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NUCLEOSYNTHESIS IN SUPERNOVAE

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ABSTRACT

The role of Type I and Type II supernovae in nucleosynthesis is treated in some detail. It is concluded that *e*-process formation of the iron-group elements takes place in Type II supernovae, while *r*-process formation of the neutron-rich isotopes of the heavy elements takes place in Type I supernovae. The explosion of Type II supernovae is shown to follow implosion of the non-degenerate core material. The explosion of Type I supernovae results from the ignition of degenerate nuclear fuel in stellar material.

I. INTRODUCTION

We regard the sudden fusion of a nuclear fuel as the source of the energy of supernova explosions. A necessary, but not sufficient, condition for the catastrophic instability of a nuclear fuel can be stated at once—the fuel must be capable of yielding an adequate energy supply in a time interval less than the explosion time scale of the star.

The implications of this condition are well illustrated by the case of hydrogen. Although a complete fusion of hydrogen would yield the large supply of 6×10^{18} ergs gm^{-1}, this supply cannot be released quickly, say in a time scale of 1–100 seconds, which is the characteristic interval associated with major stellar explosions. Pure hydrogen fuses at a slow rate even at high temperatures, because $p + p \rightarrow d + \beta^+ + \nu$ is always slow. A quick release of energy from hydrogen is best achieved through the addition of protons to light nuclei—to C^{12}, N^{14}, O^{16}, Ne^{20}—a process that takes place at an adequate rate for temperatures greater than about 10^8 degrees. The amount of energy released in this way is limited, however, by the availability of the light nuclei. Only three or four protons can be added per light nucleus, yielding not more than 10–20 Mev per nucleus. Beta-disintegration lifetimes tend to be long on the proton-rich side of the stability line, a circumstance that prevents any large number of protons from being added very quickly to any nucleus. For example, C^{12} undergoes the following chain of cyclic reactions on a short time scale:

$$C^{12}\,(p, \gamma)N^{13}\,(p, \gamma)O^{14}\,(\beta^+\nu,\ 100\text{ sec})N^{14}\,(p, \gamma)O^{15}\,(\beta^+\nu,\ 200\text{ sec})N^{15}\,(p, \alpha)C^{12}\,.$$

Beta decay must occur where indicated, since F^{15} and F^{16} are unstable to proton emission. Even so, the minimum mean cycle time is 300 sec, and C^{12} will be processed, at most, to O^{15} in stellar explosions of $\lesssim$100-second duration. The energy release in this case is 19 Mev.

In solar material the number of light nuclei is about 5×10^{20} per gram. At 10–20 Mev per nucleus, the yield per gram is therefore about 10^{16} ergs. It follows that even a solar mass of very hot material could not supply sufficient energy for a supernova explosion which requires $\gtrsim 10^{50}$ ergs. Hence we see that ordinary solar material is not potentially very explosive. If an appreciable volume within the sun were suddenly heated in some way to $\gtrsim 10^8$ degrees, the sun would probably explode, but the velocity of the outburst would only be of order 500 km sec^{-1}, considerably less than the velocities encountered in supernovae or even in novae. Only by making the optimum assumption, that the light nuclei are comparable in number to hydrogen, is it possible to obtain supernova energies from stars having masses comparable with that of the sun.

Pure helium is a very stable fuel, because Be^8 is never present in an appreciable concentration. The light nuclei are potentially very explosive, however, with or without an admixture of hydrogen or helium. Pure carbon is explosive through $C^{12}(C^{12}\ \alpha)\ Ne^{20}$ and $C^{12}(C^{12}, p)Na^{23}$ at a temperature somewhat less than 1.5×10^9 degrees, even on a time scale as short as 1 second. The energy yield is $\sim 5 \times 10^{17}$ ergs gm^{-1}. Oxygen is unstable at a somewhat higher temperature through $O^{16} + O^{16}$ reactions. Neon is unstable through photodisintegration $Ne^{20}(\gamma, \alpha)O^{16}$, followed by $Ne^{20}(\alpha, \gamma)Mg^{24}$, or by some other alpha-capture reaction. Magnesium and silicon and elements of similar atomic weight are unstable at temperatures of order 2.5×10^9 degrees. The common light elements are therefore all unstable for $T_9 \sim 2 \pm 0.5$, where T_9 is the temperature in units of 10^9 degrees. The energy yield is of order 5×10^{17} ergs gm^{-1} in all cases, so that 10^{50} ergs are released by the explosion of only one-tenth of a solar mass.

The identification of the light nuclei as the explosive agent in supernovae does not explain the cause of explosion, however. One possible cause has already been mentioned in a former paper (Burbidge, Burbidge, Fowler, and Hoyle 1957; hereafter referred to as "B^2FH"). This possibility will now be described in somewhat closer detail.

II. THE IMPLOSION-EXPLOSION CASE

Throughout the discussion of this section the material of the star will be taken to be non-degenerate. This case will be applied later to stars with masses much larger than $M_\odot$—to the Type II supernovae in fact—and for such stars degeneracy never arises.

This somewhat paradoxical result can be understood, qualitatively at least, by reference to the virial theorem, which relates the gravitational energy of a stellar mass to the thermal energy, NkT, into which the energy not radiated away is converted during stellar evolution. The magnitude of the gravitational energy per gram is proportional to GM/R, where M and R are the mass and radius of the star. Since $R \sim (M/\rho)^{1/3}$, we ultimately find $\rho \sim (NkT)^3/G^3M^2$ so that, for a given T, ρ decreases as M increases. This relation breaks down when the radiative energy content becomes important, but we can see that massive stars will not tend to develop internal densities which lead to degeneracy.

Consider the case of a star with central temperature $T_c > 5 \times 10^9$ degrees. The material of the central regions possesses the composition of the iron group, provided that T_c is not too high (cf. B^2FH). The potentially explosive lighter elements lie well out from the center at temperatures $\lesssim 1.5 \times 10^9$ degrees. Now we shall show below that continuing evolution of the central regions produces a catastrophic situation in which implosion takes place in a time of the order of free-fall—about 1 second. Then, with pressure support withdrawn, the outer regions of the star fall inward. After a short time—again ~ 1 second—the dynamical energy of this inward motion is converted into heat, thereby lifting the temperature of the light elements to an explosive state. In this connection it may be noted that the reaction rate of the explosive light elements depends steeply on the temperature—about as $\exp(-85/T_9^{1/3})$—so that only a comparatively slight change in T_9 is adequate to bring about an explosive situation. The crucial point is that implosion must take place in a time shorter than the time required for the outer part of the star to make a non-explosive reorganization of its structure. For this, some process must rob the central material of heat at an exceedingly rapid rate ($\sim 10^{18}$ ergs gm^{-1} sec^{-1}). Energy losses by radiation or by neutrino emission are both grossly insufficient for this. At the densities of interest—later considerations give about 10^7 gm cm^{-3}—the loss rate through neutrino emission is, at most, of the order of 10^{13} ergs gm^{-1} sec^{-1}, which is too small by a factor $\sim 10^5$. Loss through radiation emission is an even smaller effect.

But a refrigerating reaction of exactly the required degree is indeed available. It is a remarkable feature of statistical equilibrium for nuclear matter that at sufficiently high temperatures the equilibrium shifts decisively from the iron-group elements to helium plus neutrons. Consider the equilibrium $A \rightleftharpoons \alpha A_0 + \beta A_1$, $Z \rightleftharpoons \alpha Z_0 + \beta Z_1$ between a nucleus (A, Z) and its breakup products, a number α of nuclei (A_0, Z_0) and a number β of nuclei (A_1, Z_1). In the notation of B²FH the equilibrium ratio, then, is

$$n(A,Z) = n_0^\alpha n_1^\beta \frac{\omega(A,Z)}{\omega_0^\alpha \omega_1^\beta}\left(\frac{A}{A_0^\alpha A_1^\beta}\right)^{3/2}\left(\frac{2\pi\hbar^2}{M_0 kT}\right)^{3(\alpha+\beta-1)/2} \exp\frac{Q}{kT}, \quad (1)$$

where n with identifying bracket or subscript designates number density, ω designates a statistical factor, A designates atomic weight, $M_0 = N_0^{-1}$ is the atomic mass unit, N_0 is Avogadro's number, and Q is the energy necessary to promote the breakup of A into the products indicated. For the process $Fe^{56} \rightleftharpoons 13\alpha + 4n$, the relevant equation relating Fe^{56}—the most abundant iron-group nucleus—to alpha particles and neutrons is

$$n(56,26) = \omega(56,26)\, n_\alpha^{13} n_n^4 \left(\frac{56^{3/2}}{2^{43}}\right)\left(\frac{2\pi\hbar^2}{M_0 kT}\right)^{24} \exp\frac{Q}{kT}, \quad (1')$$

where $\omega(56, 26) \sim 1.4$ and $Q = 124.4$ Mev is the energy necessary to promote $Fe^{56} \rightarrow 13\alpha + 4n$. Putting $n_n = (4/13)n_\alpha$, as must be the case for neutrons and alpha particles derived from Fe^{56}, it is easily shown from equation (1′) that the material will be half Fe^{56} by mass and half alpha particles and neutrons when

$$\log \rho = 11.62 + 1.5 \log T_9 - \frac{39.17}{T_9}, \quad (2)$$

where ρ is the total mass density in grams/cm³ and T_9 is the temperature in 10^9 degrees. Equation (2) gives the pairs of values for $\log \rho$ and T_9 given in Table 1. A plot of these $\log \rho$, T_9 values is given in Figure 1. To the left of the curve the statistical abundance is concentrated in the iron peak, while to the right of the curve the abundance shifts rapidly toward alpha particles and neutrons. The exponential factor in equation (1′) is the cause of this rapid swing. Consider the situation near the point ($\log \rho = 7$, $T_9 = 6.69$), for example. The exponential factor at this point is equal to $\sim 10^{94}$. An increase of T_9 merely to 7 reduces this factor to $\sim 10^{90}$, showing how quickly the number density of Fe^{56} decreases with rising temperature. Indeed, a range $\Delta T_9 = \pm 0.5$ distributed about

TABLE 1

DENSITY (GRAMS/CM³) AND TEMPERATURE (10^9 DEGREES) AT STATISTICAL EQUILIBRIUM WHEN MATTER IS HALF Fe^{56} BY MASS AND HALF HELIUM AND NEUTRONS

T_9	$\log \rho$	$\log \rho$	T_9
1	−27.55	0	3.17
2	− 7.52	1	3.43
3	− 0.72	2	3.74
4	+ 2.73	3	4.10
5	+ 4.83	4	4.55
6	+ 6.26	5	5.10
7	+ 7.29	6	5.79
8	+ 8.08	7	6.69
9	+ 8.70	8	7.90
10	+ 9.20	9	9.57
11	+ 9.62	10	12.05
12	+ 9.97		

the curve of Figure 1 gives the appropriate transition strip in the log ρ, T_9 plane. To the immediate left of this strip the material is essentially all iron-group nuclei, while on the right it is essentially a mixture of alpha particles and free neutrons in the ratio 13:4. Since the material is completely ionized, we have referred, in the above, to nuclei rather than atoms. It will be clear that the 26 electrons per initial Fe nucleus keep the material neutral throughout the transition, so we can speak of a helium-neutron mixture if we wish.

Now the mean binding energy of the nucleons in Fe^{56} is 8.79 Mev, whereas the average binding of the nucleons in a mixture of 13 alpha particles and 4 neutrons is only 6.57 Mev. Energy must therefore be supplied, in order that $Fe^{56} \rightarrow 13\alpha + 4n$, the amount being 2.22 Mev per nucleon or $\sim 2 \times 10^{18}$ ergs/gm. This is greater than the thermal energy of the material, even at the highest temperature given in Table 1. At $T_9 = 12$ the thermal energy of the helium-neutron mixture amounts to ~ 1.2 Mev per nucleon (this

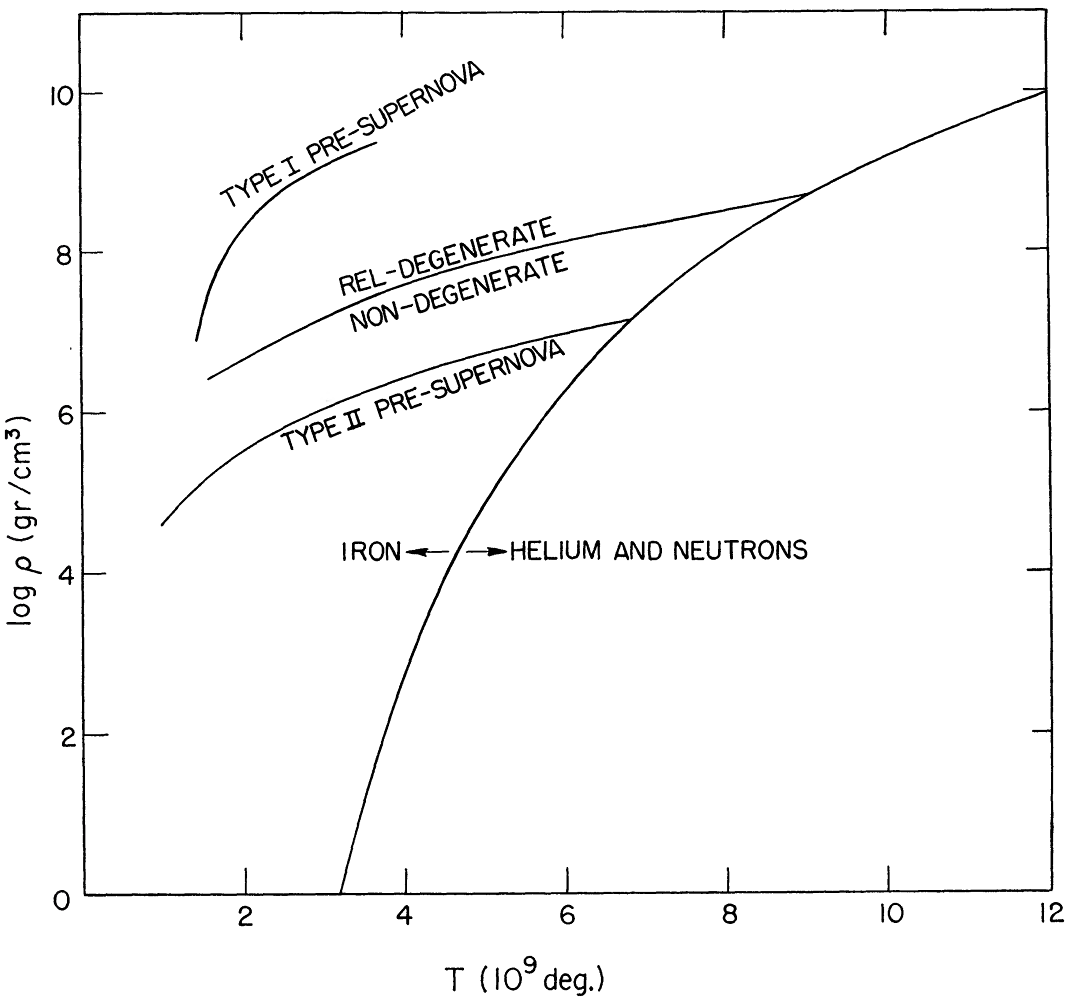

FIG. 1.—The right-hand curve illustrates eqs. (2) and (2′) (approximately), which relate the density and temperature at equilibrium when the material of a supernova core is half iron-group elements by mass and half helium plus neutrons. The upper left-hand curve shows ρ versus T (eq. [20]) throughout the interior of a star with $M \simeq 1.28\, M_{\odot}$ prior to exploding as a Type I supernova. The lower left-hand curve shows ρ versus T (eq. [24]) throughout the interior of a star with $M \simeq 30\, M_{\odot}$ prior to exploding as a Type II supernova. The central left-hand curve separates relativistically degenerate matter from non-degenerate matter.

takes account of the thermal energy carried by the electrons). The thermal and excitation energy of the material as Fe^{56} is $\sim$0.8 Mev per nucleon. The radiation energy is negligible at $T_9 = 12$, $\rho = 10^{10}$.

Returning to the material in the central regions of the star, although the energy loss by neutrino emission cannot promote a free-fall condition, it does cause the evolution to proceed at a comparatively rapid rate. For $T_9 > 3$, the central material moves to the right in the log ρ, T_9 plane in a time scale of $\sim 10^7$ seconds or somewhat less if the neutrino loss mechanism suggested by Pontecorvo (1959) is effective. We shall discuss the exact path for several values of the stellar mass in what follows. Eventually the material reaches the transition strip of Figure 1. It then becomes deflected upward along the edge of the strip; it cannot continue into the helium zone, since there is no source of energy immediately available to supply the 2.2 + 0.3 Mev per nucleon necessary to make the transformation to helium + neutrons at $T_9 = 6.69$. With continuing evolution, more and more of the stellar material reaches the helium zone. It is in turn deflected along the outer edge of the transition strip. The question now arises as to whether the whole material of the star can thus slide in an equilibrium structure along the outer edge of the helium zone, evolving to higher and higher densities as it does so.

This question is answered by remembering that the virial theorem must be satisfied by any stellar model in mechanical equilibrium, or in near equilibrium. The virial theorem requires that

$$|\text{Gravitational energy}| = n\,(\text{thermal energy})\,, \tag{3}$$

where the gravitational and thermal energies are integrals for the whole star and n is a number between 1 and 2, the precise value depending on the relative importance of radiation pressure and gas pressure. Now the left-hand side of equation (3) increases very markedly as the material of the star increases its density along the edge of the transition strip. The right-hand side increases very little, however, since T_9 remains nearly constant (e.g., T_9 increases only from 6.69 to 7.90 as ρ increases from 10^7 to 10^8 gm cm^{-3}). Hence the virial theorem cannot continue to be satisfied as the density increases, and hence the material of the star cannot slide along the outer edge of the transition strip unless the structure of the star departs markedly from mechanical equilibrium. *Such a departure implies a catastrophic implosion—a rapid increase in ρ without a compounding increase in T.*

The situation has a similarity to the well-known Chandrasekhar-Schönberg limit on the mass of the isothermal core of a star at early evolutionary stages near the main sequence—no equilibrium model exists for a core mass containing more than about 12 per cent of the total mass. In the present situation a departure from equilibrium is similarly to be expected when a fraction between 0.1 and 0.2 of the mass of the star has reached the transition strip of Figure 1.

The cause of explosion in this case is logically clear. Nuclear fuels that are potentially explosive contained in the envelope of the star fall inward in the time of free-fall. The dynamical energy so acquired is converted into heat—also in the time scale of free-fall. Then, at the resulting higher temperature, the nuclear fuel yields an explosive supply of energy before the material can expand apart significantly. Explosion of the envelope follows implosion of the core.

III. EXPLOSION WITHOUT CATASTROPHIC IMPLOSION

We come now to the more difficult question of whether explosion can ever arise in a case where there is no such sudden catastrophic release of gravitational energy. We again consider stars in which the central material has evolved to a very high temperature, but where the helium zone of Figure 1 has not yet been reached. Because of the loss of energy through neutrino emission, the evolution again proceeds at a comparatively rapid rate, with a time scale again of order 10^7 seconds. By the evolution being "comparatively rapid" we simply mean that the evolution time scale is short compared with the time required for radiation to diffuse from the inner regions to the surface of the star. Although the evolution is rapid in this sense, the evolution time scale is much longer than the time of free-fall. The star must therefore satisfy the condition of mechanical equilibrium to a high degree of approximation.

For two reasons, the temperature of the potentially explosive nuclei in the outer parts of the star will tend to rise. To provide for energy loss by neutrino emission, the inner parts of the star must contract. The effect is to increase the gravitational field that acts on the outer parts of the star, tending to cause compression, with a consequent release of thermal energy. Since radiation is, moreover, quite unable to diffuse out of the star, energy production by nuclear reactions must also tend to raise the temperature. Sooner or later the light nuclei must therefore reach an explosive condition unless some opposite tendency has the effect of lowering the temperature.

Only one possibility of lowering the temperature arises—expansion of the heated material against the gravitational field. (The refrigerating effect of neutrino emission is unimportant for $T_9 \simeq 2 \pm 0.5$, which is the approximate range of explosion temperatures for the light nuclei.) Ample time is certainly available for a non-explosive expansion to take place, since the time scale of the evolution, $\sim 10^7$ seconds, is long compared with the shortest time in which an expansion of a non-explosive character can take place —this is comparable with, but greater than, the time of free-fall, say 1–100 seconds. Because ample time is thus available, we expect that an explosive condition will not arise *provided that a sequence of expanded configurations that lower the temperature really does exist.* This seems to us to be the crucial criterion for deciding whether or not an explosion can take place in the absence of a catastrophic implosion of the inner parts of the star.

Experience of stellar models suggests that such a series of expanded configurations must exist whenever the material is non-degenerate. Consequently, we take the view that, unless a catastrophic implosion of the core of a non-degenerate star takes place, there can be no explosion.

The situation appears to be crucially different for a degenerate star. This can be understood from the equation of state (cf. eq. [12] below). The pressure depends, in the first order, only on the density and composition. The effect of temperature appears only in the second order. *Hence a first-order change of temperature provides only a second-order change in the structure of the star.* This means that a rise of temperature to the explosion temperature does not affect the structure in the first order, provided, of course, that the temperature rise is not enormous—as, indeed, it need not be—a rise of T_9 from 1.5 to 2 would be ample to induce explosion. We accordingly expect that explosion will always ultimately take place *if potentially explosive nuclear fuels become degenerate.*

To sum up, there appear to be two distinct conditions that can lead to a major stellar explosion: (1) A catastrophic implosion of the core. This condition is necessary when the nuclear fuels are non-degenerate. We shall find this to be the case in massive stars ($M \sim 30\ M_\odot$). (2) Degenerate nuclear fuels are inherently unstable. Explosion can take place during normal evolution—i.e., without a catastrophic implosion being necessary. We shall find this to be the case in stars with mass somewhat greater than $M_\odot$. The existence of two distinct conditions for explosion suggests an association with the two types of supernovae identified by observers. These will now be considered.

IV. SUPERNOVAE OF TYPES I AND II

Baade and Minkowski have identified supernovae of Types I and II with the properties listed below (references up to 1958 are given by Zwicky 1958; also see Minkowski 1960).

Type I supernovae.—These supernovae are found among populations of old stars (pop.

II). At maximum the visual magnitude is -17 to -18. Some 100 days after maximum the light-curve develops a remarkable decline, with half-lives ranging from 40 to 70 days but in many cases close to 55 days.

The initial color is very red, but later the color index settles to a steady value of about $+0.6$, implying that the bolometric correction becomes small.

The expanding envelope contains little or no hydrogen. The mass of the gaseous remnants is the order of a tenth of a solar mass (Minkowski 1960). The expansion velocity is about 2000 km sec^{-1}. The frequency of occurrence is about 1 per 400 years per galaxy (Minkowski 1960).

Type II supernovae.—These supernovae are found only in the arms of spiral galaxies (pop. I). At visual maximum, M_v is about 1 mag. fainter than for Type I. The color is very blue, however, so that the bolometric correction could be large. The light-curve does not decline exponentially. The expanding envelope is rich in hydrogen. The mass of the gaseous remnants are of the order of several solar masses (Minkowski 1960). Velocities of expansion can be as high as 5000 km sec^{-1}. The frequency is about 1 per 400 years per spiral galaxy (Minkowski 1960). Until recently this frequency was given as 1 per 50 years.

If we suppose that Type II supernovae are stars of large mass, say 30 $M_\odot$, it is immediately clear why they are very young stars, too young to have strayed far from their parent gas clouds. Also, if we suppose that supernovae of Type I are stars of comparatively small mass, not much greater than $M_\odot$, it is clear why they are found in old stellar populations.

A division of supernovae according to mass is supported by considerations of stellar evolution. In order that internal temperatures may rise to $\sim 2 \times 10^9$ degrees or more, it is almost certainly necessary that the mass M of the star will satisfy the condition

$$M \gtrsim M_{cr} , \tag{4}$$

$$M_{cr} = \frac{5.80}{\mu_e^2} M_\odot , \tag{5}$$

$$\begin{aligned} \mu_e &= \text{Mean molecular weight per electron} \\ &= \frac{\text{Number of nucleons per unit volume}}{\text{Number of electrons per unit volume}} . \end{aligned} \tag{6}$$

If relation (4) is not satisfied, the star can be supported mechanically by degeneracy pressure and can simply cool off to a white dwarf state. We shall find below that μ_e can be as large as 2.23, in which case relations (4) and (5) require $M \gtrsim 1.16 M_\odot$.

It is not sufficient that relation (4) be satisfied at the birth of a star. We must satisfy it at a stage where the inner regions of the star have reached an advanced evolutionary state. Mass loss at intermediate phases of evolution—for example, mass loss during a giant phase—could reduce M below M_{cr}, even if relation (4) were satisfied initially.

Two classes of star suggest themselves as likely to satisfy relation (4), not only initially, but at an advanced evolutionary stage: (I) stars with initial masses in the range $1.16M_\odot$–$1.5M_\odot$ and (II) stars of very large mass. It seems that stars in the range $1.16M_\odot$–$1.5M_\odot$ lose comparatively little mass in the giant phase. Probably, indeed, they lose comparatively little mass even up to an advanced evolutionary stage. On the other hand, stars of moderately large initial mass appear to suffer extensive losses in the giant phase. However, stars of very large mass undergo nuclear evolution so extremely rapidly that very high central temperatures must arise, even before the neighborhood of the main sequence is quitted. For a sufficiently rapid rate of nuclear evolution, mass loss can again become unimportant. This explains the probable situation for stars of Class II.

In this way, it is possible to understand, at any rate qualitatively, how the supernovae might divide into two groups, one a small-mass group (Type I supernovae), the other a large-mass group (Type II supernovae). Material inside stars of large mass must be non-degenerate, so that we expect Class II to be associated with catastrophic core implosion. But degeneracy occurs in Class I, as we shall see in the following section, so that catastrophic core implosion does not appear to be necessary. Indeed, it seems that the supernovae can be divided into two classes not only in mass but in terms of the mode of explosion.

The division into two mass groups suggests a plausible explanation of why hydrogen is observed to be present in great abundance in Type II supernovae but not in Type I supernovae. A high degree of nuclear evolution in the central regions of a star of small mass implies a considerable nuclear evolution throughout the whole star (except perhaps in layers near the surface). This would require the initial supply of hydrogen to have been effectively consumed throughout the whole star, during the evolution that preceded the supernova stage. Hence only very little hydrogen would be expected in Type I supernovae. In stars of very large mass, on the other hand, advanced nuclear evolution at the center does not necessarily demand advanced nuclear evolution throughout the whole star, large quantities of hydrogen could still remain. Hence hydrogen can well be present in plenty in Type II supernovae.

V. THE INTERNAL STRUCTURE OF SUPERNOVAE OF TYPE I

For stars with $M \gtrsim M_{cr}$, any discussion of degeneracy must include relativistic variations of the electron mass. The pressure within completely degenerate matter at zero temperature and higher density is given by Chandrasekhar (1938) as

$$P = K \left(\frac{\rho}{\mu_e}\right)^{4/3}, \tag{7}$$

where

$$K = \tfrac{1}{4} (3\pi^2)^{1/3} \frac{c\hbar}{M_0^{4/3}} = 1.243 \times 10^{15},$$

$2\pi\hbar$ being Planck's constant, c the velocity of light, and M_0 the atomic mass unit. In this paper we use M_0, not the proton mass H, as the mass unit.

The equilibrium structure of stars, built from material satisfying equation (7), is a polytrope of index 3 satisfying the relation (Eddington 1930)

$$M = \frac{8.060}{\mu_e^2} \left(\frac{K^3}{\pi G^3}\right)^{1/2}, \tag{8}$$

where $G = 6.67 \times 10^{-8}$ erg-cm/gm^2 is the gravitational constant. With K given as above, equation (8) yields

$$M = \frac{5.80}{\mu_e^2} M_\odot = M_{cr}.$$

But this is just the Chandrasekhar critical mass.

Although the radius apparently disappears from the polytropic calculation (in the case of index 3), the value of the radius is carried implicitly in the adoption of equation (7), for equation (7) is an asymptotic formula for $\rho \rightarrow \infty$ (ρ is the mass density). Thus define

$$x = \frac{\hbar}{m_0 c} (3\pi^2 n_e)^{1/3}, \tag{9}$$

where m_0 is the electron rest mass and n_e is the electron number density given by $n_e = \rho/\mu_e M_0$. Then, for x finite but $\gg 1$, the pressure is (Chandrasekhar 1938*a*)

$$P = K\left(1 - \frac{1}{x^2}\dots\right)\left(\frac{\rho}{\mu_e}\right)^{4/3}. \tag{10}$$

The solution for the equilibrium structure of the star tends asymptotically to equation (8) in the limit $x \to \infty$, $\rho \to \infty$; x and ρ are evidently related by

$$\begin{aligned}\rho &= \frac{\mu_e M_0}{3\pi^2}\left(\frac{m_0 c}{\hbar}\right)^3 x^3 \\ &= 9.74 \times 10^5 \mu_e x^3 .\end{aligned} \tag{11}$$

The solution in the asymptotic case is entirely formal, since in an actual star there will always be a non-zero temperature. This adds terms to the infinite series in relation (10). The leading terms of the series are (Chandrasekhar 1938*b*)

$$P = K\left[1 + \frac{1}{x^2}\left(\frac{2\pi^2 k^2 T^2}{m_0^2 c^4} - 1\right)\dots\right]\left(\frac{\rho}{\mu_e}\right)^{4/3}. \tag{12}$$

The polytrope of index 3 is evidently applicable always in the first approximation (i.e., neglecting the term in x^{-2}). It is also applicable in the second approximation in the special case where the coefficient of x^{-2} vanishes, viz., $m_0c^2 = \sqrt{2}\pi kT$, i.e., $T_9 \cong 1.33$. In such a special case equation (8) holds in the second approximation, so that the mass M must be essentially equal to the critical mass.

A similar special case, in which the polytrope of index 3 holds in the second approximation, as well as in the first approximation, also arises when M is not closely equal to M_{cr}. The relevant condition is that

$$\frac{2\pi^2 k^2 T^2}{m_0^2 c^4} - 1 = \alpha x^2 , \tag{13}$$

with alpha a constant throughout the star. Then, correct to the second order,

$$P = K(1+\alpha)\left(\frac{\rho}{\mu_e}\right)^{4/3}, \tag{14}$$

and, instead of equation (8), we have

$$M = \frac{8.060}{\mu_e^2}\left[\frac{K^3(1+\alpha)^3}{\pi G^3}\right]^{1/2} = M_{cr}(1+\alpha)^{3/2}. \tag{15}$$

Using equation (15) to eliminate α from equation (13), we obtain

$$\frac{2\pi^2 k^2 T^2}{m_0^2 c^4} - 1 = x^2\left[\left(\frac{M}{M_{cr}}\right)^{2/3} - 1\right] = \left(\frac{3\pi^2\rho}{M_0\mu_e}\right)^{2/3}\left(\frac{\hbar}{m_0 c}\right)^2\left[\left(\frac{M}{M_{cr}}\right)^{2/3} - 1\right] \tag{16}$$

or

$$\left(\frac{T_9}{1.33}\right)^2 - 1 = \left(\frac{\rho}{9.74\times 10^5 \mu_e}\right)^{2/3}\left[\left(\frac{M}{M_{cr}}\right)^{2/3} - 1\right]. \tag{16'}$$

When, for given M, the density and temperature are related by equation (16), the polytrope of index 3 is satisfied in both first and second approximations (provided, of course, that ρ is large enough for the higher terms in the series [12] to be of smaller order than the x^{-2} term).

There is the physical requirement that the polytrope of index 3 shall be satisfied in the first approximation, but there is no such requirement in the second approximation. Accordingly, there is no physical requirement that equation (16) be satisfied. The appropriate physical condition is that the virial theorem be satisfied in the second approximation (the virial theorem is automatically satisfied in the first approximation, since polytropes satisfy the virial theorem). This condition demands an integral relation between the density and temperature, the integrals being taken throughout the whole star. Relation (16) between ρ and T certainly satisfies the required integral relation (since polytropes always satisfy the virial theorem), but relation (16) is only one example of an infinite class of relations between ρ and T for given M. The member of this infinite class that must be applied for an actual star is decided by the evolutionary history of the star. Unfortunately, no quantitative information is available concerning the later stages of this history, so that, of necessity, we are forced to make an arbitrary choice of one member from the infinite class. The distribution (16) is our choice. The arbitrariness of this procedure is mitigated, however, by the consideration that, although an infinite class of relations between ρ and T is permitted by the virial theorem, the relation must always be the same in order of magnitude, otherwise the integral condition demanded by the virial theorem would not be satisfied for all members of the class.

At first sight, it might be thought that, because of the high thermal conductivity of degenerate matter, relation (16) is an unfortunate choice, since high thermal conductivity tends to produce an isothermal condition. But a time scale $\sim 10^7$ seconds for evolution, such as we are here concerned with, is much too short for the conduction of heat to be an appreciable effect.

Before we proceed to obtain results, it is worth giving some consideration to the condition $x \gg 1$, introduced above to insure that the third term in the expansion for P would be small compared to x^{-2}. The third term is, in fact, small compared to x^{-2}, provided (Chandrasekhar 1938*b*) that

$$\frac{7}{15}\left(\frac{\pi k T}{m_0 c^2}\right)^4 \ll x^2 . \tag{17}$$

Since we are concerned with explosion temperatures of order 2×10^9 degrees, the factor $\pi k T / m_0 c^2$ is of order unity for these fuels. Hence, in considering the potentially explosive nuclear material, the third term is adequately small even for x no larger than 2. Our approximations break down, however, for x of order unity.

We apply relation (16), with $x \geq 2$, to a nuclear fuel possessing an explosion temperature T_{ex}. The following condition on the mass of the star follows immediately:

$$\frac{M}{M_{cr}} \leq \left(\frac{\pi^2 k^2 T_{\text{ex}}^2}{2 m_0^2 c^4} + 0.75\right)^{3/2} = \left[\left(\frac{T_{\text{ex}}}{2.67}\right)^2 + 0.75\right]^{3/2} .$$

In the numerical expression T_{ex} is expressed in units of 10^9 degrees. Combining this with equation (4) places upper and lower limits on the mass:

$$1 \leq \frac{M}{M_{cr}} \leq \left[\left(\frac{T_{\text{ex}}}{2.67}\right)^2 + 0.75\right]^{3/2} , \tag{18}$$

thereby determining the mass range in which a supernova of Type I must lie. As an example, if we require that relation (18) be satisfied for all explosion temperatures in the range $2 \pm 0.5 \times 10^9$ degrees, the closest condition is obtained by putting $T_{\text{ex}} = 1.5 \times 10^9$ degrees in relation (18), viz.,

$$1 \leq \frac{M}{M_{cr}} \leq 1.10 . \tag{19}$$

This yields $M \leq 1.28\ M_\odot$. For $T_{ex} = 1.75 \times 10^9$ degrees, we find $M \leq 1.5\ M_\odot$. Apparently, the mass range is quite small, a circumstance that may well explain the infrequent rate of Type I supernovae.

It is implicit in the foregoing considerations that the major part of the star be degenerate—i.e., x be ≥ 2. This requires the central temperature to be appreciably greater than the explosion temperatures of the nuclear fuels. Thus the star could not explode when, during the evolution, the central temperature rose to $\sim 2 \times 10^9$ degrees, for the reason that at this stage the main body of the star was non-degenerate. But after this the inner regions must soon have become degenerate, and explosion must eventually have taken place, probably when the central temperature reached $\sim 3.5 \times 10^9$ degrees. This can be seen from the following explicit case.

First, we choose a definite value of M/M_{cr}, say 1.1. Then, from the argument leading to inequality (19), it follows that all material at $T > 1.5 \times 10^9$ degrees is effectively degenerate (i.e., $x \geq 2$). According to relation (16),

$$\frac{\rho}{\rho_c} = \left(\frac{2\pi^2 k^2 T^2 - m_0^2 c^4}{2\pi^2 k^2 T_c^2 - m_0^2 c^4}\right)^{3/2} \qquad \text{(Type I supernovae)}, \qquad (20)$$

where ρ_c and T_c are values at the center. With $T_c = 3.5 \times 10^9$ degrees, we obtain Table 2. With ρ/ρ_c worked out from equation (20), the mass fractions in the third line of Table

TABLE 2

CONDITIONS IN INTERIOR OF STAR OF MASS $M = 1.28\ M_\odot$ PRIOR TO EXPLODING AS TYPE I SUPERNOVA

	T_9				
	1.5	2	2.5	3	3.5
ρ/ρ_c	0.00951	0.0976	0.279	0.573	1.00
$M(>T_9)/M$*	0.95	0.70	0.41	0.16	0.00
x	2.00	4.35	6.17	7.84	9.44

* $M(>T_9)/M$ = fraction of mass interior to T_9 for polytrope of index 3.

2 are read off from the appropriate polytropic table (Eddington 1930). The last line of Table 2 follows from the choice of M/M_{cr} to give $x = 2$ at $T_9 = 1.5$, the value of x being proportional to $\rho^{1/3}$, as can be seen from equation (11).

With 95 per cent of the mass of the star lying at $T_9 > 1.5$, only 5 per cent of the star is non-degenerate (i.e., has $x < 2$). And with the nuclear fuels lying in the range $\Delta T_9 = 2 \pm 0.5$, about 54 per cent of the total mass consists of potentially explosive material. Remembering that the energy yield is about 5×10^{17} log gm^{-1}, it follows that the total explosive output can be as high as $2.7 \times 10^{17}\ M$ erg, where M is the total mass in grams. With $M = 1.10\ M_{cr} = 1.28\ M_\odot \simeq 2.6 \times 10^{33}$ gm, the explosive output can therefore be of order 7×10^{50} ergs. At first sight, this might seem rather large, but we shall see in a later section that this is probably not so, that there may well be many calls on the explosive output, quite apart from the generation of dynamical motions and of the emission of radiation.

Reference to equation (11) with $x = 9.44$ at $T_c = 3.5$ gives $\rho_c = 8.2 \times 10^8\ \mu_e$, and, with μ_e somewhat greater than 2, the central density is about 2×10^9 gm cm^{-3}. The point (9.3, 3.5) of the log ρ, T_9 plane lies far away to the left of the helium zone of Figure 1. *Hence we conclude that stars of the small-mass group explode well before their inner regions reach an evolutionary stage where catastrophic implosion could take place.* With $T_c = 3.5$

and $\rho_c = 2 \times 10^9$, we have plotted ρ versus T from equation (20) in Figures 1 and 2. The internal structure of a Type I pre-supernova star is represented by a segment of this curve. It can also be taken very approximately as the evolutionary track of the central region of the star.

To conclude the present section, we note that the mean density in the polytrope of index 3 is 1/54 of the central density. The radius R of the star is therefore given by

$$\frac{4\pi}{3}\,\rho_c R^3 = 54 M\,. \tag{21}$$

Writing $\rho_c = 2 \times 10^9$ gm cm^{-3}, $M \simeq 2.6 \simeq 10^{33}$ gm, gives $R \simeq 2.5 \times 10^8$ cm—less than half the radius of the earth.

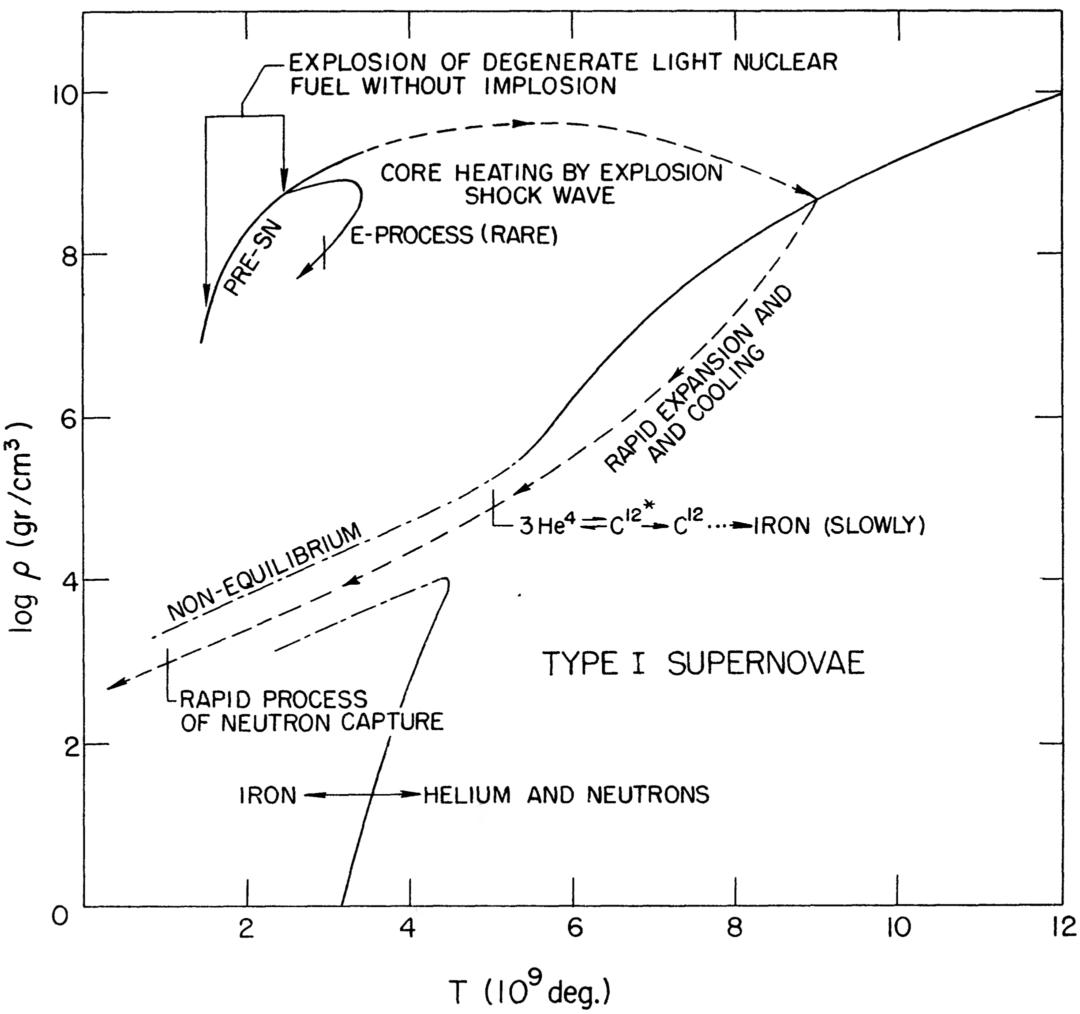

FIG. 2.—Explosion of a Type I supernova. Relativistically degenerate nuclear fuels, C^{12}, O^{16}, Ne^{20}, explode on reaching $1.5 < T_9 < 2.5$ (*upper left*). Material immediately below in the star is compressed and heated and, upon cooling and expanding, "freezes" at $\rho = 10^8$ gm/cm^3, $T_9 = 3$. The equilibrium-process elements which are produced have peak abundance at Fe^{56} (Table 3). The contribution to element abundances from this *e*-process is quite small. The core is heated and compressed by the shock wave from the explosion and rapidly enters the helium-neutron zone. The ultimate expansion and cooling are very rapid, and the slow rate of the $3He^4 \rightleftharpoons C^{12*} \rightarrow C^{12}$ process retards the return to iron via $C^{12}(\alpha,\gamma)O^{16}$, etc. Hence only a few heavy seed nuclei are produced, and the great ratio of neutrons to seed nuclei results in the *r*-process formation of the heavy nuclei by neutron capture at $\rho = 10^3$ gm/cm^3, $T_9 = 1$.

The present results have important implications for nucleosynthesis, but we shall postpone a discussion of these implications until after the structure of the large-mass group of supernovae has been considered.

VI. THE INTERNAL STRUCTURE OF SUPERNOVAE OF TYPE II

We take the supernovae of Type II to be massive stars with $M \sim 30M_{\odot}$. Since the mean molecular weight μ is close to 2 for all common elements heavier than hydrogen and helium, we expect the inner regions of these supernovae to have nearly uniform μ and to be non-degenerate. Outside the core of heavier elements we expect there to be an envelope of hydrogen and helium.

Owing to the lack of quantitative calculations, we make the following further assumptions:

1. The mass of the star is divided between the core and the envelope in the ratio 2:1.
2. The core, with nearly constant μ, possesses a structure similar to that of main-sequence stars of uniform chemical composition. The latter are known not to be greatly dissimilar from Eddington's standard model. Accordingly, we take the core to possess a structure similar to that of the standard model. We have (Eddington 1930*a*)

$$\rho = \frac{a\mu\beta}{3R(1-\beta)} T^3 , \tag{22}$$

$$1-\beta = 0.0030 \left(\frac{2M}{3M_{\odot}}\right)^2 \mu^4 \beta^4 , \tag{23}$$

where a is Stefan's constant, β is the ratio of gas pressure to total pressure, and R is the gas constant. In accordance with our first assumption we have written $2M/3$ for the core mass in equation (23).

The core may be expected to consist quite largely of normal iron-group elements, and for these $\mu \simeq 2.1$. With this value and with $M = 30M_{\odot}$, equation (23) gives $\beta \simeq 0.40$. Inserting the appropriate numerical values of a and R in equation (22), together with $\mu = 2.1$, $\beta = 0.4$, yields

$$\rho = 4.3 \times 10^4 \, T_9^3 \text{ gm cm}^{-3} \qquad \text{(Type II supernovae)} . \tag{24}$$

Reference to Table 1 shows that material satisfying equation (24) reaches the helium zone of Figure 1 near the point (7.1, 6.8) of the log ρ, T_9 plane. Catastrophic implosion occurs shortly after this stage is reached. Equation (24) is plotted in Figures 1 and 3. The internal structure of a Type II pre-supernova star with $M = 30M_{\odot}$ is represented by a segment of this curve. It can also be taken very roughly as the evolutionary track of the central region of the star. For comparison, we also show $\rho = 3.1 \times 10^5 \mu_e T_9^3$, which represents the boundary between non-degenerate and relativistically degenerate matter.

Once again we can estimate the energy released in the ensuing explosion. With the explosive fuels existing once again in the temperature range $\Delta T_9 = 2 \pm 0.5$ and with $T_9 \simeq 7$ at the center, we are evidently concerned with the portion of the standard model, in which $\sim 0.2 \lesssim T/T_c \lesssim 0.35$. We again refer to the polytrope of index 3, since the standard model satisfies this polytrope. With T proportional to $\rho^{1/3}$, in accordance with equation (24), we easily find that about 13 per cent of the core mass lies in the required temperature range. The total mass of explosive fuels is therefore $0.13(2M/3) = 2.6\,M_{\odot}$, and, with an energy release again of 5×10^{17} ergs gm^{-1}, the total potential explosive output is 2.6×10^{51} ergs.

Because of the blanketing effect of the hydrogen-helium envelope, it is reasonable to suppose that not very much of this total output escapes in the form of radiation. Then with an envelope mass of $10\,M_{\odot}$, together with the $2.6\,M_{\odot}$ of exploding material, the energy made available per gram is close to 10^{17} ergs. If this appears as the dynamical energy of expansion, the velocity of explosion is $\sim$4500 km sec^{-1}, comparable with the values actually observed in Type II supernovae.

The core radius is again given by equation (21) but with M now equal to 20 $M_\odot$ and $\rho_c \simeq 1.5 \times 10^7$ gm cm^{-3}. The resulting value of R is $\sim 3 \times 10^9$ cm. The Type II structure is therefore considerably larger than that of the Type I supernovae. The hydrogen-helium envelope can, moreover, enhance the total Type II radius very considerably. A total stellar radius before explosion greater than 10^{10} cm is entirely possible.

It has been pointed out to us by Dr. Stirling A. Colgate that he and Dr. Montgomery H. Johnson have recently investigated the mechanism of core collapse in supernovae (see Colgate and Johnson 1960). They find that the gravitational collapse "overshoots" and supplies approximately twice the energy necessary to convert Fe into helium and neutrons. Thus the surplus energy made available is about 10^{53} ergs in the collapse of a core with mass equal to 20 $M_\odot$. This energy emerges as a spherical shock wave which reverses the inward fall of the envelope material and ejects it from the star. The

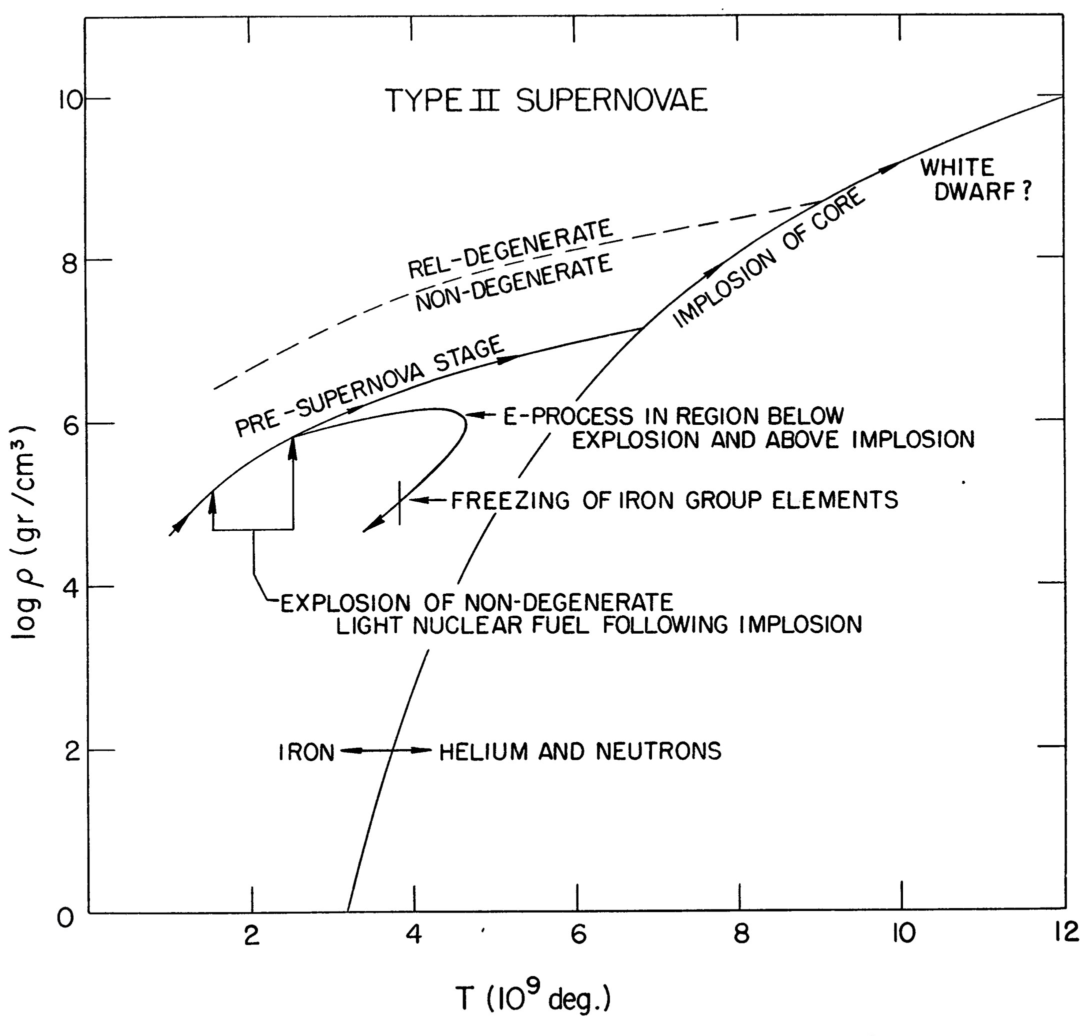

FIG. 3.—Explosion of Type II supernovae. The core of a massive star ($M \simeq 30\ M_\odot$) implodes due to the promotion of $Fe^{56} \rightarrow 13\ He^4 + 4n^1$, and a small remnant may become a white dwarf. Non-degenerate nuclear fuel, C^{12}, O^{16}, Ne^{20}, implodes, reaches $1.5 < T_9 < 2.5$, ignites, and explodes. The material immediately below, but above the core, forms the iron-group equilibrium elements freezing at $T_9 = 3.8$, $\rho = 10^5$ gm/cm^3 with peak abundance at Fe^{56}.

energy supplied in this way, 10^{53} ergs, is considerably greater than that from the nuclear explosion discussed above, namely, 2.6×10^{51} ergs. It is not clear at the present time whether observations on the energy output of supernovae demand energy supplies as great as 10^{53} ergs. In any case, nuclear energy in the range 10^{51}–10^{52} ergs will be released, with the larger of these values definitely being an upper limit for this source of energy.

We conclude this section with a discussion of the limiting lower mass for Type II supernovae. In the previous section we have found that Type I supernovae occur for the limited range of masses from $\sim 1.2\ M_{\odot}$ to $\sim 1.5\ M_{\odot}$. In this section we have taken $M \sim 30\ M_{\odot}$ as a typical value for Type II supernovae. On our view, stars of greater mass than this will become Type II supernovae. In stars with mass much less than this, mass loss will probably occur before nuclear evolution is complete and core implosion occurs. This mass loss may occur in such a way and to such an extent that the star becomes a white dwarf without experiencing either a Type I or Type II explosion. Considerations of this type lead us to estimate $M \sim 10\ M_{\odot}$ as the lower limit for the mass before explosion of Type II supernovae. Stars in the mass range $\sim 1.5\ M_{\odot}$ to $\sim 10\ M_{\odot}$ may not experience catastrophic explosions during their evolution.

VII. THE SYNTHESIS OF THE IRON-GROUP ELEMENTS BY THE *e*-PROCESS IN TYPE II SUPERNOVAE

We now turn to the problem concerning which of the two types of supernovae is the site of the *e*-process synthesis of the iron-group elements. From B²FH (1957*a*) we have the following equation for the number density $n(A, Z)$ of the nucleus A, Z relative to that for $_{26}Fe^{56}$:

$$\log \frac{n(A,Z)}{n(56,26)} = \log \frac{\omega(A,Z)}{\omega(56,26)} + 5.04\frac{A}{T_9}\left[\frac{Q(A,Z)}{A} - \frac{Q(56,26)}{56}\right]$$
$$+ \theta\left(Z - \tfrac{26}{56}A\right) + \frac{A-56}{56}\left[\log \frac{n(56,26)}{\omega(56,26)} - \tfrac{3}{2}\log T_9 - \tfrac{3}{2}\log 56 - 33.12\right]. \qquad (25)$$

A slight numerical modification has been made to incorporate a small term which was eventually neglected by B²FH. In this equation $\omega(A, Z)$ is a statistical weight factor, given by

$$\omega(A,Z) = \sum_r (2I_r + 1)\exp\left(\frac{-E_r}{kT}\right), \qquad (26)$$

where E_r is the energy of an excited state of A, Z measured above the ground state and I_r is its spin. The $Q(A, Z)$ is the binding of the ground state of the nucleus A, Z and is given by

$$Q(A,Z) = c^2\left[(A-Z)M_n + ZM_p - M(A,Z)\right], \qquad (27)$$

where M_n, M_p, and $M(A, Z)$ are the masses of the neutron, proton, and nucleus A, Z, respectively. If atomic masses are employed, replace M_p by $M_H = M_p + m_0$. The quantity θ is defined by

$$\frac{n_n}{n_p} = 10^{-\theta} = \frac{n_e}{2}\left(\frac{2\pi\hbar^2}{m_0 kT}\right)^{3/2} \exp\frac{Q_0}{kT}, \qquad (28)$$

where n_n and n_p are the free neutron and free proton densities, respectively, and n_e is the free electron density, given by

$$n_e = \sum_{A,Z} Zn(A,Z) + n_p. \qquad (29)$$

Q_0 is given by

$$Q_0 = c^2 (M_p + m_0 - M_n) = -0.78 \text{ Mev} . \tag{30}$$

As noted above, equation (28) defines the parameter $\theta = \log (n_p/n_n)$. This equation neglects the relativistic variation of the electron mass. If this is included (Chandrasekhar 1938*c*), then

$$\frac{n_n}{n_p} = 10^{-\theta} = \frac{n_e}{2} \left(\frac{2\pi\hbar^2}{m_0 kT}\right)^{3/2} \left(1 + \frac{15 kT}{8 m_0 c^2} + \dots\right)^{-1} \exp \frac{Q_0}{kT}, \tag{28$'$}$$

where kT/m_0c^2 is taken <1. We shall use equation (28′) in place of equation (28) in the following discussion.

Equations (25), (26), (27), (28′), (29), and (30) are the equations that must be satisfied by ordinary matter neglecting antimatter under statistical equilibrium (including beta-decay equilibrium). It was shown in B²FH that the well-known peak of the abundance-curve centered at $A = 56$, $Z = 26$ can be explained by these equations, the best fit to the observed abundances being given by $T_9 = 3.8$, $\theta = 2.5$.

It is immediately satisfactory that this value of T_9 is only a little higher than the explosion temperatures considered above—these fell into a range $\Delta T_9 = 2 \pm 0.5$. *Hence we may regard the iron-group elements as derived from layers of the star situated immediately below the seat of the explosion itself and just outside the imploding core.* In this layer, equation (24) holds just before the final catastrophic events. Since such layers would certainly be heated in some degree by the explosion, it is clear that their temperature before the explosion could well be less than $T_9 = 3$. Indeed, we shall not be far in error if we take the iron-group elements in which we are interested—those ejected by the explosion—as having $T_9 \simeq 2.5$ before the explosion.

We next consider the requirement $\theta = 2.5$ more closely than was done in B²FH. With $\theta = 2.5$, $T_9 = 2.5$ in equation (28′), we obtain $n_e = 1.28 \times 10^{29}$ cm^{-3}. For material mainly composed of Fe^{56}, the total mass density must be 4.5×10^5 gm cm^{-3}, in order that n_e shall take the value required to give $\theta = 2.5$.

Reference to the work of Section VI herein shows that in Type I supernovae the mass densities lie in the range 10^8–10^9 gm cm^{-3}, very much greater than the value just calculated. The discrepancy is so great that it seems possible to rule out categorically any possibility of building the iron-group elements in Type I supernovae.

The situation is otherwise for the Type II supernovae. Writing $T_9 = 2.5$ in equation (24) gives $\rho = 6.7 \times 10^5$ gm cm^{-3}, a value in excellent agreement with that deduced from the requirement $\theta = 2.5$. No precise agreement can be expected in view of the simplifying assumptions of the previous section. During the explosive stage, T at first rises rapidly from $T_9 = 2.5$, while θ remains frozen at the value 2.5 because the time scale of the explosion is so short. On expanding and cooling to $T_9 = 3.8$, the nuclear processes terminate, and the equilibrium abundance remains frozen thereafter. This is illustrated in Figure 3.

There is an approximation implicit in the above which must be investigated. Equation (29), which must be used in connection with equation (28) or equation (28′), neglects the number density of positrons as compared with that for electrons. This neglect can be justified by noting that

$$\frac{n_{e+}}{n_{e-}} = \frac{4}{n_{e-}^2} \left(\frac{m_0 kT}{2\pi\hbar^2}\right)^3 \exp - \frac{2 m_0 c^2}{kT} . \tag{31}$$

Anticipating that this ratio is small compared with unity, we can use $n_{e-} = 1.28 \times 10^{29}$ cm^{-3}, as deduced from equation (29). Then, for $T_9 = 2.5$, we find from equation (31), $n_{e+}/n_{e-} = 0.2$. This number of positrons must be balanced by an increased number of electrons, but no first-order change in mass density results, and the agreement with the Type II density-temperature relation is not affected appreciably.

At this point we are in position to revise slightly the approximation for the last term on the right-hand side of equation (25) for which B²FH wrote $\simeq 0.183(56 - A)$ for the range of temperatures and densities considered to be of interest. From the above discussion, we can estimate that, upon heating and expansion, the equilibrium mixture freezes at $T_9 = 3.8$ with ρ near 10^5 gm cm^{-3} and $n(56, 26) \sim 10^{27}$ cm^{-3}. With this for future reference, we can write equation (25) as

$$\log \frac{n(A,Z)}{n(56, 26)} = \log \frac{\omega(A,Z)}{\omega(56, 26)} + 5.04 \frac{A}{T_9}\left[\frac{Q(A,Z)}{A} - \frac{Q(56, 26)}{56}\right] + \theta(Z - 0.4643A) + 0.17(56 - A). \quad (25')$$

The above considerations provide a strong confirmation of a suggestion by Baade (1958) that the iron-group elements are produced in Type II supernovae, not in Type I supernovae. A feature of this suggestion is that the common metals begin to be produced at an early stage in the history of a galaxy, since the lifetime of a star of mass 30 $M_\odot$ is only a few million years.

On the reasonable basis that each Type II supernova ejects 3 solar masses (10 per cent of the total mass) of iron-group elements, the rate of production for a whole galaxy is $\sim 0.01\, M_\odot$ per year, accepting a supernova rate of 1 per 400 years. A galaxy condensing from a spherical volume to a disk-shape does so in about 10^9 years, in which time about $10^7\, M_\odot$ of iron-group elements could be produced. Such a production would contaminate $10^{11}\, M_\odot$ of gas with 0.01 per cent of iron-group elements, while after the elapse of $\sim 10 \times 10^9$ years the contamination would rise to ~ 0.1 per cent. This is closely comparable with the iron-group concentration found in the sun. However, note that the Type II activity could have been 10 times as great in the first 10^9 years of the Galaxy or 100 times as great in the first 10^8 years without invalidating this result.

VIII. EQUILIBRIUM ABUNDANCES IN TYPE I SUPERNOVAE

Equation (28′) is not applicable when the electrons are degenerate, as shown by Chandrasekhar (1938*d*). We must use

$$\frac{n_n}{n_p} = 10^{-\theta} = \exp\left[(1 + x^2)^{1/2} + \frac{Q_0}{kT}\right]. \quad (28'')$$

The light nuclei exist at $T_9 < 2.5$. Hence we are concerned in the *e*-process with temperatures greater than 2.5×10^9 degrees. As an example, θ can be calculated for $T_9 = 3$ by writing $x = 7.84$ in equation (28″) (cf. Table 2). The result is $\theta = -2.12$, a very different value from that required to give the observed iron-group abundances, namely, $\theta = +2.5$. The question therefore arises of what form the *e*-process abundances take in the present case.

Exactly as discussed in equation (7), but now standardizing with respect to Fe^{58} instead of Fe^{56}, we have

$$\log \frac{n(A,Z)}{n(58, 26)} = \log \frac{\omega(A,Z)}{\omega(58, 26)} + 5.04 \frac{A}{T_9}\left[\frac{Q(A,Z)}{A} - \frac{Q(58, 26)}{58}\right] + \theta\left(Z - \tfrac{26}{58}A\right) + \frac{A - 58}{58}\left[\log \frac{n(58, 26)}{\omega(58, 26)} - \tfrac{3}{2}\log T_9 - \tfrac{3}{2}\log 58 - 33.12\right]. \quad (32)$$

We again simplify the right-hand side of this equation at the expense of a slight approximation. With $\log[n(58, 26)/\omega(58, 26)]$ in the range 30–31, as it must be for mass densities in the range 10^8–10^9 gm cm^{-3}, the last term on the right-hand side can be taken with sufficient accuracy to be 0.11 $(58 - A)$. We write

$$\log \frac{n(A,Z)}{n(58,26)} = \log \frac{\omega(A,Z)}{\omega(58,26)} + 5.04 \frac{A}{T_9}\left[\frac{Q(A,Z)}{A} - \frac{Q(58,26)}{58}\right] + \theta(Z - 0.4482A) + 0.11(58 - A) . \tag{32'}$$

Table 3 gives the relative abundances of main interest when $T_9 = 3$ and $\theta = -2.12$. (The abundances of nuclei not included in the table are all small.) These results are illustrated in Figure 4.

The main feature is that the equilibrium peak occurs now at Fe^{58} instead of at Fe^{56}. The fact that the abundance ratio of Fe^{58} to Fe^{56} is less than 1:300 in the solar system suggests that the abundances of nuclei from Type I supernovae are abnormally low, a result already derived by the present authors (Fowler and Hoyle 1960) from a consideration of the *r*-process abundances in the solar system.

IX. THE CALIFORNIUM HYPOTHESIS

It was suggested in B²FH that the spontaneous fission of the nucleus Cf^{254} is responsible for supplying the energy emitted under the exponential part of the light-curves of Type I supernovae. This hypothesis proved fruitful, in suggesting the existence of a process in which neutrons are added very rapidly to nuclei of medium atomic weight. Calcula-

TABLE 3

IRON-GROUP EQUILIBRIUM ABUNDANCES ($\log_{10}$ N/Fe^{58}) PRODUCED IN TYPE I SUPERNOVAE ($T = 3 \times 10^9$ DEGREES, $\rho = 10^8$ GM CM^{-3}, $\theta = -2.12$)

	Z	A	Equilibrium Abundance	Freezing Reactions	Final Abundance
Ti.........	22	51	−4.06	....	....
V.........	23	51	−3.01	$Ti^{51}(\beta^-)$	−2.98
		53	−2.84	....	....
Cr.........	24	52	−2.39	....	−2.39
		53	−2.32	$V^{53}(\beta^-)$	−2.21
		54	−0.34	....	−0.34
		55	−3.58	....	....
Mn........	25	55	−2.03	$Cr^{55}(\beta^-)$	−2.01
		56	−3.37	....	....
		57	−3.33	....	....
Fe.........	26	56	−1.90	$Mn^{56}(\beta^-)$	−1.88
		57	−1.98	$Mn^{57}(\beta^-)$	−1.96
		58	0.00	Stan	0.00
		59	−2.04	....	....
Co........	27	59	−2.90	$Fe^{59}(\beta^-)$	−1.98
		60	−3.81	....	....
		61	−2.44	....	....
Ni.........	28	60	−3.86	$Co^{60}(\beta^-)$	−3.53
		61	−3.51	$Co^{61}(\beta^-)$	−2.40
		62	−0.76	....	−0.76
		63	−2.32	....	....
		64	−0.91	....	−0.91
		65	−3.01	....	....
Cu........	29	63	−5.80	$Ni^{63}(\beta^-)$	−2.32
		65	−3.81	$Ni^{65}(\beta^-)$	−2.96
Zn.........	30	68	−4.37	....	−4.37

tions of the abundances to be expected from this "*r*-process" gave results in good agreement with the observed abundances of a large group of heavy nuclei ($A \geq 80$).

Since the publication of B²FH, several new features concerning spontaneous fission have come to light that are relevant to the californium hypothesis. These will now be reviewed.

Johansson (1959) has made theoretical collective-model calculations which indicate that the precipitous decrease in the spontaneous fission half-lives of the isotopes of a given element occurring at neutron number $N = 152$ can be understood in terms of the Nilsson orbitals in deformed nuclei and that this decrease terminates within a few neu-

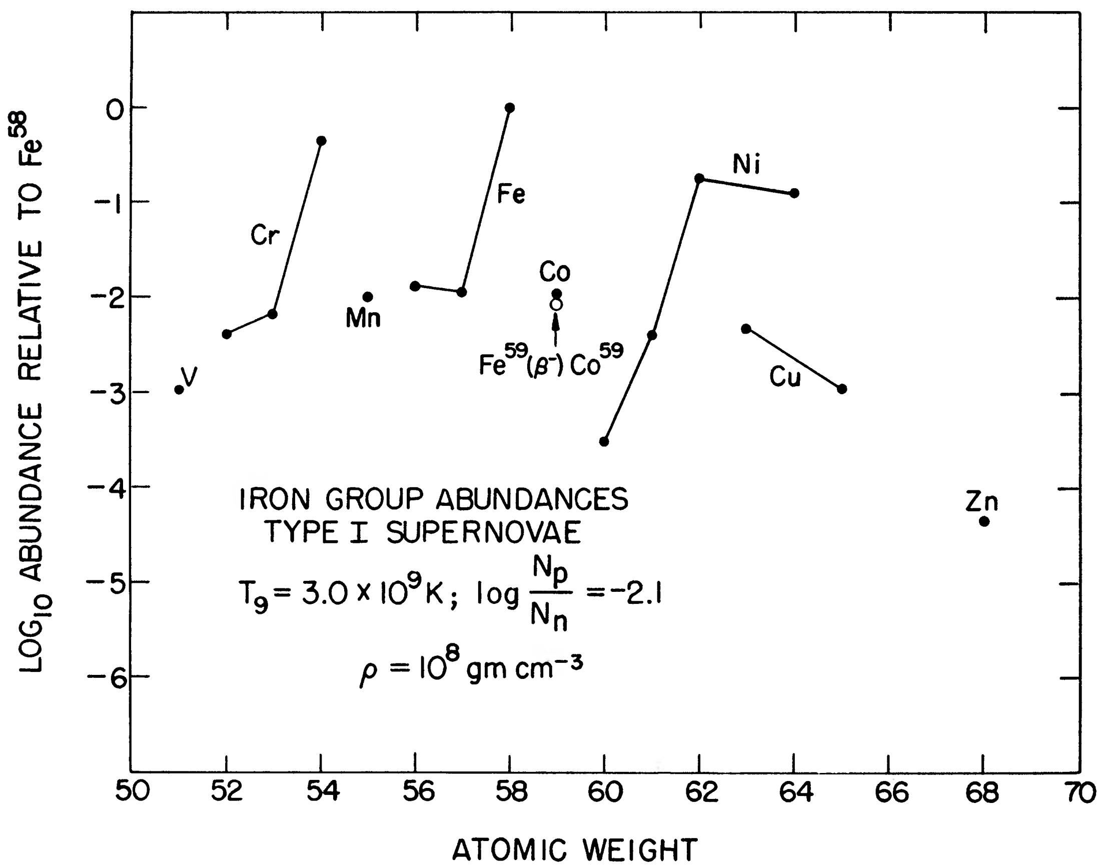

FIG. 4.—Logarithm of the abundance of the iron-group elements relative to Fe^{58} formed by the *e*-process in Type I supernovae at $T_9 = 3$, $\rho = 10^8$ gm/cm³, and $\theta = \log n_p/n_n = -2.1$. Note the great abundance in general of heavy isotopes due to the excess of neutrons. Note, however, that Fe^{59} is only 1 per cent of Fe^{58}. There is little evidence that this particular *e*-process has contributed to element abundances.

tron numbers above $N = 152$. The lifetimes increase thereafter with increasing N, as would be expected from the decrease of the fission parameter $Z^2/(Z + N)$. In Figure 5 we show a tentative revision of Figure VIII, 1, from B²FH (after Ghiorso). It is therefore possible that there are several nuclei, in addition to Cf^{254}, that possess spontaneous fission half-lives in the range from 10 days to 1 year. This is just the range of interest in relation to the light-curves of supernovae. Particularly, Johansson has estimated a lifetime of 30 days to 1 year for Cf^{256}. From an inspection of his calculations we estimate a half-life for Fm^{260} of 10–100 days. Also, Cf^{258}, Fm^{258}, and certain isotopes of 102 and heavier elements, may prove to be of importance in the supernova problem (see Fig. 5).

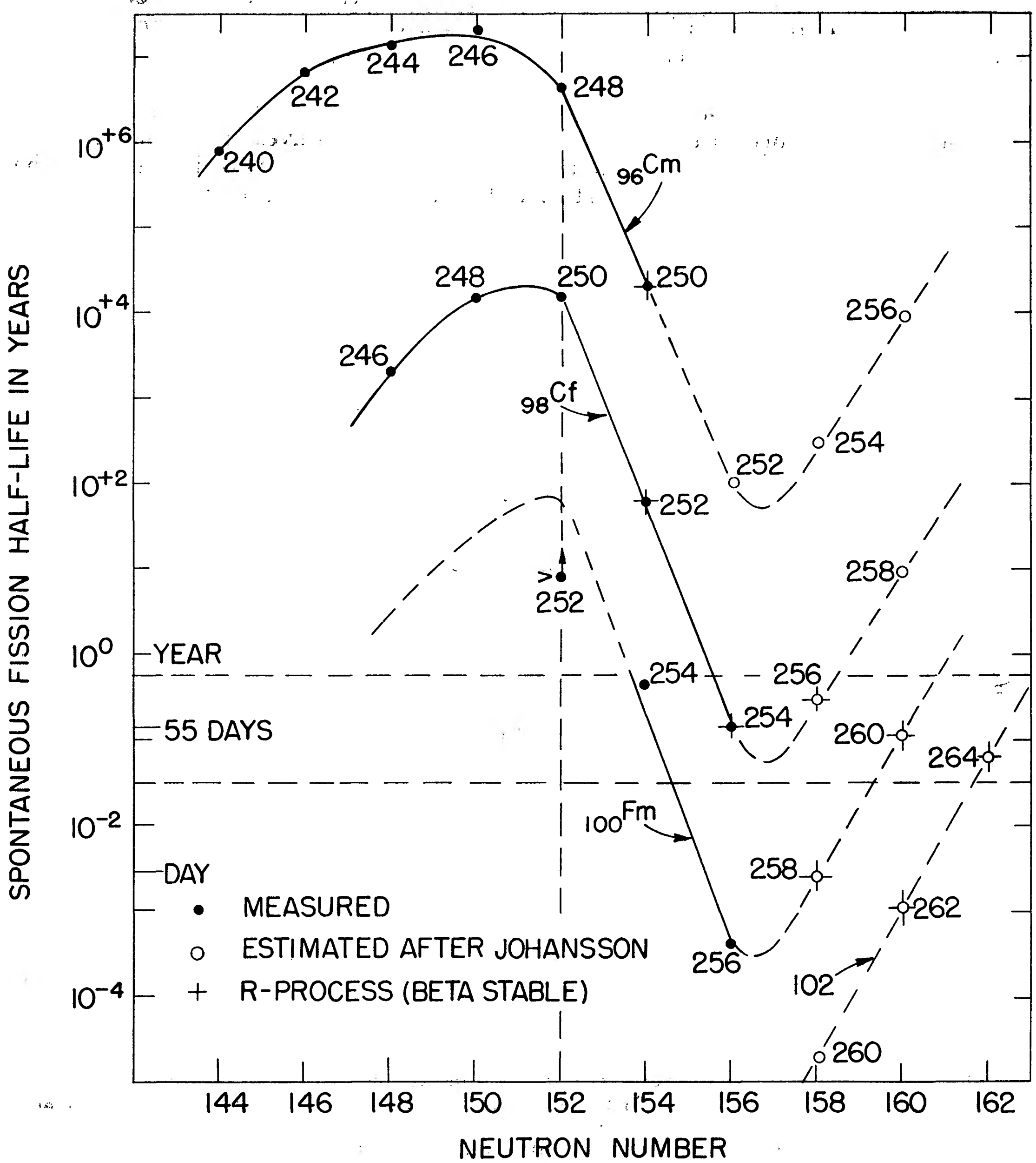

FIG. 5.—Spontaneous fission half-lives in years versus neutron number (after Ghiorso) for isotopes of Cm, Cf, Fm, and element 102. Below $N = 152$, the half-lives increase with N, since for a given element they should increase as $Z^2/A = Z^2/(Z + N)$ decreases. The precipitous drop at $N = 152$ has been explained by Johansson (1959) as a consequence of a peculiarity of Nilsson orbitals in deformed nuclei. This effect should be "healed" by $N = 156$, and we show estimates for various half-lives up to $N = 162$. Note that four beta-stable nuclei produced in the r-process may have half-lives in the range from 30 to 100 days. It is this range which is of interest in connection with the light-curves of Type I supernovae.

It therefore seems likely that the number of nuclei contributing fission energy to supernovae is limited by the maximum atomic weight produced in the r-process. If this limit were variable from one star to another, depending on the precise conditions governing the explosion, a corresponding variation in the detailed form of the supernova light-curve could arise—some variation in the apparent half-life could occur. It was suggested by B^2FH that neutron-induced fission terminates the r-process at $A \simeq 260$. In arriving at this estimate, B^2FH applied the enhancement in spontaneous fission decay rates above $N = 152$ also to neutron-induced fission. Now that Johansson's results cast doubt on the existence of any such enhancement, a reassessment of the termination obtained by B^2FH is accordingly required. However, we must first investigate another important effect due to Wheeler.

Wheeler (1958) has pointed out that the nuclear surface energy must be corrected for neutron-proton asymmetry. This leads to a fission parameter proportional to the ratio of the Coulomb energy to the surface energy,

$$\frac{E_c}{E_s} \propto \frac{Z^2}{A^{1/3}} \left[A^{2/3} \left(1 - \frac{\eta}{\gamma} \frac{I^2}{A^2} \right) \right]^{-1} = \frac{Z^2}{A} \left(1 - \frac{\eta}{\gamma} \frac{I^2}{A^2} \right)^{-1}, \tag{33}$$

where γ is the usual coefficient of the surface energy term in the Weizsäcker mass law and η is the coefficient in the asymmetry correction term $\eta I^2/A^{4/3}$. Recent semiempirical adjustments by Mozer (1959) to the mass law indicate that $\eta/\gamma \simeq 2$. It follows from this expression for the critical fission parameter E_c/E_s that neutron-rich nuclei with large $I = N - Z$ will be more susceptible to fission than would be expected from use of the old parameter $Z^2/(Z + N)$. After recovering from the precipitous drop in fission lifetimes at $N = 152$, as indicated by Johansson, still heavier nuclei eventually succumb to the Wheeler effect. With $\eta/\gamma = 2$, we estimate that neutron-induced fission will terminate the r-process in the range $270 < A < 275$. This falls just short of the abundance peak to be expected at the next closed neutron shell at $N = 184$, for which $A \simeq 280$ in the r-process.

From these considerations it appears that the number of even A nuclei produced in the r-process whose β-stable daughters disintegrate by spontaneous fission may well be of the order of 10. In the following paragraph, we assume that four of these nuclei possess half-lives in the range 30–100 days.

We are now in a position to re-evaluate the energetics of radioactive decay in supernovae.[1] The total energy emitted under the exponential part of the light-curve of Type I supernovae is of order 10^{48} ergs (taking -17 as the value of M_v at maximum). Since one solar mass of fissionable material produces 1.75×10^{51} ergs, it therefore follows that $6 \times 10^{-4} M_\odot$ of such material is required to provide 10^{48} ergs. If four activities of the same intensity as Cf^{254} are present, the required mass of C^{254} becomes $1.5 \times 10^{-4} M_\odot$ per supernova. Since it was shown by B^2FH that Cf^{254} comprises 1 per cent of the total mass of all the r-process nuclei, it follows that the latter mass must be $\sim 1.5 \times 10^{-2} M_\odot$. We shall see in the next section that $\sim 10^{-2} M_\odot$ of neutrons and $0.5 \times 10^{-2} M_\odot$ of seed nuclei can be produced and fused in Type I cores to synthesize their r-process nuclei.

[1] We emphasize once again that the exponential decline in the visible light-curve of Type I supernovae may not be connected in any simple way with the exponential decline in energy input from a radioactive nucleus of appropriate half-life. Why the visible energy output should directly follow the energy input is still an unsolved problem, as pointed out by B^2FH. On the other hand, we have considerable confidence in our order-of-magnitude estimates of the production of fissionable material in Type I supernova explosions. *The input of radioactive energy into the exploding debris of supernovae cannot be neglected.* Furthermore, the production of Cf^{254} within an interval of a few microseconds in the first hydrogen bomb test in 1952 must be taken as observational evidence for the *rapid process* of neutron capture, by which fissionable material is produced in supernova explosions. The heaviest nucleus in the bomb components was U^{238}. At least 16 neutrons were added in the short interval of the bomb explosion. It is not unreasonable to extrapolate by a factor of 10 or more in going to stellar explosions, where the iron-group elements serve as seed nuclei but the neutron fluxes are considerably enhanced.

It may be useful to end the present section by comparing this californium requirement with an alternative suggestion considered and rejected by B^2FH, but recently revised by Anders (1959)—that the 45-day activity of Fe^{59} supplies the main energy under the exponential decline of the light-curves. The decay of one solar mass of Fe^{59} produces only 4.4×10^{49} ergs. Moreover, Anders has shown that only 20 per cent of the gamma- and beta-ray energy from Fe^{59} can be converted into optical radiation emitted at the surface of the expanding supernova envelope. Hence one solar mass of Fe^{59} produces only $\sim 10^{49}$ ergs of optical energy. From this we see that Fe^{59} cannot be a major contributor to the supernova light-curve unless it is present to the extent of about 0.1 $M_{\odot}$. Reference to Table 3 shows that only $\sim 1/100$ of the *e*-process material consists of Fe^{59}. Estimating the mass of the *e*-process material as 0.5 $M_{\odot}$, the amount of Fe^{59} is of order 1/200 $M_{\odot}$. This is too small to be of consequence by a factor of 20.

X. THE *r*-PROCESS AND THE NEUTRON SOURCE IN TYPE I SUPERNOVAE

According to the previous section, the total mass of *r*-process material must be of order $1.5 \times 10^{-2} M_{\odot}$. The major part of this mass must be contributed by the neutrons that are added to the nuclei of medium atomic weight. The total mass of neutrons must therefore be of order $10^{-2} M_{\odot}$. Now the exploding fuel of a supernovae consists of light elements, and the mass of these light elements $\sim 0.5 M_{\odot}$ (cf. Sec. V).

Evidently, then, if one neutron were produced per light nucleus, the neutron supply would be closely of the required amount. This was the point of view adopted in B^2FH, the favored reaction being $Ne^{21}(\alpha, n)Mg^{24}$.

A serious objection it now seems to us can be brought against this point of view. An essential point in the former work was that the free neutrons must not be appreciably absorbed by the abundant lighter nuclei but must be taken up by the far more infrequent nuclei of medium atomic weight—e.g., by Fe^{56} or Fe^{58}. Only in this way can 100–200 neutrons be made available per absorbing nucleus, as is necessary if Fe^{56}, for example, is to be built to $A \simeq 270$.

The difficulty is that the *r*-process must take place at $T_9 \gtrsim 1$. The neutrons therefore possess thermal energies $\gtrsim 100$ kev. At such energies it now seems to us impossible that none of the common light nuclei possess appropriately placed resonance levels. Hence we cannot believe that the neutron capture cross-sections of the abundant light nuclei would all be small. (The case of the *s*-process, discussed by B^2FH, is quite different. Here the neutron energies ~ 10 kev, and, for such energies C^{12}, O^{16}, Ne^{20}, Mg^{24} all have very small $[n, \gamma]$ cross-sections, although N^{14} must be excluded even at neutron energies of 10 kev.)

This objection suggests that some entirely different neutron source be looked for, a source perhaps not immediately connected with the exploding nuclear fuel. To this end, we first notice how very nearly unstable the whole degenerate structure of a Type I supernova is, even ***before explosion***. The virial theorem requires

$$|\Omega| = 3\int P d v , \qquad (34)$$

where $|\Omega|$ is the magnitude of the total gravitational potential, the volume integral of the pressure being taken throughout the star. Writing u for the internal energy per unit volume, the total negative energy of the star is

$$\int u d v - |\Omega| = \int (3P - u) d v . \qquad (34')$$

Now, for relativistically degenerate matter, $3P - u$ is zero in first approximation. The second-order value at zero temperature depends on the parameter x. According to Table 2, throughout most of the star x lies between 5 and 10. For values in this range, $u \simeq 2.5 P$ (Chandrasekhar 1938*e*), even at zero temperature. The thermal contribution increases the ratio u/P. Clearly, then, we shall not be overestimating the binding energy of the star if we put $u = 2.5\ P$ in equation (34′). We then have

$$\text{Binding energy} \simeq 0.5 \int P dv = \tfrac{1}{6} |\Omega| . \tag{35}$$

Furthermore, since the structure of the star is the polytrope of index 3 in the first approximation, we can put $|\Omega| = 3\ GM^2/2R$. Thus

$$\text{Binding energy} \simeq \frac{GM^2}{4R} . \tag{36}$$

For $M \simeq 1.28\ M_\odot$, $R \simeq 2.5 \times 10^8$ cm—as determined in Section V—we obtain a binding energy of $\sim 1.7 \times 10^{50}$ ergs. This is appreciably less than the explosion energy obtained in that section, viz., 6×10^{50} ergs. *Hence the explosion is amply violent to shatter the whole star.* This is not the case for Type II supernovae, since the structure is not degenerate in these supernovae.

In order that total disintegration can occur, energy must be concentrated at the center of the star. A spherical shock wave converging from the region of explosion could provide such an energy source. The material at the center would then become heated. It will then eventually reach a track in the log ρ, T_9 plane of the form shown in Figures 1, 2, and 3. For $Fe^{58} \rightleftharpoons 13\ \alpha + 6\ n$, equation (2) must be modified only slightly to read

$$\log \rho = 11.64 + 1.5 \log T_9 - \frac{39.82}{T_9} . \tag{2′}$$

Finally, the material will enter the helium zone. Such an entry will demand a disintegration of the e-process elements into helium + neutrons; for example, Fe^{58} would break up into 13 alpha particles and 6 neutrons. Hence the e-process elements would yield alpha particles and free neutrons in the ratio $\sim 2.2:1$.

Suppose, further, that sufficient energy is focused in the central regions for the temperature to be at first maintained as the whole star begins to expand. In particular, suppose that the density at the center falls in the expansion to a value between 10^5 and 10^6 gm cm^{-3} *before the temperature declines sufficiently for the material to emerge from the helium zone.* The density is then so much reduced that carbon synthesis through $3\alpha \rightarrow C^{12}$ may be very incomplete. This is illustrated in Figure 2.

From a nuclear point of view, such a situation would provide ideal conditions for the operation of the r-process. The small fraction of the helium built into carbon proceeds very quickly to add further alpha particles until atomic weights of 60–70 are reached. These are then the seed nuclei for the r-process.

Let a fraction f of the alpha particles undergo $3\alpha \rightarrow C^{12}$. As just remarked, the C^{12} is built rapidly into seed nuclei, with A in the range 60–70. Since $\alpha : n \simeq 2.2:1$ on emergence from the helium zone, the ratio of seed nuclei to free neutrons is $(2.2/3)f:1$. So, for $f \lesssim 10^{-2}$, we obtain an appropriate ratio of free neutrons to seed nuclei. Hence the essential requirement for operation of the r-process is $f \lesssim 10^{-2}$. We proceed to examine this requirement.

We take the time scale for expansion of the star, through the critical phase where material emerges from the helium zone, to be ~ 10 seconds. This is obtained from $(\rho G)^{-1/2}$ with $\rho \simeq 10^5$ gm cm^{-3}. Our requirement from a nuclear point of view is, therefore, that the mean lifetime per alpha particle, $\tau_{3\alpha}$ say, against $3\alpha \rightarrow C^{12}$ must be $\gtrsim 10^3$ seconds. The expression for $\tau_{3\alpha}$ is

$$\frac{1}{\tau_{3\alpha}} = 2.37 \times 10^{-7} \rho^2 \frac{\Gamma_\gamma}{T_9^3} \exp - \frac{27.2}{T_9} , \tag{37}$$

similar to the formula given by B^2FH(1957*b*), except that C^{12} formation now takes place through the 9.63-Mev excited state of C^{12} level rather than through the 7.65 state, as it does at lower temperatures. The transition from the 9.63-Mev state to the ground state is fastest if it is electric dipole, $1^- \rightarrow 0^+$. There is reason to believe that the radiation width Γ_γ for this transition may be abnormally small, however. We therefore proceed by determining the value of Γ_γ required to give $\tau_{3\alpha} = 10^3$ seconds at $\rho \simeq 10^5$ gm cm^{-3}, $T_9 = 5$. The required value is $\Gamma_\gamma \simeq 0.01$ ev. Weidenmüller (1960) has informed us privately that this implies a 4 per cent mixture of isotopic spin $T = 1$ from the $T = 1$ state with 1^- at 17.63 Mev and that this is not an unreasonable value. Similar arguments can be advanced for states at 9.0 and 10.1 Mev.

The value of ρ at which the material emerges from the helium zone is more uncertain than the value of T_9, which cannot be much different from 5. For other values of ρ, the condition on the radiation width is, therefore,

$$\Gamma_\gamma \sim 10^8 \rho^{-2} \text{ ev} . \tag{38}$$

Thus, if Γ_γ were as low as 10^{-4} ev, the density at emergence from the helium zone could be as high as 10^6 gm cm^{-3}. In this case it would also be necessary for Γ_γ to be abnormally small for all levels of C^{12} immediately above the 9.63-Mev level, say $\Gamma_\gamma \lesssim 10^{-3}$ ev for such levels.

The neutrons do not add themselves immediately to the seed nuclei. There are waiting points along the *r*-process chain (B^2FH 1957*b*). Starting at $Z = 28$, for example, there are waiting points at $A = 78$, $Z = 28$; $A = 79$, $Z = 29$; $A = 80$, $Z = 30$; $A = 81$, $Z = 31$; $A = 84$, $Z = 32$; $A = 85$, $Z = 33$; $A = 90$, $Z = 34$. . . . The mean lifetimes at these points are 1.04, 0.90, 12.3, 8.96, 6.96, 4.91, and 4.50 seconds, respectively. Thereafter, at higher A, the lifetimes at the waiting points become markedly less. About 50 seconds are therefore needed before an appreciable fraction of the seed nuclei have been built to $A \sim 270$. This is about five times longer than the time required for appreciable expansion of the star (with $\rho \simeq 10^5$ gm cm^{-3} at emergence from the helium zone, the time of expansion—$\sim(\rho G)^{-1/2}$—is about 10 sec.). During the expansion, both the temperature and the density fall, with $\rho \propto T^3$, since the energy density is mainly in the form of radiation, which has a ratio of specific heats of 4/3. A fivefold increase of radius, occurring in the 50 seconds, implies a decline of ρ by 5^3, and hence of T by 5. The temperature therefore declines, during the process of neutron addition, from about 5×10^9 degrees to about 10^9 degrees (see Fig. 2). *We conclude that the upper cyclic part* ($A \sim 110$) *of the r-process chain must be built at about* $T_9 = 1$, a conclusion in excellent agreement with the temperature inferred by B^2FH (1957*b*). The neutron number density is $\sim 10^{26}$ cm^{-3}, larger than the value 10^{24} cm^{-3} used by B^2FH but consistent with their detailed findings for the conditions under which the *r*-process abundance peaks at $A = 130$, $N = 82$ and $A = 196$, $N = 126$ were synthesized. The higher value does not alter the values calculated by B^2FH at all significantly, however. The *r*-process abundance peak at $A = 80$, $N = 50$ is to be ascribed to a synthesis under conditions different from those in which the two heavier peaks were produced. The main difference would seem to be that the total integrated neutron flux was considerably smaller than when cycling was attained, as required for synthesis of the two heavier peaks.

The present considerations must satisfy an important energy requirement. If material is disrupted into helium + neutrons in the manner suggested above and if only a small fraction of the helium recombines through $3\alpha \rightarrow C^{12}$, energy is eventually lost. Energy of about 2×10^{18} ergs gm^{-1} is needed to disintegrate the *e*-process elements. Only about half this energy expenditure is recovered in the *r*-process. The loss must evidently be borne by the energy source, namely, by the shock waves propagated inward from the region of nuclear explosion. The important question therefore arises of whether the energy released in the explosion is adequate to bear the loss.

We require a mass of $\sim 10^{-2}\, M_\odot$ as neutrons. Together with $0.5 \times 10^{-2}\, M_\odot$ of seed nuclei from $3\alpha \rightarrow C^{12}$, etc., this will just produce the required fissionable *r*-process nuclei. The mass of Fe^{58} that must be disintegrated into He + *n* is therefore $\sim 10^{-1}\, M_\odot$. At an ultimate energy cost of $\sim 10^{18}$ ergs cm^{-1}, the total energy expenditure is $\sim 2 \times 10^{50}$ ergs. The energy released in the explosion $\sim 7 \times 10^{50}$ ergs (Sec. V).

We are now in a position to draw up a tentative energy budget for a Type I supernova (see accompanying table). A star of mass 1.28 M with a total dynamical expansion energy of $\sim 10^{50}$ ergs possesses an explosion velocity of order 3000 km sec^{-1}, which is certainly of the correct order. It will be as well to emphasize in conclusion the following points:

Expenditure	Ergs
Disintegration of Fe^{58}	$\sim 2\times 10^{50}$
Disruption of star (from eq. [36])	$\sim 1.5\times 10^{50}$
Dynamical energy of explosion	$\sim 10^{50}$
Radiant energy emitted	$\sim 10^{50}$
Total	$\sim 5.5\times 10^{50}$
Explosive release of energy (Sec. V).	$\sim 7\times 10^{50}$

(i) There is strong evidence from the empirical abundance-curve that a process of rapid neutron capture occurs in the stars, presumably in exploding stars—the supernovae.

(ii) No process appears to be available for providing an intense neutron flux unless the temperature is at least as high as 10^9 degrees.

(iii) At this temperature we expect free neutrons to be readily captured by the common light nuclei, *with the exception of* He^4.

(iv) The necessary condition for the operation of the *r*-process would therefore seem to be either a mixture of He^4 and neutrons plus a comparatively small concentration of seed nuclei, or simply pure neutrons plus seed nuclei.

(v) We have not been able to find any plausible process for producing pure neutrons. It is true that at very high densities $\sim 10^{11}$–10^{12} gm cm^{-3} a great concentration of free neutrons does arise, but other astrophysical processes always appear to prevent any such high-density condition from being attained. It is, moreover, not sufficient to consider the problem at high densities. During expansion the density declines, and there is difficulty in preserving conditions appropriate to the *r*-process during this decline.

(vi) The mixture of He^4 + neutrons has the further advantage that at sufficiently low density the process $3\alpha \rightarrow C^{12}$, followed by subsequent alpha captures, automatically provides a supply of seed nuclei for the *r*-process.

We conclude that the cores of Type I supernovae are the sites for nucleosynthesis by the *r*-process.

REFERENCES

Anders, E. 1959, *Ap. J.*, **129**, 327.
Baade, W. 1958, *Pontificiae Academiae Scientiarum Scripta Varia*, No. 16.
Burbidge, E. M., Burbidge, G. R., Fowler, W. A., and Hoyle, F. 1957, *Rev. Mod. Phys.*, **29**, 547.
———. 1957*a*, *ibid.*, p. 578.
———. 1957*b*, *ibid.*, p. 593.
Chandrasekhar, S. 1938, *Stellar Structure* (Chicago: University of Chicago Press), p. 357.
———. 1938*a*, *ibid.*, p. 360.
———. 1938*b*, *ibid.*, p. 392.
———. 1938*c*, *ibid.*, p. 396.
———. 1938*d*, *ibid.*, p. 388.
———. 1938*e*, *ibid.*, p. 361, table.
Colgate, S. A., and Johnson, M. H. 1960, *Phys. Rev.* (Letters), **5**, 235.
Eddington, A. S. 1930, *The Internal Constitution of the Stars* (Cambridge: Cambridge University Press), p. 83, eq. 57.3.

Eddington, A. S. 1930*a*, *ibid.*, pp. 117, 120.
Fowler, W., and Hoyle, F. 1960, *Ann. Phys.*, **10**, 280.
Johansson, S. A. E. 1959, *Nuclear Phys.*, **12**, 449.
Minkowski, R. 1960, *Proc. Nat. Acad. Sci.*, **46**, 13.
Mozer, F. 1960, *Phys. Rev.*, **116**, 970.
Pontecorvo, B. 1959, *J. Exper. Theoret. Phys.* (U.S.S.R.), **36**, 1615.
Weidenmüller, H. 1960, private communication.
Wheeler, J. 1958, private communication.
Zwicky, F. 1958, *Hdb. d. Phys.*, ed. S. Flügge (Berlin: Springer-Verlag), **51**, 766.