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THE USE OF A PENTAG WITH A TRANSIT-CIRCLE

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Summary

Preliminary work with a four-inch quartz pentag, mounted on the seven-inch Cooke T.C. to assess its usefulness for flexure determinations, showed that if the mounting is kept completely free of the telescope tube (as seems desirable), small orientation-errors are difficult to avoid. The second-order effects in declination which may be introduced by these errors, and by their interactions with any (permanent) error in the pentag-angle itself, are therefore all investigated; account is taken of the curvature of the diurnal circles, and of the need to observe off-axis in the telescope. It is found that of the three small angles which specify the pentag's mal-alignment, one has strictly zero effect. The second can readily be determined, at all pointings, by auto-collimation observations, and, with the Herstmonceux mounting, has been found to be so stable that these observations are not burdensome. If this angle is known, as a function of the pointing, the third one can be determined for each star by rough observations in R.A. The complete corrections for mal-alignment, to be added to the observational equation already published, are given explicitly.

It was shown in a previous paper (1) that there are serious deficiencies in all the methods so far used, or tried, for determining the flexure of a transit-circle; a new method was then proposed which involved the use of a "pentag". This is the name for a device consisting essentially of two plane mirrors, very rigidly connected so as to enclose an angle P say, which remains constant with high accuracy. It is easily verified that if a ray of light is incident on one mirror of the pair, in the plane containing both their normals, and is then reflected in the second, it is deviated through precisely $2P$ whatever the angle of incidence; this property has many important applications. Most commonly, P is designed to be 45°, so as to give a 90° deviation. If such a pentag is mounted in front of a transit-circle objective, with the plane of its normals vertical, stars observed in it will have declinations differing by 90° from those for which the circle is set; if the same star is observed (presumably on different nights) with and without the pentag, differencing the two observations leads to an equation of condition for the harmonic coefficients of the flexure as a function of zenith distance. This equation was given explicitly, as far as the pair of third harmonics.

After the 7-inch Cooke T.C. had been moved from Greenwich and brought into operation at Herstmonceux, a fused quartz pentag of four inches clear aperture was obtained and mounted on it for a trial period. The mounting, mainly of aluminium alloy angle-rods and plates, was carried entirely by thermoplastic shoes riding in the machined grooves, on the E–W axis, in which the lifting and reversing gear also engages when required; it had no direct contact with the telescope tube at all. It was counterpoised about the

E–W axis, so that it could be swung out of the beam and back again, and the pentag itself could rotate independently about a parallel axis (at A, in Fig. 1), this motion also being counterpoised. The complete dissociation from the telescope tube seemed necessary, because the "flexure" to be investigated is really differential flexure; if the tube, even though it did not carry the pentag's full weight, were used to constrain its orientation in any way, the reaction might perhaps double, or neutralize, the differential flexure even though the force involved was relatively small.

It is, however, somewhat difficult to preserve this independence and yet to guarantee that the plane of the pentag's normals is not displaced from the meridian, either by becoming inclined to the telescope-axis or by being rotated about it. If either of these faults occurs, second-order errors may be introduced; the square of $\sin 1'\cdot 72$ (i.e. of $\sin 1/2000$ radian) is $\sin 0''\cdot 05$, and since $0''\cdot 05$ would be somewhat objectionable as a random declination-error, and intolerable as a systematic one, either the mounting must orientate the pentag to appreciably better than one part in 2000 in all respects, or else the second-order terms must all be investigated and the orientation-errors determined experimentally, where this is shown to be necessary. The mounting actually made at Herstmonceux can keep one of the errors down to a few tenths of a minute, but not the others.

The present paper is therefore concerned with all the second-order errors which might, *a priori*, be involved; in addition to the two obvious orientation-errors, it considers the effect of varying the angle of incidence (irrelevant when the adjustment is otherwise perfect), and of having an "error of manufacture" in the pentag-angle P itself; the effects of curvature of the diurnal star-trails, and of observing at a number of points (and therefore off-axis) in the telescope field, are also investigated.

The investigation is confined to the case of the "hollow" pentag, i.e. one which really is fabricated from two plane mirrors; the "true" or "solid" pentag, an actual five-sided prism of optical glass in which the two reflections are internal, may possess additional errors of manufacture, and cannot be adjusted by the same auto-collimation method as the hollow one.

Fig. 1 shows a hollow pentag of 45° angle, mounted in front of the telescope objective in one of its two ideal operating positions (i.e. with the angle of incidence $= 22\frac{1}{2}°$, which results in symmetry and no vignetting); the other position can be obtained by rotating it clockwise through 90°, about an axis perpendicular to the plane of the figure, and this axis must pass through A if the beam is to be central on the objective in both positions. The rays marked T, N_1, I, N_2 and S represent the directions of the telescope-axis (looking outwards), the first normal, the "intermediate" ray, the second normal, and the ray to the star, respectively; in the ideal case these all lie in the plane of the figure; and (with the signs of the second and third reversed, as is here done) the angle between each one and the next is $22\frac{1}{2}°$. The light is deviated through the angle TAS and not, of course, through its supplement (2).

Fig. 2 shows the intersections of the above ideal rays with the celestial sphere, and also the intersections T′, I′ and S′, of the actual telescope, intermediate, and star rays respectively, when these are not in the N_1N_2 plane and the angle of incidence $S'N_2$ is not $P/2$. (P also need not be 45°.) Since angles of incidence and reflection are always coplanar and equal, $T'N_1I'$ is a great circle, and $T'N_1 = N_1I'$. Similarly $I'N_2 = N_2S'$. If $T'T''$, $I'I''$ and $S'S''$ are drawn

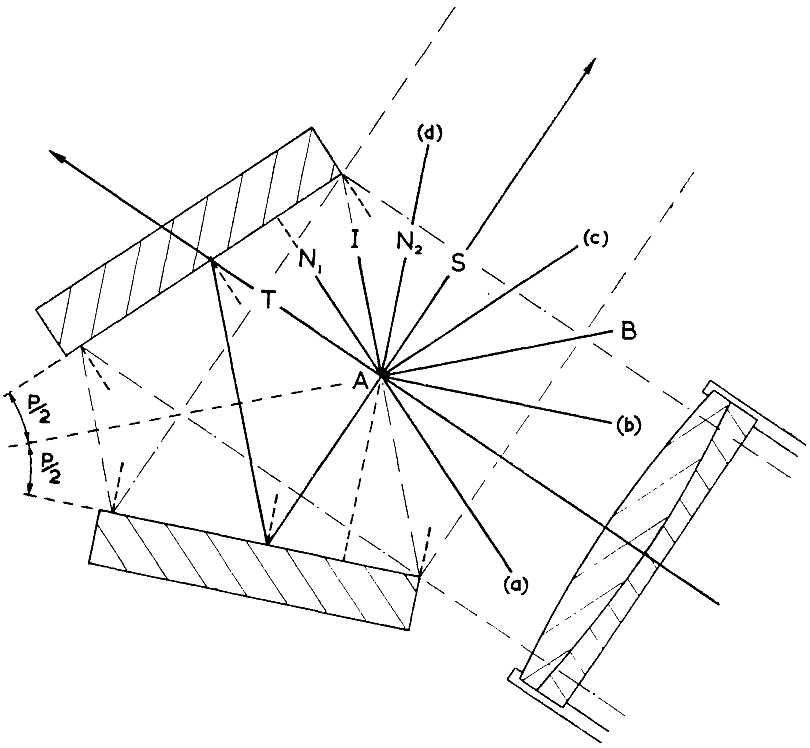

FIG. 1.

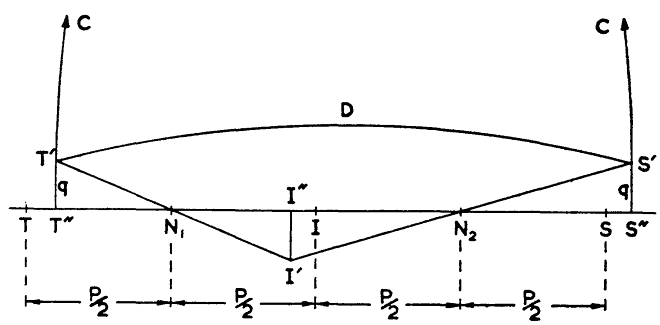

FIG. 2.

perpendicular to the great circle through N_1 and N_2, the triangles $T'T''N_1$ and $I'I''N_1$ are then equal in all respects, and similarly for the triangles $I'I''N_2$ and $S'S''N_2$. Thus $T'T''=S'S''$; the point seen, S′, departs therefore by strictly the same amount, q say, from the plane of the normals (N_1N_2) as the telescope-axis T′ does. Also, $T''I''=2N_1I''$ and $I''S''=2I''N_2$, so that $T''S''=2N_1N_2=2P$. Since $T''T'$ and $S''S'$ if produced will meet at a pole, C, which is $90°-q$ from T′ and S′, the spherical triangle $T'CS'$ yields at once the relation

$$\cos D \equiv \cos T'S' = \sin^2 q + \cos 2P \cos^2 q, \tag{1}$$

where D is the total (great circle) deviation produced by the pentag. This equation is rigorous, and shows that the magnitude of the deviation is completely independent of the angle of incidence, so long as changes in that angle do not alter q. If the pentag is rotated strictly in the plane of the normals, keeping the telescope T′ fixed, N_1 and N_2 will move along the line N_1N_2 but S′ will remain

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fixed in both coordinates, to all orders of accuracy. Since this statement would also hold for any adjacent star S‴, together with the associated image, T‴, in the telescope-field, varying the angle of incidence (at constant q) will also not rotate the field; q may even be large.

Analytically, we shall not find it necessary to consider the angle of incidence, as such, any further; and instrumentally, it clearly is not necessary to fit the axis through A with a divided circle. Unless q varies very rapidly indeed as the pentag is rotated, any good mechanical stop will be quite adequate, and no correction need be applied for the position of the Z.D. micrometer (which does, of course, enter into T′).

We may re-write (1) as

$$2 \sin (P-\tfrac{1}{2}D) . \sin (P+\tfrac{1}{2}D) = \sin^2 q(1-\cos 2P) \tag{2}$$

or

$$\sin (2P-D)[1-\cot 2P . \tan (P-\tfrac{1}{2}D)] = \sin^2 q . \tan P \tag{3}$$

(since neither $\sin 2P$ nor $\cos (P-\frac{1}{2}D)$ will vanish). P, indeed, will never be at all near either 0 or $\pi/2$; and, as q will also never be large, $P+\frac{1}{2}D$ cannot be near either 0 or π, so that $\sin (P+\frac{1}{2}D)$ must be of order unity, very roughly, rather than of order $\sin q$. It follows, then, from (2) that $\sin (P-\frac{1}{2}D)$ must be not merely of order q (radians) but of order q^2, so that (3) becomes

$$2P-D = q^2 \tan P + O(q^4) \tag{4}$$

with no terms of order q^3. Terms of order q^4 will in all cases be completely negligible, and may be dropped at this point.

We shall not in this investigation consider further any values of P except those very close to 45°. Other values would indeed be needed if it were proposed to investigate harmonics of order $4m$ in the flexure (**3**), and there is no real difficulty in extending the analysis to cover such cases; but it does make it more cumbrous. If $P = \pi/4 + v$, where v is small, (4) becomes

$$D - \frac{\pi}{2} = 2v - q^2(1+2v+\ldots), \tag{5}$$

showing explicitly that except for the very small term $2vq^2$ the effects of error of manufacture (v) and error of alignment (q) simply add; in particular, there is no interaction-term of order vq, and there are no higher powers of v that are not multiplied by q^2. A pentag designed for $P = \pi/4$ should result in $|v| < 10^{-3}$, and if we also have $|q| < 10^{-3}$, the term $2vq^2$ (0″·0004) can be neglected in all cases; dropping it, and writing $D-90°$ and v in seconds of arc, and q *in minutes*, (5) becomes

$$D - 90° = 2v - 0''\cdot 01745\, q^2. \tag{6}$$

The correction for the mal-alignment q is thus less than 0″·01 provided $|q| < 0'\cdot 757$, and less than 0″·001 if $|q| < 0'\cdot 240$.

One must clearly have a means of determining the value of q, in both of the positions in which the pentag will be used and in a variety of telescope-pointings for each. A number of unsatisfactory attempts to do this were made, before the possibility of the following simple procedure was noticed. It seems worthwhile, therefore, to describe this, as it is of general applicability.

With a hollow pentag there are four telescope-positions, necessarily all of them exactly in the plane of the normals, where auto-collimation in some part of the pentag is possible. The vignetting is substantial, but the images are surprisingly good; much more than adequate for the present purpose. Taking them in order, these positions involve normal incidence on

(*a*) mirror 1 only
(*b*) mirror 2 as seen in mirror 1
(*c*) mirror 1 as seen in mirror 2
(*d*) mirror 2 only;

the angle between each and the next is 45°, and the successive directions to the *telescope* (180° from T′) are shown in Fig. 1 (and also in Fig. 3) as (*a*), (*b*), (*c*) and (*d*) respectively. If the pentag is rotated, with respect to the telescope, about the rotation-axis A, the successive auto-collimation images will appear at these four positions, though at a first attempt it may be necessary to search through a considerable range of R.A. settings in order to find them, since the plane of the normals may not initially be closely perpendicular to the axis of rotation.

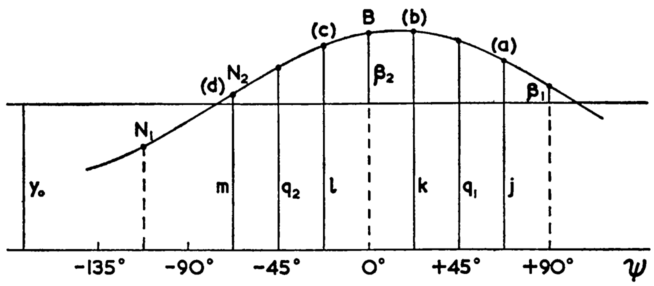

FIG. 3.

Fig. 3 is a Mercator projection of the equatorial region of that sphere whose poles are the rotation-axis through A; the plane of the normals thus defines a sine-curve (of some unknown phase and amplitude) if the adjustment is slightly imperfect. The abscissae, ψ, are angles of rotation, reckoned from the telescope tube to the bisector, B, of the angle between the mirrors; the ordinates are R.A. micrometer readings for points in the plane of the normals; and the equation of the curve is evidently

$$y = \beta_1 \sin\psi + \beta_2 \cos\psi + y_0. \tag{7}$$

If the four micrometer readings for auto-collimation are j, k, l and m respectively (minutes of arc) we readily obtain

$$\beta_1 = \frac{j-m}{2\sin 67\tfrac{1}{2}^\circ} = \frac{k-l}{2\sin 22\tfrac{1}{2}^\circ} \tag{8}$$

$$\beta_2 = \frac{(k+l)-(j+m)}{2(\cos 22\tfrac{1}{2}^\circ - \cos 67\tfrac{1}{2}^\circ)} \tag{9}$$

$$y_0 = \frac{(j+m)\cos 22\tfrac{1}{2}^\circ - (k+l)\cos 67\tfrac{1}{2}^\circ}{2(\cos 22\tfrac{1}{2}^\circ - \cos 67\tfrac{1}{2}^\circ)}. \tag{10}$$

A manufacturing error v will very slightly alter the nominal $22\frac{1}{2}^\circ$ and $67\frac{1}{2}^\circ$ angles, but the effect of this would be quite undetectable, for any reasonable values of v.

Since we have here four equations but only three unknowns, there is an identical relation between the coefficients; as is obvious from (8), this is

$$\frac{j-m}{k-l} = \tan 67\tfrac{1}{2}^\circ = 2{\cdot}41\ldots \tag{11}$$

and this may be used to throw the other equations into various equivalent forms.

The angle β_1 is that part of the pentag's maladjustment with respect to A which is anti-symmetrical about the bisector B; it may be made small by a rotation about this bisector, until $j-m$ and $k-l$ are both small, and the sign of the necessary movement will be readily seen in any trial. β_2, the symmetrical component, may be made small by moving the east end (say) of the pentag towards or away from the axis A; this affects k and l more than j and m, and the sign can be inferred from this. If the readings all reckon from the ordinary "collimation" reading as zero, y_0 is the angle by which the telescope-axis (so defined) departs from the equator of the pentag's rotation-axis; it can be made small by moving one end of the rotation-axis towards or away from the telescope, and this affects all four readings equally. Thus these four observations completely separate the three unknowns in question, and the adjustments can be made in any order.

In actual use on stars, one is concerned only with pentag-positions close to the two ideal operating ones, given by $\psi = \pm 45°$. We thus have

$$y = (q_1 \text{ or } q_2 =) \pm \beta_1/\sqrt{2} + \beta_2/\sqrt{2} + y_0. \tag{12}$$

Taking the upper sign, eliminating β_1, β_2 and y_0, and using (11) to make the coefficients of j and k equal, we obtain

$$q_1 = \tfrac{1}{2}[j + k + \tan 11\tfrac{1}{4}°(l-m)], \tag{13}$$

$$\approx \tfrac{1}{2}(j+k) + 0{\cdot}1(l-m). \tag{14}$$

The upper sign in (12) corresponds to the case shown in Fig. 1, i.e. with the telescope half-way between (*a*) and (*b*). For the other positions we have similarly

$$q_2 \approx \tfrac{1}{2}(m+l) + 0{\cdot}1(k-j). \tag{15}$$

Both equations will of course appear in other forms if (11) is applied differently; in particular, it might be preferred to eliminate m from (14) and j from (15), getting

$$q_1 = 0{\cdot}401j + 0{\cdot}740k - 0{\cdot}141l \tag{16}$$

and

$$q_2 = -0{\cdot}141k + 0{\cdot}740l + 0{\cdot}401m. \tag{17}$$

If ψ is incorrectly set (with individual stars), equations (12)–(17) will be slightly invalidated unless both β_1 and β_2 are zero; i.e. varying the angle of incidence (by rotation about the actual axis A) can have a slight effect on the deviation D, through its effect on q. But if ψ is in degrees, we have

$$\Delta q_1 = -0{\cdot}01745(q_2 - y_0)\Delta\psi, \tag{18}$$

$$\Delta q_2 = +0{\cdot}01745(q_1 - y_0)\Delta\psi, \tag{19}$$

and even a 1° error of setting will thus have negligible effects, for all reasonably small values of $q - y_0$.

If the mounting yields under gravity, the readings for auto-collimation may be different at different telescope-pointings; if they are repeatable over a reasonable period, at any one pointing, it is clearly permissible to plot q_1 and q_2 as functions of the circle-reading, and to read off the value appropriate for each star observed. It may be noted that although all fairly high *stars* can be observed in both positions of the pentag, there is no *telescope*-pointing in which both positions can be used. If circle-readings reckon from the zenith, positive to the north, with c_1 denoting (as in the previous paper) the value when the pentag is in use, and if q_1 refers to the case when $c_1 \approx z + 90°$ and q_2 to the case $c_1 \approx z - 90°$, then q_1 will be required only for positive c_1-values, i.e. north pointings of the telescope; the stars observed will be south of the zenith if the pointing is above the horizon ($c_1 < +90°$), and north if it is below. Similarly, q_2 will be required only for $c_1 < 0$. Near-zero values of c_1 involve near-horizon stars, i.e. no observations. It should thus be sufficient to observe only j, k and l at north pointings (assuming that Fig. 1 is drawn looking eastwards) and only k, l and m at south pointings, using (16) or (17) accordingly.

If (11) fails to hold, even though the readings are repeatable, errors involving higher harmonics of ψ are presumably present. They may be either pivot-errors of the axis A, or perhaps flexure between the pentag and A. Proper care in manufacture should keep the total pivot-errors below $0'\cdot 1$, and flexure which is symmetrical (east–west) will not matter; if the failures of (11) are in fact significant, or if the measurements are seriously non-repeatable in any case, a complete re-design of the mounting may be necessary.

With the preliminary mounting made at Herstmonceux, it has been found possible to keep q_1 and q_2 within about $\pm 0'\cdot 4$ for days on end, in spite of swinging the pentag out from the beam and back, and moving the telescope from one pointing to another. If the direct zenith-distance error due to q were the only one, (6) shows that this would then never reach $0''\cdot 003$, and even though it would still be systematic in z it would hardly be necessary to correct for it at all; but in fact there are also cross-products of q with other errors.

The most troublesome maladjustment to prevent is rotation of the pentag, together with the axis A, about the telescope axis. It has been found very difficult to swing the pentag out of the beam and back without slewing the mounting so as to cause such a rotation, since the telescope itself interferes with almost any diagonal stays which one might wish to incorporate. Even variations of the pointing, without removing the pentag, are somewhat liable to result in the same trouble. Fortunately, however, it proves unimportant for this error to be constant, or repeatable, as long as q is so.

We consider the general case in which we have a small error of manufacture v (supposed precisely constant, but not necessarily known), a small alignment-error q (supposed sufficiently known from auto-collimation results), and a small rotation, p, about the telescope-axis (unknown as yet). For simplicity, we shall suppose the ordinary azimuth and level errors to be zero; their interactions with p, q and v need watching, but the interactions of p, q and v with each other (and with the curvature of the diurnal circles) should sufficiently illustrate the position. All R.A. micrometer readings are reckoned from the ordinary collimation-reading, which therefore does not appear explicitly; they will be defined as positive when they carry the micrometer to the east, and the point seen to the west. We also suppose, for the moment, that the flexure is zero and the zenith is therefore recognizable without ambiguity; this, however, is only for convenience in wording and does not affect the final equations. (The flexure is, in any case, less than $1''$.)

Let the telescope be pointed horizontally to the north, so that c_1 is $+90°$; and let the pentag be set for stars at zenith-distance $z \approx c_1 - 90°$, i.e. near the zenith. If the adjustment is perfect, the point seen is then on the meridian, at the distance $2v$ south of the zenith. If the telescope and pentag were now rotated as a unit, clockwise about the vertical (looking upwards), i.e. so that the eye-end moved east, through a small angle q, the point seen would move round the small circle of radius $2v$, to a position $2vq$ east of the meridian, if all angles are in radians, and the star-field would appear to rotate, with respect to the wires, through the angle q, counterclockwise; the pentag, having an even number of reflections, does not reverse the field. (The opposite apparent rotation will be obtained if a single plane mirror at $45°$ is substituted for the pentag.) If q is small, the same net effect can be produced, without moving the telescope tube or objective, by rotating the pentag alone about the vertical, and then advancing the R.A. micrometer alone from zero through the angle $+q$, so as to place it again in the plane of the normals. Now let the micrometer alone be returned to zero; this evidently does not further rotate the field, and the point seen moves along the image of the Z.D. wire on the sky, eastwards through the angle $q\cos q$, or q to the second order, and northwards through the angle $q\sin q$, i.e. q^2, since the image of the Z.D. wire has been rotated clockwise. The distance of the point seen from the north point of the horizon is thus $\pi/2+2v-q^2$, in agreement with (5). Its distance from the meridian is $q+2vq$; from the plane of the normals it is of course q, the same as for the telescope-axis. If we repeat the argument, with the telescope pointed horizontally southwards ($c_1=-90°$) and the pentag set for $z\approx c_1+90°$, we find that a positive q again displaces the point seen to the east, but the wires are now rotated counterclockwise with respect to the field, i.e. negatively, if the former rotation was defined as positive. The distance east of the meridian is readily seen to be $q+2vq$ in this case also, and the distance from T′, now the south point, is $\pi/2+2v-q^2$, agreeing with (5) as before.

The argument is completely independent of the observer's latitude (in particular, of its sign) and can also be generalized immediately to stars at all zenith distances; it is merely necessary to replace "rotation in azimuth" by "rotation about a line which is in the meridian and perpendicular to the telescope", and to make other obvious minor changes. Thus we conclude that (with the above sign-conventions) a mal-alignment $+q$ and a pentag error $+v$ produce a total deviation from T′ of $\pi/2+2v-q^2$, a displacement to the east of $+q(1+2v)$, and a rotation $\pm q$, the upper sign being taken when c_1 is approximately $z+\pi/2$ (the same convention as in the previous paper).

We now superimpose a rotation of the pentag, through the small angle p, about the telescope-axis. We define the sign of p to be positive if it displaces the point seen towards the same side as a positive q does, i.e. towards the east, and it may then be verified in various ways that the field-rotation produced by a positive p is opposite to that produced by a positive q. If the point seen is at a distance D from T′ (the point seen without the pentag), a rotation p obviously produces a displacement $p\sin D$ along the "almost great" circle of radius D and centre T′, so that the magnitude of the distance from T′ is unchanged. Neglecting third-order quantities, it may then be shown as before that the total displacement east is now $p+q+2vq$, and the total rotation is $\pm(q-p)$, irrespective of whether p or q is performed first; there are no interaction terms of order p^2, pq, or pv. The second-order quantity $2vq$, which appears in E–W angles, but not in N–S ones, may be dropped at this stage; it will probably lie below the probable error of q, and it will in any case never be used except when multiplied by quantities comparable with p.

Fig. 4 represents a portion of the sky (looking outwards) which is viewed through a pentag orientated so that $c_1\approx z+90°$. T′, the point seen in the absence of the pentag, is distant $90°+2v$ from O, in the N direction, and all great circles through T′ are parallel lines in this figure, which may be thought of as a Mercator projection with T′ as its pole. The projections of the micrometer wires are tangents to great circles, and practically coincide with them throughout the field of view. The displacement from Q to P, caused by the rotation p, is at right angles to the N-direction, as shown. The displacement q, from O to Q, is inclined, but all inclinations are of course enormously magnified here.

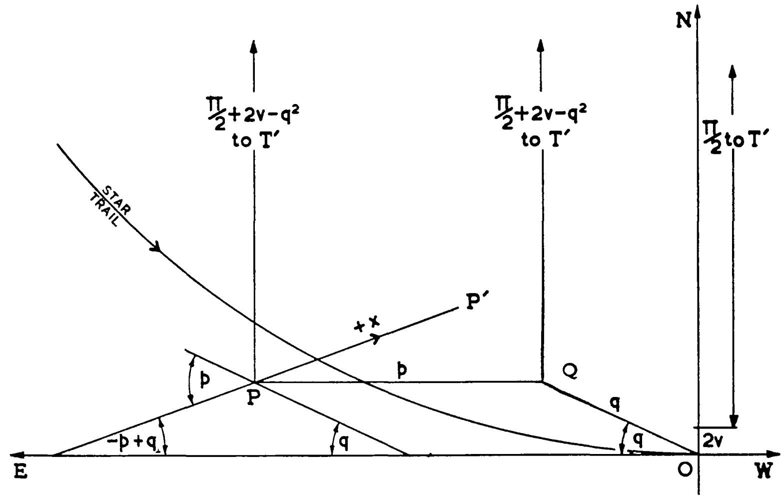

FIG. 4.

O is the point which would be seen, if the pentag were perfectly adjusted, when the circle-reading is c_1, and the R.A. micrometer-reading is zero. (The "circle-reading" includes that part arising from the Z.D. micrometer, and is adjusted to be zero at the zenith.) P is the point actually seen, with the R.A. micrometer still at zero, and it is north of the E–W great circle through O by $0''\cdot01745\, q^2$ if q is now in minutes once more. PP′ is the image of the Z.D. wire, inclined at the angle $+(q-p)$ to the east-west great circle through O; and points on this image, west of P by x (minutes of arc) are north of the great circle by $0''\cdot01745\, [q^2+(p-q)x]$. The declination-circle through O is north of the same great circle by $0''\cdot00872\, (p+q-x)^2 \tan\delta$; and the actual circle-reading, if the Z.D. wire bisects a star of the declination of O, when it is x minutes west of P, must therefore be increased by

$$\Delta c_1 = 0''\cdot01745[\pm q^2 \pm (p-q)x - \tfrac{1}{2}(p+q-x)^2 \tan\delta] \qquad (20)$$

in order to obtain the c_1 for use in eqn. (55) of the previous paper. The upper signs are those appropriate to the argument just given, i.e. to the case where the pentag is set for $c_1 \approx z+\pi/2$, and the lower ones correspond to $c_1 \approx z-\pi/2$, as in the previous paper. The term $2v$ is shown (2) explicitly in (55) and is therefore not included in Δc_1.

So far as the term in $\tan\delta$ is concerned, there is no need to determine p explicitly; the complete coefficient $p+q-x$ can be obtained directly, for each Z.D. bisection, by using the R.A. micrometer as well as the Z.D. one. It is not, of course, possible to bisect the star with both wires at once, but no real precision is required in R.A. and it is sufficient to set the R.A. wire slightly ahead of the star and to set the R.A. drive to the right speed as closely as practicable. The R.A. difference between star and wire will then be small and will vary linearly with time; if actual values are required at all, they can be estimated by eye at known times before and after the set of Z.D. bisections, and interpolated. If, then, an audible signal is sounded at the times when the R.A. micrometer is recorded, the observer merely has to make his Z.D. bisections when he hears the signal. The star's distance east of the meridian is $\frac{1}{4}(\alpha-t)\cos\delta$ (in minutes of arc), at the true local sidereal time t, if α is its apparent right ascension and $\alpha-t$ is in seconds of time; we thus have

$$\tfrac{1}{2}(\alpha-t)\cos\delta=p+q-x, \tag{21}$$

where x is the recorded R.A. micrometer reading, corrected for the star–wire difference. For α, it will be sufficient to take the tabular apparent value. Accordingly,

$$\Delta c_1=0''\cdot 01745\left[\pm q^2\pm(p-q)x-\left(\frac{\alpha-t}{8}\right)^2\sin 2\delta\right]. \tag{22}$$

Since this correction will in practice be averaged for a number of different Z.D. bisections at any one transit, the term which still contains p will vanish if the observations are so arranged that $\sum x=0$. If the observing procedure does not permit this, it will probably be simplest and most accurate to evaluate a mean p for the transit from (21), using all the t and x values recorded (as well as the *mean* star–wire difference), and to insert this in (22).

At high declinations, small errors in the instants of bisection (or in the estimated star–R.A. wire separations) will tend to become serious, in spite of the coefficient $\sin 2\delta$, since $(\alpha-t)\cos\delta$ will be roughly independent of δ. It will also be necessary, if p is large and of unknown sign, to set for each star well before true transit time and perhaps observe well after it. For both these reasons, high declinations are disadvantageous. Fortunately, it is not necessary to include them, in order to obtain a sufficient separation of the unknowns in (55); the observations always give only the difference in flexure, as between two pointings 90° apart, and one should get exactly the same result from observing equatorial stars with and without the pentag as from observing near-polars with and without it. Incidentally, this means that we may somewhat further restrict the range of pointings over which auto-collimation readings are needed; within, say, ±15° either way from the equator, we may perhaps omit them altogether.

The pentag-mounting as used in this experimental work was inconvenient even when not in use, and was removed again to facilitate the AGK3R programme. When that has been completed, it is planned to make a sufficient series of flexure determinations, with a more fully engineered mounting. The use of tubing instead of angle-rods should diminish the tendency to slew, and so enable p to be kept small more readily.

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References

(1) R. d'E. Atkinson, *M.N.*, **115**, 427, 1955.
(2) This point was overlooked in the previous paper, and wherever the quantity $2v$ occurs there (p. 440, l. 8, l. 19, and eqns. 52, 53, 54 (twice), and 55), its sign (or double sign) should be reversed.
(3) *L.c.*, p. 440.