1963ApJ...137..945P

# THE LOWER SOLAR CORONA: INTERPRETATION OF THE ULTRAVIOLET SPECTRUM*

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*Received April 30, 1962; revised October 26, 1962*

## ABSTRACT

An analysis of the resonance lines of nine elements (27 ions) formed in the chromosphere and corona of the sun yields the following results: (*a*) the chemical composition of this region of the sun can be determined, without any knowledge of the detailed temperature-density structure in this region; (*b*) a further clue concerning the detailed structure of this region may be obtained. A prediction of the expected emission of the sun in radio frequencies can be used both as a check on the correctness of the present work and as a means of obtaining the ratio of the observed elements to hydrogen.

## I. INTRODUCTION

The ultraviolet solar spectrum is important in the interpretation of the solar atmosphere because it contains the resonance lines of at least eight elements, other than hydrogen and helium, whose observable ions span the complete range of ionization potential up to at least 500 electron volts. At present, however, only the integrated spectrum, over all heights in the solar atmosphere, is known. Because no height resolution is available, the problem of interpreting these observations is rendered extremely difficult.

Let us assume for the present that the observations are accurate and the oscillator strengths of the lines known. The problem can be stated in the following way. We do not know the electron density or the electron temperature as a function of height in the region of the formation of these resonance lines, and we wish to use the observations to determine these quantities. Further, the abundances of the various elements enter into the analysis, and, since we would prefer to make an independent determination of the abundances, these must also be regarded as unknown quantities. Can all these quantities be determined from the spectrum, integrated over height, of the wide range of ions observed? If not, what definite conclusions can be obtained regarding the distribution of temperature and density with height and the abundances of the elements?

Several approaches to this problem have already been attempted. Allen (1961) assumed initially that he knew both the distribution of density and temperature with height in the solar atmosphere and the abundance of the elements. He then computed the intensities of the ultraviolet lines to be expected from such an atmosphere and found rough agreement with the observed intensities, although the $f$-values he used for the resonance lines were probably an order of magnitude too low. Allen then concluded that he could discriminate between the model that he used, which may be correct, and one with a very sharp temperature transition, since the observations showed all ionization stages to be about equally present, while a very sharp temperature transition would presumably eliminate those ions normally present at those temperatures through which the transition occurs. Such a conclusion is unwarranted.

The model Allen used (taken from Oster 1956) is but a single possibility, and other models can presumably reproduce the observations as well. Further, this particular model does not reproduce the continuum measured at λ 4700 as a function of height during the 1952 eclipse (see Athay, Menzel, Pecker, and Thomas 1955). A comparison of the density distribution used by Allen and that derived from the continuum measure-

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* *Publications of the Goethe Link Observatory*, No. 51.

ments is shown in Table 1. One would like to know whether either density distribution could have been used, together with some suitable temperature distribution, to produce the observable ultraviolet spectrum or whether the density distribution derived from observations of the continuum is somehow incompatible with the observed ultraviolet emission.

A somewhat different approach has been used by Ivanov-Kholodnyi and Nikolskii (1961). They attempted to derive the temperature and density distributions in the atmosphere, using the observed ultraviolet intensities supplemented by measurements of the Balmer continuum radiation as a function of height to 2400 km and an extrapolation of these above this height. Unfortunately, this extrapolation, which they carried to 20000 km is very uncertain. Furthermore, there is a fundamental flaw in this kind of analysis: the results appear to depend markedly on just what the second piece of information is. Had the observed density as a function of height been used instead of the Balmer continuum, the derived temperature distribution would have been different. Had the radio emission in centimeter wavelengths been used, still different temperature and density distributions would have been obtained. This is not a reflection on the quality of the observations but may be an indication that the atmosphere is not spherically symmetric and that a single temperature-density distribution will not completely specify the atmospheric structure.

TABLE 1

ELECTRON DENSITY

| Height above Limb (km) | Used by Allen (1961) (From Oster 1956) | Derived from Continuum Measurements by Athay, Menzel, Pecker, and Thomas (1955) |
|---|---|---|
| 6000 | 79 $\times 10^8$ cm$^{-3}$ | 77$\times 10^8$ cm$^{-3}$ |
| 8000 | 18 | 42 |
| 10000 | 5 7 | 24 |
| 12000 | 2 4 | 18 |

It is clear that all the observations of the lower corona must be considered together to obtain a coherent model of this region of the atmosphere. We do not do this in the present paper but, instead, limit ourselves to a discussion of the ultraviolet lines and to the deductions that can be drawn from their study. In particular, we attempt to show that the following two important conclusions may be drawn: (1) that the chemical composition of the lower corona can be determined from the ultraviolet observations, *independently* of an exact knowledge of the temperature-density distribution in the atmosphere, and (2) that a common characteristic of all models which will produce the observed ultraviolet spectrum can be obtained.

## II. THE OBSERVATIONAL DATA

Ultraviolet measurements extend to about 200 A with reasonable resolution and calibration. In this region all the expected resonance lines of oxygen, nitrogen, carbon, and many of the silicon and sulfur lines have been observed. (Unfortunately, the resolution is insufficient to separate the O II and O III resonance lines.) In addition, the resonance lines of Ne VII, Ne VIII, Mg IX, and Mg X almost certainly are detected, and the Fe XV and Fe XVI resonance lines are probably observed as well. A summary of the observed lines and their intensities is given in Table 2. Longward of $\lambda$ 1300, great weight is placed on the observations of Detwiler, Garrett, Purcell, and Tousey (1961) for the quiet sun on April 19, 1960. Concerning their accuracy these authors state: "In the

## TABLE 2

### OBSERVED LINES AND THEIR INTENSITIES

| ION (1) | IONIZ. POTENT. (2) | TRANSITION (3) | WAVELENGTH (A) (4) | $f_{lu}$ (5) | INTENSITY (ergs cm$^{-2}$ sec$^{-1}$) | | | $N$(element)× $\int_R N_e^2 dh \div N$(H) (cm$^{-5}$) (9) | $T_{\max}$ (° K) (10) | RANGE OF TEMPERATURE (° K) (11) | $N$(O) $\int_R N_e^2 dh$ $\div N$(H) (cm$^{-5}$) (12) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Detwiler *et al.* (1961) (6) | Hinteregger (1961) (7) | Adopted (8) | | | | |
| C II | 24.4 | $^2P^0_{1/2,3/2}-^2D_{3/2,5/2}$ | 1335.7 | 0.42 | 0.050 | | 0.050 | 4.8×$10^{23}$ | 31000 | 20000–40000 | 1.9×$10^{23}$ |
| | | | 1334.5 | .21 | .050 | | .050 | | | | |
| C III | 47.9 | $^1S_0-P^0_1$ | 977 0 | 74 | .050 | 0.11 | .09 | 1.5×$10^{23}$ | 55000 | 37000–80000 | 6.0×$10^{22}$ |
| C IV | 64.5 | $^2S_{1/2}-^2P^0_{1/2,3/2}$ | 1550.7 | .1 | 06 | | .06 | 1.7×$10^{23}$ | 90000 | 60000–120000 | 6.6×$10^{22}$ |
| | | | 1548.2 | .2 | .11 | | .11 | | | | |
| N II | 29.6 | $^3P_{0,1,2}-^3D^0_{1,2,3}$ | 1085.7 | .14 | .006 | 011 | 009 | 4.2×$10^{22}$ | 34000 | 25000–43000 | 1.6×$10^{23}$ |
| | | | 1085.5 | .05 | | | | | | | |
| | | | 1084.6 | .34 | | | | | | | |
| | | | 1084.0 | .17 | | | | | | | |
| N III | 47 4 | $^2P^0_{1/2,3/2}-^2D_{3/2,5/2}$ | 991.5 | .35 | 010 | .01 | .012 | 2.5×$10^{22}$ | 60000 | 40000–85000 | 9.8×$10^{22}$ |
| | | | 989.8 | .18 | .006 | | | | | | |
| N IV | 77.5 | $^1S_0-^1P^0_1$ | 765.1 | .59 | | .0077 | .0077 | 9 7×$10^{21}$ | 100000 | 75000–140000 | 3.8×$10^{22}$ |
| N V | 97.9 | $^2S_{1/2}-^2P^0_{3/2,1/2}$ | 1238.8 | .165 | 004 | | .004 | 9.3×$10^{21}$ | 145000 | 100000–220000 | 3.8×$10^{22}$ |
| | | | 1242.8 | .08 | 003 | | .003 | | | | |
| O II | 35.1 | $^4S^0_{3/2}-^4P_{1/2,3/2,5/2}$ | 832.8 | .25 | .012 | 016 | .016 | 1.5×$10^{23}$ | 43000 | 30000–60000 | 1.5×$10^{23}$ |
| | | | 833.3 | .25 | | | | | | | |
| | | | 834.5 | .25 | | | | | | | |
| O III | 54.9 | $^3P_{0,1,2}-^3D^0_{1,2,3}$ | 833–835 | 61 | | .016 | .016 | 4.2×$10^{22}$ | 80000 | 45000–95000 | 4.1×$10^{22}$ |
| O IV | 77 4 | $^2P^0_{1/2,3/2}-^2D_{3/2,5/2}$ | 790.1 | .29 | | .013 | 013 | 3.8×$10^{22}$ | 125000 | 75000–150000 | 3.8×$10^{22}$ |
| | | | 787 7 | .15 | | .012 | 012 | | | | |
| O V | 113.9 | $^1S_0-^1P^0_1$ | 629 7 | 48 | | .045 | 045 | 5.3×$10^{22}$ | 160000 | 130000–240000 | 5.3×$10^{22}$ |
| O VI | 138.1 | $^2S_{1/2}-^2P^0_{3/2,1/2}$ | 1037 6 | .07 | .025 | 027 | .026 | 1.1×$10^{23}$ | 220000 | 170000–350000 | 1.1×$10^{23}$ |
| | | | 1031.9 | .135 | .020 | 0 039 | 039 | | | | |
| Si II | 16.3 | $^2P^0_{1/2,3/2}-^2D_{3/2,5/2}$ | 1817.2 | 74 | 45 | | 45 | 2.0×$10^{24}$ | 17000 | 14000–27000 | 6.6×$10^{24}$ |
| | | | 1808.0 | 0.36 | 0.15 | | 0.15 | | | | |

TABLE 2—*Continued*

| ION<br>(1) | IONIZ. POTENT.<br>(2) | TRANSITION<br>(3) | WAVELENGTH (A)<br>(4) | $f_{lu}$<br>(5) | INTENSITY (ergs cm$^{-2}$ sec$^{-1}$)<br>Detwiler *et al.* (1961)<br>(6) | Hinteregger (1961)<br>(7) | Adopted<br>(8) | $N$(element)× $\int_R N_e^2dh - N$(H) (cm$^{-5}$)<br>(9) | $T_{\max}$ (° K)<br>(10) | RANGE OF TEMPERATURE (° K)<br>(11) | $N$(O)$\int_R N_e^2dh$ $-N$(H) (cm$^{-5}$)<br>(12) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Si III. | 33.5 | $^1S_0$–$^1P^0_1$ | 1206.5 | 1.62 | 0.030 | 0.082 | 0.048 | 4.0×10$^{22}$ | 38000 | 27000–55000 | 1.3×10$^{23}$ |
| Si IV | 45.1 | $^2S_{1/2}$–$^2P^0_{3/2,1/2}$ | 1402.7<br>1393 7 | 0.28<br>0.57 | 013<br>.030 | . . . | 013<br>030 | 2.5×10$^{22}$ | 49000 | 45000–82000 | 8.2×10$^{22}$ |
| Si XII. | 523.2 | $^2S_{1/2}$–$^2P^0_{1/2,3/2}$ | 521.1<br>499.3 | 0.035<br>0.075 | . . . | .010<br>021 | .010<br>021 | 1.3×10$^{23}$ | 1600000 | 1000000–2400000 | 4.4×10$^{23}$ |
| S II. | 23.3 | $^4S^0_{3/2}$–$^4P_{1/2,3/2,5/2}$ | 1250.5<br>1253.8<br>1259.5 | 0.19<br>0.37<br>0.54 | .004 | . . . | .004 | 2.2×10$^{22}$ | 26000 | 19000–36000 | 3.8×10$^{23}$ |
| S III. | 34.7 | $^3P_{0,1,2}$–$^3D_{3,2,1}$ | 1190–1202 | 1.13 | .005 | 016 | 008 | 9.8×10$^{21}$ | 42000 | 30000–60000 | 1.6×10$^{23}$ |
| S IV | 47.3 | $^2P^0_{1/2,3/2}$–$^2D_{3/2,5/2}$ | 1073.3<br>1062.7 | 0.48<br>0.24 | .002 | .004 | 003 | 3.2×10$^{21}$ | 66000 | 49000–90000 | 5.3×10$^{22}$ |
| S VI. | 88.0 | $^2S_{1/2}$–$^2P^0_{3/2,1/2}$ | 944.5<br>933 4 | 0.29<br>0.58 | 0 003 | 005 | 0044 | 2.8×10$^{21}$ | 140000 | 100000–200000 | 4.4×10$^{22}$ |
| Ne VII. | 207.3 | $^1S_0$–$^1P^0_1$ | 465.1 | 0.5 | . . . | 013 | 013 | 1.8×10$^{22}$ | 350000 | 280000–470000 | 1.5×10$^{23}$ |
| Ne VIII. | 239 0 | $^2S_{1/2}$–$^2P^0_{1/2,3/2}$ | 780.3<br>770.4 | 0.5<br>0.105 | . . . | 0065<br>013 | 0065<br>013 | 4.7×10$^{22}$ | 480000 | 350000–650000 | 3.8×10$^{23}$ |
| Mg IX. | 327.9 | $^1S_0$–$^1P^0_1$ | 368.1 | 0.50 | . . . | .034 | 034 | 4.7×10$^{22}$ | 700000 | 500000–920000 | 3.1×10$^{23}$ |
| Mg X. | 367 4 | $^2S_{1/2}$–$^2P^0_{1/2,3/2}$ | 625.3<br>609.8 | 0.045<br>0.085 | . . . | .013<br>025 | 013<br>025 | 1.1×10$^{23}$ | 820000 | 620000–1300000 | 7.2×10$^{23}$ |
| Fe XV | 457 | $^1S_0$–$^1P^0_1$ | 284.3 | 1.18 | . . . | 029 | .029 | 2.2×10$^{22}$ | 1300000 | 930000–1800000 | 2.7×10$^{23}$ |
| Fe XVI. | 489 4 | $^2S_{1/2}$–$^2P^0_{3/2,1/2}$ | 361.7<br>336.62 | 0.13<br>0.25 | . . . | .018<br>027 | .018<br>027 | 5.7×10$^{22}$ | 1600000 | 1000000–2400000 | 6.9×10$^{23}$ |
| He I. | 24.6 | $1^1S_0$–$2^1P^0_1$ | 584.3 | 0.44 | . . . | 054 | 054 | 5.3×10$^{25}$ | 32000 | 25000–48000 | 1.8×10$^{23}$ |
| He II. | 54.4 | $1^2S_{1/2}$–$2^2P^0_{3/2,1/2}$ | 303.8 | 0.42 | . . . | 0.24 | 0.24 | 2.0×10$^{25}$ | 85000 | 65000–130000 | 6.6×10$^{22}$ |

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region 2000 A to 1400 A we believe that the accuracy is better than ±20 per cent, and are fairly sure that there are no errors greater than a factor of ±1.5. Below 1300 A there is the possibility of errors as great as a factor of ±1.5." Between 300 and 1100 A the work of Hinteregger (1961 and unpublished) referring to observations taken on August 23, 1960, and August 23, 1961, has been used. There measures average over the entire surface of the sun, and, while lacking the resolution of the longer-wave observations, their accuracy in intensities is probably equivalent. The intensities reported in Table 2 refer to the 1961 date, which is presumed to be the more accurate of the two observations, but there are no important differences between the two sets of data.

In the region of overlap between the measurements of Detwiler, Garrett, Purcell and Tousey and those of Hinteregger, a sliding scale is used to average their results. Above 1300 A, only the results of the former authors are used, and below 900 A, only the observations of the latter author are considered, while at 1100 A equal weight is given to each observation.

Hinteregger has found that, in the region below 800 A, absorption in the earth's atmosphere was still appreciable even when the rocket was at a height of 225 km, its highest point. Hinteregger (1962) has estimated the correction factors to convert observed intensities to intensities that would be observed at the top of the atmosphere, as given in the accompanying table. Not enough information is available at present to be certain of the precise values of these factors.

| Wavelength | Correction Factor | Wavelength | Correction Factor | Wavelength | Correction Factor |
|---|---|---|---|---|---|
| 833 | 1 14 | 610 | 1 7 | 370 | 1 5 |
| 770 | 1 2 | 500 | 1 7 | 280 | 1 4 |

When two lines in a multiplet could be resolved, their relative intensities were generally proportional to their theoretical oscillator strengths, within the errors of observation. Since collisional excitations are proportional to the oscillator strengths for permitted transitions, it can be argued that collisional excitation followed by radiative de-excitation is the important process in producing these lines. This is discussed in the following sections.

The oscillator strengths, or *f*-values, given in Table 2 are taken from the following sources. For very highly ionized particles the work of Varsavsky (1961) was used. These computations are based on a theoretical method advanced by Layzer (1959). Varsavsky (1958) has carried out a detailed comparison between the results obtained with this method and those obtained by more elaborate procedures. For transitions in which the principal quantum number undergoes no change, he found a remarkably close correspondence between the two sets of predictions. In this paper we are concerned only with transitions for which there is no change in the principal quantum number.

For the less highly ionized systems we have used other sources. The lithium and sodium isoelectronic sequences seem to pose no problem because there is only a single electron in the outer shell. The work of Bely (unpublished) for the lithium sequence and that of Tully (unpublished) for the sodium sequence show no differences greater than 10 per cent, compared with Varsavsky's results. For C III and C IV the results of Vainstein (1960) were used. For the first five members of the beryllium, boron, and carbon isoelectronic sequences, the results of Bolatin, Levinson, and Levin (1956) were used.

There seems to be no reason to suspect that the *f*-values used here will be in error by more than a factor of 2.

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### III. THE FORMATION OF THE ULTRAVIOLET LINES

All the lines we consider in this paper have the following two common characteristics: (1) They are resonance lines of the ion in question, i.e., they connect the ground state with the lowest energy level from which an electric dipole transition can take place; (2) this excited state has the same total quantum number $n$ as the ground state. This is specified because $f$-values are better known for these transitions (helium and hydrogen are excepted, since $f$-values are known for these elements).

Further, we assume that the diffusion length over the region of formation of the line is less than unity, i.e., most of the radiation produced in the atmosphere escapes from it. If this is true and if we are concerned only with an atmosphere of constant density and temperature, we no longer have to consider a complicated transfer equation. In the actual case where the source terms vary through the atmosphere, in general we must consider the transfer problem to determine how much of the total radiation (which we know) will be directed away from the sun and will be observable and how much will be directed toward the photosphere and will be absorbed there. This problem is considered further in the appendix, where it is shown that, for conditions that we expect in the outer solar atmosphere, we may assume that the outward-directed radiation field is equal to the inward-directed field, as long as an error of 30 per cent is tolerable. Thus we do not consider the transfer equation any further. In the final section we shall compute optical depths and diffusion lengths for each line we consider through the atmosphere.

All the lines to be discussed, which range in ionization potential from 16.3 ev for Si II to 523 ev for Si XII, are formed in a limited region of the atmosphere. We assume here that the temperature range in which a given line is formed is determined when the product of the collisional excitation rate and the amount of the ion in question has a maximum value. This will not be true if the atmosphere has no material, or only a negligibly small amount of material, in this temperature range. This assumption is justified in the final section, where it is shown that the resultant knowledge of temperature and density as a function of height is consistent with the assumption and also leads to reasonably accurate predictions of the radio-frequency spectrum.

At electron densities below $10^{10}$ cm$^{-3}$, the principal means of recombination from an ionized state is by radiative recapture rather than by a three-body collision; the ultraviolet continuous radiation in the sun is so small that if high-ionization stages exist, they must be collisionally ionized. Thus the ratio of the population of a single stage of ionization $N_{i+1}$ to that of the immediately lower stage $N_i$ is determined by

$$\frac{N_{i+1}}{N_i} = \frac{C_{i,\, i+1}}{\alpha}, \tag{1}$$

where $C_{i,\, i+1}$ is the rate of collisional ionization and $\alpha$ is the recombination coefficient. The collisional ionization rate has been given by Burgess (1960) and Seaton (1962) as

$$C_{i,\, i+1} = 1.15 \times 10^{-8} \left(3.1 - \frac{1.2}{Z} - \frac{0.9}{Z^2}\right) \frac{T^{1/2}}{(I)^2}\, \zeta\, 10^{-5040 I/T} \text{ cm}^3 \text{ sec}^{-1}, \tag{2}$$

where $\zeta$ = number of electrons in the outer shell, $T$ is the electron temperature, $I$ is the ionization potential in electron volts, and $Z$ is the ionic charge after ionization.

The recombination coefficient has been given by Elwert (1952) as

$$\alpha = 3.0 \times 10^{-12}\, I n_0\, T^{-1/2} \text{ cm}^3 \text{ sec}^{-1}, \tag{3}$$

where $n_0$ is the total quantum number of the ground state of the $i$th stage of ionization. (Elwert increased the numerical factor by 30 per cent for iron.) For helium, the recombination coefficient given by Burgess and Seaton (1960) was used. From these three equations it is possible to find the state of ionization of a given element as a function of temperature.

The question arises as to whether the lines are formed in that region in which the stage of ionization associated with the line is a maximum or in the region in which the next higher stage of ionization is a maximum, i.e., is the λ 833 line of O II formed by collisional excitation in the $O^+$ region or by recombination in the $O^{++}$ region? The collisional excitation rate given by van Regemorter (1962), is

$$C_{1u} = 1.7 \times 10^{-3} N_e T^{-1/2} W^{-1} f_{1u} 10^{-5040W/T} P\left(\frac{W}{kT}\right) \sec^{-1}, \tag{4}$$

where $W$ is the excitation energy in electron volts, $P(W/kT)$ is a correction factor listed in Table 3, and $f$ is the oscillator strength. Since all collisional excitations are radiatively de-excited, the total number per $cm^3$ per sec of O II quanta produced by collisions in the $O^+$ region is

TABLE 3

VALUES OF $P(W\text{-}kT)$

| $W/kT$ | $P$ (Neutral Atoms) | $P$ (Positive Ions) | $W/kT$ | $P$ (Neutral Atoms) | $P$ (Positive Ions) |
|---|---|---|---|---|---|
| 0 01 | 1 16 | 1 16 | 0 4 | 0 209 | 0 290 |
| 02 | 0 956 | 0 977 | 1 | 100 | 214 |
| 04 | 0 758 | 0 788 | 2 | 063 | 201 |
| 1 | 0 493 | 0 554 | 4 | 040 | 200 |
| 0 2 | 0 331 | 0.403 | 10 | 0 023 | 0 200 |

$$\int C_{1u} N(O^+) dh ,$$

where the integral extends over the region where $O^+$ exists. Since $O^+$ is present in the temperature range around $T = 43000°$ K (see Table 2), this integral has the value

$$5.0 \times 10^{-10} A \int_{O^+} N_e^2 dh ,$$

where $A$ is the ratio of oxygen to hydrogen by number. It is assumed, as a first approximation, that all the oxygen is in the form of $O^+$ in this region. The integral in this expression will be evaluated presently. Similarly, the number per $cm^3$ per sec of quanta produced by recombination from $O^{++}$ to the $^4P$ level of $O^+$ (the upper level of the resonance line) is

$$\tfrac{1}{3} \int a N_e N(O^{++}) dh = 3.9 \times 10^{-13} A \int_{O^{++}} N_e^2 dh .$$

Using the values of $\int N_e^2\, dh$, which we shall determine later (the results are given in Table 2), we find that the upper level of the resonance line is populated $10^4$ times more rapidly by collisions from the ground state than by recombinations from the ionized state; thus these collisions determine the upper-level population.

This result is generally true for all the ions with which we deal. Recombinations are most important in the case of the resonance line of He II because $He^+$ exists only in a small region of the atmosphere, while $He^{++}$ exists in the entire region above this. Even in this extreme case, however, collisions from the ground state of $He^+$ are about eight times more important than recombinations in populating the upper level of the resonance line. In this single case a small correction to the assumption that only collisional excitations are important might be contemplated, but, at present, a 10 per cent correction is not significant. There is also the possibility that a contribution to the ultraviolet resonance radiation is made by collisional excitation to some higher level and subsequent cascade to the upper level of the resonance line, from which a resonance quantum is emitted. The collisional rate for such a process is substantially smaller than that to the lowest excited level (at a given temperature) both because the $f$-value may be smaller and because $W$ is larger ($W^{-1}\,10^{-5040W/T}$ may decrease substantially). If one knew the $f$-values more accurately, it would be possible to make a numerical estimate of this effect. For the purposes of our present approach to the problem, we neglect these collisions to higher levels. In helium, where the $f$-values are known, this approximation is valid. For other elements we may look for emission lines which involve transitions with higher levels, and the absence of such lines seems to confirm that the neglect of the higher levels is a good approximation.

We assume that, for a given ion, all the ultraviolet resonance-line radiation is formed by collisional excitation and radiative de-excitation in a volume whose temperature range is appropriate for the existence of the ion. To estimate whether each quantum so formed eventually leaves this volume, we must know the optical depth and the diffusion length for each line. As shown later, typical values of the optical depth ($\tau$) through the shell for a given line range between 0.1 and 3, with the helium values somewhat higher. Because spontaneous radiation from the upper state of a resonance line is much more likely than any other transition, almost all the radiation will be scattered. The quantum can be destroyed either by a collision of the excited ion with an electron or by radiative ionization or by further excitation. On the average, the ratio of a collisional transition to reradiation of the initial quantum will be

$$\frac{10^{-8}N_e}{10^{8}} < 10^{-6}\,.$$

Absorption of radiation from the excited level is even less likely than a collisional transition if a quantum with energy of more than 7 ev is involved. Thus it seems fairly safe to assume that $\lambda$, the fraction of ions which absorb the resonance radiation and eventually reach their ground state without re-emitting (scattering) this radiation, is less than $10^{-6}$. The diffusion length, $\sqrt{(\lambda)}\tau$, is always less than $10^{-1}$, and almost all the radiation produced in the shell leaves the shell.[1]

Half this radiation is directed away from the sun and can be observed (see Appendix 1). The other half is directed toward the photosphere, where it is absorbed. This assumes that the shell is reasonably close to the photosphere (within 100000 km), which is a good approximation.

It seems unlikely that any effects of continuous absorption, especially in the Lyman continuum, are important. We have shown (Pottasch and Thomas 1960) that the optical depth in the Lyman continuum becomes less than unity for the entire atmosphere above 1000 km. Since the temperature is rising and the density is falling, the opacity drops rapidly above this height. Unfortunately, since the actual values of the temperature and density are unknown, we cannot make a more precise estimate of the Lyman continuum optical depth.

## IV. ANALYSIS OF THE OBSERVATIONS

As has been discussed in Section III, we assume that each collisional excitation from the ground state of an ion to the lowest excited state from which electric dipole radiation is allowed results in the emission of a resonance-line quantum. Half the radiation pro-

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[1] For He I, which has the highest diffusion length, $\tau$ is estimated to be 100 at the line center (see final section) and $\lambda = 10^{-6}$. This is a borderline case.

duced leaves the sun, while the remainder is emitted downward and is absorbed in the photosphere. Thus the intensity, $E$(ergs/cm$^2$/sec), emitted from the sun is

$$E=\tfrac{1}{2}h\nu\int N(\text{ion})C_{1u}d\,h\,, \tag{5}$$

where $h\nu$ is the energy of a single quantum, $C_{1u}$ is the collisional rate given in equation (4), $N$(ion) is the density of the particular ion in question, and the integration is performed over all heights $h$ above the solar limb. If we assume that the density of both neutral and ionized hydrogen, $N(\text{H}) = 0.8\ N_e$, then

$$E=0.8\,\frac{h\nu}{2}\,\frac{N(\text{element})}{N(\text{H})}\int\frac{N(\text{ion})}{N(\text{element})}N_eC_{1u}d\,h\,, \tag{6}$$

where $N$(element) is the total density of all the stages of ionization of a given element and it is assumed that the abundance of an element relative to hydrogen does not depend on the position in the solar atmosphere. Substituting equation (4) in equation (6), we have

$$E=1.1\times10^{-15}Pf_{1u}\,\frac{N(\text{element})}{N(\text{H})}\int 10^{-5040W/T}\,\frac{N(\text{ion})T^{-1/2}}{N(\text{element})}\,N_e^2d\,h\,. \tag{7}$$

The intensity $E$ measured at the earth's distance from the sun is found by multiplying this quantity by $2.18\times10^{-5}$.

For a given stage of ionization, the function

$$g(T)=T^{-1/2}10^{-5040W/T}\,\frac{N(\text{ion})}{N(\text{element})}$$

can be shown from equations (1), (2), and (3) to have a sharp maximum at a given temperature and to fall off rapidly for lower or higher temperatures. To a reasonable approximation, we may assume that this function, $g(T)$, is zero when it is less than one-third its maximum value, since the function decreases very quickly outside this region. As we have stated before, this will be a good approximation if there is a reasonable amount of material that has a temperature within this temperature range. We can get into difficulty, however, if only a negligible amount of material is within this temperature range. We cannot show that we have made the correct assumption at this point in the investigation, but it will be considered in detail in the final section.

Inside this region we assume that the function has a constant value, equal to 0.7 times its maximum value, i.e.,

$$\langle\, g(T)\,\rangle=0.7\ \text{Max}\ [\,g(T)\,]\ ,$$

which is found from normalizing $\Delta T\langle g(T)\rangle$ to $\int g(T)\,dT$. Then the intensity measured at the earth's distance becomes

$$E=7.6\times10^{-21}Pf_{1u}\,\frac{N(\text{element})}{N(\text{H})}\,\langle\, g(T)\,\rangle\int_R N_e^2d\,h\,, \tag{8}$$

where the integral is now limited to the temperature range where $\langle g(T)\rangle$ is not zero, i.e., to the region of formation of the particular observed line.

Since $\langle g(T)\rangle$ can be computed, $f_{1u}$ is known, and the intensity $E$ has been measured, we may find for each ion the quantity

$$\frac{N(\text{element})}{N(\text{H})}\int_R N_e^2d\,h\,.$$

All these quantities are listed in Table 2 for each ion. The range of temperature over which the line is formed and over which the integral $\int N_e^2 dh$ is taken is also given in the table, as well as the temperature $T_{\max}$, where the function $g(T)$ reaches a maximum.

Notice in Table 2 that the temperature ranges in which a given ion is produced are very similar from one ion to another. For example, S II is produced in the temperature range 19000° and 36000° K; C II in the range 21000° and 40000° K; N II in the range 25000° and 50000° K; etc. If we assume that the temperature is increasing as a reasonably smooth function of height, this would imply that the height range for the formation of a given line is a reasonably smooth function of the ionization potential. Since it is generally assumed that $N_e$ is also a smooth function of height, this implies that

$$\int_R N_e^2 dh$$

is also a smooth function of height and therefore of ionization potential.

Consider the quantity

$$\frac{N(\text{element})}{N(\text{H})} \int_R N_e^2 dh$$

given in Table 2, for a single element, as a function either of the temperature range over which the integral is taken or of some temperature near the middle of the range where the function $g(T)$ has its maximum value. We find that, for a single element, the series of points obtained from each ion of that element form a smooth curve. Since we assume that the composition does not vary with temperature or height in the atmosphere, the form of this curve must be determined only by the variations of

$$\int_R N_e^2 dh$$

with temperature. Thus the curves determined from two elements are, in reality, a single curve but are displaced by an amount that is directly proportional to the relative abundances of the two elements. By fitting all the elements to a single curve, we may determine the relative abundances of the elements, and we have chosen to refer all the abundances to that of oxygen. Figure 1 is a plot of

$$\frac{N(\text{O})}{N(\text{H})} \int_R N_e^2 dh$$

for each ion against $T_{\max}$, roughly the average temperature of the region in which the observed line is formed. The temperature range over which the ion is formed is also indicated. In the region of $T_{\max}$ less than 300000° K, there is a sufficient number of ions of each element that the relative abundances are fairly well determined, and the curve

$$\int_R N_e^2 dh$$

is reasonably well defined. That such a smooth curve can be obtained, with none of the individual points differing by as much as a factor of 2 from the curve demonstrates the internal consistency of the arguments used and the observational data. This does not necessarily mean that the abundances found are better than this factor, because a systematic effect might influence all ionization stages of a single element, although it is not immediately apparent what such an effect might be.

There is a reliable point (O VI) at $T_{\max} = 220000°$ K, and another one (Si XII) exists at $T_{\max} = 1500000°$ K. The curve between is interpolated. Given these two points and the slope of the curve on the low-temperature side, the interpolation is not too uncertain.

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It is less reliable than the previous portion of the curve simply because of its dependence on the accuracy of the Si XII point. We may obtain further information on how high the curve can go in the intervening region by noting that the Si VIII, IX, X, and XI lines have not yet been identified, indicating that the curve probably cannot be higher than shown. Thus, although the neon, iron, and magnesium abundances are not so reliable as the others, they are not far behind.

Small systematic effects are observable. For example, the five ions whose ground-state electron configuration is $1s^2\ 2s^2$, have values of

$$\int_R N_e^2 dh$$

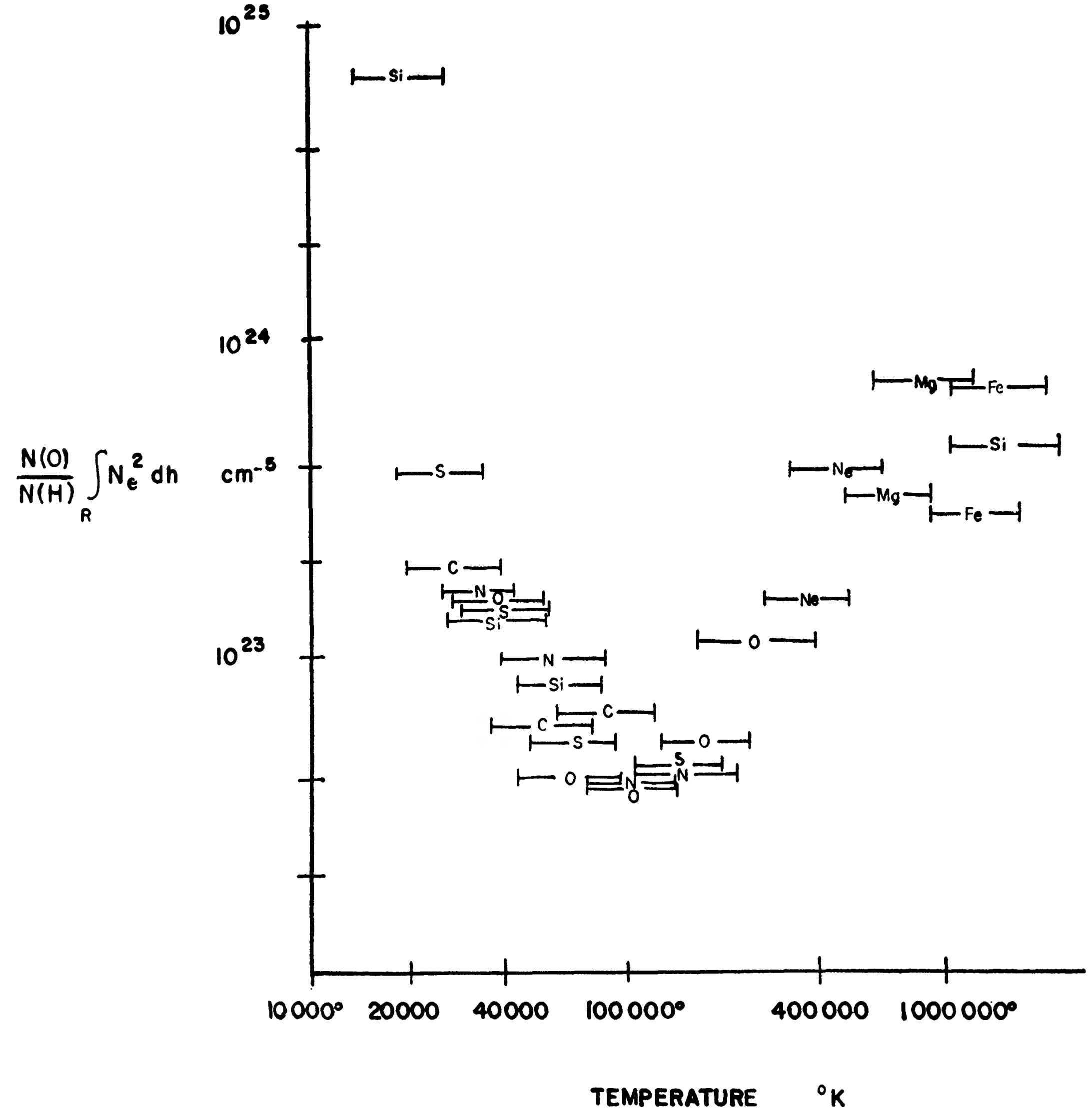

FIG. 1.—The value of $N(O)/N(H)\int_R N_e^2 dh$ cm$^{-5}$, where the integral is taken over the temperature range in which the resonance line of the observed ion is formed, is plotted against the temperature of that region. For a given element, each plotted point represents a different stage of ionization. The relative chemical composition is adjusted so that as little scatter as possible from a common curve is obtained.

which are, on the average, 30 per cent below the average curve drawn in Figure 1. Similarly, those five ions (excluding Si XII, because it alone determines its portion of the curve) whose ground-state electron configuration is $1s^2\ 2s$ have values of

$$\int_R N_e^2 dh$$

which average 25 per cent above the average curve. It is probable that such differences as appear along an isoelectronic sequence are attributable to errors in the $f$-value determinations. If it is true that errors in the $f$-values used here appear systematically in one direction or another along an isoelectronic sequence, this will minimize any systematic errors in the relative abundances due to $f$-value inaccuracies, when more than one stage of ionization of a given element is observed.

A further word should be said about the determination of the helium abundance because the energy-level structure of helium is somewhat different from that of the other elements. In neutral helium there is a triplet system, so that collisional excitations from the ground state (singlet) to the $2^3P$ level return to the ground state by way of collisional transitions to the $2^1P$ level, where they radiate the λ 584 resonance line. Thus one must include in the collisional term the collisions from the ground state to the $2^3P$ level, which was done by using the ratio of $1^1S$–$2^1P$ to $1^1S$–$2^3P$ rates summarized by Athay and Johnson (1960). Furthermore, in H, He, and $He^+$ one must include the collisional transitions to the 2S levels from the ground state, since from the 2S state a collisional transition to the 2P level again leads to emission of a resonance-line quantum. As a first approximation to this effect, we have increased the collision rates by 50 per cent.

It is useful to point out that the relative abundances are obtained without any knowledge of the detailed density and temperature distributions in the outer atmosphere. The initial assumption that

$$\int_R N_e^2 dh$$

is a smooth function as it varies with temperature or height in the atmosphere has been borne out in the region below $T = 300000°$ K, and, while the possibility remains of rapid changes in this function in the higher-temperature regions, it would now appear unlikely.

If a particular region of the sun were especially brighter than its surroundings, it still would not influence the abundance determination, since its effect would simply be to raise a portion of the curve

$$\left(\int_R N_e^2 dh \text{ versus } T\right).$$

This would change the results, however, if one combined the observations for a day in which such a region appeared with those for a quiet day, using some lines from one day and some from the other. Of necessity, we have made such a combination; the observations above 900 A were taken about 16 months before those below. The earlier observations refer specifically to the quiet portion of the sun, while the latter observations refer to the integrated disk. However, reasonable agreement was obtained in the overlapping wavelength regions, and the spectrum below 1300 A remained the same as that found a year earlier (Hinteregger 1961, 1962). It is thus felt that no serious error has been made by combining the two sets of observations in this case, but, if possible, one would prefer to work with data that were taken simultaneously.

### V. DETERMINATION OF RATIO OF THE SEVERAL ELEMENTS TO HYDROGEN

In the previous section we have shown how it is possible to determine the relative abundances of nine elements. Unfortunately, the method used may not apply to hydrogen for two reasons: (*a*) the region in which Lyman-$\alpha$ is formed probably has a diffusion length in Lyman-$\alpha$ greater than unity; and (*b*) even if this were not true, the value of

$$\int_R N_e^2 dh$$

for Lyman-$\alpha$ could be fitted onto the curve (Fig. 1) only with difficulty because we have not established the shape of the curve very accurately in the temperature region in which Lyman-$\alpha$ is formed, below $T = 20000^\circ$ K.

We must determine the ratio of hydrogen to the other elements in some other manner. For convenience, we continue to use oxygen as a standard. Such an alternative method exists, using an entirely different set of observations—the radio-frequency measurements between 1 and 50 cm.

As shown in Figure 1 or in Table 2 (cols. 9 and 12), we have determined the quantity

$$\frac{N(\mathrm{O})}{N(\mathrm{H})}\int_R N_e^2 dh \tag{9}$$

as a function of the average temperature of the region over which the integral extends. The average values of this integral are shown again in the second column of Table 3, while the temperature range over which the integral extends is shown in the first column. We may determine the quantity

$$\frac{N(\mathrm{O})}{N(\mathrm{H})}\int_R N_e^2 T_e^{-3/2} dh \tag{10}$$

simply by dividing by some appropriate average temperature. We do not know precisely what temperature to use, but we do know that it must lie within the temperature range over which the integral extends and probably more toward the lower end of this range because, presumably, $N_e$ is greater at the lower temperature and thus the greater part of the integral is contributed by the lower-temperature regions. The temperature ranges are small enough that little significant error is made by the choice of a temperature. The average temperatures chosen are shown in the fourth column of Table 4. We may now find this same quantity, with the same lower limit on the integral, but with the upper limit extended to infinity, i.e., far out in the solar corona. We do this simply by summing all the higher-temperature ranges. Thus we determine the quantity

$$\frac{N(\mathrm{O})}{N(\mathrm{H})}\int_{h(T_L)}^{\infty} N_e^2 T_e^{-3/2} dh, \tag{11}$$

where $T_L$ is the lower temperature of the range considered and is given in the first column of Table 4.

The observed radio emission at a given wavelength at the center of the solar disk comes chiefly from a region of the atmosphere where the optical depth at that wavelength is approximately unity. Because the temperature increases strongly outward, the observed emission corresponds very nearly to the temperature at the optical depth $\tau = 0.5$, as one can readily ascertain by assuming several reasonable models, computing the brightness temperature, and determining at exactly what optical depth this temperature actually occurs. For radio radiation it seems reasonably certain that the source function is equal to the Planck function, since the radiation is produced by a transition between two continuum states and a Maxwellian velocity distribution has been assumed.

Using the absorption coefficient given by Oster (1961) or Scheuer (1960) and assuming a hydrogen:helium ratio of 7, we find, in the region where the refractive index is close to unity,

$$\tau = \int_{h(T_L)}^{\infty} \kappa_\nu d\nu$$

and

$$0.5 = \lambda^2\, 1.33 \times 10^{-23} \log_e A \int_{h(T_L)}^{\infty} N_e^2 T_e^{-3/2} dh\,, \tag{12}$$

where $T$ is the temperature found in the atmosphere at $\tau = 0.5$ and is the observed brightness temperature as well. The quantity $A = 1.65 \times 10^{-3} T_e^{3/2} \lambda$, and $\lambda$ is the wavelength in centimeters. Since we now know the integral in equation (12), except for the ratio $N(\mathrm{O})/N(\mathrm{H})$, we can solve equation (12) to determine the wavelength at which a given brightness temperature $T$ will be observed. In this way we can predict $\lambda$ as a function of $T$. The results found are plotted in Figure 2 for five different assumptions of the ratio $N(\mathrm{O})/N(\mathrm{H})$.

TABLE 4

CONVERSION OF ULTRAVIOLET DATA FOR USE IN PREDICTING RADIO-FREQUENCY CONTINUUM

| TEMPERATURE RANGE | | $N(\mathrm{O})/N(\mathrm{H}) \int_R N_e^2 dh$ | $T_{\mathrm{av}}$ | $N(\mathrm{O})/N(\mathrm{H}) \int_R N_e^2 T_e^{-3/2} dh$ | $N(\mathrm{O})/N(\mathrm{H}) \int_{h(T_L)}^{\infty} N_e^2 T_e^{-3/2} dh$ |
|---|---|---|---|---|---|
| $T_L$ | $T_u$ | | | | |
| $10^6$ | $2\ 4\times10^6$ | $3\ 7\times10^{23}$ | $1.4\times10^6$ | $2\ 2\times10^{14}$ | $2\ 2\times10^{14}$ |
| $6\ 0\times10^5$ | $10^6$ | $4\ 0\times10^{28}$ | $7.6\times10^5$ | $6\ 0\times10^{14}$ | $8\ 0\times10^{14}$ |
| $3\ 5\times10^5$ | $6\ 0\times10^5$ | $2\ 6\times10^{23}$ | $4\ 6\times10^5$ | $8\ 3\times10^{14}$ | $1\ 6\times10^{15}$ |
| $2\ 0\times10^5$ | $3\ 5\times10^5$ | $1\ 1\times10^{23}$ | $2\ 7\times10^5$ | $7\ 8\times10^{14}$ | $2\ 4\times10^{15}$ |
| $1\ 1\times10^5$ | $2\ 0\times10^5$ | $4\ 5\times10^{22}$ | $1\ 3\times10^5$ | $9\ 6\times10^{14}$ | $3\ 4\times10^{15}$ |
| $7\ 0\times10^4$ | $1\ 1\times10^5$ | $4\ 1\times10^{22}$ | $8\ 7\times10^4$ | $1\ 6\times10^{15}$ | $5\ 0\times10^{15}$ |
| $3\ 5\times10^4$ | $7\ 0\times10^4$ | $9\ 0\times10^{22}$ | $4\ 5\times10^4$ | $9\ 4\times10^{15}$ | $1\ 4\times10^{16}$ |
| $2\ 0\times10^4$ | $3\ 5\times10^4$ | $3\ 3\times10^{23}$ | $2\ 3\times10^4$ | $9\ 5\times10^{16}$ | $1\ 1\times10^{17}$ |
| $1\ 4\times10^4$ | $2\ 0\times10^4$ | $6\ 5\times10^{24}$ | $1\ 7\times10^4$ | $2\ 9\times10^{18}$ | $3\ 0\times10^{18}$ |

We can now use the observed values of the brightness temperature $T$ as a function of $\lambda$ to determine the correct value of $N(\mathrm{O})/N(\mathrm{H})$. The observed values are plotted as points in Figure 2.[2] There seems to be a noticeable difference between the radio observations taken at sunspot maximum and those made at sunspot minimum, which indicates a slight change in the structure of the atmosphere between maximum and minimum. At the time of the ultraviolet measurements used in this paper, 1960 and 1961, the structure was probably closer to that at sunspot maximum than minimum, so that $N(\mathrm{O})/N(\mathrm{H})$ is approximately $7 \times 10^{-4}$, with an error of a factor of 2 considered probable.

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[2] The observations referring to sunspot maximum are from the following investigators: 62, 10.2 cm (1946), 50, 25, 10.2 cm (1947), Pawsey and Yabsley (1949) *Australian J. Sci. Res. A*, **2**, 198; 88 cm (1957), Firor (1959); 21 cm (1957), Waldmeier (1959), Newkirk (1959). Where apparent temperatures are given, these have been converted to central temperature. The observations for sunspot minimum are 0.86 cm (1954), Coates (1958), *Ap. J.*, **128**, 83; 3 cm (1954) Hachenberg, Furstenberg, and Prinzler (1956), *Zs. f. Ap.*, **39**, 232; 3 cm (1952) Blum, Denisse, and Steinberg (1952), *Ann. d'ap.*, **15**, 184; 10 cm (1954), Haddock (1957); 10.3 cm (1954), Covington, Medd, Harvey, and Broten (1955), *J. R. Astr. Soc. Canada*, **49**, 235; 21 cm (1952–1954), Christiansen and Warburton (1955), *Australian J. Phys.*, **8**, 474; 60 cm (1954), O'Brien and Tandberg-Hanssen (1955), *Observatory*, **75**, 13; Hewish (1957). Where no references are given, (1957) = *I.A.U. Symposium*, No. 4, ed. H. C. van de Hulst; (1959) = *I.A.U. Symposium*, No. 9, ed. R. N. Bracewell.

The similarity of the shape of the predicted $T$ versus $\lambda$ curve and that observed is striking. The values of $N(\mathrm{O})/N(\mathrm{H}) = 7 \times 10^{-4}$ are reasonably close to the values determined from photospheric analyses: Osterbrock and Rogerson (1961) find $N(\mathrm{O})/N(\mathrm{H}) = 1.4 \times 10^{-3}$ and Goldberg, Müller, and Aller (1960) find $N(\mathrm{O})/N(\mathrm{H}) = 9 \times 10^{-4}$.

### VI. DISCUSSION OF THE ABUNDANCE

The relative abundances of the chemical elements in the outer atmosphere of the sun derived in the present analysis from the ultraviolet resonance lines are summarized in the first two columns of Table 5. Oxygen is used as a standard, and the number of atoms of a particular element relative to the number of oxygen atoms is adjusted so the oxygen is 100. It is hard to estimate the error that should be placed on these figures, but, to obtain a general idea, one would estimate that the mean error of the observations for the lines of the several ions of a given element is 40 per cent and that the uncertainties due to the $f$-values along an isoelectronic sequence are, on the average, 50 per cent. This gives an over-all uncertainty of about a factor of 2 for the first five elements in the list and a somewhat higher uncertainty for neon, magnesium, and iron because of the interpolation of the curve for these elements, as discussed above. Helium is also uncertain because it is more sensitive to knowledge of the ionization equilibrium than are the other elements.

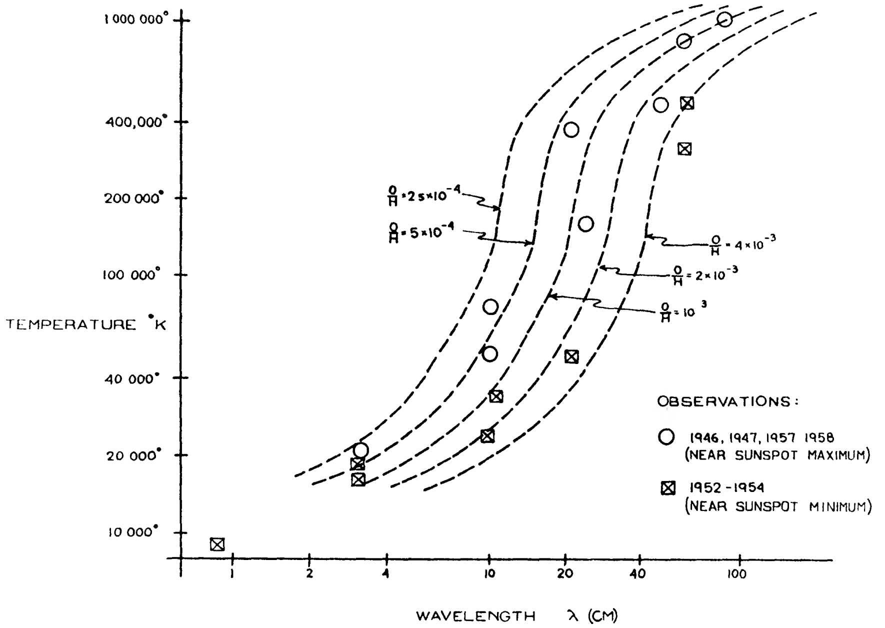

FIG. 2.—Using the atmospheric structure derived from the ultraviolet lines, the radio-frequency spectrum of the sun can be predicted as a function of the oxygen-to-hydrogen ratio. The dashed curves show the predicted central brightness temperature against wavelength. The circles and squares indicate the observed brightness temperatures. The comparison is used to derive the ratio of hydrogen to the other elements.

In the third column of Table 5 the results of the curve-of-growth analysis of the abundances in the solar photosphere by Goldberg, Müller, and Aller (1960) are given. These results agree with the present determinations within a factor of 2, if one uses hydrogen as a standard, for the elements oxygen, nitrogen, and sulfur. Helium and neon, which have no observable lines in the solar photosphere spectrum, agree within a factor of 2 with the abundances found in the planetary nebulae (Aller 1961). Carbon is found to be a little more than a factor of 3 more abundant in the present analysis, but this may not be significant, especially in view of the fact that the photospheric carbon determination is quite uncertain. Thus agreement seems quite good for six of the nine elements observed. Magnesium, however, is a factor of 5 higher, and silicon and iron are 8 and 20 times higher, respectively. The possible errors in the Goldberg, Müller, and Aller analysis, insofar as the observations go, are much less than ours, and the *f*-values they have used for neutral lines have, in several cases, more experimental bases than those used here. One must look outside these sources of error to explain the difference.

The photospheric analysis contains two additional possible sources of error: (1) the model of the photosphere used may be wrong; (2) the assumption of local thermodynamic equilibrium may be wrong. It is to be noted that the present abundance determination from the ultraviolet lines is independent of a detailed knowledge of the temperature and density distributions in the outer atmosphere, and it makes no assumption of the existence of local thermodynamic equilibrium.

TABLE 5

RELATIVE ABUNDANCES OF CHEMICAL ELEMENTS

| ELEMENT | SUN | | | PLANETARY NEBULAE (ALLER) | COSMIC RAYS (AIZU *et al*; PETERS; NEY) |
|---|---|---|---|---|---|
| | Ultraviolet Lines (Present Analysis) | Photospheric Analysis (Goldberg, Müller, and Aller) | Coronal Analysis (Woolley and Allen) | | |
| Oxygen | 100 | 100 | | 100 | 100 |
| Nitrogen | 25 | 11 | | 40 | 65 |
| Carbon | 250 | 58 | | 80 | 130 |
| Sulphur | 6 | 2 2 | . . | 10 | 5 |
| Calcium | 0 5 (:)* | 0 16 | 0 3 | | 9 |
| Silicon | 30 | 3 5 | | | 13 |
| Sodium | 0 5 (:) | 0 22 | | . .. | 2 |
| Neon | 12 | . . . | | 20 | 13 |
| Magnesium | 15 | 2 8 | . . | | . . |
| Iron | 10 | 0 4 | 10 | ..... | 25† |
| Helium | 30000 | ..... | ...... | 32000 | 5000 |
| Hydrogen | 150000 | 110000 | 150000 | 160000 | 63000 |

* (:) These are upper limits on the abundances of these elements

† This refers to the "iron group," i e, $23 \leq Z \leq 30$, of which it is assumed that iron is the principal constituent

The last two columns of Table 5 give the chemical composition of planetary nebulae and of cosmic rays. It is interesting to note that, with the exception of carbon, there is a much closer agreement between the planetary nebulae and our present abundances than with the abundances determined from the photospheric analysis. The abundance of carbon in planetary nebulae is most uncertain because it is not determined from an observation of a forbidden line. No conclusion can be drawn from this agreement, however, as there is no compelling reason that the sun and planetary nebulae should have the same abundances and the controversial elements iron, silicon, and magnesium are not observed. One must also bear in mind that our present knowledge of planetary nebulae is crude, and this may well affect the abundance determinations.

The cosmic-ray abundances given in the last column of Table 5 are probably also uncertain because of the difficulty of separating the heavy elements from each other. Again, however, the agreement with the ultraviolet line analysis is much closer than with the photospheric analysis, and here silicon and iron are observed, but again there need be no relation between the cosmic-ray abundances and those of the sun. (Aizu *et al.* report on the cosmic-ray abundances between 200 and 700 Mev and state that there appear to be no abundance variations with energy.)

A more direct and meaningful comparison is with the hydrogen:iron ratio, which has been determined from the forbidden coronal iron lines by several authors (Woolley and Allen 1948; Shklovskii 1951). These authors, using different data but assuming spherical symmetry, find the same hydrogen:iron ratio, $8 \times 10^{-5}$. This is very close to the hydrogen:iron ratio found in the present ultraviolet line analysis, but a factor of 20 more than the photospheric analysis.

One notices, in examining Table 5, that, apart from oxygen, most of the elements we have analyzed are more abundant than found in the photospheric analysis. This may be partly a selection effect, since we can observe only the most abundant elements. This suggests that we look for other elements for which the photospheric analysis has found an abundance similar to that of iron and make a determined effort to look for their lines. Calcium and sodium both have a photospheric abundance within a factor of 2 of iron, and we should investigate them in some detail.

The sodium resonance line most likely to be observed is the line at $\lambda$ 411.1 due to Na VIII. This line has a large $f$-value (0.56) and arises from the same electron configuration as the observed lines of Ne VII and Mg IX. It is in a region of the spectrum that was observed with high sensitivity, but no strong line was found in the immediate vicinity. Attributing the intensity at this wavelength entirely to the Na VIII line, we find an upper limit to the abundance of sodium (see Table 5) which is only a factor of 2 greater than that observed in the photospheric analysis.

The calcium resonance line most likely to be observed in the available wavelength range is the line at $\lambda$ 466.2 due to Ca IX. This line has a very large $f$-value (1.7) and arises from the same electron configuration as the Fe XV line at $\lambda$ 284. This line is blended with Ne VII, but if we attribute 20 per cent of the observed intensity to the Ca IX line, we find an upper limit to the abundance of calcium, which is shown in Table 5. We find that the ratio of iron to calcium must be at least 20, while the photospheric analysis finds a ratio of only 2.5. This immediately recalls to mind the results of the coronal abundance analysis, using the forbidden lines of iron and calcium (Wooley and Allen, 1948), where an iron-to-calcium ratio of 30 was found.

The agreement between the abundances found from the forbidden-line coronal analysis and our present analysis leads us to conclude that the results presented here are reasonable and must be considered seriously. If these conclusions are assumed to be correct, it still leaves open the question as to whether we are dealing with differences in abundances between the photosphere, on the one hand, and the chromosphere and corona, on the other, or whether the photospheric analysis, especially with regard to iron, magnesium, silicon, and carbon, is in error.

One may then ask whether the chemical composition of the low corona actually differs from that of the photosphere, perhaps because interplanetary dust and meteoritic material fall into the sun. Whipple (1961) has estimated that between $10^8$ and $10^9$ gm of matter fall into the sun per second from the interplanetary medium. Using the higher estimate of infall of matter and assuming the mean atomic weight of this matter to be 20, then $5 \times 10^8$ particles/cm$^2$/sec are falling into the sun. Then, in order for the infalling matter to reach the observed abundance, we require that its drift velocity be $10^2$ cm/sec at a point where $N_e = 10^{10}$ cm$^{-3}$ and $10^3$ cm/sec at $N_e = 10^9$ cm$^{-3}$. However, actual computation of the drift velocity shows that it is $10^4$ cm/sec and $10^5$ cm/sec at $N_e = 10^{10}$ cm$^{-3}$ and $10^9$ cm$^{-3}$, respectively. Thus the material proceeds much too rapidly through the outer atmosphere to affect substantially the abundance determination. It will require further work to ascertain whether or not an abundance difference between the photosphere and the outer atmosphere can be maintained.

## VII. THE TEMPERATURE AND DENSITY DISTRIBUTIONS IN THE OUTER SOLAR ATMOSPHERE

Using the hydrogen:oxygen ratio found in Section V, we now may determine the average value of the integral

$$\int_R N_e^2 dh$$

over a given range of temperature. This value is shown in the last column of Table 2 and refers in each case to the region whose temperature range is given in column 11 of the same table. This seems to be the maximum information about the temperature and density distributions that can be derived from the present ultraviolet observations. It is not an unexpected result, since we have observed only the radiation integrated over the entire atmosphere.

In the preceding section we have shown that this information is consistent with the temperature and density distributions that can be obtained from the continuous radio-frequency spectrum. The results derived here are quite similar but somewhat more detailed because the integral found is taken over shorter increments in height. Even so, if we wish to determine the density and temperature as a function of height, we must have other information. Such a determination is a subject for future work.

We must now justify our assumption that the diffusion length $[\sqrt{(\lambda)}\tau]$ is less than unity for all the lines we have considered. For example, consider the Ne VII resonance line λ 465. Here

$$\int_R N_e^2 dh = 7 \times 10^{26}$$

over the region where Ne VII exists. If we assume that $N_e = 5 \times 10^9$ in this region, which is only an educated guess, then

$$\int_R N_e dh = 1.4 \times 10^{17} .$$

Since we have found $N(\mathrm{Ne})/N(\mathrm{H}) = 7 \times 10^{-5}$ and all neon is in the form of Ne VII in this region,

$$\int_R N(\mathrm{Ne\ VII})\, dh = 10^{13} .$$

Now the absorption coefficient at the line center is $\kappa_0 = 2.8 \times 10^{-14}$ cm$^2$, so that $\tau = 3 \times 10^{-1}$. The scattering parameter $\lambda = 10^{-8}$, so that the diffusion length at the line center is about $3 \times 10^{-5}$, substantially less than unity. This same qualitative result is true for all the lines we have considered; the optical depth $\tau$ varies from $10^{-2}$ to 20, and the diffusion length is always substantially less than unity.

Let us now return to the assumption that most of the observed radiation from a given ion comes from a temperature region in the atmosphere that is most favorable for the production of this radiation, i.e., that the radiation originates from a region in the atmosphere where $g(T)$ (see Sec. III) is within one-third of its maximum value. As we have previously mentioned, this assumption will be in error if there is negligible material present in the region of favorable temperature. Now that we have determined $\int N_e^2\, dh$ for each temperature region, we can compute how much radiation would have come from the surrounding temperature region which we did not include by assuming that $g(T)$ was zero in those temperature regions. As an example we shall compute the errors made for O IV, which is the most unfavorable case, since $\int N_e^2\, dh$ has its smallest value in this temperature region. We can now use the actual values of $g(T)$ that correspond to each particular temperature region instead of zero, as previously used. The results are shown in Table 6. From this table we see that 87 per cent of the radiation is indeed formed in the region in which we had assumed it to be. This is, of course, only a consistency check, since it is possible that the assumption was wrong and led to wrong values of $\int N_e^2\, dh$, although the values are consistent with the assumption. But we have a further check on the values of $\int N_e^2\, dh$ that is given by the radio-frequency continuum measurements discussed in the previous section. There it was shown that the values of $\int N_e^2\, dh$ found by the two methods are the same, within the errors of measurement, from which one may conclude that the consistency check is an adequate justification of the assumption.

## VII. SUMMARY

It has been the purpose of this paper to show that the following two pieces of information can be obtained from the present observations of the ultraviolet emission resonance lines of ions whose ionization potential is greater than hydrogen:

1. A common characteristic of all models that will produce the observed ultraviolet spectrum can be obtained. This is a relationship between the temperature range in which a given ion chiefly contributes to its resonance-line emission and $\int N_e^2\, dh$ over that temperature range. This relationship is shown in columns 11 and 12 of Table 2. Not enough information is contained in the present observations completely to determine the temperature-density distribution in the outer solar atmosphere, but this relationship severely limits the choice of a model.

TABLE 6

AMOUNT OF O IV RESONANCE-LINE RADIATION FORMED OUTSIDE MOST FAVORABLE TEMPERATURE REGION

| Temperature Region (° K) | Per Cent of O IV Radiation Formed in Region | Temperature Region (° K) | Per Cent of O IV Radiation Formed in Region | Temperature Region (° K) | Per Cent of O IV Radiation Formed in Region |
|---|---|---|---|---|---|
| 14000–27000 . | $10^{-8}$ | 28000–55000 | 0 023 | 76000–150000 | 87 0 |
| 19000–35000 | $10^{-6}$ | 45000–82000 | 9 6 | 170000–350000 | 3 3 |

2. The chemical composition of the outer atmosphere of the sun can be determined for 10 elements from the ultraviolet observations *without* a detailed knowledge of the temperature and density distributions. The results are given in Table 5. These results show important differences from the chemical composition found for three elements in the photosphere, which can mean one of two things, if we accept the methods and results given in this paper. Thus either (*a*) the results using the curve-of-growth method may be in error for several elements, because either the model of the photospheric regions used was inadequate or the assumption of local thermodynamic equilibrium is too far from the truth or (*b*) the chemical composition in the solar atmosphere differs from the photosphere to the corona. No plausible mechanism for causing such a difference has yet been presented, but little thought has been directed toward this end. In this paper we have considered the influx of meteoric material from the interplanetary medium, but it seems insufficient to account for a difference in chemical composition.

There are two criticisms that can be applied to the present work. The first is that the errors that have been used are too low. For example, the errors in observations of the intensities may be greater than 50 per cent, in spite of the observer's estimates of the probable errors; the corrections which must be made for atmospheric absorption may be underestimated; or the errors in the collisional cross-sections may be greater than a factor of 2, in spite of Seaton's and van Regemorter's estimates. We may have introduced some errors in other assumptions made. Is it possible that these errors are great enough to account for the abundance differences noted between the present analysis and the photospheric curve-of-growth analysis? This is considered extremely unlikely, at least for C, O, N, Si, and S, for the following reason. To reduce the abundance of carbon by a factor of 6, we must have an error of that magnitude for each of the three stages of ionization observed—C II, C III, and C IV—and all these errors must be in the same direction. In general, the errors we expect are either random or systematic along an isoelectronic sequence, and thus we should regard it as unlikely that an error of such magnitude could affect all the observed stages of ionization of only a single element, such as carbon, in precisely the same manner.

The other criticism to which this analysis is subject is more fundamental in nature. Although we contend that it is possible to derive the abundances without detailed knowledge of the temperature-density structure of the region of the atmosphere where these lines are formed, we have implicitly assumed that conditions are reasonable. We never can be certain of this until much more is known of the chromosphere and corona. To this end, the fact that we can predict the radio-frequency spectrum from this analysis is encouraging. Probably the uncertainty regarding this point could be removed if one could observe the ultraviolet spectrum as a function of height in the solar atmosphere and successfully interpret this spectrum.

The author is indebted to Drs. J. T. Jefferies, D. Morton, H. van Regemorter, M. Schwarzschild, L. Spitzer, Jr., B. Strömgren, and R. N. Thomas for their comments and to the National Science Foundation for supporting the latter phases of this investigation under contract G 24278.

## APPENDIX

It was assumed in an early section of this paper that, in a shell through which the diffusion length $\tau\sqrt{(3\lambda)} \ll 1$ and in which conditions correspond to those in the outer solar atmosphere, the amount of radiation emitted by each side of the shell will be equal. It is clear that if all the radiation is produced at one side of an atmosphere in which the optical depth is very large, most of the radiation will be emitted from that side, and only a little will diffuse through to the other side of the atmosphere. Is this true for the case we considered in this paper? It seems probable that, for the ultraviolet lines we are considering here, the source term does not vary by more than a factor of 10 in a given line in that part of the solar atmosphere that is appreciably deep in that line, and the optical depth for any given line is not more than 30. We shall investigate here the error involved in our assumption for this particular case.

We shall assume that the transfer equation in the Eddington approximation,

$$\tfrac{1}{3}\frac{d^2 J}{d\tau^2} = \lambda J - \lambda B \,,$$

is an adequate representation for a first approximation. We shall assume that the scattering parameter $\lambda$ is constant in the atmosphere but that the source term $B$ varies,

$$B = B_0 (1 + V e^{\sqrt{3q\tau}}) \,.$$

By allowing only $B$ to vary, the problem is simplified but still meaningful, since it is probable that, in the actual atmosphere, $B$ varies more than $\lambda$.

We assume a solution in the form

$$J = A + C e^{-\tau\sqrt{(3\lambda)}} + D e^{\tau\sqrt{(3\lambda)}} + E e^{-\sqrt{3q\tau}}$$

and subject to the boundary conditions that there is no incident radiation on either side of the shell, so that at

$$\tau = \tau_0 , \qquad J = -\frac{1}{\sqrt{3}}\frac{dJ}{d\tau} .$$

The constants then are, assuming $\tau_0\sqrt{(3\lambda)} \ll 1$:

$$A = B_0 ; \qquad E = -\frac{\lambda V B_0}{q^2 - \lambda} ;$$

$$\begin{aligned}(\mathrm{Den})C = & -A\,[2\sqrt{\lambda} + \tau_0(1+\sqrt{\lambda})\sqrt{(3\lambda)} + \tfrac{3}{2}\lambda\tau_0^2(1+\sqrt{\lambda})] \\ & + E\,[e^{-q\tau_0\sqrt{3}}(q-1)(\sqrt{\lambda}-1) - (q+1)(1+\sqrt{\lambda}) \\ & - \tau_0(q+1)(1+\sqrt{\lambda})\sqrt{(3\lambda)} - \tfrac{3}{2}\lambda\tau_0^2(q+1) \\ & \times (1+\sqrt{\lambda})] ;\end{aligned}$$

$$\begin{aligned}(\mathrm{Den})D = & -A\,[2\sqrt{\lambda} + \tau_0(1-\sqrt{\lambda})\sqrt{(3\lambda)} - \tfrac{3}{2}\lambda\tau_0^2(1-\sqrt{\lambda})] \\ & + E\,[e^{-q\tau_0\sqrt{3}}(q-1)(1+\sqrt{\lambda}) + (q+1)(1-\sqrt{\lambda}) \\ & - \tau_0(q+1)(1-\sqrt{\lambda})\sqrt{(3\lambda)} \\ & + \tfrac{3}{2}\lambda\tau_0^2(q+1)(1-\sqrt{\lambda})] ;\end{aligned}$$

and

$$(\mathrm{Den}) = 4\sqrt{\lambda} + 2\tau_0(1+\lambda)\sqrt{(3\lambda)} + 6\lambda\tau_0^2\sqrt{\lambda} .$$

The solutions for the emergent intensity from each side of the shell then are

$$\begin{aligned}(\mathrm{Den})J(\tau = 0) = & A\,[2\lambda\tau_0\sqrt{(3\lambda)} + 3\lambda\tau_0^2\sqrt{\lambda}] + 2E\sqrt{\lambda}\,[e^{-q\tau_0\sqrt{3}}(q-1) \\ & + (1-q) + \tau_0(\lambda-q)\sqrt{3} + \tfrac{3}{2}\lambda\tau_0^2(1-q)] ;\end{aligned}$$

$$\begin{aligned}(\mathrm{Den})J(\tau = \tau_0) = & A\,[2\lambda\tau_0\sqrt{(3\lambda)} + 3\lambda\tau_0^2\sqrt{\lambda}] + 2E\sqrt{\lambda}\,\{e^{-q\tau_0\sqrt{3}} \\ & \times [(1+q) + \tau_0(\lambda+q)\sqrt{3} + \tfrac{3}{2}\lambda\tau_0^2(1+q)] \\ & - (1+q)\} .\end{aligned}$$

We must now specify the way in which $B$ varies through the shell, i.e., what the values of $V$ and $q$ should be. The value of $V = 20$ was chosen, but the results would be almost the same had any larger value of $V$ been used, and smaller values of $V$ would have reduced the variation of $B$. The value of $q$ was chosen so that $q\tau_0\sqrt{3} = 3$, which will allow $B$ to vary by a factor of 10, regardless of what the total optical depth depth of the atmosphere, $\tau_0$, is.

In the following table the ratio of the mean intensity of radiation $J$ at $\tau = \tau_0$ to half the total amount of emitted radiation is shown as a function of $\tau_0$, the total optical depth of a shell. We have assumed that $\lambda = 10^{-8}$. It is noted that for optical depths of the order of 30, which is about the highest we are dealing with in the present discussion, the error made by assuming that half the total radiation is emitted in each direction is about 30 per cent.

TABLE A

FRACTION OF TOTAL RADIATION EMITTED ON ONE SIDE OF ATMOSPHERE

| $\tau_0$ | $\frac{J(\tau=\tau_0)}{\frac{1}{2}[J(\tau=0)+J(\tau=\tau_0)]}$ | $\tau_0$ | $\frac{J(\tau=\tau_0)}{\frac{1}{2}[J(\tau=0)+J(\tau=\tau_0)]}$ |
|---|---|---|---|
| 0 3. | 0 92 | 30 . | 0 64 |
| 1 | 83 | 100 | 62 |
| 3 | 72 | 300 | 62 |
| 10 . . . | 0.66 | 1000 | 0 62 |

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