33141	5.49	0.818	72.1	198.9	18.4	181.9	+81.4	43.5	0.10
	52 Oct 19.44602	3.14	0.683	77.4	206.0	24.3	167.3	+76.0	45.0	0.10
	Mean values	*4.32*	*0.750*	*74.8*	*202.4*	*21.4*	*174.6*	*+78.7*	*44.2*	*0.10*
ϵ Geminids	52 Oct 17.40034	10.37	0.919	173.8	203.9	71.4	102.4	+26.3	70.6	0.11
	52 Oct 22.26372	26.77	0.971	173.0	208.8	85.6	104.2	+26.5	70.6	0.14
	53 Oct 16.42579	22.87	0.965	173.3	202.7	76.3	98.6	+26.8	70.5	0.03
	Mean values	*20.00*	*0.952*	*173.4*	*205.1*	*77.8*	*101.7*	*+26.5*	*70.6*	*0.09*
η Taurids	52 Dec 9.26560	2.60	0.682	2.1	257.1	130.3	51.0	+24.0	18.3	0.10
	53 Jan 15.21244	3.98	0.763	0.7	114.7	140.7	65.3	+19.3	16.0	0.10
	Mean values	*3.29*	*0.722*	*0.7*	*275.9*	*135.5*	*58.2*	*+21.6*	*17.2*	*0.10*

TABLE 3.—*Spurious streams found in the artificial sample*

Stream	Greenwich Date	a (a.u.)	e	i (deg)	Ω (deg)	π (deg)	Original Classification	$D(M,N)$
(A)	52 Mar 21. 36111	3. 23	0. 713	4. 3	43. 6	257. 2	Sporadic	0. 13
	52 Sept 25. 17361	1. 79	0. 510	11. 5	25. 1	256. 4	Sporadic	0. 14
	53 Aug 13. 42525	2. 50	0. 617	4. 8	46. 4	258. 2	ϑ Herculid	0. 06
	53 Aug 15. 42275	2. 06	0. 518	4. 6	47. 4	246. 4	Sporadic	0. 12
	Mean values	*2. 40*	*0. 590*	*6. 3*	*40. 6*	*254. 6*		
(B)	52 Apr 2. 38125	2. 80	0. 644	1. 9	21. 1	210. 4	Sporadic	0. 07
	53 May 13. 16348	2. 04	0. 512	4. 9	0. 6	196. 8	ω Ursa Majorid	0. 13
	54 June 25. 24461	2. 81	0. 642	1. 2	16. 6	209. 7	τ Herculid	0. 07
	Mean values	*2. 55*	*0. 599*	*2. 7*	*12. 8*	*205. 6*		
(C)	52 June 24. 20301	2. 79	0. 720	8. 6	39. 6	283. 2	Sporadic	0. 15
	52 Sept 16. 33958	1. 57	0. 422	4. 4	48. 4	276. 7	ξ Capricornid	0. 20
	52 Sept 19. 23462	2. 70	0. 713	7. 6	30. 0	273. 2	α Pegasid	0. 14
	53 Oct 3. 29383	1. 73	0. 501	4. 2	46. 3	280. 5	α Pegasid	0. 12
	54 Mar 1. 36968	2. 01	0. 639	4. 8	25. 1	276. 8	σ Leonid	0. 10
	Mean values	*2. 16*	*0. 599*	*5. 9*	*37. 9*	*278. 1*		
(D)	52 Sept 16. 33723	3. 22	0. 937	8. 9	63. 1	13. 6	Sporadic	0. 07
	53 Apr 13. 25916	1. 71	0. 914	3. 9	46. 5	8. 4	ζ Librid	0. 07
	Mean values	*2. 96*	*0. 926*	*6. 4*	*54. 8*	*11. 0*		
(E)	52 Sept 25. 17579	15. 94	0. 947	7. 0	48. 4	275. 9	Sporadic	0. 08
	53 Nov 2. 35675	6. 87	0. 872	13. 3	51. 2	271. 7	Sporadic	0. 07
	Mean values	*11. 41*	*0. 910*	*10. 2*	*49. 8*	*273. 8*		
(F)	52 Aug 18. 25543	20. 92	0. 952	0. 2	24. 0	207. 4	Sporadic	0. 08
	53 Oct 9. 31647	19. 45	0. 949	9. 7	30. 9	207. 9	Sporadic	0. 08
	Mean values	*20. 18*	*0. 950*	*5. 0*	*27. 4*	*207. 6*		
(G)	53 Jan 13. 45411	2. 52	0. 798	1. 9	23. 2	298. 4	χ Geminid	0. 06
	54 Feb 6. 38914	2. 63	0. 810	2. 4	32. 1	308. 1	σ Leonid	0. 07
	Mean values	*2. 58*	*0. 804*	*2. 2*	*27. 6*	*303. 2*		
(H)	53 Apr 7. 28372	1. 98	0. 809	4. 2	53. 0	346. 0	Sporadic	0. 09
	54 Feb 10. 40343	3. 69	0. 871	0. 9	77. 9	354. 2	Sporadic	0. 09
	Mean values	*2. 84*	*0. 840*	*2. 6*	*65. 4*	*350. 1*		

TABLE 4.—*Further data on streams*

Stream	Hourly rate	Number of members	Inclina-tion	Mean $D(M,N)$	Radiant $\lambda-\lambda_\odot$	β
MAJOR STREAMS						
Quadrantids	4.30	5	71°6	0.04	278°	64°
Orionids	3.69	12	164.9	.08	247	−7
Perseids*	*1.67	9	113.8	*.11	283	37
Geminids	5.92	16	23.3	.02	208	10
KNOWN MINOR STREAMS						
S. ι Aquarids	0.46	4	1°2	0.22	201°	−1°
S. δ Aquarids	1.80	3	28.9	.03	210	−7
N. δ Aquarids	0.37	3	21.2	.10	207	7
α Capricornids	0.29	2	8.0	.08	175	12
S. Taurid-Arietids	0.71	11	5.0	.19	197	−5
N. Taurids	0.54	8	3.8	.14	195	2
Andromedids	0.67	14	0.4	.20	172	−1
KNOWN MINOR STREAMS HAVING ONE MEMBER IN OUR SAMPLE						
N. Aquarids	—	1	15°0	—	202°	7°
Draconids	—	1	24.6	—	78	70
Monocerotids	—	1	39.8	—	205	−14
NEWLY FOUND POSSIBLE MINOR STREAMS						
χ Geminids	0.63	4	6°3	0.16	182°	8°
ρ Geminids	0.89	4	5.2	.08	166	10
σ Leonids	0.74	27	2.2	.29	171	2
ξ Orionids	0.43	3	0.9	.09	119	−2
ψ Ursae Majorids	0.20	2	9.4	.08	144	36
α Bootids	0.44	2	18°5	.10	176°	31°
γ Bootids	0.67	2	27.8	.08	171	49
δ Librids	0.24	2	0.2	.09	201	0
ν Draconids	1.24	2	45.4	.09	204	82
φ Bootids**	0.29	3	20.2	.09	159	60
ξ Cepheids	0.32	2	19°2	.10	344°	68°
ω Ursae Majorids	0.38	3	8.5	.12	79	31
Cyclids	—	5	1.9	.10	—	—
ϑ Ophiucids	0.37	5	4.0	.11	189	−4
μ Draconids**	1.31	6	33.5	.15	148	74
τ Herculids	0.84	2	25°1	.10	140°	68°
μ Ophiucids	1.76	3	3.6	.12	128	15
φ Herculids**	0.92	3	18.2	.08	129	58
β Cetids	0.71	2	13.8	.09	186	−20
ξ Capricornids	0.24	3	3.0	.12	134	18
α Pegasids	0.56	9	4°5	.18	148°	15°
λ Draconids	0.57	2	74.8	.10	273	68
ε Geminids**	1.09	3	173.4	.09	256	3
η Taurids	0.15	2	0.7	.10	145	2

TABLE 4.—*Further data on streams*—Continued

Stream	Hourly rate	Number of members	Inclination	Mean $D(M,N)$	Radiant $\lambda-\lambda_\odot$	β
SPURIOUS STREAMS						
(A)	—	4	6°3	0.11	136°	22°
(B)	—	3	2.7	.09	98	13
(C)	—	5	5.9	.14	161	14
(D)	—	2	6.4	.07	202	6
(E)	—	2	10.2	.08	152	22
(F)	—	2	5.0	.08	90	16
(G)	—	2	2.2	.06	183	2
(H)	—	2	2.6	.09	192	2

*Incomplete observations; shower peak missed.
**The most probable streams.

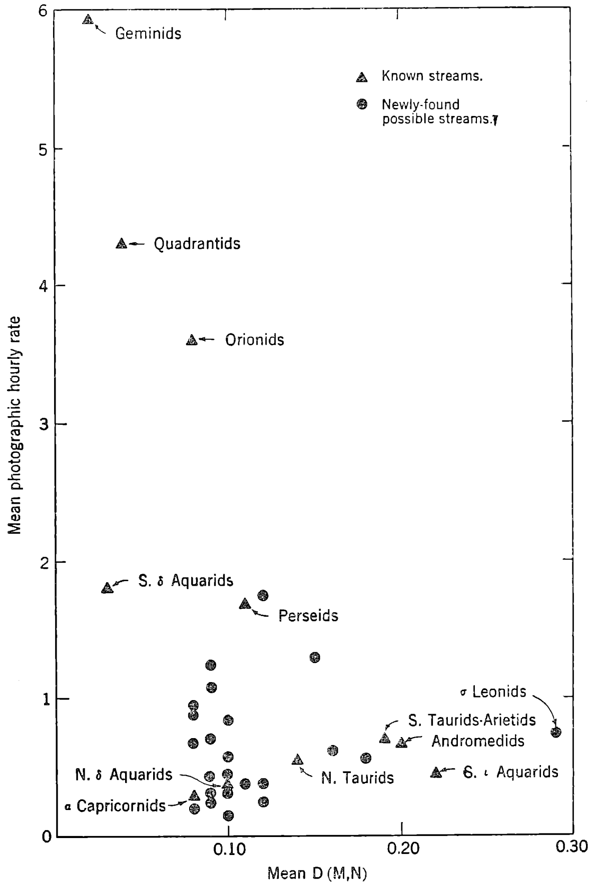

FIGURE 3.—Relation between hourly rate and internal dispersion of streams.

Thus neither our value of the rate nor of the mean dispersion within the stream is representative of the stream. Accordingly, no further use is made of values of D from the Perseid stream.

Streams with a photographic rate less than three per hour we shall call the minor streams. The Southern δ Aquarids are the only minor stream in our sample with a mean $D(M,N)$ less than 0.08. Because the duration of a shower is correlated with its mean $D(M,N)$, the total number of meteors in the sample is correlated with the product of ordinate and abscissa in figure 3; thus a stream in the lower left corner is comparatively unlikely to be detected. Nevertheless, the sharp boundary of the points at $D=0.08$ seems meaningful as an approximate lower bound to the mean dispersion within the majority of minor streams.

On the right edge of figure 3, the "σ Leonids" are doubtless a composite of two or more streams. The Southern Taurid-Arietid stream is known to be composite, as its name indicates. Either of these composite streams, if it could be divided into its originally independent parts, would be represented by two or more points nearer to the majority of streams in the diagram.

Nature of the newly found minor streams

Figure 4 shows histograms of the normalized distribution of values of $D(M,N)$ in each of four classes of streams. The data for major streams and previously known minor streams are drawn from table 1, Perseids being omitted; for newly found minor streams from table 2; and for spurious streams from table 3. For each class of stream the ordinate is the percent

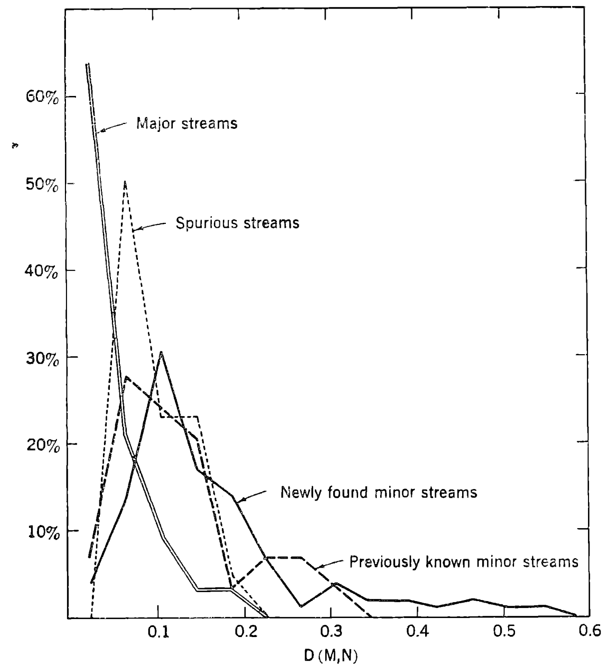

FIGURE 4.—Normalized distribution of $D(M,N)$ in different classes of streams.

of values of D within an interval 0.04 unit wide.

All these curves indicate high central condensation in the streams. D may be interpreted as distance in a four-dimensional phase

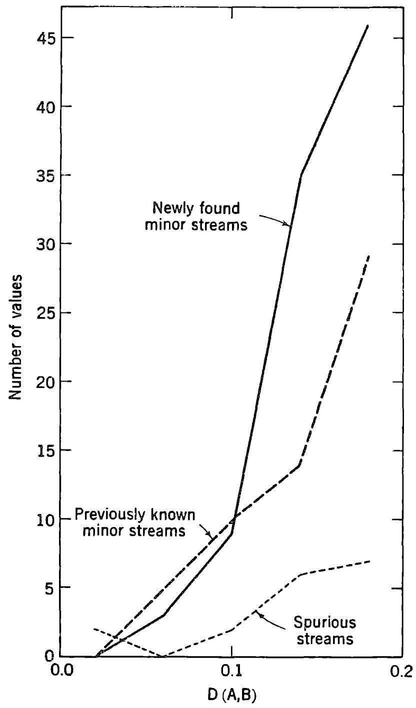

FIGURE 5.—Distribution of $D(A,B)$ in different classes of streams.

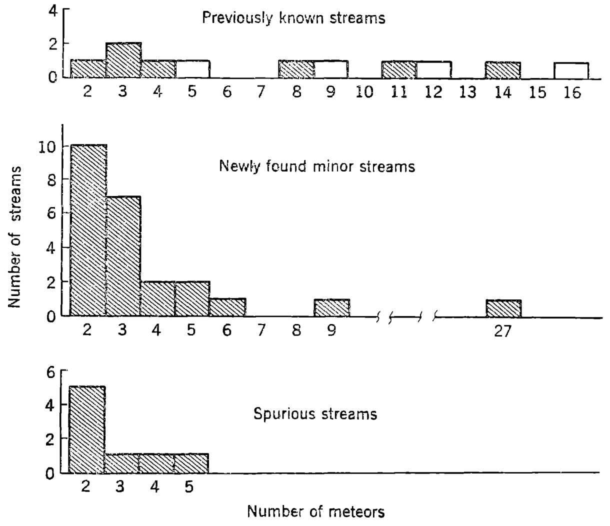

FIGURE 6.—Distribution of number of meteors in a stream. Major streams are unshaded.

space from the mean elements of the stream. A stream with members uniformly distributed through a volume in this phase space would have a histogram in which the ordinate varied as the cube of the abscissa. Thus, the histogram for newly found minor streams indicates, so far as the numbers are significant, that the density of points at $D(M,N)=0.1$ has actually decreased from the density at lower values of D.

Figure 5 shows the distribution of values of $D(A,B)$ within streams. It is clear in both figures 4 and 5 that the newly found streams are less compact than the previously known minor streams. Figure 6 shows the distribution of the number of meteors in a stream. The comparatively low values of $D(M,N)$ for spurious streams occur because such a large proportion of the spurious streams have two members only. We may also conclude from figure 6 that it is the newly found minor streams with two or few members that are more likely to be chance groupings.

Figure 7 shows the distribution of the inclinations of the mean orbits of streams. From the concentration of all the spurious stream inclinations to low values we conclude that the newly found minor streams with low inclinations, like those with few members, are less likely to be genuine streams.

The distribution of radiants of meteors and streams in our sample is shown in figures 8 and 9, referred to a system of coordinates defined by the ecliptic plane and the direction to the sun.

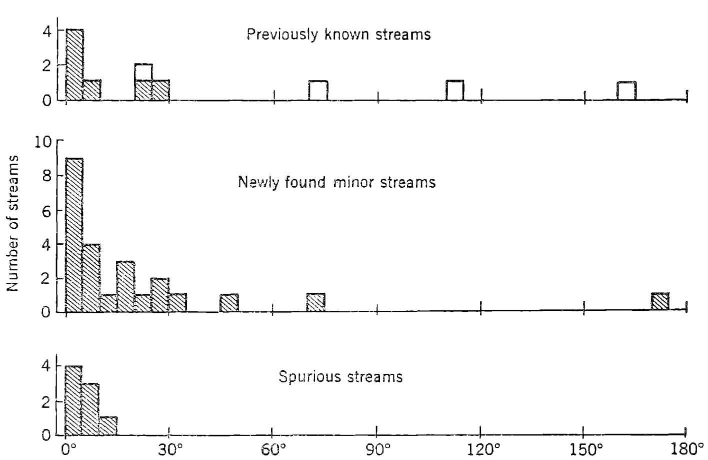

FIGURE 7.—Distribution of inclinations. Major streams are unshaded.

These radiants have been corrected for zenith attraction and diurnal aberration. We found the ecliptic latitude β of each radiant and also the ecliptic longitude $\lambda-\lambda_{\odot}$ measured from the sun's direction as origin. The radiants of major streams and of all other meteors in the sample are so plotted in figure 8. The apex of the earth's motion is in the center of the diagram, the antapex at the right and left ends. The large cluster of points at the right is nearly centered on the anti-sun at $\lambda-\lambda_{\odot}=180°$, $\beta=0°$; while the direction to the sun at left center is $\lambda-\lambda_{\odot}=\beta=0°$.

The last two columns of table 4 list the mean values of $\lambda-\lambda_{\odot}$ and β for each stream radiant. Figure 9 repeats figure 8, substituting mean radiants of minor streams for the individual radiants of their members. The "Cyclid" radiant spread is too large to permit plotting.

In figure 9, the minor streams are seen to cluster principally around the ecliptic in longitude 120° to 210° with respect to the sun. Table 4 also lists the mean radiants of the spurious streams, computed from their orbital elements neglecting the eccentricity of the earth's orbit. These spurious stream radiants are all in or close to this cluster of stream radiants. All the streams with low inclination are in this cluster, and no stream with high inclination; thus we again conclude that the newly found streams with high inclination are much more probably genuine.

All the information that we have found useful in estimating the probability of a stream has been assembled in table 4, although some of it appears also in the other tables. By means of the criteria described above, we have selected four of the provisional streams as the most probable, the φBootids, μDraconids, θHerculids, and ϵGeminids. We have indicated above that the Cyclids are quite probably not a stream, and that the σLeonids are certainly not a single stream. More definite conclusions can hardly be drawn from our sample.

Comparison with other results

McCrosky and Posen (1959) have examined 2538 meteors photographed by the Super-Schmidt cameras, and by comparing radiants, velocities, and dates, have found seven new streams. Substantially all of our meteor sample is included in theirs. Four of the streams in each list are associated. The ϵ-Geminid stream bears the same name in both lists. Their δ Arietids are the same as our ηTaurids, and their μPegasids are included in our γPiscids. Finally, their αVirginids are included within our overgrown σ-Leonid association. This result emphasizes the difficulty of deciding the validity of a possible stream in our sample for, while the ϵGeminids had already been selected as one of the most probable, there was hardly a less promising stream than the ηTaurids.

Associations with comets

We have compared the mean orbits of the possible streams with the orbits of comets observed since 1700, seeking similarities. The nearest associations found appear in table 5. The comets listed have been seen on only one revolution, except Comet De Vico-Swift, for which we list all three observed perihelion passages. The difference D between each stream and its provisionally associated comet is tabulated.

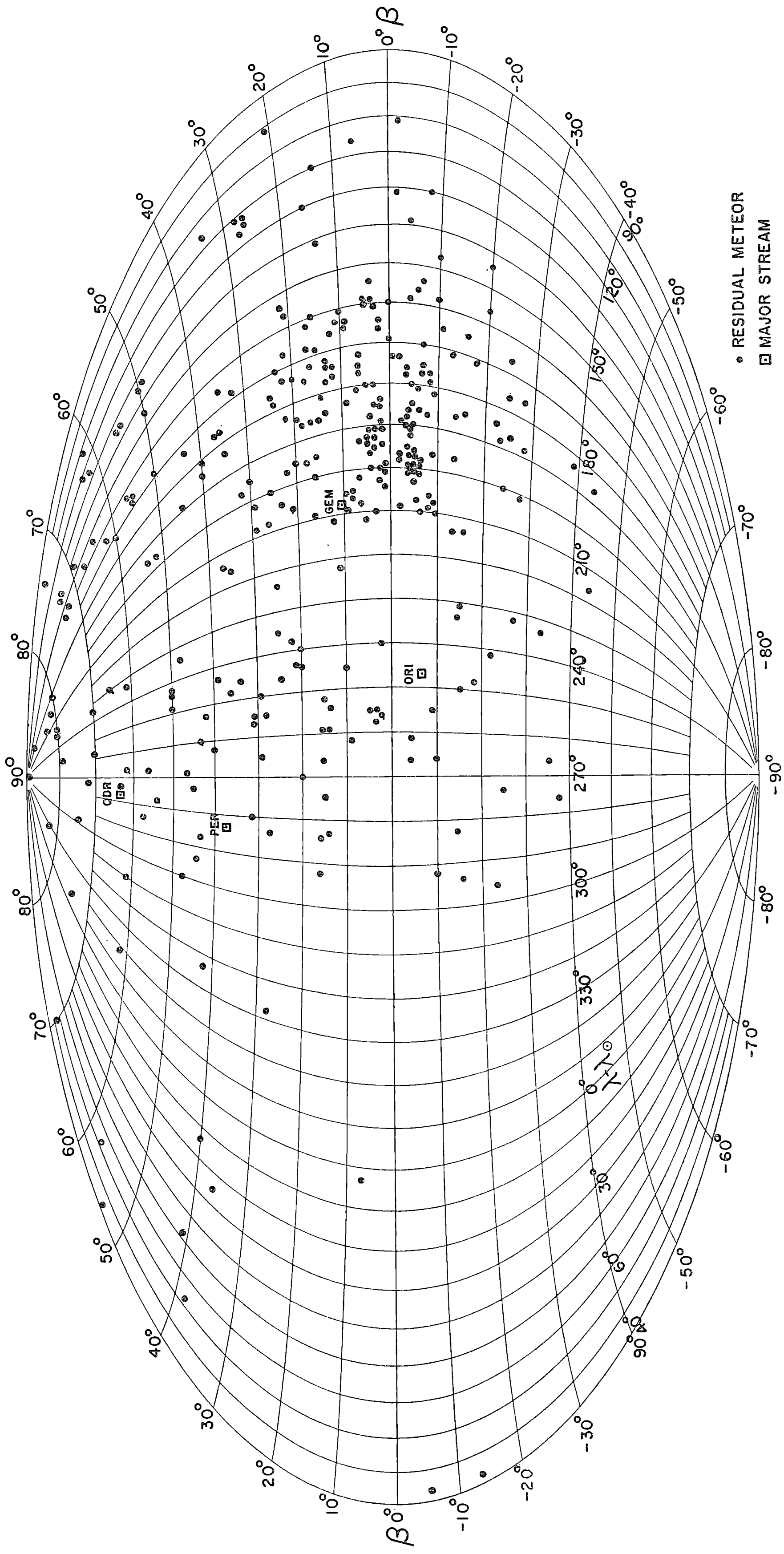

Figure 8.—Distribution of radiants of major streams and residual meteors.

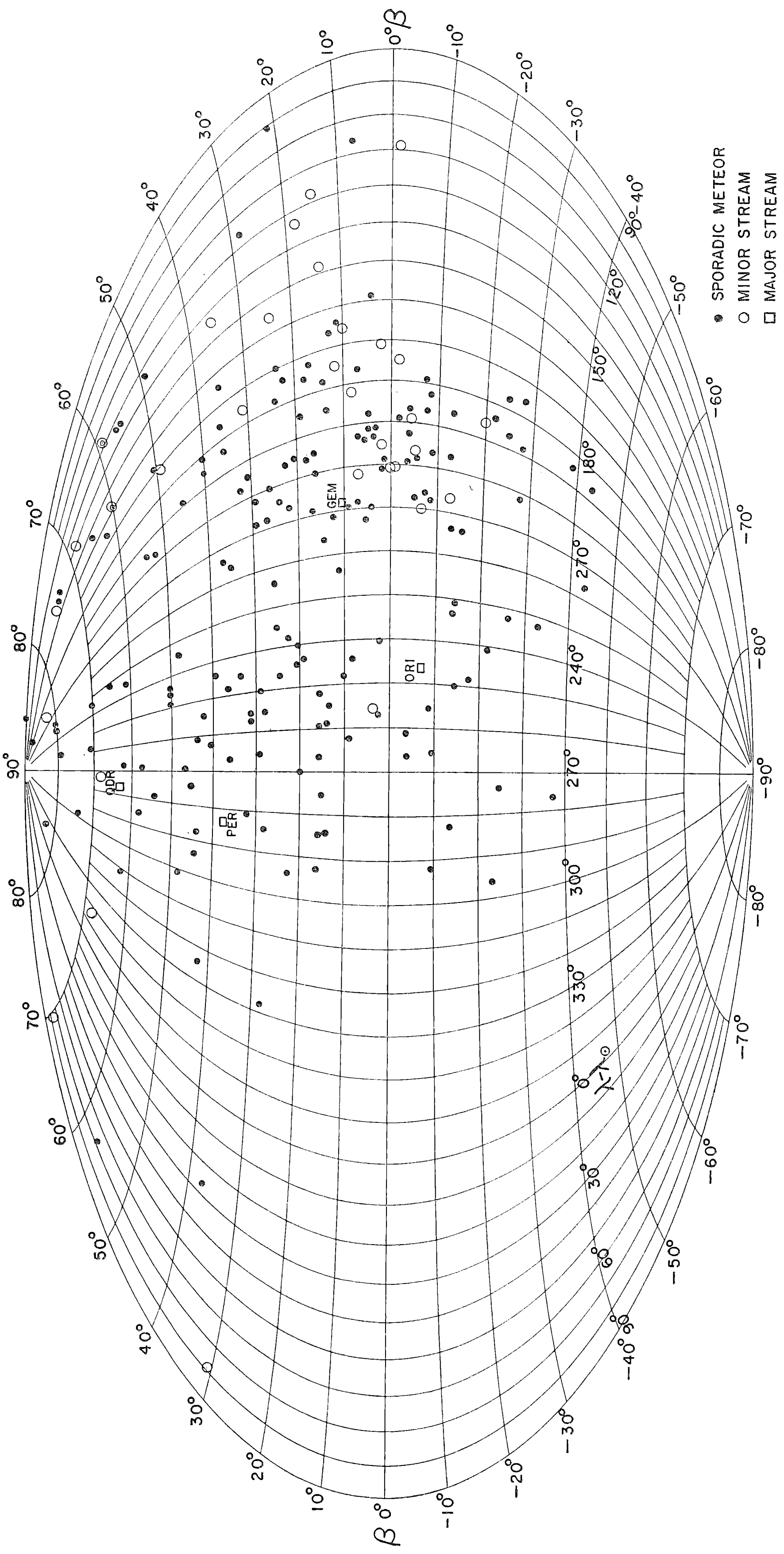

FIGURE 9.—Distribution of radiants of major streams, minor streams, and sporadic meteors.

TABLE 5.—*The closest correspondences between newly found possible streams and known comets*

Stream	a	e	q	i	☊	ω	π	Equinox	D
ξ Orionids	1.86	.480	.947	0°9	159°8	16°4	176°2	1950	
Comet 1928 III	3.43	.71	.995	1.4	196.8	345.2	182.0	1928	.24
τ Herculids	2.44	.582	.998	25°1	90°9	197°7	288°6	1950	
Comet 1930 VI	3.09	.672	1.011	17.4	76.8	192.3	269.1	1930	.28
μ Ophiucids	2.32	.575	.975	3°6	135°0	205°3	340°3	1950	
Comet De Vico-Swift 1678	3.07	.627	1.145	2.9	163.3	159.5	322.8	1678	.26
Comet De Vico-Swift 1844 I	3.10	.617	1.186	2.9	63.8	278.7	342.5	1844	.23
Comet De Vico-Swift 1894 IV	3.25	.572	1.392	3.0	48.8	296.6	345.4	1900	.43
α Pegasids	2.52	.629	.872	4°5	206°1	225°0	71°1	1950	
Comet 1819 IV	2.96	.699	.892	9.1	77.4	350.1	67.5	1820	.23
η Taurids	3.29	.722	.885	0°7	275°9	219°6	135°5	1950	
Comet 1894 I	3.80	.698	1.147	5.5	84.4	46.2	130.6	1894	.29

Only the connection between the α Pegasids and Comet 1819 IV, which was previously found by McCrosky and Posen (1959), is fully convincing. We should expect a parent comet to differ from the mean of an observed stream by more than the average observed meteor because the observed meteors, all of which strike the earth, are a restricted sample of the stream. Since all these comets except 1819 IV differ from the streams by more than 0.2, however, and since some of the streams themselves may not exist, anything more than the possibility of association between these remaining streams and comets remains to be shown.

Space density of minor streams

In this discussion we shall use the symbols defined as follows:

V_H: Heliocentric velocity of meteors.
V_G: Geocentric velocity of the meteors, neglecting the earth's attraction.
V_c: Accelerated geocentric velocity of the meteors, at the top of the atmosphere.
V_∞: Velocity of the meteors with respect to the earth's surface, at the top of the atmosphere.
L: Angular elongation of the corrected radiant from the apex of the earth's way.
γ: Angle between V_G and V_H.
w: Width of meteor stream in the plane of the figure.
ρ: Density of meteors in the stream.
X: Earth's collisional cross-section for meteors.
R: Earth's radius including the atmosphere.
V_E: Earth's orbital velocity.

The number of meteors swept up by the earth in one pass through the stream is $N_1 = X\rho w \operatorname{cosec} \gamma$. The number of meteors in the stream passing within any given small distance h from the earth's orbit per unit time is $N_2 = 2hw\rho V_H$. It is well known (see, for example, Schiaparelli, 1871, p. 116) that $X = \pi R^2 V_c^2/V_G^2$. From figure 10 we see that $V_E \operatorname{cosec} \gamma = V_H \operatorname{cosec} L$. Thus the ratio of the number of meteors striking the earth to those crossing its orbit,

$$\frac{N_1}{N_2} = \frac{\pi R^2}{2hV_E} \frac{V_c^2}{V_G^2 \sin L},$$

is independent of w and ρ. Following Whipple (1954), we approximate by V_∞^2 the velocity dependence of the ratio of the number of meteors brighter than a given magnitude on photographs to the number above a given mass, but omit his special factor for velocities

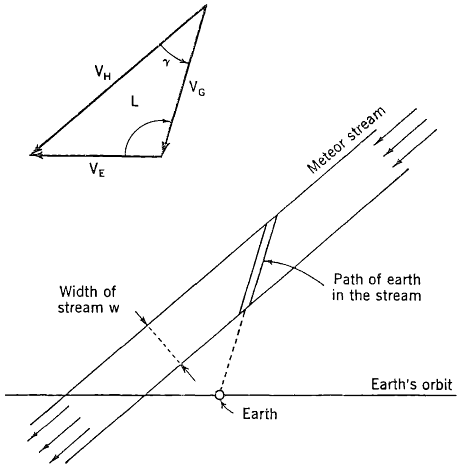

FIGURE 10.—Geometry of the intersection of the earth with a meteor stream.

below 19 km/sec. The qualitative result is not sensitive to a fractional change in the power of V_∞. We approximate V_∞ by the closely similar V_c. Thus (neglecting constants) we find that the ratio of the number of meteors passing near the earth's orbit to those photographed is given by

$$W=\frac{V_G^2 \sin L}{V_c^4}, \tag{5}$$

which may be interpreted as a weight of a photographic meteor in determining the present strength of its stream at the earth's orbit. Figure 11 shows W as a function of L for various values of the stream major semi-axis a. For all except nearly circular orbits, W has a strong maximum between $L=90°$ and $L=130°$.

Figure 9 contains, in addition to the cluster of stream radiants already discussed, a smaller group in high latitudes which includes three of the most probable streams. All of this group, and most of the larger group fall within the range of high W, $90° \leq L \leq 130°$. The minor streams, therefore, are perhaps twice as important numerically in the earth's neighborhood with respect to sporadic meteors, as the observed numbers indicate.

Origin of the newly found minor streams

As shown above, the newly found streams are broader and more diffuse than the major streams, or than most of the previously known minor streams. By this characteristic, we readily recognize that in the dynamical sense they are old streams, which have been more disturbed and perturbed than previously known streams. The absence of closely associated comets corroborates their relatively great age.

There are some interesting relationships among certain of these streams. The μ Ophiucids and ϑ Herculids form a pair of very similar streams differing only in inclination. In contrast to known pairs of streams on either side of the ecliptic, the μ Ophiucids are nearly in the ecliptic while the ϑ Herculids have an inclination of 18°. The ϵ Geminids bear a somewhat similar relation to the Orionids, but in this case the longitudes of perihelion also differ.

In figure 12 the orbits of all the newly found minor streams with inclinations less than 15° are projected onto the plane of the ecliptic. Although the majority of the streams plotted here doubtless are chance associations, some conclusions can be drawn. The tendency of these orbits to be approximately tangent to the earth's orbit is clearly a selection effect. The tendency to have aphelia at or near Jupiter's orbit appears to be significant. Jupiter will strongly affect these streams, and they will age rapidly.

Conclusions

The differences D between meteors may be used as a means of determining membership in known streams, or of finding new streams. As we have applied D to the search for streams, the result is certainly complete in the sense that all streams with two or more members in the sample doubtless were found. Unfortunately, the method as applied was not very discriminating, since many streams were "found"

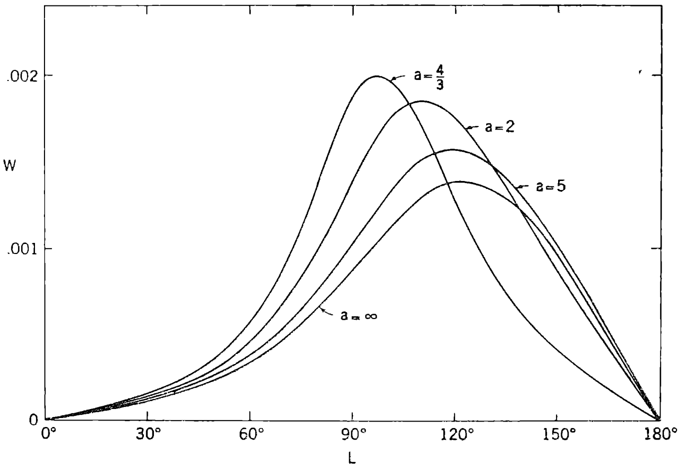

FIGURE 11.—Weight W of a photographic meteor in determining the present strength of its stream at the earth's orbit, as a function of the angular elongation L of the radiant from the apex, and the major semi-axis a.

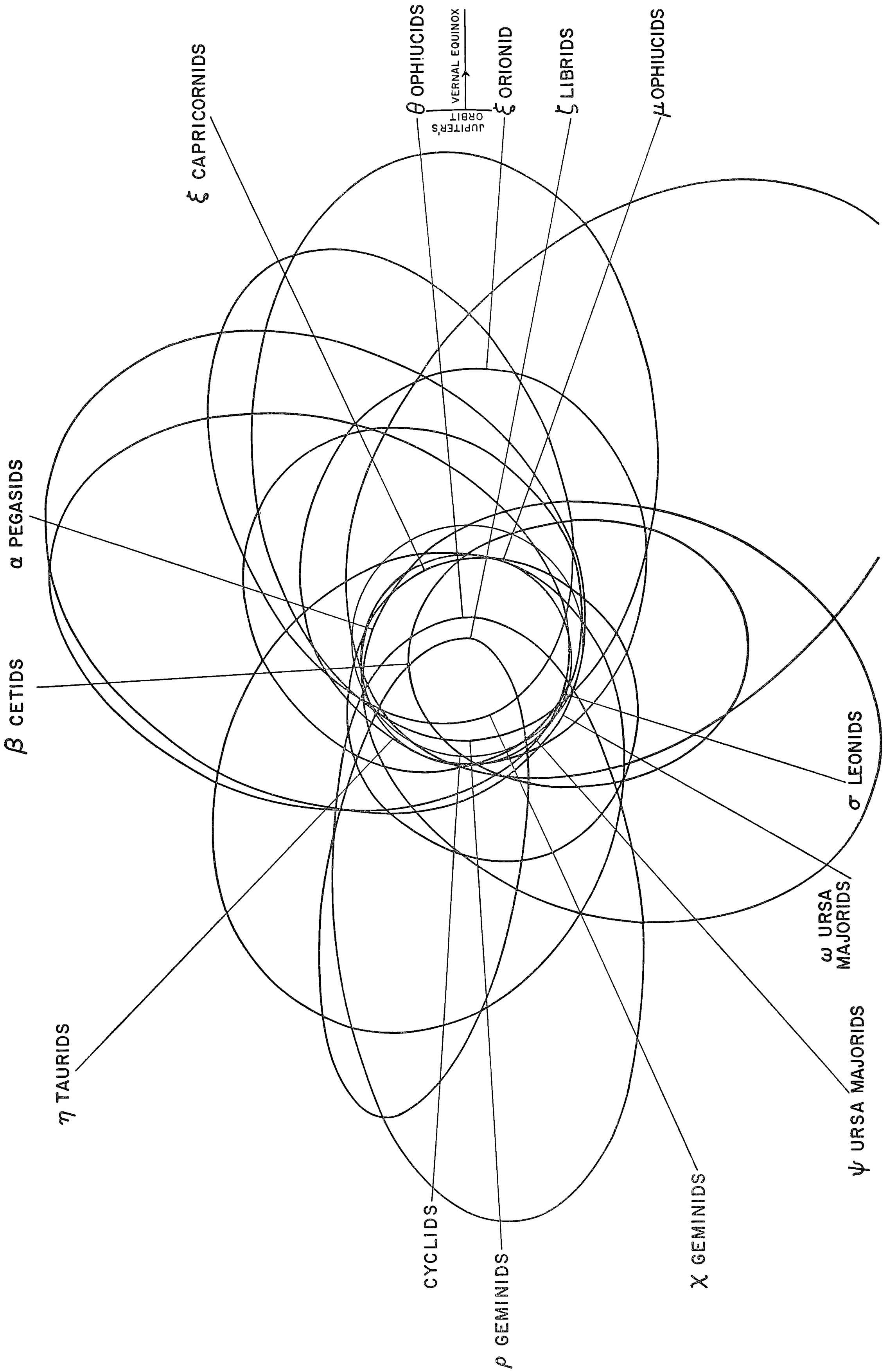

Figure 12.—Orbits of all the newly found minor streams with inclinations less than 15° are projected onto the plane of the ecliptic.

which no doubt do not exist, but the number of spurious streams may be estimated. Streams too small to have at least two meteors in our sample, streams with brief showers during a gap in the observing, variable and temporary streams: all may or will be missed.

Of the 359 meteors in the sample, 42 or 12 percent belong to major streams, i.e., those with mean photographic hourly rates exceeding three; 48 or 13 percent belong to previously known minor streams, and approximately 39 or 11 percent to newly found streams. About 64 percent of the sample is sporadic. In the immediate neighborhood of the earth's orbit, the minor streams, compared to other meteors, are roughly twice as numerous as in the photographic sample.

The newly found streams are in fact old streams; they have been much perturbed, and some show obvious relationships with Jupiter. In overall orbital distribution, the minor streams do not differ, in any way that has been conspicuous to us without special scrutiny, from the sporadic meteors with which they are intermingled.

Appendix

To compute the average gradients of orbital elements with respect to perturbations, let the units be the astronomical unit, the solar mass, and $\frac{1}{2\pi}$ year. Consider perturbations at a point P in a meteor orbit, at a distance r from the sun. Resolve the meteor's heliocentric velocity $\mathbf{V}_H$ into components as in figure 13: V_R directed away from the sun, and V_S directed forward in the orbit plane normal to V_R. Resolve the perturbations in velocity into components: δV_R and δV_S along V_R and V_S, respectively, and δV_T normal to the orbit plane forming a right-handed set with δV_R and δV_S.

Let G denote any orbital element, and $|\nabla G|$ the magnitude of the gradient of G with respect to perturbations in velocity. Then

$$|\nabla G|=\left[\left(\frac{\partial G}{\partial \delta V_R}\right)^2+\left(\frac{\partial G}{\partial \delta V_S}\right)^2+\left(\frac{\partial G}{\partial \delta V_T}\right)^2\right]^{1/2}. \tag{A1}$$

It is evident that differentiation with respect to δV_R and δV_S may be replaced by differentiation with respect to V_R and V_S, respectively.

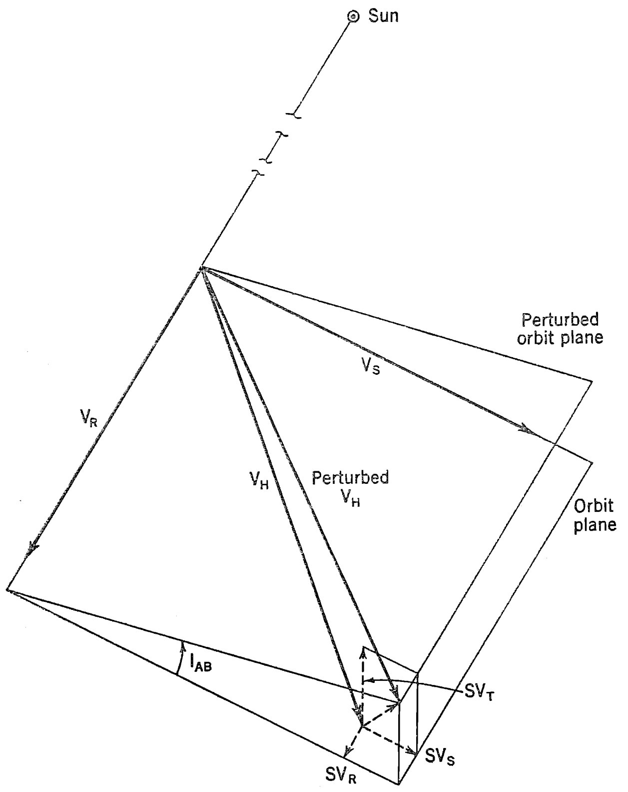

FIGURE 13.—Components of velocity and of perturbations in velocity.

Let the perturbations be expressed in units of the circular speed $U=r^{-1/2}$; the magnitude of the gradient of G with respect to such perturbations is

$$|\nabla G|_c=r^{1/2}|\nabla G|. \tag{A2}$$

Let Grad G denote the average of $|\nabla G|_c$ over the orbit with respect to time. Let t be the time of the meteor's arrival at P, counted from perihelion passage. Then, since the period of revolution is $2\pi a^{3/2}$,

$$\text{Grad } G=\frac{1}{2\pi a^{3/2}}\int_0^{2\pi a^{3/2}} r^{1/2}|\nabla G|dt \tag{A3}$$

$$=\frac{1}{2\pi a^{3/2}}\int_0^{2\pi a^{3/2}} r^{1/2}\left[\left(\frac{\partial G}{\partial V_R}\right)^2+\left(\frac{\partial G}{\partial V_S}\right)^2+\left(\frac{\partial G}{\partial \delta V_T}\right)^2\right]^{1/2} dt. \tag{A4}$$

This is the desired measure of the importance of G in D.

Of the components of D, differences within the orbital plane are measured by δe, δq, Π_{AB}. Obviously, Grad δe=Grad e, and Grad δq=

Grad q. Let v be the true anomaly at P. Any perturbation δv in true anomaly implies a perturbation $-\delta v$ in the direction of perihelion; thus

$$\text{Grad } \Pi_{AB} = \text{Grad } v. \tag{A5}$$

By standard formulae, if $p=a(1-e^2)$ is the parameter of the orbit, then

$$\left.\begin{aligned} r &= p/(1+e\cos v), \\ V_R &= (e\sin v)/p^{1/2}, \\ V_S &= (1+e\cos v)/p^{1/2}, \\ dv/dt &= V_S/r. \end{aligned}\right\} \tag{A6}$$

If r, V_R, V_S are given at any point, then by equally standard formulae:

$$\left.\begin{aligned} e\sin v &= rV_RV_S, \\ e\cos v &= rV_S^2-1, \\ q &= r^2V_S^2/(1+e). \end{aligned}\right\} \tag{A7}$$

Straightforward analysis yields:

$$\left.\begin{aligned} \partial e/\partial V_R &= r^2V_RV_S^2/e, \\ \partial e/\partial V_S &= r^2V_S(V_S^2-1/a)/e, \\ \partial v/\partial V_R &= rV_S(rV_S^2-1)/e^2, \\ \partial v/\partial V_S &= -rV_R(rV_S^2+1)/e^2, \\ \partial q/\partial V_R &= -V_Rq^2/e, \\ \partial q/\partial V_S &= V_S(r-q)^2/e. \end{aligned}\right\} \tag{A8}$$

Since e, v, q are functions of r, V_R, V_S only,

$$\partial e/\partial\delta V_T = \partial v/\partial\delta V_T = \partial q/\partial\delta V_T = 0. \tag{A9}$$

Expressions for Grad e, Grad v, Grad q may now be found by substitution of equations (A6), (A8), (A9) into equation (A4). We have not found it possible to evaluate those expressions in closed form. Since an approximate result is sufficient for the purpose, we take advantage of the following lemma.

Lemma: If y and z are functions of x which are finite throughout $a\leq x\leq b$, and if

$$y\geq 0, \tag{A10}$$

$$z\geq 0, \tag{A11}$$

$$Y=\int_a^b y\,dx, \tag{A12}$$

$$Z=\int_a^b z\,dx, \tag{A13}$$

$$R=\int_a^b (y^2+z^2)^{1/2}dx, \tag{A14}$$

then

$$(Y^2+Z^2)^{1/2}\leq R\leq Y+Z. \tag{A15}$$

If y bears a constant proportion to z for all x in $a\leq x\leq b$, then $R=(Y^2+Z^2)^{1/2}$. Any deviation from constant proportionality between y and z increases R; for example, if $y(x_1)$ is increased by Δy over a range dx about x_1, and then $y(x_2)$ is decreased by Δy over a range dx about x_2 in order to satisfy equation (A12), some algebra will show that R increases. If $y=0$ for all x such that $z>0$ and vice versa, then $R=Y+Z$; if there is any range in x in which both $y>0$ and $z>0$, then $R<Y+Z$ because $(y^2+z^2)^{1/2}<y+z$.

By substituting equation (A9) into (A4), and taking the mean of the limits in equation (A15), we find that an approximation to equation (A4) when G is δe, Π_{AB}, or δq is

$$\text{Grad } G=\frac{1}{2}\left\{\left[\left\langle\frac{\partial G}{\partial V_R}\right\rangle^2+\left\langle\frac{\partial G}{\partial V_S}\right\rangle^2\right]^{1/2} +\left\langle\frac{\partial G}{\partial V_R}\right\rangle+\left\langle\frac{\partial G}{\partial V_S}\right\rangle\right\}, \tag{A16}$$

where

$$\left\langle\frac{\partial G}{\partial V_R}\right\rangle=\frac{1}{2\pi a^{3/2}}\int_0^{2\pi a^{3/2}} r^{1/2}\frac{\partial G}{\partial V_R}\,dt, \tag{A17}$$

and $\left\langle\frac{\partial G}{\partial V_S}\right\rangle$ is similarly defined.

With the aid of a particular set of elliptic integrals,

$$F'(\phi)=\int_0^\phi [1-2e(1+e)^{-1}\sin^2 u]^{-1/2}du, \tag{A18}$$

$$E'(\phi)=\int_0^\phi [1-2e(1+e)^{-1}\sin^2 u]^{1/2}du, \tag{A19}$$

we may now evaluate Grad G. By substituting equations (A6) and (A8) into (A17), and performing the necessary algebra, we find:

$$\langle\partial e/\partial V_R\rangle=2(1-e^2)[(1+e)^{1/2}-(1-e)^{1/2}]/\pi e, \tag{A20}$$

$$\begin{aligned}\langle\partial e/\partial V_S\rangle=8(1-e^2)\{&(1-e)^{1/2}[2F'(A)-F'(\pi/2)] \\ &-(1-e)^{-1/2}[2E'(A)-E'(\pi/2)]+[2 \\ &-2\,(1-e^2)^{1/2}]^{1/2}[2(1-e^2)^{-1/2}+1/2]\}/3\pi e,\end{aligned} \tag{A21}$$

where

$$A=\frac{1}{2}\text{ arc cos}\{[-1+(1-e^2)^{1/2}]/e\}\cdot \quad \text{(A22)}$$

Also,

$$\langle\partial v/\partial V_R\rangle=2(1-e^2)(1-e)^{1/2}\{2F'(\pi/4)-F'(\pi/2)$$
$$-(1-e)^{-1}[2E'(\pi/4)-E'(\pi/2)]$$
$$+2e(1-e)^{-1}(1+e)^{-1/2}\}/\pi e^2, \quad \text{(A23)}$$

$$\langle\partial v/\partial V_S\rangle=2\{(1+e)^{3/2}-(1-e)^{3/2}$$
$$+3(1-e^2)[(1+e)^{1/2}-(1-e)^{1/2}]\}/3\pi e^2, \quad \text{(A24)}$$

$$\langle\partial q/\partial V_R\rangle=2a(1-e^2)[(1+e)^{1/2}-(1-e)^{1/2}]/\pi e, \quad \text{(A25)}$$

$$\langle\partial q/\partial V_S\rangle=4a(1-e)^{1/2}[2(1+e)E'(\pi/2)$$
$$-(2-e)(1-e)F'(\pi/2)]/3\pi e. \quad \text{(A26)}$$

The values of Grad e, e Grad Π_{AB}, and of $1/a$ Grad q in figure 1 were computed from equations (A5) and (A16) through (A26). The largest possible error, computed by equation (A15), is 15 percent.

Grad I_{AB} remains to be discussed. Since I_{AB} measures the change of orbital plane, it is independent of velocity changes within the plane; hence

$$\partial I_{AB}/\partial\delta V_R=\partial I_{AB}/\partial\delta V_S=0. \quad \text{(A27)}$$

From figure 13 we see that

$$\partial I_{AB}/\partial\delta V_T=1/V_S. \quad \text{(A28)}$$

Thus Grad I_{AB} may be found exactly from equations (A4), (A6), (A18), (A19), (A27), (A28), as

$$\text{Grad } I_{AB}=2(1-e)^{-1/2}[4E'(\pi/2)$$
$$-(1-e)F'(\pi/2)]/3\pi, \quad \text{(A29)}$$

and the values in figure 1 were so computed.

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