

Astrometric Study of Barnard's Star from Plates Taken with the 24-inch Sproul Refractor

PETER VAN DE KAMP

Sproul Observatory, Swarthmore College

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Twenty-five consecutive years of photographic observations of Barnard's star show deviations from uniform proper motion and secular acceleration which can be represented by Keplerian motion with a period of 24 yr and semi-axis major of $''0245 \pm ''002$ (p.e.). Assuming a value of $0.15\odot$ for the mass of Barnard's star, the mass of the companion proves to be $0.0015\odot$, or 1.6 times the mass of Jupiter.

INTRODUCTION

BARNARD'S star, BD $+4^{\circ}3561$ ($17^{\text{h}}52^{\text{m}}9$, $+4^{\circ}25'$, 1900), visual magnitude 9.54, spectrum dM5, is the second nearest known stellar system, with a parallax of $+''545 \pm ''003$, a proper motion of $10''.31$ in 356° , and a radial velocity of -108 km/sec (Gliese 1957). The first Sproul parallax series includes 24 plates taken over the years 1916–1919. Beginning 1938, a large number of plates was taken each year for the purpose of improving the parallax, determining the secular acceleration, and testing the star for possible variable proper motion. A tentative interpretation of the latter, made almost two decades ago (van de Kamp 1944) is superseded by the results of the present investigation.

A determination of the secular acceleration was made in 1961 and an abstract reported in this Journal (van de Kamp 1962d). After correcting for this acceleration, a perturbation appears to be established; an orbit is derived and interpreted. This investigation is being continued. Color and magnitude equations are being further studied; after improving our knowledge of these systematic errors a determination of the parallax will be made. Adjusted values for secular acceleration and orbital elements will be determined when warranted.

MATERIAL AND MEASURES

The procedure described in previous Sproul papers has been followed (van de Kamp and Lippincott 1949).

The basic exposure time in the years 1916–1919 was 15 minutes, 1938–9: 2–3 minutes; 1940–9: 1 minute; 1950 on: $3/4$ minute. The actual exposure time varied, mostly upward, depending on seeing, transparency and temperature. Cramer–isochromatic plates were used in the years 1916–1919, Eastman G-emulsion in the

years 1937–1939 and from 1949 on, Eastman C-emulsion in the years 1940–1948.

Fifty observers participated in the photographing of the 2413 plates with 8260 exposures, obtained on 609 nights over the interval 1916–1962, resulting in a total weight of 1621. The outstanding observer was Roy W. Delaplaine, who over the years 1938–47 observed on 162 nights, or 26.6% of the total number. During the war years 1943 to 1945, while the only observer, he photographed Barnard's star on 75 nights, or 12.3% of the total. The next largest contribution was by Laurence W. Frederick, who, over the years 1951–59, observed on 75 nights, or 12.3% of the total. Armstrong Thomas (1939–41; 1946), Hans K. Roth (1948–51), A. Wayne Conger (1949–1954), John H. Pitman (1916–19; 1938–41), Leonard F. Barcus (1938–40; 1942), Leendert Binnendijk (1947–50) and Arne A. Wyller (1954–57) added a combined total of 157 nights, or 25.7% of the total. These eight observers contributed, therefore, 64.6% of all material. The remaining 215 nights were taken care of by 42 observers, contributing an average of 5.1 nights, ranging anywhere from one night in one year to 13 nights over 3 years.

Three reference stars were chosen as a suitable system which was not unduly large but would yet serve for several decades (Fig. 1). The choice of reference stars and the possible transfer to another reference system has been discussed elsewhere (van de Kamp 1947).

Table I gives the dependences and their annual changes, as well as the standard frame and position for Barnard's star in 1940.000.

The rapid changes in the dependences and in the geometric accuracy are illustrated in Table II, and

TABLE I. Reference system.

No.	diameter	m_{pv}	Sp	Standard frame		Dep. 1940	$\Delta D/\text{yr}$
				x_s	y_s		
1	$''077$	10.2	G0	-25.12	-0.66	$+0.128$	$+0.0114$
2	$''150$	9.0	B8	-9.65	-31.55	$+0.689$	-0.0144
4	$''097$	9.8	F2	$+34.77$	$+32.21$	$+0.183$	$+0.0030$
Barnard's star	$''115$	9.54	dM5	-3.10	-21.40		

FIG. 1. Path of Barnard's star. Reference stars 1, 2, 4 and their proper motion displacements in 10 000 yr.

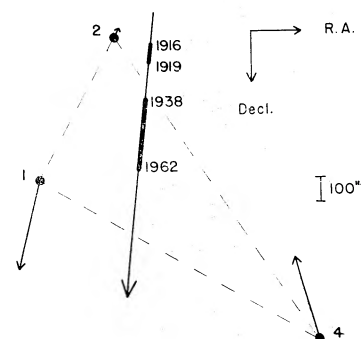


TABLE II. Dependences and inverse weight.

Epoch	1	Dependences 2	4	Inverse weight $1+[D^2]$
1910	-.213	+1.121	+.092	2.31
20	-.099	+.977	+.122	1.98
30	+.014	+.833	+.153	1.72
40	+.128	+.689	+.183	1.52
50	+.242	+.545	+.213	1.40
60	+.356	+.401	+.243	1.35
70	+.469	+.258	+.273	1.36
80	+.583	+.114	+.303	1.44

Figs. 2 and 3 (see also Tables II and III, van de Kamp 1947).

Configuration 124 reaches its highest geometric accuracy in 1962.

MAGNITUDE AND COLOR EQUATION

The reference configuration is, unavoidably, not well balanced with respect to diameters and spectra, and errors may be expected accordingly.

Magnitude, i.e., diameter compensation, is poor, star 2 being bright, star 1 faint, the difference amounting to over one magnitude. All diameters increase not only with increased exposure time but also with poor seeing, when the diameters for star 1 become uncertain, and those for star 2 grow out of proportion. There is always a slight excess of diameter in right ascension x as compared with declination y due to guiding error. Both the diameters and the elongation—diameter in x minus diameter in y —vary appreciably, from year to year as well as within any one year. The diameter of Barnard's star ranges from .06 to .19 mm; the average diameter is .115 mm, .124 mm in x , .105 mm in y .

The rapid changes in dependences, particularly for stars 1 and 2, in opposite senses, imply an ever changing magnitude compensation, which changes sign about 1955, but which depends not only on the epoch but also

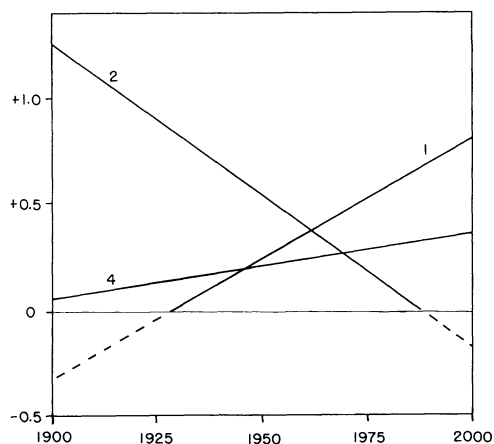


FIG. 2. Barnard's star. Dependences for reference stars 1, 2, 4 over twentieth century.

on the diameters for any one year, night or plate. Adopting a set of "standard" values for the diameters of the reference stars and of Barnard's star, the dependence average diameter decreases by .0010 mm per year. Table III gives the relevant data.

Positions obtained with the Sproul refractor are subject to color equations in right ascension due to adjustments of the objective; all positions in right ascension are reduced to one and the same system by subtracting an amount γ_x for color equation from positions obtained over the interval 1937–1941.822 and again after 1949.21, respectively (Lippincott 1957). The color equations are well established; it has been found convenient and sufficiently accurate to consider their amounts as a linear function of the spectral type expressed as an increasing color parameter with unit differences between the spectral types B0, A0, F0, G0, K0, M0, and M5. The effect γ_{x0} per unit color is $+^{mm}00061$ over the interval 1937–1941.822 and one fourth of this amount, i.e., $+^{mm}00015$ after 1949.21.

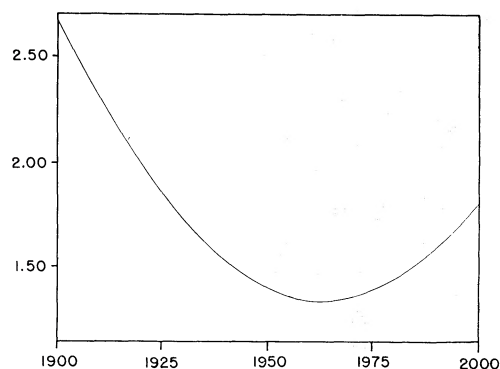


FIG. 3. Inverse weight $1+[D^2]$ of reduced positions of Barnard's star over twentieth century.

For Barnard's star the spectral type is dM5, that of the dependence average background is A3, resulting in a color difference of 4.65 units (for 1940). The corresponding color equation γ_x over the interval 1937–1941.822 is $+^{mm}0028$, after 1949.21 it is $+^{mm}0007$.

PERSONAL EQUATION OF MEASUREMENT

The plates have been measured on the Gaertner long-screw measuring machine over the interval 1938–1963, principally by Sarah Lee Lippincott, assisted by Mrs. Mary Jackson, and Mrs. Dorothy B. Allen. Nine other measurers participated. To simplify further study at this stage, the material has been grouped into 99 normal points, one for the interval 1916–1919, and 98 over the interval 1938–1962. The normal points are of average weight 16, and average number of plates 24; their number in any one year beginning with 1938 ranges from 2 to 8. Three-fourths of the normal points were measured twice. An inter-comparison of the 12 measurers and their times of

measurements revealed no definite time trend in the behavior of the measuring machine. Remeasurements by the same measurers show a satisfactory agreement.

Intercomparison of measurers of the same plates by different measurers establishes personal equation, which is more pronounced in right ascension than in declination, undoubtedly partly due to guiding error, but also to the spacing of the images in right ascension. Oil creep along the screw as related to personal pattern of measurement is considered to be among the causes of personal equation. Remeasurements of plates with small images show a much better agreement than those with large images. Large systematic differences are found for the plates taken in 1951 and 1952 when the magnitude compensation is comparatively small but frequent inferior seeing increased the sizes of all images, particularly star 2 with a corresponding increased elongation due to guiding error.

It was decided to allow for personal equation, using steps of $^m00025$. The average absolute corrections

TABLE III. Diameter compensation.

Epoch	Dependence average diameter	Diameter Barnard's star minus reference system
1910	$^{mm}.160$	$^{mm}-.045$
20	$^{mm}.150$	$^{mm}-.035$
30	$^{mm}.140$	$^{mm}-.025$
40	$^{mm}.130$	$^{mm}-.015$
50	$^{mm}.120$	$^{mm}-.005$
60	$^{mm}.110$	$^{mm}+.005$
70	$^{mm}.100$	$^{mm}+.015$
80	$^{mm}.090$	$^{mm}+.025$

amounted to $\pm ^m00064$ in x and $\pm ^m00041$ in y , attesting to both a reasonable constancy of the Gaertner machine over two decades and the generally satisfactory small degree of personal equation.

Table IV lists the corrections used to reduce all measurers to the Lippincott system of measurements. After these corrections were applied straight averages of the measurements were used.

SECULAR ACCELERATION

Barnard's star has the largest known secular perspective acceleration in proper motion due to the large values of radial velocity, proper motion, and parallax. This acceleration amounts to

$$-2''.05V_{\mu p} \times 10^{-6} \text{ or } ^m00124$$

(van de Kamp 1962b). The probable error $\pm ^m00003$ of the predicted acceleration could be reduced primarily by a better determination of the radial velocity. The components of the secular perspective acceleration are $^m00010$ and $^m00123$ in right ascension and declination, respectively (van de Kamp 1962 a,e).

TABLE IV. Corrections to Lippincott system.

Measurer	unit $^{mm}.0001$	
	x	y
Sarah Lee Lippincott	0	0
Mary Jackson	+ 7.5	+10.0
Dorothy Allen	+10.0	+ 2.5
Marion Wolff	+ 7.5	+10.0
Judith Damkoehler	- 5.0	- 5.0
Laurence W. Frederick	-15.0	- 5.0
Marion Heaney	+10.0	- 2.5
Charlotte Garceau	+ 2.5	0
Martha Binnendijk	+ 5.0	+ 2.5
Betty Ann Mateer	+ 5.0	+ 5.0
Richard Woltman	+ 2.5	- 2.5
Leendert Binnendijk	0	0

The determination of the secular acceleration was made before adjusting all measurements to the Lippincott system. Subsequent applications of personal equation will hardly change this study, first reported in 1962, which will now be presented in somewhat more detail. All measurements were reduced to the standard frame (Table I) by the usual method. The yearly mean positions were corrected for an adopted parallax value of $^m0290$ and were represented by equations

$$X - \gamma_x - ^m0290P_\alpha = c_x + \mu_x t + h_x t^2,$$

$$Y - ^m0290P_\delta = c_y + \mu_y t + h_y t^2,$$

where the symbols have the usual meaning; h_x and h_y are half the acceleration in x and y , respectively; t is counted from 1950.000.

A least-squares solution yields

$$c_x = -3.923304, \quad c_y = -10.508272,$$

$$\mu_x = -.0424699, \quad \mu_y = +.5445396,$$

$$h_x = +.000003166, \quad h_y = +.000013461.$$

Expressed in seconds of arc (1 mm = $18''.87$) the observed acceleration is therefore

$$\text{in } x \quad +''00012 \pm ''00005,$$

$$\text{in } y \quad +''00051 \pm ''00002.$$

The probable errors of unit weight are $\pm ''099(x)$ and $\pm ''045(y)$. The abnormally high value in x will be explained later on in terms of a perturbation. The secular perspective acceleration is derived by correcting the observed acceleration for the effect of spurious acceleration introduced by the proper motions of the reference stars and the changing dependences of Barnard's star (van de Kamp 1935a). The proper motions of the reference stars on a background of 24 faint stars (including the reference stars but excluding Barnard's star) were obtained from two pairs of plates with an interval of 44 yr. The epochs of the two early plates are 1916.634 and 1917.611, that of the two recent plates is 1960.686. Linear plate constants were calculated from 16 stars having small proper motions; the

TABLE V. Proper motions of 24 stars in field of Barnard's star. 1, 2 and 4 are the reference stars used in the present astrometric study. Periods indicate the stars used for the calculation of plate constants.

	<i>x</i> in mm	<i>y</i> in mm	<i>m_{pe}</i>	annual <i>μ_x</i>	annual <i>μ_y</i>
.	28.8	113.9	10.3	−".0006	+".0080
.	32.6	96.8	11.0	+ 20	− 2
.	36.7	52.4	10.9	− 46	− 54
.	37.7	114.0	10.7	+ 170	− 9
.	49.6	52.0	11.0	+ 51	− 6
.	51.3	66.4	8.6	− 42	− 42
.	68.2	131.1	10.7	+ 375	− 474
1	80.8	109.3	10.2	− 78	+ 365
.	87.1	105.2	11.2	+ 34	+ 20
.	89.2	106.4	10.9	− 170	− 180
.	95.1	90.8	10.7	+ 50	+ 104
2.	96.3	78.5	9.0	+ 9	− 18
.	102.6	129.2	10.4	− 22	− 64
.	108.9	62.8	10.8	+ 31	+ 50
.	113.2	75.8	10.9	+ 44	− 34
.	125.2	90.5	10.9	− 4	− 76
.	126.1	109.6	10.8	− 40	+ 26
.	128.6	82.8	9.7	− 575	− 434
.	133.8	92.9	10.9	+ 94	− 85
.	134.1	66.4	10.9	− 29	+ 79
.	137.8	67.7	11.0	− 12	+ 16
4	140.7	142.3	9.8	− 96	− 350
.	147.5	81.9	11.1	− 31	− 14
.	153.4	107.9	10.9	+ 42	+ 6

resulting proper motions are given in Table V (see also Fig. 4).

The observed absolute value of the average (relative) proper motion of the 16 background stars is

$$\pm".00318 \quad (\pm".00289 \text{ in } x, \pm".00347 \text{ in } y)$$

corresponding to a probable *observed* "cosmic" error (van de Kamp 1962c) of

$$\pm 0.8453 \times ".00318 \left(\frac{16}{16-3} \right)^{\frac{1}{2}} = \pm".0029.$$

The absolute value of the average difference between the proper motions derived from the two pairs of plates amounts to $\pm".0022$, corresponding to a probable error of $(0.8453/2) \times ".0022 = \pm".0009$ for the proper motion derived from the two pairs of plates. This error is, of course, included in the above value of the observed cosmic error.

The spurious acceleration of Barnard's star is calculated from the following data:

Star	ΔD	μ_x	μ_y
1	+ .0114	−".0078	+".0365
2	− 144	+ 9	− 18
4	+ 30	− 96	− 350

which yield the following values for the spurious acceleration and its probable error resulting from the effect of the observed cosmic error, i.e., a linear dis-

tortion of the reference system (van de Kamp 1962c)

$$\text{in } x \quad -2[\Delta D \mu_x] = +".00026 \pm ".00003,$$

$$\text{in } y \quad -2[\Delta D \mu_y] = -.00067 \pm .00003.$$

The correction of the observed acceleration for the spurious acceleration and the resulting value for the true perspective acceleration are summarized below:

	(<i>x</i>)	(p.e.)	(<i>y</i>)	(p.e.)
observed acceleration	+".00012 ± ".00005		+".00051 ± ".00002	
correction for spurious acceleration	−.00026	3	+ .00067	3
true perspective acceleration	−".00014	6	+".00118	4
predicted perspective acceleration	−.00010	1	+ .00123	3.

The agreement is excellent and it can be said that the perspective secular acceleration of Barnard's star at last has been accurately measured (Fig. 4). The present determination supersedes the earlier results (van de Kamp 1935b, 1951).

PERTURBATION

The positions for all normal points, adjusted to the Lippincott system, were reduced by adding the "negative ephemeris"

$$-X_c = -\gamma_x + 3.923304 + .0424699t - .000003166t^2 - .0290P_a, \quad (A)$$

$$-Y_c = +10.508272 - .5445396t - .000013461t^2 - .0290P_b,$$

based on the solution for secular acceleration. The resulting remainders

$$X - X_c,$$

$$Y - Y_c,$$

are now available for further analysis. The 98 remainders over the interval 1938–1962 indicate a curious yearly behavior in declination, namely, a systematic decrease with the time within each season (March to September). This apparent seasonal effect is strongly correlated to the brightness, or rather the diameter of the image of

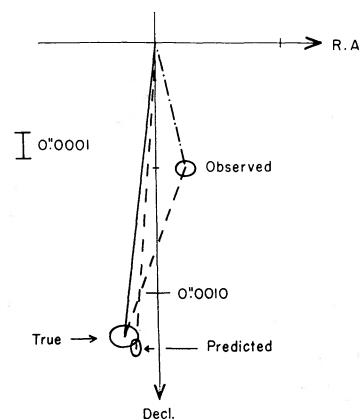


FIG. 4. Perspective secular acceleration of Barnard's star. The true acceleration is the vectorial sum of the observed acceleration and the correction for spurious acceleration. The small ovals represent probable errors.

TABLE VI. Observational data. Yearly mean remainders R_x and R_y based on (A) and (B) and corrected for diameter effect; residuals v_x and v_y for orbit: $P=24$ yr, $e=.6$, $T=1950$.

Epoch	nights	Number of		Total weight	Remainders		Residuals	
		plates	exposures		R_x	R_y	v_x	v_y
					unit $^{\text{mm}}$.0001		unit $^{\text{mm}}$.0001	
1918.048	21	24	50	24	- 9.6	+14.2	- 2.3	+11.0
38.489	23	77	171	48	- 1.0	+ 3.4	+ 2.6	+ 2.9
39.511	21	62	199	48	- .5	- 4.7	+ 4.2	- 6.0
40.480	28	108	381	79	- 3.3	+ 8.5	+ 2.4	+ 6.4
41.511	46	192	656	137	- 6.8	+ 8.8	- .4	+ 6.0
42.538	29	142	526	94	-12.3	+ 1.0	- 5.4	- 2.5
43.517	21	130	506	73	- 7.9	- 1.0	- .9	- 5.1
44.519	29	114	427	83	-10.6	+ 2.2	- 3.8	- 2.4
45.516	25	93	333	68	- 2.5	+ 6.1	+ 3.6	+ 1.1
46.545	41	230	820	124	- 2.3	+ 3.8	+ 2.3	- 1.4
47.511	40	183	538	105	- 7.2	+ 3.3	- 4.8	- 1.7
48.550	25	95	350	67	+ 0.9	+ 6.5	- .4	+ 2.5
49.426	32	152	528	88	+11.4	+ 3.6	+ 5.5	+ 1.2
50.462	19	65	230	44	+ 3.7	+ 1.2	- 7.6	+ 1.8
51.552	24	92	315	64	+10.3	- 9.3	- 3.0	- 6.4
52.532	29	108	391	80	+14.3	- 3.8	+ 1.4	0
53.510	19	64	194	46	+19.7	+ 5.4	+ 8.1	+ 9.6
54.536	19	65	183	43	+12.3	- 0.7	+ 2.4	+ 3.5
55.509	15	57	176	41	- 0.4	- 8.3	- 8.5	- 4.3
56.530	13	46	174	36	- 6.0	- 1.6	-12.2	+ 1.9
57.529	15	54	206	41	+ 7.4	- 9.1	+ 3.0	- 6.1
58.500	17	63	231	48	+11.5	- 4.6	+ 8.9	- 2.2
59.486	14	55	184	37	+ 4.7	- 5.6	+ 3.9	- 3.9
60.506	6	21	80	16	0	- 7.6	+ .8	- 6.6
61.444	12	48	183	34	+ .8	+ 0.4	+ 3.0	+ .6
62.468	19	73	278	55	- 9.9	+ 2.0	- 6.3	+ 1.5
Total	609	2413	8260	1621				

Barnard's star, since with increasing seasonal temperature the star image tends to be smaller due to decreased sensitivity of the photographic emulsion. Systematic errors depending on color and brightness have been noticed and studied before (Schlesinger 1937). We have assumed the present effect to be linearly related to the diameter. A correlation diagram shows an increase in declination of $+^{\text{mm}}0010$ for an increase in diameter (in y) of $^{\text{mm}}050$.

By 1956 we definitely excluded a short-period perturbation but realized that a perturbation with a period of 16 yr or more was indicated. We can now see that the earlier near-year period (van de Kamp 1944) was spurious and was caused by the above-mentioned magnitude error. The measurements in right ascension could be adjusted to confirm this notion by simply changing the value of the parallax. To analyze the long-period perturbation we used yearly mean positions, averaging 96 plates (average weight 64) over the interval 1938–1962. These positions were corrected for the diameter effect, using a diameter (in y) of $^{\text{mm}}110$ for Barnard's star as a standard. The mean diameters (in y) for the yearly means range from $^{\text{mm}}076$ to $^{\text{mm}}151$, corresponding to extreme corrections of $+^{\text{mm}}0007$ and $-^{\text{mm}}0008$.

The resulting yearly means clearly exhibit a long-period systematic run, mainly in x , but also in y , whether diameter equation is applied or not, which admit of no simpler explanation than that of a perturbation caused by an unseen companion of Barnard's star.

Because of the early observations (1916–19) both proper motion and secular acceleration of Barnard's star are known with high accuracy. Although it is still possible to admit small changes both in the proper motion and the period of the perturbation, it appears unlikely that the period could be shorter than 23 yr or much over 25 yr. A period of 24 yr has been adopted, together with minor adjustments in proper motion and quadratic time term. The following terms were added to the remainders:

$$\left. \begin{array}{l} \text{in } x \quad +0.6t - 0.01t^2 \\ \text{in } y \quad -0.4t \end{array} \right\} \text{unit } ^{\text{mm}}.0001 \quad (\text{B})$$

Table VI lists the yearly mean remainders R_x and R_y (corrected for diameter effect) based on reductions (A) and (B), together with other relevant information.

The time-displacement relations R_x/t and R_y/t (Fig. 5) are well represented by a Keplerian motion with the

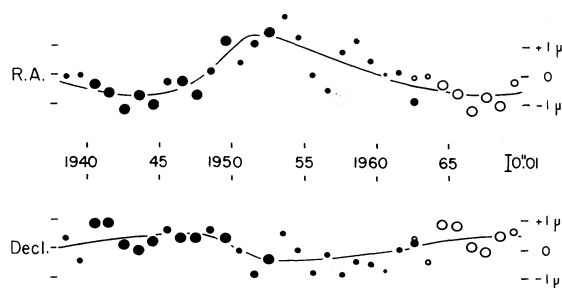


FIG. 5. Barnard's star—yearly means, averaging 96 plates and weight 64. Time displacement curves for $P=24$ yr, $e=0.6$, $T=1950$. Circles are early means transferred 24 yr forward. The scale of the displacements is shown both in terms of $0''.01$ and of 1μ (.001 mm) on the Sproul plates.

dynamical elements

$$\begin{aligned} P &= 24 \text{ yr,} \\ e &= .6, \\ T &= 1950. \end{aligned}$$

The resulting values for the geometric elements are (van de Kamp 1962d)

$$\left. \begin{aligned} (B) &= +6.06 \\ (A) &= +.30 \\ (G) &= +10.24 \\ (F) &= -5.87 \end{aligned} \right\} \text{unit } {}^{\text{mm}}.0001,$$

while the probable errors of unit weight, derived from the yearly residuals v_x and v_y (Fig. 6) of average weight 64, are

$$\begin{aligned} \text{p.e. } 1_x &= \pm {}^{\text{mm}}.0027 = \pm ''050, \\ \text{p.e. } 1_y &= \pm {}^{\text{mm}}.0023 = \pm ''044. \end{aligned}$$

The largest residuals are $-{}^{\text{mm}}00122$ (weight 36) in x and $+{}^{\text{mm}}00096$ (weight 46) in y .

The size of the reference configuration $[x_s^2] = 19.33 \text{ cm}^2$, $[y_s^2] = 20.33 \text{ cm}^2$, indicates an expected probable error of unit weight of $\pm {}^{\text{mm}}0020$ (Lippincott 1957). Hence, in view of the likely existence of systematic errors, varying from year to year, the agreement of the observed and the expected p.e. 1 is satisfactory.

A test of the reality of the perturbation lies in a comparison of the above p.e. 1 and the p.e. 1 calculated

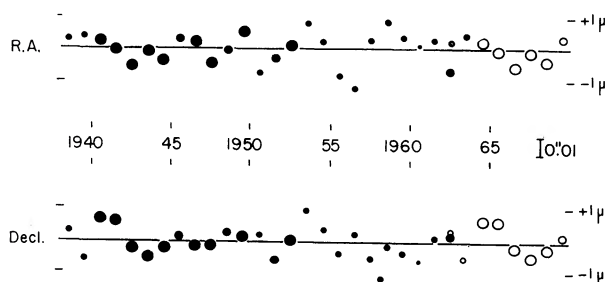


FIG. 6. Barnard's star: yearly means. Residuals from $P=24$ yr, $e=0.6$, $T=1950$.

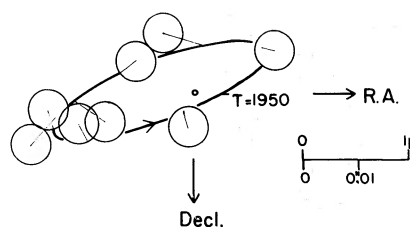


FIG. 7. Barnard's star—apparent orbit. Eight normal points of average weight 203. $\circ$ center of mass. Radii of circles indicate probable errors. $\alpha = 1.30\mu = 0''.0245$, $i = \pm 77^\circ$.

on the assumption that the observed data contain no perturbation but are ascribed to errors of observations:

p.e. in	ignoring perturbation	allowing for perturbation
R.A.	$\pm ''089$	$\pm ''050$
Dec.	$\pm ''053$	$\pm ''044$

The geometric elements yield a value of $\alpha = {}^{\text{mm}}0013 = ''0245$ for the semi-axis major of the observed photocentric orbit relative to the center of mass. The probable errors of the geometric elements as well as of α , are slightly below $\pm {}^{\text{mm}}0001$, or $\pm ''002$. The inclination of the orbit is $\pm 77^\circ$. The observed orbit is illustrated in Fig. 7; the yearly means have been combined into eight normal points of average weight 203.

It is excluded that the observed perturbation could be a reflex of a perturbation in one of the reference stars. The relevant data are given below.

Reference star No.	Dep. 1950	Sp.	App. vis. mag.	Adopted abs. mag.	parallax	α	Upper limit $D\alpha$
1	+ .25	G0	10.2	+4.5	''007	''028	''007
2	+ .53	B8	9.0	-0.5	''001	''005	.003
4	+ .22	F2	9.8	+3.2	''005	''022	.005

Assuming a 24-yr period and reasonable upper limits of the values for the sum of the masses for any one of the reference stars and an upper limit of $B-\beta=0.4$, values of the semi-axis major α of the photocentric orbit are derived. Multiplying these by the dependence (taken for 1950), upper limits $D\alpha$ are obtained for the semi-axis major of any possible reflex effect on Barnard's star of a photocentric orbit of any one of the reference stars. The largest value ''007 is *well below* the value ''0245 found from observations.

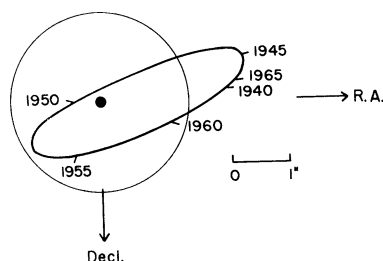
DYNAMICAL INTERPRETATION

A dynamical interpretation of the perturbation requires knowledge of the mass of Barnard's star. Both in absolute magnitude (13.27) and infrared-red color index (Kron *et al.* 1957), Barnard's star is close to Ross 614A and to Krüger 60B, which have masses of $0.14\odot$ and $0.16\odot$, respectively (van de Kamp 1959).

We have adopted a mass of $0.15\odot$ for Barnard's star, which for a period of 24 yr yields a semi-axis major of

$$[0.15 \times (24)^2]^{\frac{1}{3}} = 4.42 \text{ a.u., or } 2''.41$$

FIG. 8. Barnard's star and companion
—Apparent relative orbit. Circle represents extreme size of image of Barnard's star.



for the relative orbit of Barnard's star and a comparison of negligible mass.

The relative apparent orbit is similar to the apparent orbit of Barnard's star (but differs 180° in phase). We note that the apparent separation is over $2''$ from 1939 to 1946, and again from 1963 to 1970 (Fig. 8). On the average, the radius of an image of Barnard's star is $^m058$, or $1''.0$, while there are few images with a radius larger than $^m080$, or $1''.5$. The companion image should therefore be clear of the image of Barnard's star, certainly on the Sproul plates taken over the interval 1937–1947, and again after 1961. An inspection of the best plates reveals no sign of any companion, as indeed is to be expected because of the small mass, indicated by the small amplitude of the perturbation. Were a companion seen in the predicted positional angle, its very visibility would imply a mass sufficiently large to cause a much larger perturbation than is observed. In other words, somewhat paradoxically, a visible companion would disprove the very prediction which would have led to its discovery.

Assuming, therefore, no blending effect of Barnard's star and its companion because of (1) wide separation and (2) low luminosity, we may write

$$\frac{m}{M} = \frac{a}{a} = 0.0102.$$

Hence, for $M = 0.15 \odot$ we find

$$m = 0.0015 \odot.$$

CONCLUSION

The orbital analysis leads, therefore, to a perturbing mass of only 1.6 times the mass of Jupiter. We shall interpret this result as a companion of Barnard's star, which therefore appears to be a planet, i.e., an object of such a low mass that it would not create energy by the conventional nuclear conversion of hydrogen into

helium. Apart from the eccentricity, the spatial relationship of Barnard's star A and its companion B is not very different from that of the sun and Jupiter. The surface temperature of Jupiter is in the neighborhood of 120°K . Barnard's star has an absolute visual magnitude of 13.27, or visual luminosity of $0.0004 \odot$. The surface temperature of Barnard's star (dM5) may be taken as 2900°K (cf. Krüger 60B, Limber 1958). The surface temperature of the companion of Barnard's star may be estimated to be in the neighborhood of 60°K on the average as a result of the radiation from Barnard's star. As seen from Barnard's star, our planet Jupiter at greatest elongation from the sun would appear as an object of the 24th visual magnitude. The companion of Barnard's star would appear to us as an object fainter than the 30th visual magnitude, beyond the reach of existing instruments.

Judging from the probable errors of unit weight the present astrometric study appears to confirm our initial hope and expectation that a substantial increase in the amount of observational material—number of images per plate, number of plates per night, number of nights per year—would result in a substantial increase in accuracy. In other words, the law of diminishing returns has not seriously inhibited the attainable accuracy expected from the intensive observational effort. Special mention should be made of Roy W. Delaplaine, who photographed more than one-fourth of the material; Sarah Lee Lippincott, who measured the great majority of the plates; and Mrs. Mary Jackson, and Mrs. Katherine Gray, who assisted particularly with the recent measurements and calculations, respectively. The continued assistance from the National Science Foundation is gratefully acknowledged.

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