

THE NATURE OF THE GRAINS IN THE TAILS OF COMETS 1956h AND 1957d

WILLIAM LILLER*

University of Michigan Observatory

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ABSTRACT

Photoelectric spectrophotometric observations of the tails of Comets 1956h (Arend-Roland) and 1957d (Mrkos) show that, at a solar distance of about two-thirds of an a.u., most of the radiation between λ 3400 and λ 6400 was of a continuous nature and redder than sunlight. Comparison of the observed energy distributions with theoretical curves of light-scattering by small particles strongly suggests that spherules of iron in the tails with average diameters of 0.6μ and masses of 8×10^{-13} gm produced this radiation. Considerations of the brightnesses of the comet tails permit estimates to be made of the total masses of these particles in each of the comets.

I. INTRODUCTION

Very little is known about the particles which scatter sunlight in the tails of comets. They undoubtedly exist in all cometary tails, but their numbers vary greatly from one comet to another. Orlov (1942) has classified as type II those tails which shine primarily by scattered light; type I tails, which have spectra dominated by molecular bands, contain relatively few scattering particles. From studies of *syndynames* and *synchrones*, Orlov (1935) has probably contributed the major portion of our meager knowledge of the solid particles in cometary tails. His results are nicely summarized by Bobrovnikoff (1951), but they have recently been challenged by Vsekhsvyatsky (1959).

The appearance of two bright comets in 1957—Comet 1956h (Arend-Roland) and Comet 1957d (Mrkos)—provided two of the few opportunities to study comet tails with modern instruments. Because the discovery of Comet 1956h occurred nearly 5 months before perihelion passage, it was possible to plan an observational program for this comet well in advance. It was a most pleasant surprise to have the opportunity to repeat this program a few months later for Comet 1957d.

This paper presents a description and a discussion of some observations of the tails of these comets made by the writer with a direct recording photoelectric spectrophotometer. This instrument, which was used at the Newtonian focus of the 24–36-inch Curtis Schmidt telescope has been described in detail elsewhere (Liller 1957), and only a brief description of its performance will be given here.

The spectrophotometer consists of two small identical $f/5$ Newtonian systems, which collimate and focus the light. By rotating the 600-line/mm replica grating which provides the dispersion, a spectrum can be swept over an analyzing slit directly before a photocell. Approximately 10 minutes are required to make one complete trace. All the observations described in this paper were made with a 20-stage Lallemand photomultiplier ("Patrice") in the wave-length interval from λ 3400 to λ 6400.

The following two sections contain a description of the observations and their reduction. An analysis of the results appears in Section IV, and a discussion of the forces producing the observed tail motions follows in Section V. Section VI discusses how the comet masses can be estimated and presents the results. The paper concludes with a summary in Section VII.

II. THE OBSERVATIONS OF COMET 1956h

A summary of the observations of the tail of Comet Arend-Roland is given in Table 1, which is largely self-explanatory. The values of r , the comet-sun distance, and Δ , the

* Present address: Harvard College Observatory.

comet-earth distance, are taken from the ephemeris of Hasegawa (1957). Because of the higher weight which must be given to the spectral traces of April 30, far greater emphasis will be given to these observations.

The following observational procedure was used on April 30: As soon as the sky became dark enough, two scans of the spectrum of α Persei were obtained. This star was only some 7° away from the comet head at the time, and it is one of the stars for which Code and Whitford (1955) have measured the spectral energy distribution. Immediately following the observations of α Persei, the spectrum of the comet head was traced twice as a part of another photometric program. These head observations will be described in a later paper, and further reference to them will not be made here. These scans were succeeded by the tail measures, which are summarized in Table 2. The quantities $\Delta\alpha$ and $\Delta\delta$ refer to the position in the tail where the trace was made, relative to the nucleus

TABLE 1
SUMMARY OF OBSERVATIONS

Date (U.T.) April, 1957	No. of Scans	Quality of Night	Δ (a.u.)	r (a.u.)
26.1	2	Fair	0.627	0.599
30.2	5	Excellent	0.713	0.687

TABLE 2
THE OBSERVATIONS OF THE TAIL OF COMET 1956h, APRIL 30, 1957

Trace No. (1)	U.T. (2)	$\Delta\alpha$ (3)	$\Delta\delta$ (4)	Distance from Head (5)	Area of Tail Admitted (sq. min. of arc) (6)	$\Delta\lambda$ (Å) (7)	Weight (8)
1	30.135	+3 ^m 7	+0 ^s 50	0 ^s 72	16.0	47	10
2	30.156	+3.7	+0.50	0.72	28.5	99	4
3	30.164	+5.2	+1.00	1.23	28.5	99	5
4	30.190	+7.2	+1.50	1.80	28.5	99	3
5	30.195	+9.7	+2.00	2.41	28.5	99	3

of the comet. The sixth and seventh columns give the area of the tail admitted to the photocell and the width of the scanning slot expressed in angstrom units. The relative positions and sizes of the entrance apertures are shown in Figure 8. The rectangles in the photograph correspond to the trace numbers given in Table 2.

Between the first and second traces summarized in Table 2 and immediately following the last trace, the telescope was moved to an adjacent area of the sky, where the sky spectrum was scanned. During the time elapsed between the two sky traces, the sky brightened approximately 0.1 mag. The N_2^+ auroral band at λ 3914 and the night-sky radiation of [O I] at λ 5577 can be seen faintly on both scans of the sky.

The first of the five tail traces is reproduced in Figure 1. The solid line shows the extrapolated brightness of the sky derived from the two traces just described.

An attempt was made to trace again the spectrum of α Persei between the third and fourth tail scans, but by this time the star had moved behind trees. Therefore, a second Code-Whitford star, η Ursae Majoris, was selected as a new standard. The spectrum of this star was scanned immediately after the final sky trace and then numerous times

during the remainder of the evening by R. C. Bless, who was engaged in another investigation with the same equipment. From these observations an excellent set of atmospheric extinction-curves was derived by Dr. Bless, one for each of the ten wave lengths chosen by Code and Whitford. I am most grateful to Dr. Bless for making these data available to me.

As can be seen from Figure 1, there is no sign of the CO^+ bands which often dominate the spectra of comet tails. However, on the preceding night Stecher and McLaughlin (1957) obtained a spectrogram of the comet showing several bands of this molecular ion, but these radiations were very faint and were present in only a few areas of the tail. Except for the auroral and night-sky features, the spectra of the tail present a perfectly continuous appearance, demonstrating that the light is almost entirely scattered sun-

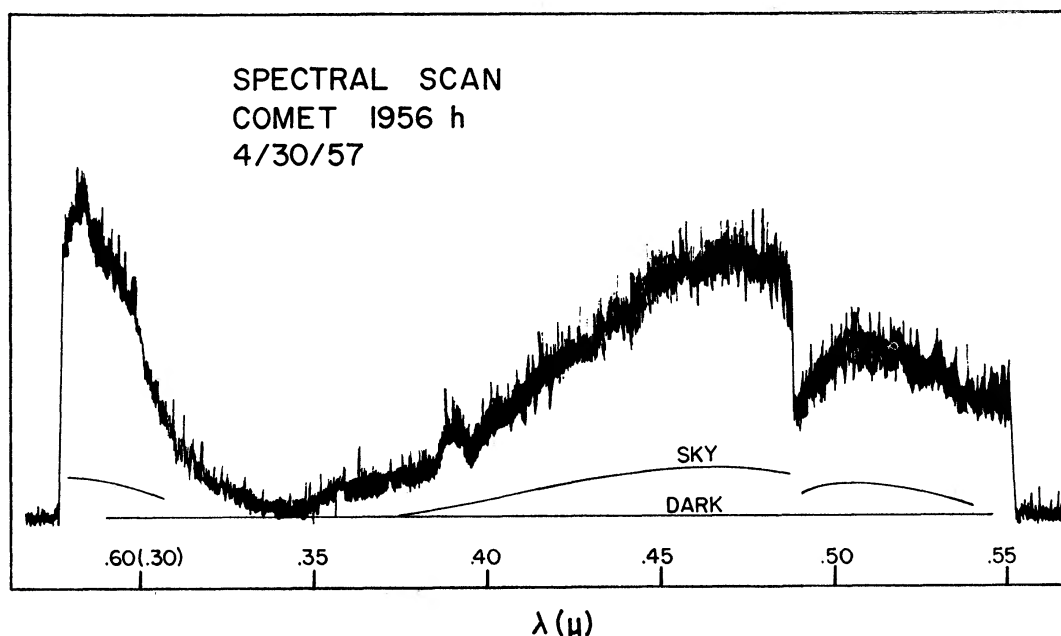


FIG. 1.—Spectral scan of the tail of Comet 1956h, $0^{\circ}72$ away from the nucleus. Overlapping orders produce the duplicity of wave lengths at the left; a yellow filter inserted in the light-path between $\lambda = 0.49$ and 0.55 eliminates contamination at these wave lengths. The vertical deflections are proportional to intensity. Scan made at the focus of the Curtis Schmidt telescope.

light. The familiar solar spectrum is greatly washed out because of the wide scanning slot which had to be used, and even the most prominent absorption lines have disappeared in the noise of the trace.

In the reduction a total of six wave lengths, at which the monochromatic brightnesses of the tail were to be measured, were chosen. To avoid the possibility of systematic errors entering into the results, the wave lengths were carefully selected to avoid the brighter auroral and night-sky emission lines and bands. A further effort was made to work between possible comet emissions, such as the CO^+ bands mentioned already, and the strong C_2 , CN , and C_3 emissions present in the head spectrum. The final wave lengths that were used throughout the reduction are listed in the first column of Table 3.

The contribution of the sky was subtracted from the traces and the comet-tail deflection at each of the six wave lengths was reduced to outside the atmosphere by means of the extinction curves derived by Dr. Bless. Then the scans of the spectrum of the tail were compared directly with those of the two standard stars, α Persei and η Ursae Majoris, whose traces were also reduced to zero atmosphere. Since Code and Whitford

(1955) have also measured the energy distribution in the spectrum of a solar-type star, 16 Cygnus A (Sp. = G2.5 V), the spectrum of the tail can be readily compared with that of the sun.

It should be emphasized here that there was no observable systematic change whatsoever in the distribution of energy in the continuous spectrum of the tail at points farther and farther from the head of the comet. In general, the energy-curves derived from the later observations were somewhat less well determined because they were made at points farther removed from the head and at lower altitudes also. In the reductions, each tail scan was assigned a weight proportional to the tail brightness and inversely proportional to the secant of the zenith distance. These weights are listed in the last column of Table 2. Then a weighted mean of these observations was taken to give the final results a higher precision. The energy distributions, expressed in monochromatic magnitudes relative to $m(\lambda 4790) = 0$, are listed in the second column of Table 3, together with their probable errors. The angular distance of the weighted mean point from the nucleus is $1^{\circ}.14$, which, according to the ephemeris of Hasegawa (1957), corresponds to a linear distance of 2.12×10^6 km perpendicular to the line of sight. For comparison,

TABLE 3
ADOPTED MONOCHROMATIC MAGNITUDES OF 16 CYGNI A AND
TAILS OF COMETS 1956h AND 1957d

λ (1)	m_{λ}			
	1956h, 4/30 (2)	1956h, 4/26 (3)	1957d, 8/23 (4)	16 Cyg A (5)
3760.....	$+1^m13 \pm 0^m18$		$+0^m70$	$+0^m65$
4080.....	$+0.60 \quad .033$	$+0^m94$	$+ .59 \pm 0^m16$	$+ .24$
4460.....	$0.00 \quad .028$	$+0.22$	$+ .14 \quad .05$	$- .01$
4790.....	0.00	0.00	$.00$	$.00$
5280.....	$-0.07 \quad .021$	-0.15	$+ .04 \quad .08$	$+ .04$
6090.....	$-0.51 \quad 0.082$	-1.36	$-0.42 \quad 0.30$	$+0.18$

the energy-curve of the solar-type star 16 Cygni A, according to Code and Whitford (1955), is also given in Table 3 (last column).

Figure 2 displays these results in graphical form. The vertical lines through the observed points indicate the sizes of the probable errors; the dashed line represents the energy distribution in the spectrum of 16 Cygni A.

It is obvious that the light scattered by the particles in the comet's tail is definitely redder than sunlight. Indeed, the energy distribution comes closest to matching the Code-Whitford energy distribution for α Bootis (Sp. = K2 III), which has a $B-V$ color of $+1.23$ (Johnson and Morgan 1953).

The single spectral trace which was made on April 26 was obtained in much the same way as the observation of April 30. The scan was made $0^{\circ}.3$ (5×10^5 km) from the nucleus. α Persei was again the standard star, and, although an extinction-curve was not obtained, the nearness of the comet to α Persei (5°) kept the extinction corrections small at all times, despite their low altitudes. The derived energy distribution is tabulated in column 3 of Table 3 and is plotted as a dotted line in Figure 3. No probable errors are listed for these single observations.

Once again the tail was quite a bit redder than the incident sunlight—in fact, slightly more so than on April 30. However, this single observation on April 26 made at sec $z \cong 7$ should be given far less weight than those made three nights later.

It should be reported here that on the night of April 26 a trace was secured of the sunward-pointing "spike," which was visible for a short while after perihelion passage. Spectroscopically, no difference can be seen between the spike and the main tail on this night.

III. THE OBSERVATIONS OF COMET 1957d

The spectrum of the tail of Comet 1957d was observed on only one night because of rather poor weather. On August 23.2 U.T., 1957, two scans were made when, according to the ephemeris of Hoeg, Laustsen, and Thernöe (cf. Thernöe 1957), the values of Δ and r were 1.145 and 0.684, respectively.

The atmospheric transparency at the time the observations were made was quite high. Because the comet was shortly to pass behind trees, an initial trace on a standard star was omitted, and work was begun immediately on the comet. First, the spectrum of the head of the comet was scanned twice. Then followed two traces of the tail 1^h33 from the nucleus ($\Delta\alpha = +1^m7$; $\Delta\delta = +1^{\circ}28$), a distance which corresponded to

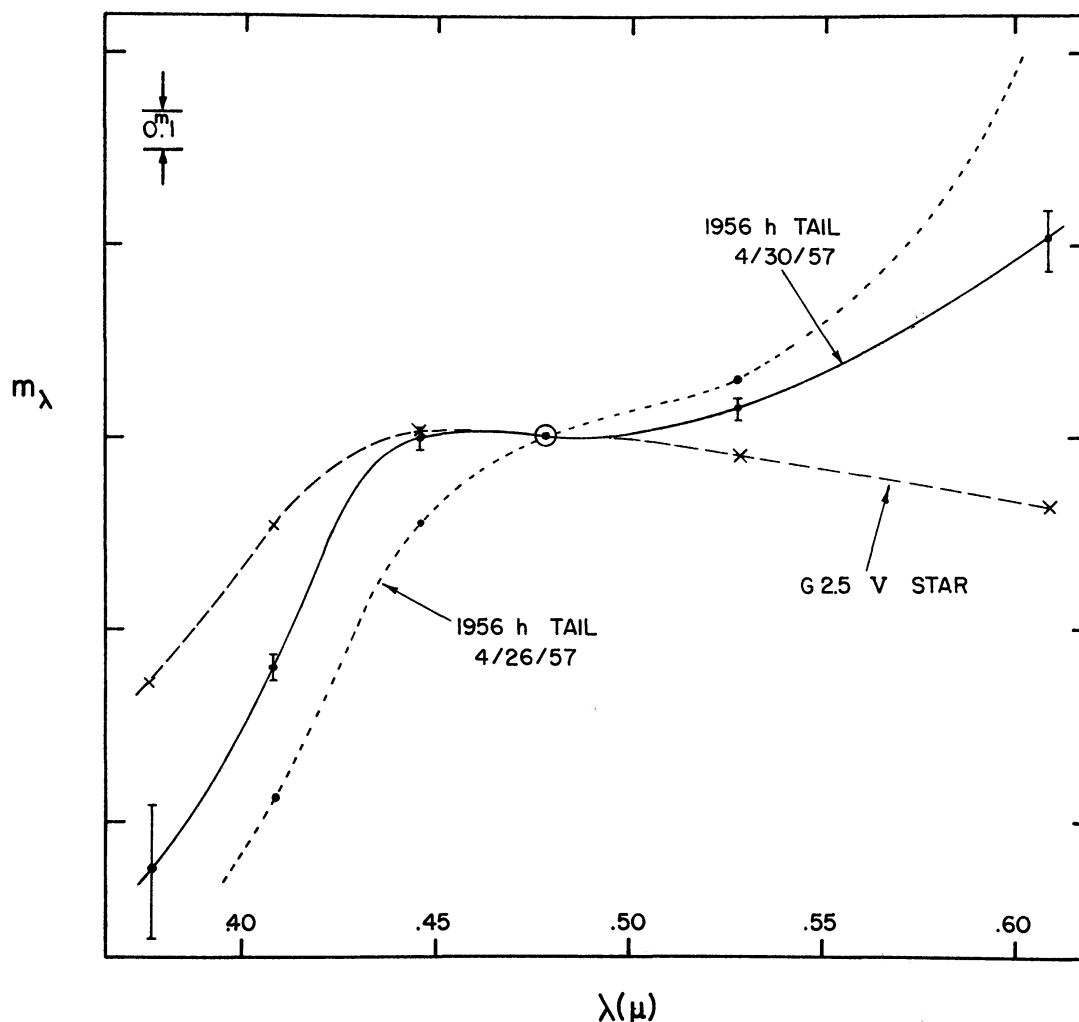


FIG. 2.—Distribution of energy in the continuous spectrum of the tail of Comet 1956h and of a solar-type star, 61 Cygni A. The energy units are monochromatic magnitudes relative to $\lambda = 0.479 \mu$. Vertical lines indicate sizes of probable errors.

4.42×10^6 km. Sixteen square minutes of arc of the tail were admitted to the photocell; the scanning aperture was 47 Å wide. The relative position and size of the entrance diaphragm are shown in Figure 9.

Following the passage of the comet behind trees after the second tail trace, two scans of the spectrum of the sky just above the trees were obtained. Again the [O I] emission at λ 5577 was visible in both the sky and the tail deflections. However, the auroral band at λ 3914 did not appear on either sky trace.

The first of the two tail traces is reproduced in Figure 3. Again the brightness of the continuous spectrum of the sky is indicated by the smooth curve.

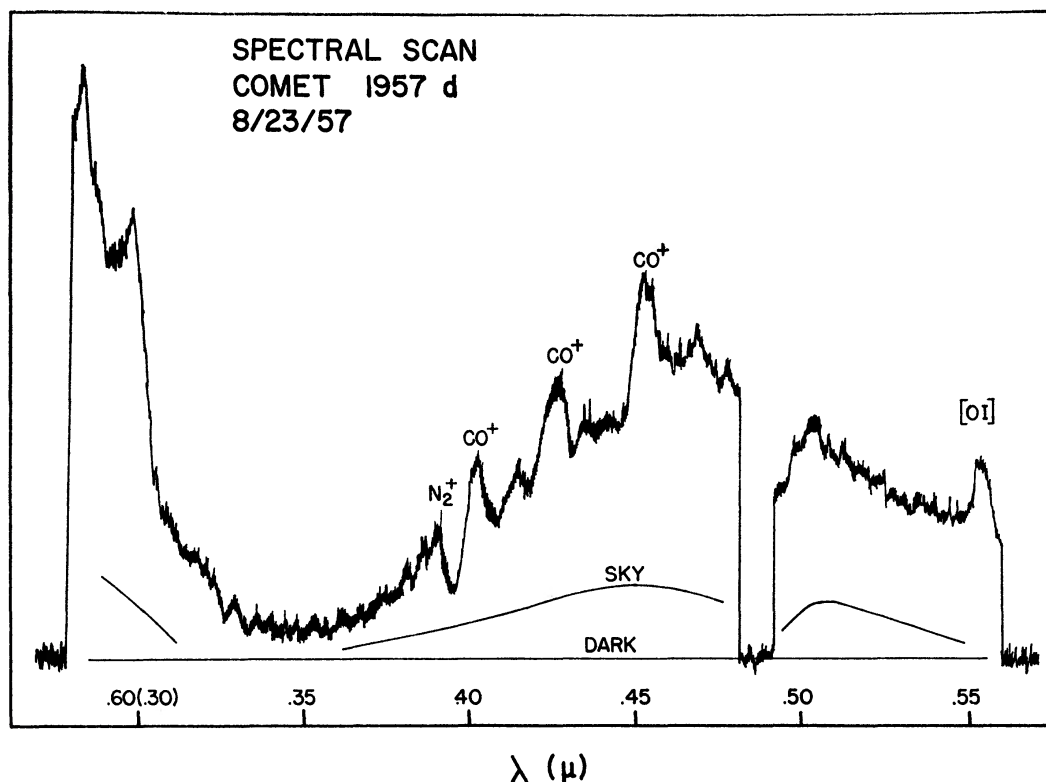


FIG. 3.—Spectral scan of the tail of Comet 1957d, 1.33 away from the nucleus. Made in the same manner as the scan shown in Fig. 1.

Following the sky observations, the spectrum of η Ursae Majoris was traced twice. This star was only 26° away from the comet and therefore served as an excellent standard star. A star near the zenith, α Lyrae, was then selected as an atmospheric extinction star. However, only two scans of its spectrum could be made before clouds appeared and halted work for the night. Therefore, it was necessary to use a mean extinction-curve in the reductions. This approximation has been used successfully in Michigan upon other occasions and has been found to be quite good, since only differential color extinction corrections are needed. The mean extinction-curve used is one derived by Bless (1958) during a period of time extending over a year.

Unlike Comet 1956h, the tail of Comet 1957d shows conspicuous CO^+ bands and probably the N_2^+ band at λ 3914. This latter band falls in the wing of the Ca II K absorption line of the solar spectrum, and it appears, therefore, to have an abnormally low intensity. Nevertheless, the contribution of scattered sunlight to the spectrum is quite strong and accounts for the major percentage of the light detected by the photocell.

The reduction procedure followed was identical with that described in Section II. Because the first of the two traces was made when the comet was at a higher altitude and because there was a slight possibility that trees might have interfered with the second trace, the former was arbitrarily given a weight 1.5 times the latter in the final reductions.

As was true for Comet 1956h, the light scattered by the tail of Comet 1957d is conspicuously redder than sunlight. The weighted mean values of the energy-curves, expressed in monochromatic magnitudes relative to $m(\lambda 4790) = 0$, are given in the fourth column of Table 3 with the probable errors. These results appear graphically in Figure 4, with the probable errors and the energy curve of a G2.5 V star indicated as

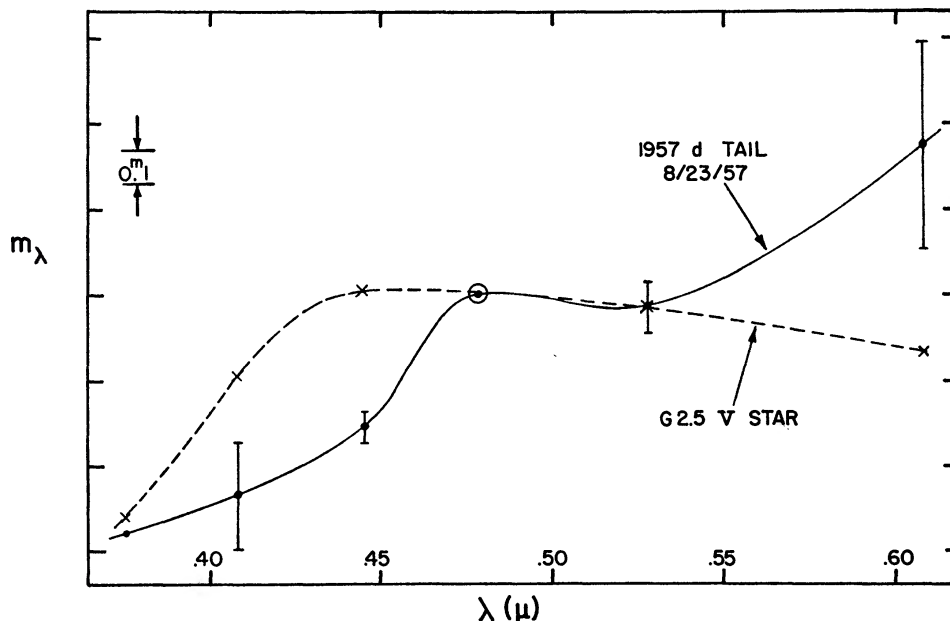


FIG. 4.—Distribution of energy in the continuous spectrum of the tail of Comet 1957d and of a solar-type star, 61 Cygni A. The energy units are monochromatic magnitudes relative to $\lambda = 0.479 \mu$. Vertical lines indicate sizes of probable errors. The point on the comet curve at $\lambda = 0.375 \mu$ is based on one observation.

before. The observational errors are larger for Comet 1957d because of the smaller number of traces and possibly because a mean extinction-curve had to be used.

The color of the tail of Comet 1957d on August 23.2 was only slightly less red than that of Comet 1956h on April 30.2. The similarities in color of the tails of the two comets is striking and suggest that the radiation was produced in the same manner.

IV. DISCUSSION OF THE RESULTS

It should be noted here that the tails of a number of past comets have been reported as being red or pink. According to Bobrovnikoff (1951), Comet 1874 III (Coggia) was observed to have a greenish nucleus and a pink tail for several days; Comet 1862 III (Tuttle) reportedly had an orange-colored tail; red plates of Comet 1942a (Whipple) showed the tail almost a full magnitude brighter than ultraviolet plates (Classen 1943). However, the evidence is that the colors of comet tails vary greatly with both time and position in the tail. Cep-lecha (1958) found photographically that the mean color index (blue minus orange) of the tail of Comet 1956h rose to a maximum of $+0.93$ on April 29.5 and then fell to -0.05 by May 2.9. Variations of color throughout the tail at any one time were as large as 0.45.

Obviously, many, if not all, of the color differences can be attributed to the presence or absence of molecular emission bands. The CO^+ radiations usually dominate the emission spectra, when they exist, and, since the CO^+ bands fall mainly between λ 4000 and λ 5000, they can be expected to add a bluish cast to cometary tails.

That small particles can scatter away red light from astronomical objects was convincingly demonstrated in the fall of 1950. As many astronomers who were in the north-eastern United States and eastern Canada at that time will remember, smoke from forest fires in the Canadian province of Alberta blew over the area for a few days and caused the sun and the moon to be conspicuously blue. This phenomenon, which is described more fully by Hogg (1950), was the result of small particles which scattered away red light to a greater extent than blue light. As Mrs. Hogg suggests, such fires are possibly responsible for the expression "once in a blue moon."

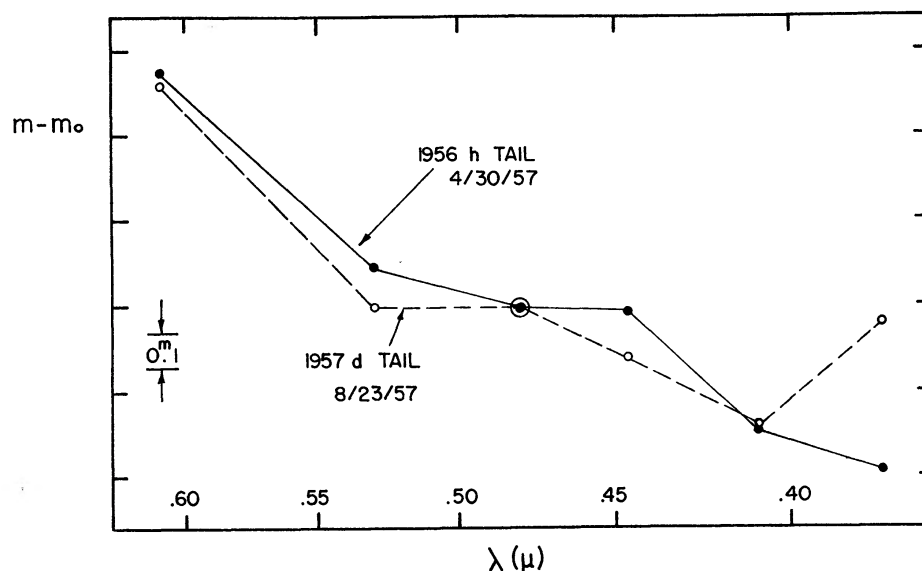


FIG. 5.—The scattering excess as a function of wave length for the particles in the tails of the two comets. The excess is expressed in magnitudes relative to $\lambda = 0.479 \mu$. Colorless scattering would be represented by a horizontal line.

Figure 5 illustrates the results of the preceding two sections in a somewhat different way. Here plotted against $\log 1/\lambda$ for each comet tail is the *scattering excess*, defined as the difference in monochromatic magnitude between the sun and the comet tail. A material which has the property of scattering radiation of all wave lengths equally efficiently would be represented in this graph by a horizontal line. The presence of Rayleigh scattering, for example, would be indicated by a line having a slope of $+10$. The material of which the two comet tails is composed scatters light by an amount which is approximately proportional to λ^2 (slope = -5). The problem now is to find what characteristics this scattering material must have to produce the observed spectrum.

H. C. van de Hulst in his book, *The Scattering of Light by Small Particles* (1957), describes thoroughly our present knowledge of the subject. He points out that for spherical particles of known index of refraction, with a circumference larger than a few hundredths the wave length of the impinging light, the Mie theory (1908) is exact in predicting the resulting scattering effects. However, the appropriate formulae are exceedingly difficult to evaluate because of the complexity of the theory. It has been only recently that systematic computations made with large electronic calculators have become available.

However, because the scattering characteristics often change very rapidly with scattering angle and with relative circumference size ($2\pi r/\lambda$), even very ambitious programs are apt not to be complete.

To illustrate this problem, the scattering diagram for spherical particles of index of refraction, $n = 1.50$, with a scattering angle of 90° is reproduced in Figure 6. Here the scattering excess possessed by a cloud of such particles all of radius r is plotted against $2\pi r/\lambda$. The positions of the plotted points were calculated by Gumprecht and Sliepcevitch (1951) with the ENIAC at the Aberdeen Proving Ground. There is some question as to whether or not they selected a sufficiently large number of points to illustrate completely the trend of scattering with varying particle size or wave length of light. For example, Penndorf (1957) finds a regularly varying damped oscillation pattern in similar diagrams constructed from many more points than used by Gumprecht and Sliepcevitch.

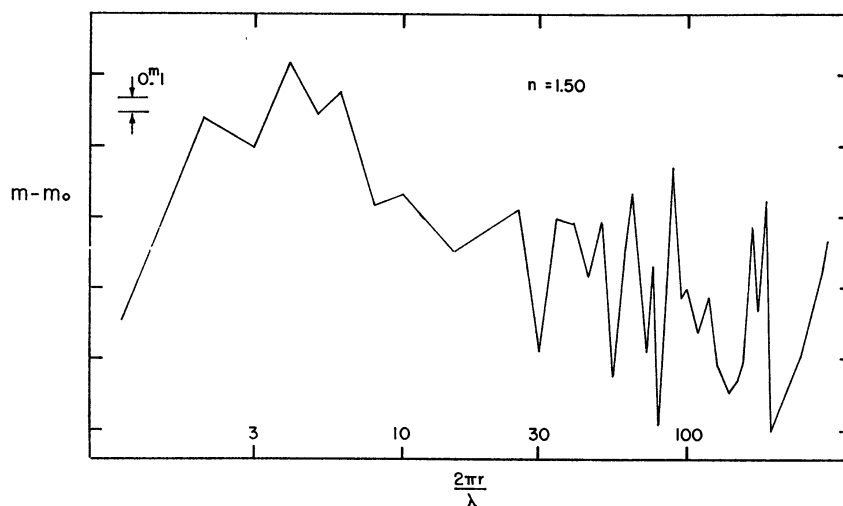


FIG. 6.—Calculated scattering excess as a function of wave length for transparent spheres of radius r and index of refraction 1.50, according to Gumprecht and Sliepcevitch. The excess is expressed in magnitudes. Colorless scattering would be represented by a horizontal line.

In practice, matters are further complicated when one considers the effects of a mixture of particle sizes, shapes, and compositions. It is some consolation to know that the scattering characteristics of cylindrical particles are very similar to those of small spheres (van de Hulst 1957). Furthermore, a small range of particle sizes will do nothing more than smooth out the small-scale variations in the scattering diagram. In principle, it should be possible to learn something about the distribution of particle sizes from a precise measurement of a scattering diagram.

The calculations of scattering coefficients by Gumprecht and Sliepcevitch (1951) are among the most complete of any yet made. Their tables give quantities which are proportional to the intensities of the two incoherent, plane-polarized components of light scattered at a 90° angle by transparent spheres. Forty values of $2\pi r/\lambda$ from 1 to 400 were selected and six indices of refraction: 1.20, 1.33, 1.40, 1.44, 1.50, and 1.60. Their values for $n = 1.50$ are those plotted in Figure 6; the ordinate used here is the scattering excess, defined above.

Additional diagrams for a scattering angle of 90° have been published for $n = 1.27$ – $1.37 i$ (Schalén 1946), the index of refraction of iron. The asymptotic solution to the Mie theory for $n = \infty$ (totally reflecting particles) is given by van de Hulst (1957).

Through a fortunate coincidence, the scattering angle (sun-comet-earth) at the weighted mean position of the observations of the tail of Comet 1956h on April 30 was

$90^\circ 0 \pm 0^\circ 1$ the same angle as that used in the computations of Gumprecht and Sliepcevitch. It must be remembered, however, that the light is scattered from many particles along the line of sight. Therefore, this value is only a mean scattering angle. It will be assumed that multiple scattering is non-existent. The corresponding values for the April 26 observation of this comet and for the August 23 trace of Comet 1957d were $109^\circ 2$ and $61^\circ 2$, respectively. The shape of the Mie scattering diagram for these scattering angles is not known, but perhaps in the future it will be possible to calculate them.

The problem now is to find the proper mixture of spherical particles which will produce the observed dependence of scattering on wave length (Fig. 5). To begin with, it was assumed that the distribution of particle sizes was such that the maximum value of $\log 2\pi r/\lambda$ exceeded the minimum value by 0.10. It was further supposed that all sizes of spheres within this range occurred equally frequently. In Figure 7 the resulting smoothed scattering diagrams are displayed. These are to be compared with the observed scattering diagram derived from the April 30 observations of Comet 1956h (reproduced on the same scale in the inset at the upper right). Possible values of the index of refraction, n , and the size parameter, $2\pi r/\lambda$, can then be selected by matching the observed and theoretical curves. Considerable freedom is allowed, of course, by changing the particle size distribution or by considering mixtures of n -values. However, it can be seen that the observed scattering characteristics indicate a fairly unique value of $2\pi r/\lambda$ for each index

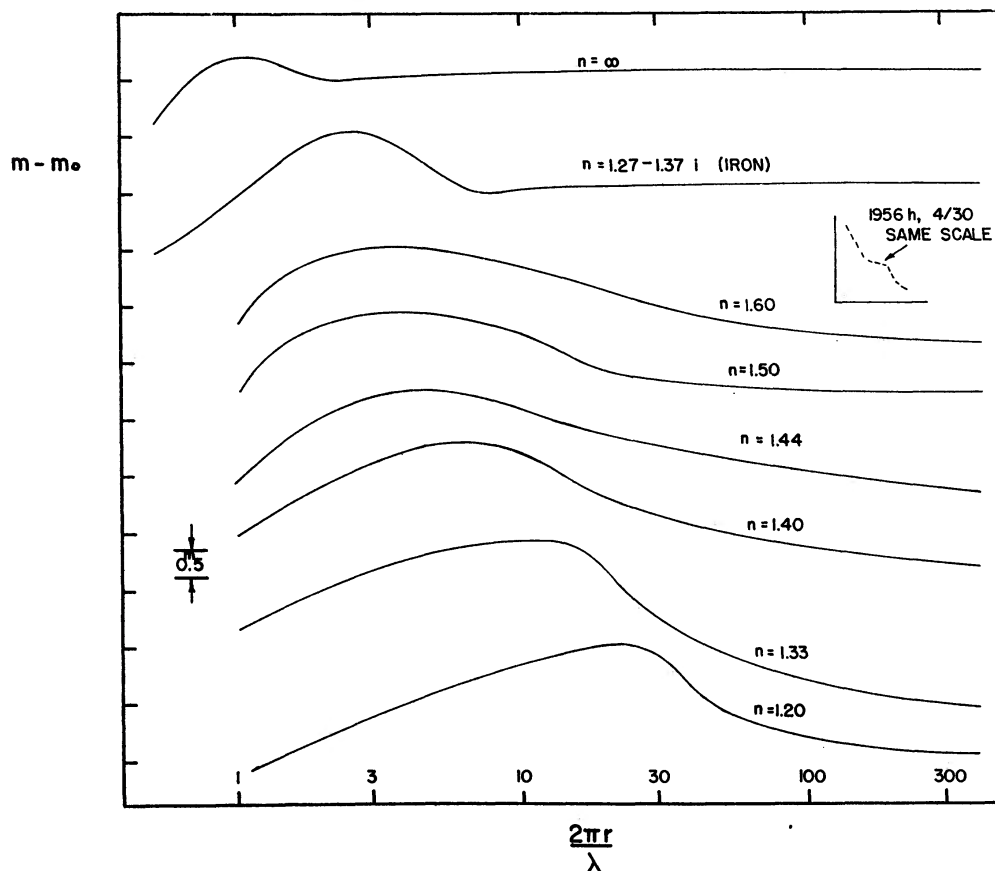


FIG. 7.—Calculated scattering excess as a function of wave length for transparent spheres with index of refraction n and smoothed, assuming a frequency distribution flat over a range of radius r from $\log r - 0.05$ to $\log r + 0.05$ and zero everywhere else. Colorless scattering would be represented by a horizontal line. The insert at the upper right shows on the same scale the scattering curve for Comet 1956h on April 30.

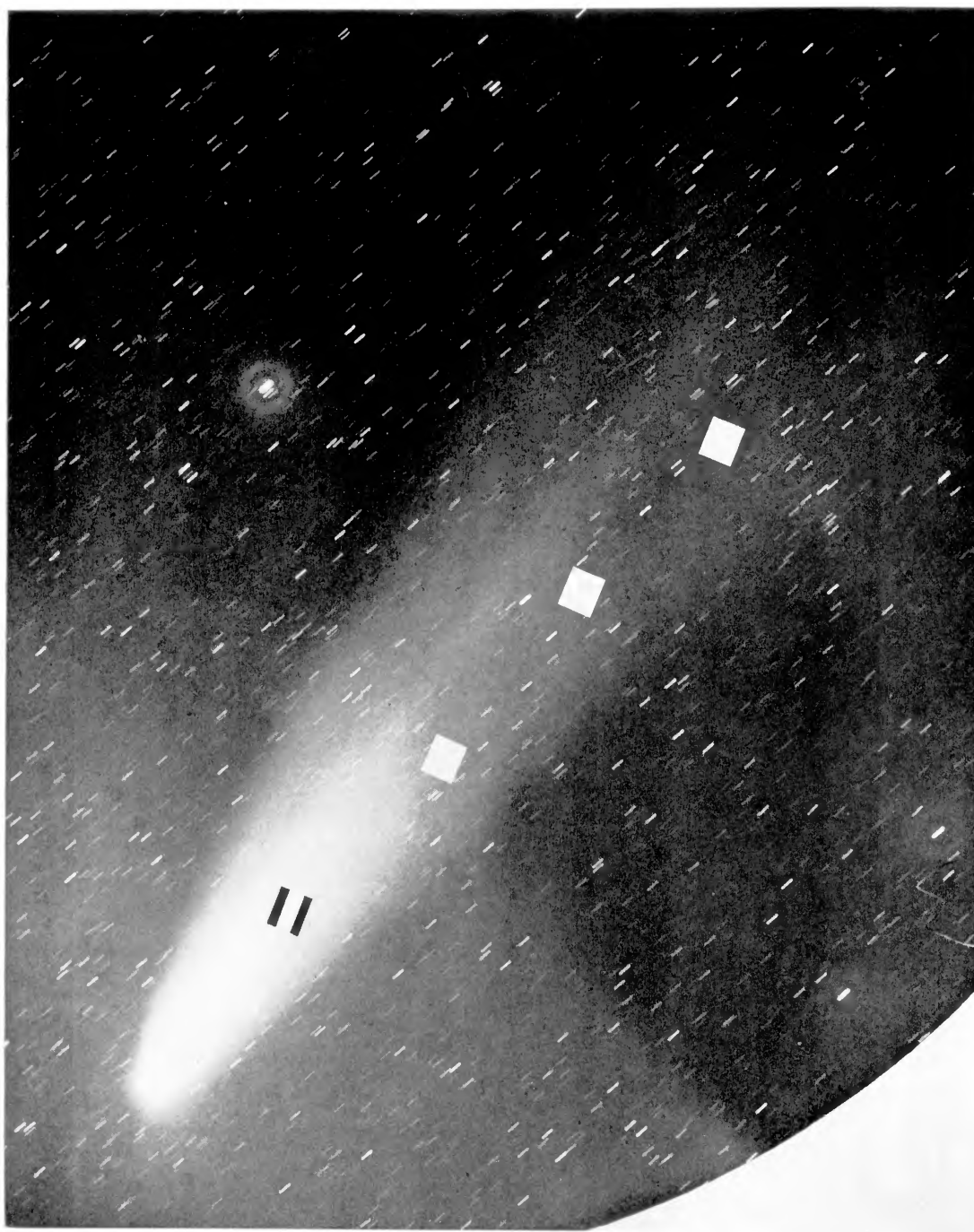


FIG. 8.—Photograph of Comet 1956h (Arend-Roland) taken by F. D. Miller with the Curtis Schmidt telescope, May 1.2, 1957, 103a-O emulsion plus GG5 filter; 20-minute exposure. The rectangles show the projected size, shape, and position of the entrance slit for each spectral scan listed in Table 2.



FIG. 9.—Photograph of Comet 1957d (Mrkos) taken by Dorritt Hoffleit with the 7.5-inch Cooke triplet at the Maria Mitchell Observatory, August 23.2, 1957. Blue emulsion, no filter; 15-minute exposure. The rectangle shows the projected size, shape, and position of the entrance slit for the spectral scan described in the text.

of refraction. Since the values of λ are known, it is a simple matter to evaluate r , the mean particle radius.

These solutions have been tabulated in Table 4. Since the index of refraction of a solid or liquid is roughly proportional to its density, it is possible to make an estimate of the mass of each scattering sphere. Table 4 also lists materials representative of some of the n -values. Columns 4 and 5 give, respectively, the expected density and mass of each type of particle.

The correlation which exists in Table 4 between the index of refraction and the radius of the scattering particles is not surprising. A similar relationship exists between n and the size of absorbing particles (van de Hulst 1957), and van de Hulst has told this investigator privately that such a correlation might exist.

For all these particles, data are given by those who have derived scattering diagrams

TABLE 4
CHARACTERISTICS OF SCATTERING PARTICLES OF VARIOUS
INDICES OF REFRACTION

n (1)	r (μ) (2)	Representative Composition (3)	Density (gm/cc) (4)	Mass (gm) (5)	P (6)
1.20.....	2.2	Liquid O ₂	0.92	4.1 (-11)	+0.20
1.33.....	2.0	Liquid H ₂ O	1.00	3.4 (-11)	— .35
1.40.....	0.80		1.4	3.0 (-12)	— .48
1.44.....	0.63		2.1	2.2 (-12)	— .50
1.50.....	0.57	Glass	2.7	2.2 (-12)	— .48
1.60.....	0.63	Quartz	2.9	3.0 (-12)	— .25
2.00.....		Diamond	3.3	3.0 (-12)	
Iron.....	0.29	Iron	7.9	8.0 (-13)	+ .13
∞	0.13	Perfect reflector	8.0	7.0 (-14)	+0.68

for calculating both the degree and the sense of polarization. Listed in the last column of Table 4 is the polarization, defined as

$$P = \frac{i_{\perp} - i_{\parallel}}{i_{\perp} + i_{\parallel}},$$

where $i_{\perp}$ and $i_{\parallel}$ are proportional to the intensities of the electric vector components, which are perpendicular and parallel to the plane of scattering, respectively. A comet tail directed along a radius vector from the sun through the nucleus would lie in this plane. Therefore, a positive value of P means that the electric vector perpendicular to the tail is the larger component.

Because of the desirability of knowing both the degree and the sense of polarization of the continuous radiation from the tail of each comet, an effort was made to learn of all polarization measures made throughout the world. Summarized below are those observations which have been found by, or kindly communicated to, the author.

Comet 1956h.—Unfortunately, all polarization measures were made within a few minutes of arc of the nucleus. All reports agree, however, that on April 30 the degree of polarization of this radiation was very nearly +0.20. Bappu and Sinval (1957) and Blackwell and Willstrop (1957) restricted their observation to the comet head; H. M. Johnson (1960) measured the polarization a few minutes of arc behind the head; Boiko and Kharitonov (1957) found that the polarization about 1'.5 from behind the head amounted to about +0.23, while observations centered on the nucleus gave +0.17 as

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1.33.	2.0	Liquid H ₂ O	1.00	3.4 (-11)	— .35
1.40.	0.80		1.4	3.0 (-12)	— .48
1.44.	0.63		2.1	2.2 (-12)	— .50
1.50.	0.57	Glass	2.7	2.2 (-12)	— .48
1.60.	0.63	Quartz	2.9	3.0 (-12)	— .25
2.00.		Diamond	3.3	3.0 (-12)	
Iron.	0.29	Iron	7.9	8.0 (-13)	+ .13
∞	0.13	Perfect reflector	8.0	7.0 (-14)	+0.68

for calculating both the degree and the sense of polarization. Listed in the last column of Table 4 is the polarization, defined as

$$P = \frac{i_{\perp} - i_{\parallel}}{i_{\perp} + i_{\parallel}},$$

where $i_{\perp}$ and $i_{\parallel}$ are proportional to the intensities of the electric vector components, which are perpendicular and parallel to the plane of scattering, respectively. A comet tail directed along a radius vector from the sun through the nucleus would lie in this plane. Therefore, a positive value of P means that the electric vector perpendicular to the tail is the larger component.

Because of the desirability of knowing both the degree and the sense of polarization of the continuous radiation from the tail of each comet, an effort was made to learn of all polarization measures made throughout the world. Summarized below are those observations which have been found by, or kindly communicated to, the author.

Comet 1956h.—Unfortunately, all polarization measures were made within a few minutes of arc of the nucleus. All reports agree, however, that on April 30 the degree of polarization of this radiation was very nearly +0.20. Bappu and Sinvaal (1957) and Blackwell and Willstrop (1957) restricted their observation to the comet head; H. M. Johnson (1960) measured the polarization a few minutes of arc behind the head; Boiko and Kharitonov (1957) found that the polarization about 1'.5 from behind the head amounted to about +0.23, while observations centered on the nucleus gave +0.17 as

the value of the polarization; and Blaha *et al.* (1958) observed an increase in the percentage polarization with increasing distance from the nucleus, reaching about 20 per cent some 8' from the head.

It is impossible with the present observations to untangle the contributions to the measured polarization by light which is scattered by small particles, emitted by molecules, and often even reflected from a hard nucleus. However, the photoelectric spectral scans of the head of Comet 1956h described in Section II show that less than one-fourth of the light from the head is emitted in molecular bands. Furthermore, the observations of Johnson, of Boiko and Kharitonov, and of Blake *et al.* probably do not include the nucleus of Comet 1956h. It would, therefore, be most surprising if scattering by small particles were not the primary source of the polarized radiation. Also, it seems not too unreasonable to assume that the scattering particles a few minutes of arc (about 10^5 kilometers) from the nucleus were similar in character to those well back in the tail and that the degree and sign of polarization of the radiation from the tail was in the vicinity of $+0.20$.

Comet 1957d.—At the writer's request, Hoag (1958) obtained an excellent set of measures of the polarization of light from the head and from several points as far as 15' from the comet nucleus. On August 23 the polarization amounted to $+0.14$ at a point 2.5' from the head and seemed to be somewhat more pronounced but of the same sign 15' into the tail. However, trees may have interfered with the latter observation. Bappu and Sinvhal (1960,) using interference filters to eliminate the contribution of major emission features, found the polarization of radiation from the head of Comet 1957d to be close to $+0.13$ on the same night. At 8' from the head, the polarization amounted to 11 per cent, according to Blaha *et al.* (1958). Their observations suggest a somewhat greater polarization farther back in the tail. Lipski (1957) found the polarization to be $+0.14$ at 2' or 3' from the head.

As was noted in Section III, the scattering angle for Comet 1957d was far from 90° , the angle to which the results in Table 4 are restricted. However, the observations of Comet 1956h by Bappu and Sinvhal and by Blackwell and Willstrop show no change in sign of polarization as the scattering angle changed from 90° to nearly 40° , and only a gradual decrease of the amount of polarization.

The agreement in both the size and sign of the polarization values for both comets is striking. From a comparison of these polarization observations with Table 4, it is immediately seen that all particles except those with indices of refraction of approximately 1.20 or those of iron scatter light which is polarized in a negative sense. Again, it should be stressed that a mixture of particle compositions may be present. For example, if half the observed radiation were scattered from quartz ($n = 1.60$) and half from totally reflecting spheres ($n = \infty$), the resulting polarization would agree well with that which is observed. However, it will be assumed here that there exists a predominance of spheres of one size and composition. That such a predominance actually occurs is suggested by studies of micrometeorites (see the last paragraph of this section).

Many medium-weight gases, such as CH_4 , NH_3 , CO , CO_2 , O_2 , and N_2 , when liquified, have indices of refraction near 1.20. However, they all have such low melting points and such small heats of vaporization that their existence within 1 a.u. of the sun and several thousand kilometers away from a comet nucleus would be extremely doubtful. The measured velocities of motion of the particles in comet tails rarely exceed a few hundred kilometers per second (Bobrovnikoff 1951). Therefore, the observed particles in the tails of the two comets had been subjected to solar radiation in a near vacuum for many hours, if not days. Even if these particles existed at first as small ice crystals, it is most unlikely that they would resist melting or subliming for this period of time. Rough calculations by the writer indicate that a pellet of H_2O ice 5μ in diameter would melt within a few minutes at a distance of 1 a.u. Therefore, it is not inconsistent with the observed properties of the tails of Comets 1956h and 1957d that small spheres of iron

the size of this charge because the necessary physical parameters are poorly known. However, it can be assumed that further accelerations would be produced as other electrons and ions from the sun moved rapidly through a comet tail and exerted electrostatic forces on the charged particles of iron.

It is doubted that accelerations as large as observed for knots in the tails of Comets 1956h and 1957d could be shared by the scattering particles. McClure and Liller (1958), from a study of the rectilinear streaks photographed in the tail of Comet 1957d, estimate that the solid grains experience accelerations of the order of 1 cm/sec^2 , which is a not unreasonable value to expect on the basis of the preceding paragraph.

VI. THE MASSES OF THE COMETS

It is not difficult to estimate the density of the small iron spheres in the comet tails necessary to produce the observed surface brightnesses, since the spectral intensity distributions of the tails were compared with those of stars of known spectral type and brightness. At a distance of 1.14 from the head ($2.12 \times 10^6 \text{ km}$) of Comet 1956h, an average separation of the particles of 4.2 meters would produce the observed surface brightness. The point in the tail of Comet 1957d at which the observations were made was farther from the head ($4.42 \times 10^6 \text{ km}$), but the surface brightness was somewhat greater. The calculated average particle separation was 3.1 meters.

With these average separations and taking the particle mass to be $8.0 \times 10^{-13} \text{ gm}$ (see Table 4), one can readily calculate the mass densities of the iron particles at the positions where the observations were made. These values are 1.1×10^{-20} and $2.5 \times 10^{-20} \text{ gm/cm}^3$, respectively. For a rough estimate of the tail masses, consider the tails to be cones of height $8 \times 10^6 \text{ km}$ and base diameter $2 \times 10^6 \text{ km}$ (volume = $8 \times 10^{23} \text{ cm}^3$) with mean densities equal to those observed. The total masses of the particles in the tails come out to be 9×10^{13} and $2 \times 10^{14} \text{ gm}$ (10^8 and 2×10^8 tons), respectively. The true masses are, of course, somewhat higher because of the contribution of atoms, molecules, and solid particles too small to contribute much to the scattered light. For example, Biermann's (1953) estimate of the maximum number of molecules/ cm^3 (10^2) corresponds to a density of $5 \times 10^{-21} \text{ gm/cm}^3$.

Knowing the rate of acceleration of the tail particles (here we take 1 cm/sec^2), their masses, and their number density, one can calculate the rate of mass loss rather quickly. For Comet 1956h, if one uses the numbers already estimated for these quantities, the rate of mass loss was $8 \times 10^7 \text{ gm/sec}$ (90 tons/sec); for Comet 1957d, this quantity was $1 \times 10^9 \text{ gm/sec}$ ($1 \times 10^3 \text{ tons/sec}$). If it is assumed that the comets lost mass continuously at this rate while within 1 a.u. of the sun, but lost no further mass beyond this solar distance, we find that the masses of the particles lost by the two comets during one complete revolution were $5 \times 10^{14} \text{ gm}$ ($6 \times 10^8 \text{ tons}$) and $7 \times 10^{15} \text{ gm}$ ($8 \times 10^9 \text{ tons}$), respectively. And, finally, if we assume that these comets will survive one hundred more revolutions about the sun (Halley's Comet has passed perihelion at least thirty times, Comet Encke fifty-two times), we find that the present masses of these comets must be $5 \times 10^{16} \text{ gm}$ ($6 \times 10^{10} \text{ tons}$) and $7 \times 10^{17} \text{ gm}$ ($8 \times 10^{11} \text{ tons}$).

It is again emphasized that these figures apply only to those iron particles from which the light we see is scattered and can be taken only as rough lower limits to the total masses. It is impossible to do more than guess at what the total masses might be. In Table 5, where the results of these calculations are summarized, a factor of about 20 was taken as appropriate.

As approximate as these numbers are, it seems probable that they are as reliable as any other estimates of comet masses. The masses of the iron particles alone set a lower limit on the total masses of the comets, which can be contrasted with the upper limit of $6 \times 10^{23} \text{ gm}$ set by observations of comets passing near to planets and satellites (Bobrovnikoff 1951).

VII. SUMMARY

Spectrophotometric observations of the tails of Comets 1956h and 1957d show that the greater part of the visible light was scattered by solid particles. The color, definitely redder than sunlight, establishes that the probable diameters fall within the range from 0.25 to $5\ \mu$ (see Table 4). The observed polarization values would be anticipated if the scattering agents were iron spheres with diameters near $0.6\ \mu$ and masses about 8×10^{-13} gm. This result is supported by studies of micrometeorites collected at high altitudes, which show large numbers of particles closely resembling these small iron spheres. The scattering particles are accelerated away from the comet nuclei at rates probably near $1.0\ \text{cm/sec}^2$. These quantities make it possible to calculate the rates of mass loss for the two comets and ultimately the comet masses. The measured surface brightnesses of the tails permit estimates to be made of the masses of the tails. These figures are presented in Table 5.

TABLE 5
DATA LEADING TO THE MASSES OF THE COMETS

	Comet 1956h	Comet 1957d
Date of observations on which results are based.	April 30.2, 1957	August 23.2, 1957
Distance from nucleus at which observations were made.....	$2.12 \times 10^6\ \text{km}$	$4.42 \times 10^6\ \text{km}$
Average separation of scattering particles.....	4.2 meters	3.1 meters
Mean number density of scattering particles.....	$1.35 \times 10^{-8}/\text{cm}^3$	$3.4 \times 10^{-8}/\text{cm}^3$
Mean mass density of scattering particles.....	$1.1 \times 10^{-20}\ \text{gm}/\text{cm}^3$	$2.5 \times 10^{-20}\ \text{gm}/\text{cm}^3$
Assumed volume of tail.....	$8 \times 10^{33}\ \text{cm}^3$	$8 \times 10^{33}\ \text{cm}^3$
Mass of scattering particles in tail.....	$9 \times 10^{13}\ \text{gm}$	$2 \times 10^{14}\ \text{gm}$
Rate of mass loss of comet (scattering particles).	$8 \times 10^7\ \text{gm/sec}$	$1 \times 10^9\ \text{gm/sec}$
Mass of scattering particles lost by comet per revolution.....	$5 \times 10^{14}\ \text{gm}$	$7 \times 10^{15}\ \text{gm}$
Estimated lifetime of comet (number of revolutions).....	100	100
Total scattering particle mass (1957).....	$5 \times 10^{16}\ \text{gm}$	$7 \times 10^{17}\ \text{gm}$
Total mass (guess).....	$10^{18}\ \text{gm}$	$10^{19}\ \text{gm}$

The scattering particles are accelerated away from the comet nuclei at rates probably near $1.0\ \text{cm/sec}^2$. These quantities make it possible to calculate the rates of particle mass loss for the two comets, namely, 8×10^7 and $10^9\ \text{gm/sec}$, respectively, for Comets 1956h and 1957d. Assuming that each comet can endure one hundred more revolutions about the sun, we find that at present the total masses of the particles in the comets are 5×10^{16} and $7 \times 10^{17}\ \text{gm}$, respectively.

Future work on comets with conspicuous tails should include spectral scans such as those described here, to see whether the scattering agents are similar to the ones in Comets 1956h and 1957d. If possible, the observations should extend into the infrared with the hope that a bending-over of the scattering curves might be observed (see Fig. 7). A further check on the results given here would result if the continuous absorption spectrum were observed as the tail swept over a bright star. Best of all, of course, would be a properly instrumental space probe sent through the tail of a bright comet.

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