COLOR EXCESSES FOR SUPERGIANTS AND CLASSICAL CEPHEIDS
I. CALIBRATION OF THE G-BAND PHOTOMETRY*

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ABSTRACT

The determination of spectral types for cepheids on the MK system is critically examined. It is concluded that the anomalous strengthening of certain strong lines in cepheid spectra near maximum light results chiefly from a kinematical disturbance rather than from an excitation or ionization effect Spectral types for cepheids that are consistent with those of non-variable supergiants of the MK system are derived from a study of the G band. G-band strengths are measured using interference-filter techniques, and the measurements are correlated with intrinsic $(B-V)$ color through observations of U Sgr and EV Sct, which are members of open clusters, and δ Cep, which has a blue, presumably physical, companion. Supplementary spectral types are obtained for these and other cepheids from visual inspection of the G band on classification spectrograms. Intrinsic colors are given for class *Ib* stars from F0 to K0. Color excesses of high precision are derived for 19 cepheids; somewhat less precise excesses are given for 19 other cepheids and 7 non-variable supergiants. It is concluded that the Magellanic Cloud cepheids with periods between 2.5 and 45 days might have the same relation between $(B-V)^{\circ}_{\max}$ and log P as do the galactic cepheids if the Cloud cepheids are reddened, on the average, by about 0.15 mag. owing to interstellar material in the Clouds themselves.

I. INTRODUCTION

The period-luminosity relation for the classical cepheids provides a powerful tool not only for the determination of extragalactic distances but also for the derivation of critical intragalactic distances by means of which the large-scale structure of the galaxy can, in part, be described. The latter problem divides naturally into a study of galactic rotation and a study of the space distribution of cepheids. Previous investigations have usually proceeded statistically; i.e., cepheids were grouped according to position in the galaxy and a mean color excess derived from studies of presumably nearby B-type stars (Miss Torgård 1956) or from the condition that the absorption, based on some model, should lead to a distribution of cepheids such that, if z is the distance from the galactic plane, $\langle z \rangle$ is independent of distance from the sun (Stibbs 1956). However, in view of the spotty nature of the interstellar absorbing material and the rarity of cepheids in space (there are only 13 within 500 pc of the sun [Gascoigne and Eggen 1957]), it is not surprising that rather little concentration into spiral structure has been found for these stars (cf. especially Stibbs 1956), even though the radial velocities show a fairly well-defined "double sine wave" (Joy 1937, 1939; Stibbs 1955, 1956).

Other discussions have involved a statistical treatement of the colors rather than the space geometry. Thus Gascoigne and Eggen (1957) assumed that, at a given period, the colors of galactic cepheids were identical with those of the cepheids in the Magellanic Clouds; the latter were assumed to be unreddened. However, since the period-color relation of the galactic cepheids at maximum light shows a scatter of about 0.2 mag. in $(B-V)$ (see Sec. VI), this approach can, at best, assign an "average" color at maximum light for a certain period. Thus, for any particular cepheid, it is possible to make an error of 15 per cent in the distance. A similar procedure was adopted by Walraven, Muller, and Oosterhoff (1958). These investigators found, nevertheless, some fairly convincing evidence of spiral structure among southern cepheids. On the other hand, Fernie (1958) studied the distribution of northern cepheids, assuming that all cepheids have the same

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color at maximum light independent of period, the value of the color being taken from studies of cepheids in open clusters (summarized by Kraft 1957*a*). Fernie concluded that the cepheids showed no particular preference for spiral structure, except perhaps those of long period. It is possible, of course, that this conclusion might not be substantiated if accurate colors and magnitudes were known for a larger number of northern cepheids.

The foregoing discussion makes clear the need for a method of deriving color excesses for individual cepheids that is as free as possible from a statistical treatment. A step in this direction has been made by Kron and Svolopoulos (1959), who derived color excesses based on multicolor photometry. From studies of nearby F and G supergiants, a limiting envelope was found in an appropriate color-color plot that is supposed to correspond to unreddened stars (Kron 1958). At any phase, a cepheid may be moved along the color trajectory to the "unreddened" envelope, and the excess thereby derived. As these authors point out, the main weakness of the method is the assumption that there are, in fact, any unreddened supergiants, though some allowance for this was made in drawing the envelope. It seems to the writer that this is unlikely in view of the fact that so apparently bright a star as α Per (F5 I*b*) is reddened by 0.12 mag. on the U, B, V system (Eggen 1955; Harris 1956). It must be admitted, of course, that observational evidence on this point is not readily obtainable because of the paucity of F and G supergiants in open clusters. Recently, however, assuming that the F0 I*a* star ϕ Cas is a member of NGC 457, Pesch (1959) has derived a color excess $E(B-V)$ of 0.50; Kron's (1958) value, transformed to the B, V system, is 0.45. This suggests, but naturally does not prove, that Kron's intrinsic colors may be a little too red, but it seems best to withhold final judgment until more supergiants are found in open clusters.

In the present paper it is proposed that color excesses for cepheids be derived star by star, using the classical method of relating intrinsic colors to appropriate spectral features. For reasons cited in the next section, we have calibrated the strength (or in certain cases, strength and appearance) of the G band as a function of intrinsic $(B-V)$. The method is feasible because of the discovery of several cepheids in open clusters (van den Bergh 1957; Kraft 1957*b*; Irwin 1958*b*). U, B, V photometry and color excesses are now available for a number of these stars (Arp 1958; Irwin 1958*a*, 1958*b*; Sandage 1958*b*; Arp, Sandage, and Stephens 1959). Available also are intrinsic colors for δ Cep and Polaris, based on the assumption that the visual companions of these stars are, in fact, physically related to the variables (Gascoigne and Eggen 1957).

Finally, we wish to point out that, even if the color excesses for individual cepheids can be determined with high precision (say, ± 0.03 mag.), this does not insure an equally good distance determination because of the intrinsic spread in the P-L law. Sandage (1958*a*) has convincingly demonstrated that the $P(\bar{\rho}/\bar{\rho}_{\odot})^{1/2} = Q$ equation implies a functional relation between P, M_v, and $(B-V)$ rather than between P and M_v alone. However, if the intrinsic color at mean light can be found (and this is insured as soon as the excess is known and light- and color-curves are available), M_v can be found by solving the equation on the assumption of a common Q-value for all cepheids. The derivation of cepheid distances by this technique will be the subject of a later paper in this series.

II. A CRITIQUE OF THE DETERMINATION OF SPECTRAL TYPES FOR CEPHEIDS

The first determination of spectral types for cepheids on the Yerkes (then, MKK) system was made by Struve (1944, 1945), who studied the cyclical variation of the spectra of seven classical cepheids with periods between 5 and 17 days. Moderately low dispersion was employed (76 A/mm at Hγ). The results, later confirmed and extended by Code (1947), were essentially as follows:

1. At minimum light the spectra of cepheids are nearly normal and, as a function of period, show a systematic increase in luminosity from class II to class I*a*. Most cepheids, i.e., those with periods between 4 and 25 days, belong to class I*b*.

2. With increasing period, the spectra at minimum light become later.
3. The spectral types at maximum light are nearly the same, ranging from F5 to F8, with increasing period.
4. The spectra at maximum light (and perhaps a short interval before maximum) are, however, not normal. The types are based primarily on the appearance of the G band, and the ratios of the intensities of the Fe I and Fe II lines. The hydrogen lines are much too strong, certain Ti II lines are enhanced, and Fe II lines are slightly strengthened in comparison with Yerkes standards in the range F5 *Ib* to F8 *Ib*. These peculiarities are almost absent in the cepheids of short period (<5 days) but become increasingly conspicuous with increasing period.

Code (1947) emphasized the importance of this result for problems in galactic research. If all cepheid spectra are nearly the same at maximum light, one need only find the normal colors of a few nearby (presumably) unreddened cepheids and assign these colors at maximum light to all cepheids. Setting aside for the moment the question of whether, in fact, any cepheids are free from reddening, the main drawback to this approach would seem to be the uncertain significance of the spectral "peculiarities." The problem may be stated this way: Are the unreddened $(B-V)$ colors of cepheids at maximum light consistent with the assignment of spectral types in the manner of Struve and Code? This is a question of more than academic interest, since, in some cases, the G band gives a spectral type at maximum in the F5–F8 range, whereas the hydrogen lines suggest a type as early as F0. As will be seen in Section III, $(B-V)$ increases by nearly 0.2 mag. in the interval from F0 to F5 among supergiants.

Three lines of evidence bear on this problem. First, virtually the entire metallic-line spectrum is in agreement with the assignment of spectral types from the appearance of the G band. In Figure 1, T_{ion}, T_{exc}, and $\log P_e$ are plotted for a number of non-variable stars of luminosity class *Ib* as a function of spectral type on the MK system. The method of analysis is by curves of growth and in all cases follows the treatment by Greenstein (1948). The data were obtained from the following sources: α Per (Greenstein 1948); ν Aql, 35 Cyg, and HR 690 (Abt 1958); β Aqr and 9 Peg (Kraft, Camp, Fernie, Fujita, and Hughes 1959). In Figure 2, corresponding data at maximum light for the cepheids T Vul and S Sge (Lüst-Kulka 1954), FF Aql (Miss Hack 1955), δ Cep (Walraven 1948), and SV Vul (Kraft *et al.* 1959) are given as a function of $\log P$. Since, in Figure 1, $\log P_e$ shows almost no variation with spectral type, only the T_{ion} and T_{exc} curves of Figure 1 were entered with the values of these quantities for the five cepheids of Figure 2. The temperatures of the normal supergiants were thereby used to calibrate the spectral types of cepheids at maximum light. The results are summarized in Table 1, in which the spectral type is given as a straight mean between the T_{ion} and T_{exc} determinations. Also shown are the spectral types given by Code (1947) and the writer's redetermination of these types (see Sec. III) from Code's spectrograms, as well as types derived from some spectrograms obtained by the writer at the McDonald Observatory. In all cases the type derived from the curves of growth is between F5 and F9, and is certainly never as early as F0 or F2. This test is rather decisive; it indicates that the intensities of many lines are in agreement with the assignment of spectral types from a single feature, viz., the G band.

Second, from Figure 2 we see that the variations of T_{ion}, T_{exc}, and $\log P_e$ (at maximum light) with period are essentially flat. Admittedly, these three constructs are only rather rough indicators of conditions in a stellar atmosphere; furthermore, we have no stars with periods longer than 10 days for which these parameters are known, except SV Vul. Nevertheless, unless there are serious differences in blanketing (owing perhaps to differences in microturbulence) between short- and long-period cepheids, the evidence of Figure 2 suggests that the mean $(B-V)$ color at maximum light cannot be a steep function of the period.

Finally, and most significantly, the unreddened $(B-V)$ colors of two cepheids are known, both of which are F5 I*b* stars at maximum light (on the basis of the G band). These are U Sgr, which is a member of the open cluster M25 (Feast 1957; Wallerstein 1957; Irwin 1958*a*, *b*), and δ Cep, which has a blue, presumably physical, companion (Gascoigne and Eggen 1957). The unreddened $(B-V)$ colors at maximum light of these stars are compared in Table 2 with α Per (F5 I*b*), a member of the α Per association (Eggen 1955; Harris 1956). The three stars have closely the same color. It is shown later in this paper (see Fig. 7) that the spectral-type–color gradient near F5 I*b* is 0.05 mag. $(B-V)/0.1$ spectral class; clearly, the colors of these two cepheids at maximum light are just what would be expected if the spectral type is assigned on the basis of the appearance of the G band.

If the excessive strength of the hydrogen, and to some extent Ti II, lines at or before maximum light bears no relation to the colors or to the parameters of the stellar atmosphere, the presumption is strong that the phenomenon is kinematical, i.e., it arises from some effect of differential mass motion in the cepheid atmosphere. High-dispersion studies show that this is very probably the case. As a typical example, we consider Sanford's (1956) study of the spectra of T Mon ($P = 27.0$ days) and SV Vul ($P = 45.1$ days) from Mount Wilson coudé spectrograms taken largely at a dispersion of 10 A/mm. It was found that the widening of H, Sr II, and Ti II lines near maximum light is ac-

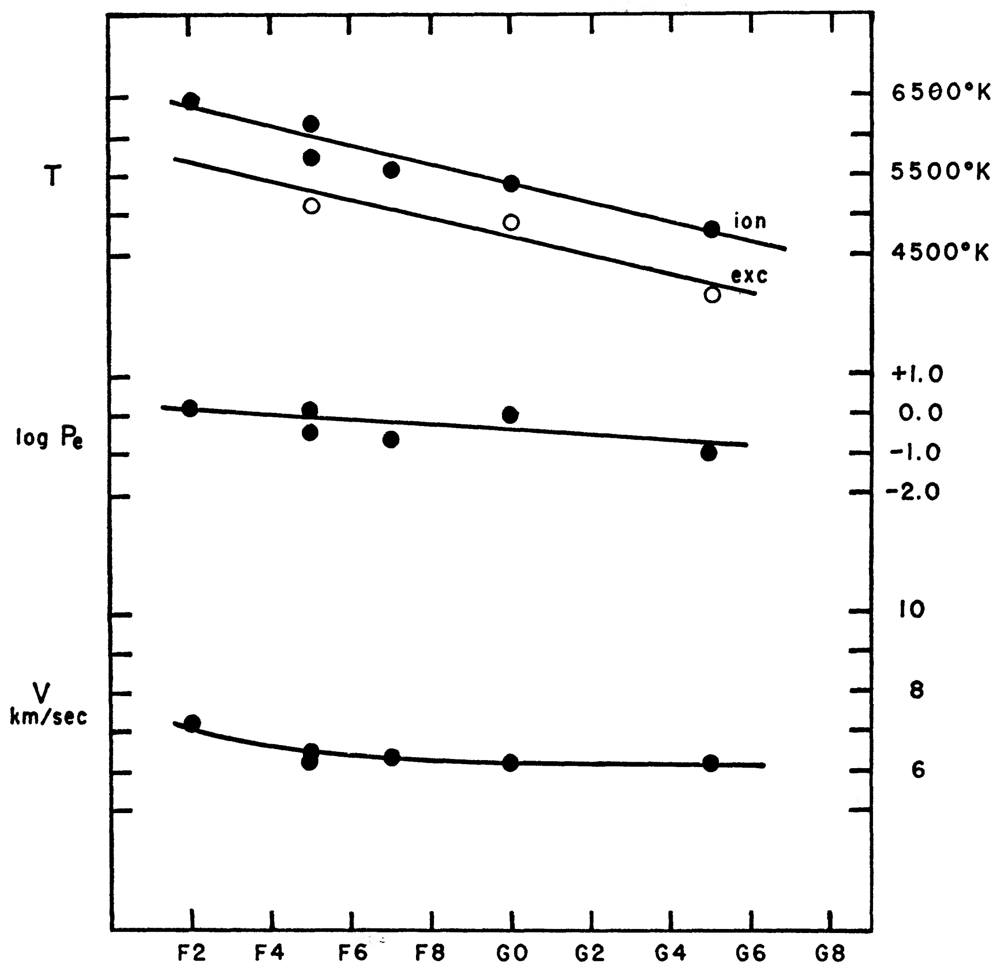

FIG. 1.—T_{ion}, T_{exc}, log P_e, and microturbulent velocity as a function of spectral type for non-variable supergiants. The method of analysis is by curves of growth.

companied by a large positive displacement of the radial velocity in comparison to lines of Fe I. The H displacement amounted at most to +55 km/sec in one of the stars; more typical displacements were perhaps +20 km/sec; the displacements were smallest (<9 km/sec) for the Ti II lines. The most reasonable explanation of this phenomenon is the so-called "level effect," which, among the shorter-period cepheids, is discernible most clearly in the radial-velocity displacement of the core of the K line compared with the Fe I lines (Petrie 1932; Adams and Joy 1939; Jacobsen 1949, 1950, 1956). Effects similar

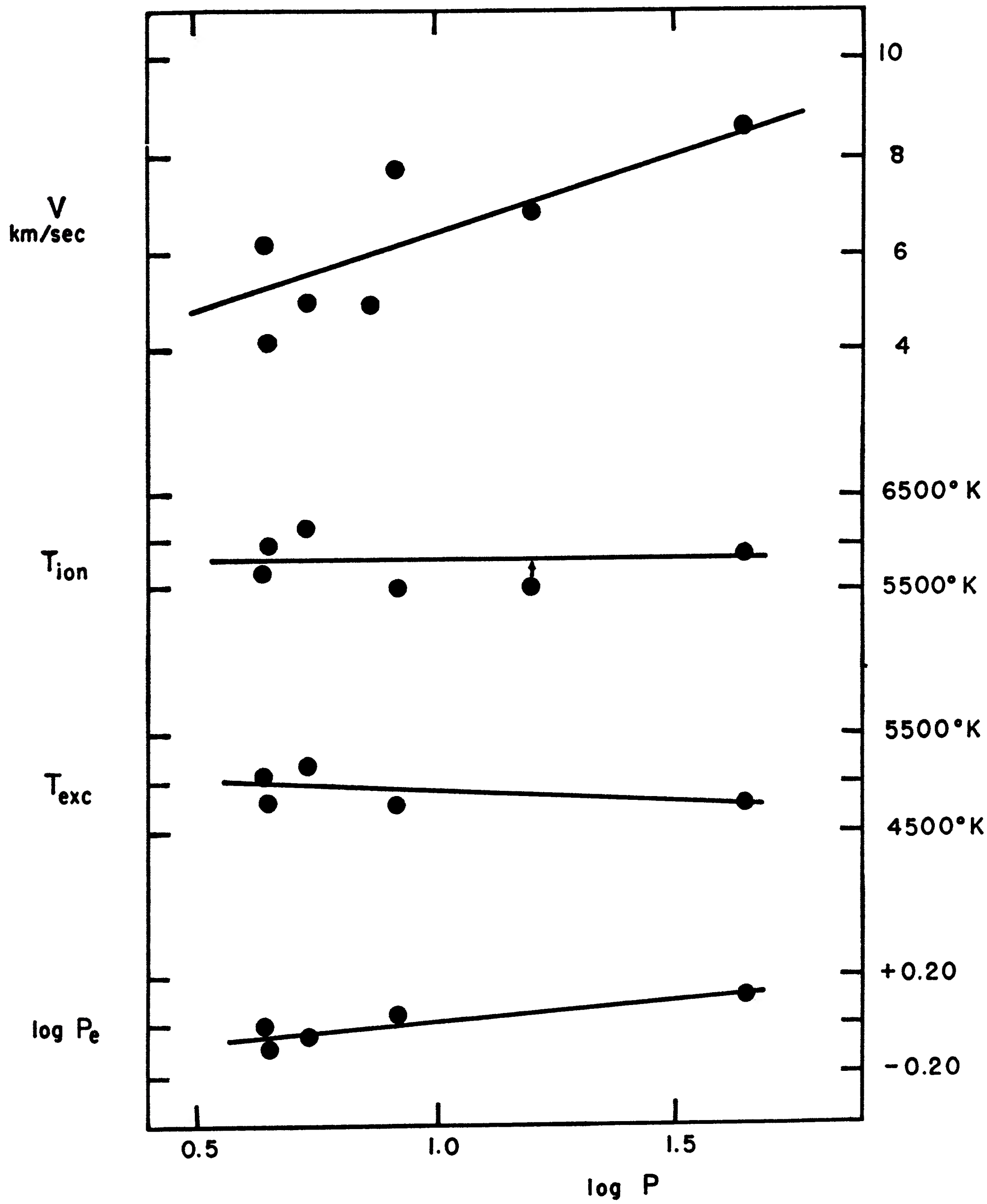

FIG. 2.—Atmospheric parameters for cepheids at maximum light as a function of log P. The point with the arrow marks X Cyg not long after maximum light (Abt, private communication); the position is presumably a lower bound to the value at maximum light.

to those described by Sanford have been found also in X Cyg (Kraft 1954). These studies indicate that the atmosphere is moving differentially; near maximum light, the higher layers are still falling while the lower layers are already advancing, presumably in response to an outward-passing running wave. Since the acceleration of the atmosphere is most pronounced just before maximum light, it is not surprising that the maximum differential velocity is found at these phases. Related to these effects may be the complicated line splitting of certain zero-volt lines of Fe I (shortward of K, and therefore not seen on Struve's or Code's spectrograms), Hα, and low-excitation lines of Ti II discovered by the writer (1956) in X Cyg ($P = 16.4$ days). The split lines, which appeared

TABLE 1

SPECTRAL TYPES OF CEPHEIDS AT MAXIMUM LIGHT DERIVED FROM EXCITATION AND IONIZATION TEMPERATURES

Star	T_{ion}	T_{exc}	Mean	Sp (Code 1947)	Sp (Reclassification, Present Paper)
T Vul .	F7 5	F7	F7	. . .	F5
FF Aql...	F5	F8 5	F6 5	. ..	F5
δ Cep	F4	F6	F5	F5	F5
S Sge .	F9	F9	F9	F6	F7
SV Vul	F6	F8 5	F7	F7	F7

TABLE 2

INTRINSIC COLORS OF α PER, U SGR, AND δ CEP

Star	$(B-V)$	Sp	Reference
α Per. ..	0 36	F5 I*b*	Eggen (1955); Harris (1956)
δ Cep .	34	F5 I*b**	Gascoigne and Eggen (1957)
U Sgr.. .	0 37	F5 I*b**	†

* At maximum light

† The color excess of U Sgr was derived as 0 50 by Kraft (1957*a*) from Irwin's (1958*b*) data on M25; this was confirmed spectroscopically by Wallerstein (1957) The color of U Sgr at maximum light is taken as $(B-V) = 0.87 \pm 0\ 01$ (p e), an average of the measurements by Irwin (reported by Kraft 1957*a*), Eggen *et al* (1957), and Walraven *et al* (1958)

on the mid-rising branch of the light-curve, generally consisted of one component having a velocity in agreement with that of the Fe I lines, and another component displaced 30–40 km/sec longward. Similar phenomena in Hα were found in TT Aql ($P = 13.8$ days) and WZ Sgr ($P = 21.8$ days) (Kraft 1957*c*).

Regardless of the details of the mechanism responsible for the widening, or splitting, and longward displacement of the H, Sr II, and Ti II lines, these kinematical effects will act to increase the equivalent widths of saturated lines, i.e., lines on the flat and square-root portions of the curve of growth, whereas lines on the linear portion will have their equivalent widths unaffected. In cepheids, especially near maximum light, the strongest spectral lines in the photographic region (excluding H and K of Ca II) are, in fact, those of H and Sr II. It follows that meaningful spectral types can be derived only from lines which suffer minimally from kinematical effects. At low (classification) dispersion, lines on the linear portion of the curve of growth are generally unresolved. The G-band complex comes close to having the desired properties: it is moderately strong, but it is produced to a considerable extent by the individual rotational lines in an electronic transition of the CH molecule. These individual structures are themselves moderately weak lines. However, the complex is not altogether ideal, since the region contains an admixture of fairly weak lines of Ca I, Fe I, and certain rare earths, and two moderately strong lines of Ti II (cf. Swensson 1946; Pannekoek 1949). On the other hand, the enhancement of Ti II lines, especially those in the G-band complex, is relatively small (cf. Sanford 1956) compared with H or Sr II λ 4077.

It is of interest, therefore, to calculate η_0, the ratio of optical depth at the center of the line to the continuous opacity, for a typical rotational line in the CH band. If α_0 is the absorption coefficient per molecule, $\bar{\kappa}$ the continuous absorption coefficient per gram, and ρ the density,

$$\eta_0 = \frac{N\alpha_0}{\bar{\kappa}\rho},$$

where N is the number of CH molecules per cubic centimeter. Taking $f = 2 \times 10^{-3}$ (Bates and Spitzer 1951) and the atmospheric parameters given by Greenstein (1948) for α Per (F5 I*b*) and by Kraft *et al.* (1959) for β Aqr (G0 I*b*), we find $\eta_0 = 1$ for the former and 10 for the latter. These stars bracket the range of spectral types for cepheids near maximum light. From Wrubel's (1949) curves of growth, we find that a line is on the linear portion of the curve of growth if $\eta_0 \leqq 1$, and is definitely on the flat portion when $\eta_0 \geqq 40$. From these rough calculations, we conclude that the individual rotational lines of the CH molecule are either on the linear portion of the curve of growth or on the transition between the linear and flat portions. The requirement that the kinematical effects shall have minimal influence on the spectral-type indicator seems to be reasonably well satisfied. We conclude that spectral types for cepheids derived from an examination of the G band will be consistent with spectral types assigned to non-variable supergiants of types F and G.

III. PHOTOMETRY OF THE G BAND

Inspection of the Yerkes *Spectral Atlas* (Morgan, Keenan, and Kellman 1943) shows immediately that, among class I stars, the G band increases rapidly in strength from F5 to a maximum about G5, with a subsequent decline. Using a narrow band-pass interference filter (half-width 10 A centered at λ 4305) and a wider filter (half-width 200 A, centered at λ 4290), an index Γ has been measured, which expresses, in magnitudes, the depression of the G band relative to the local continuum. Measurements were made in two ways: (1) on nights of good photometric transparency, a conventional photometer was employed. Measurements were made in the order *a*, *b*, *a*, where *a* refers to a deflection through the narrow filter and *b* to a deflection through the wide filter. (2) On poorer nights and otherwise whenever available, a photometer employing two photocells behind a beam splitter was used; narrow and wide deflections were measured simultaneously. The photometer was that described by Crawford (1958) as "Model II"; the reader is referred to this paper for a detailed description of the device. Conventional d.c. amplification was used with method 1; a pair of integrators was employed with 2. In all cases, observations were made with either the 82-inch or the 36-inch McDonald reflectors.

Measurements of several stars at various zenith distances show that, within the errors of observation, there is no dependence of the index Γ on atmospheric extinction. A typical set of observations of non-variable MK supergiants is shown in Figure 3. All but four points were obtained on a single good night; each point represents one observation only. The remaining four points are single observations taken from two other nights. It should be emphasized that the nights represented in Figure 3 were photometrically excellent; on poorer nights the scatter was somewhat larger. Standards were observed each night in order to obtain the nightly corrections to the basic system.

Figure 3 confirms the visual impression of the change in strength of the G band with type obtained when one inspects the Yerkes *Atlas*. Among the intermediate F types, there is evidently no systematic difference between stars of classes II, I*b*, and I*ab*, but later types show some departures which may be systematic. For example, among late F's and early G's the I*a* stars have slightly weaker G bands than do those of class I*b*.

The measurement of Γ for any star gives the spectral type from the mean curve of Figure 3; such types are obtained from the photometric strength of the G band rather than from visual inspection of spectrograms and are therefore denoted by German script letters. Spectral types determined from visual inspection of the G band (and among stars later than G5, from the inspection of some other features, noted below) will be designated in the normal fashion.

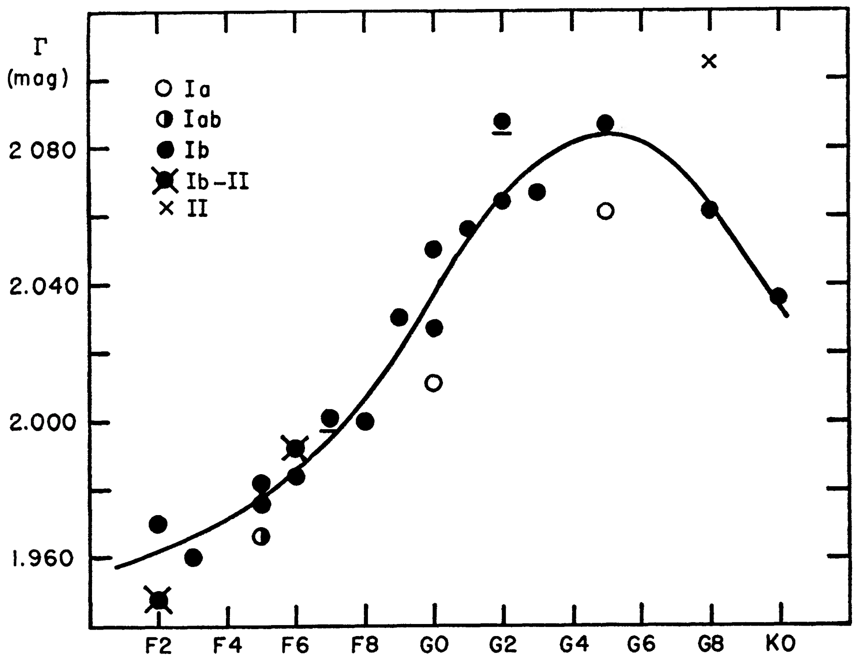

FIG 3.—The index Γ as a function of spectral type for non-variable supergiants of the MK system. Underscored symbols have half-weight

Normal colors are known for the cepheids CF Cas (Sandage 1958*b*), EV Sct (Arp 1958), DL Cas (Arp, Sandage, and Stephens 1959), and U Sgr (Irwin 1958*a*, *b*, 1959) from membership in open clusters, and for Polaris and δ Cep (Gascoigne and Eggen 1957) from excesses based on photometry of companions. Γ has been measured around the cycle of U Sgr and at isolated phases of EV Sct and δ Cep. The measurements of U Sgr have been converted to "spectral types," and these are given as a function of phase in Figure 4; U Sgr evidently varies from $\mathfrak{F}$ 5 to $\mathfrak{G}$ 1 between maximum and minimum light. Points marked with an *X* in Figure 4 are ordinary spectral types derived from inspection of the G band on classification spectrograms; there is evidently no systematic difference between the German script and ordinary spectral types. From the unreddened $(B-V)$ color-curve of U Sgr, the colors were read out corresponding to each observation of spectral type; these are shown in Figure 5, along with corresponding points for δ Cep and EV Sct.

IV. THE SPECTRAL-TYPE—INTRINSIC COLOR RELATION FOR SUPERGIANTS

Since the German script and ordinary spectral types are in agreement from F5 to G1, we shall make the assumption that agreement persists throughout the later spectral types. Since all cepheids lie between F5 and F8 at maximum light, we can extend the color–spectral-type relation for supergiants beyond G1 in the following way.

First, the entire body of spectrograms taken by Code and Struve was reclassified by the writer in order to eliminate any possibility of systematic differences. Also available to the writer were a number of additional classification spectrograms of cepheids taken at the McDonald Observatory, as well as a number of supergiant standards at many intermediate spectral types not available either to Struve or to Code. The latter were selected from the list by Bidelman (quoted by Kron 1958). Spectral types at maximum and minimum light for all available stars are given in Table 3 and are compared with those given by Struve and Code. Because the appearance of the G band changes rather slowly from G5 to K0, the strength of the blend at λ 4406 has been used as a supplementary indicator; Ca I λ 4227 has been avoided—it is a strong line that may suffer from kinematical disturbance. The table indicates that the present classifications agree very

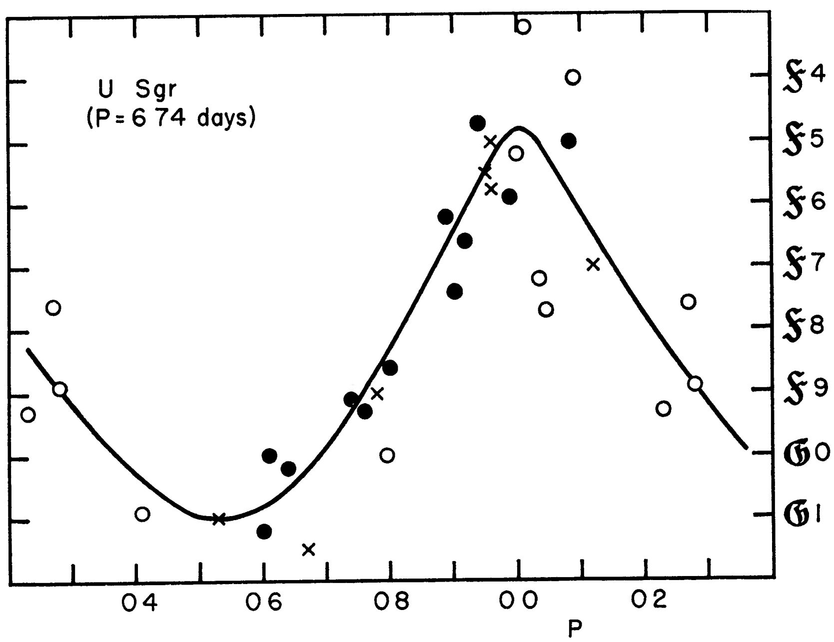

FIG. 4.—Spectral types based on Γ-measurements as a function of phase for U Sgr Points marked with an *X* are ordinary spectral types derived from inspection of the G band on spectrograms Open circles have half-weight

well with those of Code and Struve at maximum light but are systematically somewhat earlier at minimum. The differences probably result mostly from the use, in the present case, of more standards and the fact that some of the standards used by Struve and Code have subsequently been changed slightly in spectral type; e.g., the *Atlas* gives 56 Peg as G8 I*b*, but Miss Roman (1952) reclassified the star as K0 II*p*. The presently classified spectral types at maximum and minimum light are shown in Figure 6, as a function of log *P;* comparison is made with the corresponding figure given by Code (1947).

Second, a plot of spectral type versus intrinsic $(B-V)$ color was made for all available spectrograms of U Sgr, δ Cep, EV Sct, Polaris, and DL Cas. These are shown as dots or circles in Figure 7, and a curve best fitting these points was drawn up. Also shown as a dashed line is the curve derived from the G-band photometry (Fig. 5); there is evidently no significant difference between these curves. With the spectral types at maximum light, we entered the curve of Figure 7 for each of the Code-Struve stars and read out the intrinsic color; the color excess $E(B-V)$ was thereby determined for those stars having color-curves given by Eggen (1951) or Eggen, Gascoigne, and Burr (1957). This excess was applied to the observed color at minimum light, taken from the same sources; comparison with the observed spectral type at minimum then "manufactures" the normal color corresponding to that spectral type. Points of Figure 7 so determined are marked by the symbol *X*. Also shown are the non-variable stars α Per (Harris 1956) and φ Cas (Pesch 1959). As already mentioned, the agreement of the variables with the former is excellent; the latter star is of class I*a*. The extension of the I*b* relation through this star is therefore supposititious but probably not far wrong.

Interesting features of Figure 7 are the following: (1) Earlier than F8, class I*b* stars are bluer than main-sequence stars for a given spectral type. This is in qualitative agreement with the conclusions of Feinstein (1959). (2) Later than G8, stars of class I*b* are redder than ordinary giants. (3) Later than F5, our I*b* sequence is systematically bluer, by 0.05–0.1 mag., than the sequence given by Feinstein (1959), who simply drew an envelope line through the least reddened F, G, and K supergiants as a function of MK spectral type. This is in agreement with our statement (Sec. I) that there are probably no unreddened supergiants of these spectral types. The I*b* color sequence is given as a function of spectral type in Table 4.

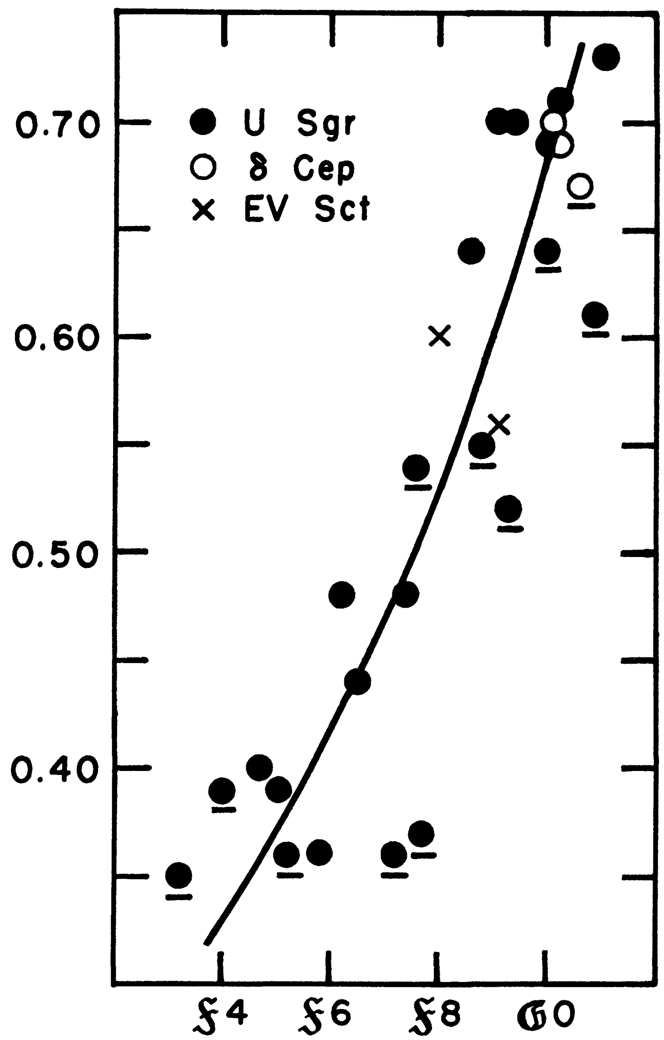

FIG. 5.—Spectral types from Γ-photometry as a function of intrinsic color Underscored symbols have half-weight.

V. COLOR EXCESSES FOR CEPHEIDS AND SUPERGIANTS

Once the spectral type of any supergiant star is given, its color may be read out of Figure 7. Strictly speaking, cepheids with periods longer than about 20 days tend to have I*ab* spectral characteristics; thus the portion of our supergiant-curve with $(B-V) > +1.00$ may reflect more nearly the colors of I*ab* rather than those of I*b* stars. For want of better information, we shall assume the two luminosity classes to have the same colors.

Spectral types for cepheids have been obtained from Γ-photometry, and from visual inspection of classification spectrograms. The normal color is then read out of Figure 7; comparison at the appropriate phase with the color-curves of Eggen (1951) or Eggen, Gascoigne, and Burr (1957) gives the excess. The estimated probable error of a single determination of spectral type among I*b*'s from 𝔉 3 to 𝔊 3 is 0.05 spectral class on good nights and about 0.07 on poorer nights. This corresponds to an error in the intrinsic color of 0.03 and 0.05 $(B-V)$, respectively, if we are free from systematic effects. We can also estimate the mean error of our excess determinations resulting not only from possible errors in the Γ-photometry but from read-out errors in the color-curves of the cepheids. Clearly, the excess determination must be independent of phase. We can compare excesses determined at different phases of several cepheids for which we have more than one observation of Γ. These are given in Table 5. Only in the case of WZ Sgr do the individual determinations differ by more than 0.05 mag. One observation of this star was made, however, on the steep rising branch of the color-curve where a small phase error makes a large change in the observed color.

TABLE 3

SPECTRAL TYPES OF TWENTY-TWO CEPHEIDS

Star	Period (days)	Log P	Spectral Type (Code or Struve)	Spectral Type (Present)	ΔSp (Present)	$\Delta(B-V)$ (Eggen *et al* 1957)
SU Cas	1 95	0 29	F5 I-II—F7 I-II	F5 I*b*-II—F6.5 I*b*-II	0 15	0 14
DT Cyg	2 50	0 40	F5 5 I-II—F7 I-II	F5 5 I*b*-II—F7 5 I*b*-II	0 2	14
SZ Tau	3 15	0 50	F6 I*b*—F9 I*b*	F5 I*b*—F9.5 I*b*	0 45	17
RT Aur	3 73	0 57		F5 5 I*b*—G0 I*b*	0 45	37
α UMi	3 97	0 60		F7 I*b*-II—F8 I*b*-II	0 1	05
T Vul	4 44	0 65	. . . . . .	F5 I*b*—F9.5 I*b*	0 45	30
V 386 Cyg	5 24	0 72	F5 I*b*—G1 I*b*	F5 5 I*b*—G0 I*b*	0 45	. .
δ Cep	5 37	0 73	F5 I*b*—G2 I*b*	F5 I*b*—G1 I*b*	0 6	37
MW Cyg	5 96	0 78	F8 I*b*—G1 I*b*	F8 5 I*b*—G2 I*b*	0 35	. .
U Sgr	6 74	0 83	. .	F5 I*b*—G1 5 I*b*	0 65	39
η Aql	7 18	0 86	F6 I*b*—G4 I*b*	F7 I*b*—G2 I*b*	0 5	41
VY Cyg	7 86	0 90	F6 I*b*—G1 I*b*	F5 5 I*b*—G0 5 I*b*	0 5	. .
S Sge	8 38	0 92	F6 I*b*—G5 I*b*	F7 I*b*—G3 I*b*	0 6	40
BZ Cyg	10 14	1 01	F8 I*b*—G5 I*b*	F7 I*b*—G4 I*b*:*	0 7:	. .
ζ Gem	10 15	1 01	F7 I*b*—G3 I*b*	F7 5 I*b*—G1 5 I*b*	0 4	30
Z Lac	10 89	1 04	F6 I*b*—G6 I*b*	†	. . .	51
TX Cyg	14 71	1 17	F5 I*b*—G6 I*b*	F5 5 I*b*—G5 I*b*:	0 95:	
SZ Cyg	15 11	1 18	F8 I*b*—G8 I*b*	F7 5-F8 I*b*—G6 5 I*b*	0 9	.
X Cyg	16 39	1 21	F7 I*b*—G8 I*b*	F7 I*b*—G9 I*b*:	1 2:	64
CD Cyg	17 07	1 23	F8 I*b*—K0 I*b*	F8 I*b*-I*ab*—G5 I*b*-I*ab*	0 8	
T Mon	27 01	1 43	F7 I*a*-I*b*—K1 I*a*-I*b*	F7 5 I*b*-I*ab*—G8 I*b*-I*ab*	1 05	55
SV Vul	45 13	1 65	F7 I*a*—K0:	F7 I*ab*—K0 I*ab*:	1 3:	0 70

* Colons indicate uncertainty

† In the writer's opinion, there are not enough spectrograms to make a satisfactory determination of spectral types

A comparison can also be made of the excesses determined from Γ-photometry and from ordinary classification. This is shown in Figure 8. There are no systematic differences; for high weight points, the maximum deviation is 0.07 mag. We conclude that, from a single high-quality Γ-observation, the maximum accidental error in the excess is about 0.05 mag. in $(B-V)$; this can be reduced considerably if two or three observations are made—perhaps to 0.03.

An estimate of spectral type from a single spectrogram will naturally lead to a larger error in the excess than that obtained from the Γ-photometry, since a spectral type cannot generally be determined to better than 0.1 spectral class. An excellent determination is possible only if a number of spectrograms are available; this is the case for all the Struve-Code stars, as well as a few others.

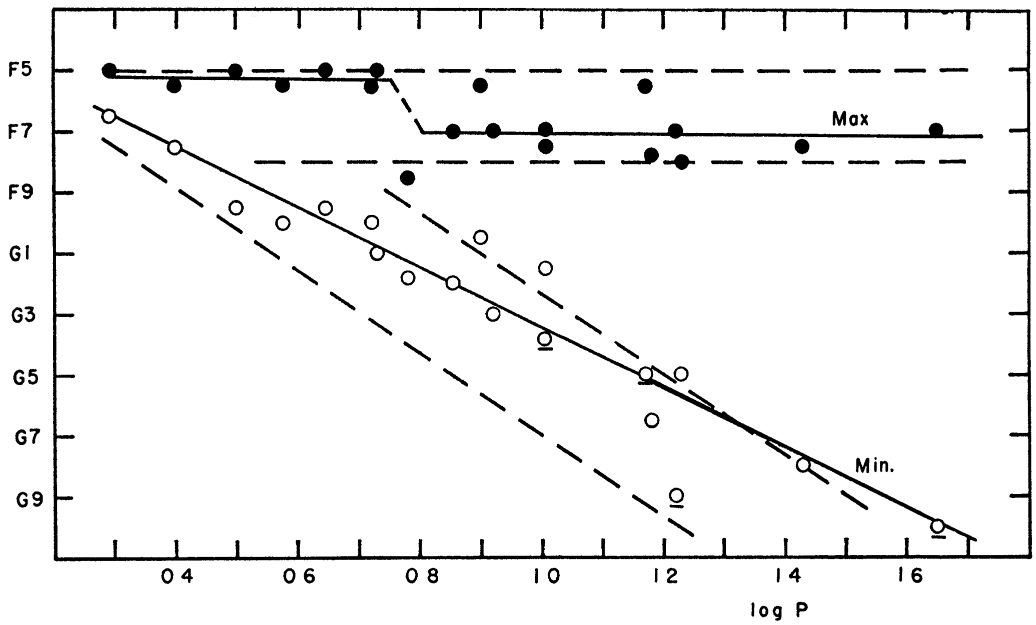

FIG. 6.—Spectral types at maximum and minimum light for cepheids as a function of log P Underscored symbols have half-weight. The dashed lines are the boundaries originally outlined by Code (1947) The tendency for a break in the spectral types at maximum around log P = 0 8 may not be real but rather may result only from a paucity of points.

As a test of the internal consistency of our spectral-type determinations, we have plotted the range (from maximum to minimum light) in the spectral types against the range in $(B-V)$ color (Eggen 1951; Eggen, Gascoigne, and Burr 1957). Internal consistency is assured if the spread about the mean correlation-curve is small. From Figure 9, it can be seen that the probable error of a determination of the color amplitude from the spectral-type amplitude is ± 0.03 mag. $(B-V)$. This rather tight result indicates only that the spectral types given for cepheids at minimum light are self-consistent but naturally does not insure complete freedom from systematic errors.

The estimates of color excess for a number of cepheids are summarized in Table 6. High weight has been given to those stars from which spectrograms were obtained during the entire cycle or to stars for which more than one Γ-measurement have been made. Comparison is also made with the excesses given by Gascoigne and Eggen (1957), Walraven, Muller, and Oosterhoff (1958), and Kron and Svolopoulos (1959). This is shown also in Figure 10. Table 7 contains excesses for some bright, non-variable supergiants, the $(B-V)$ colors of which are known.

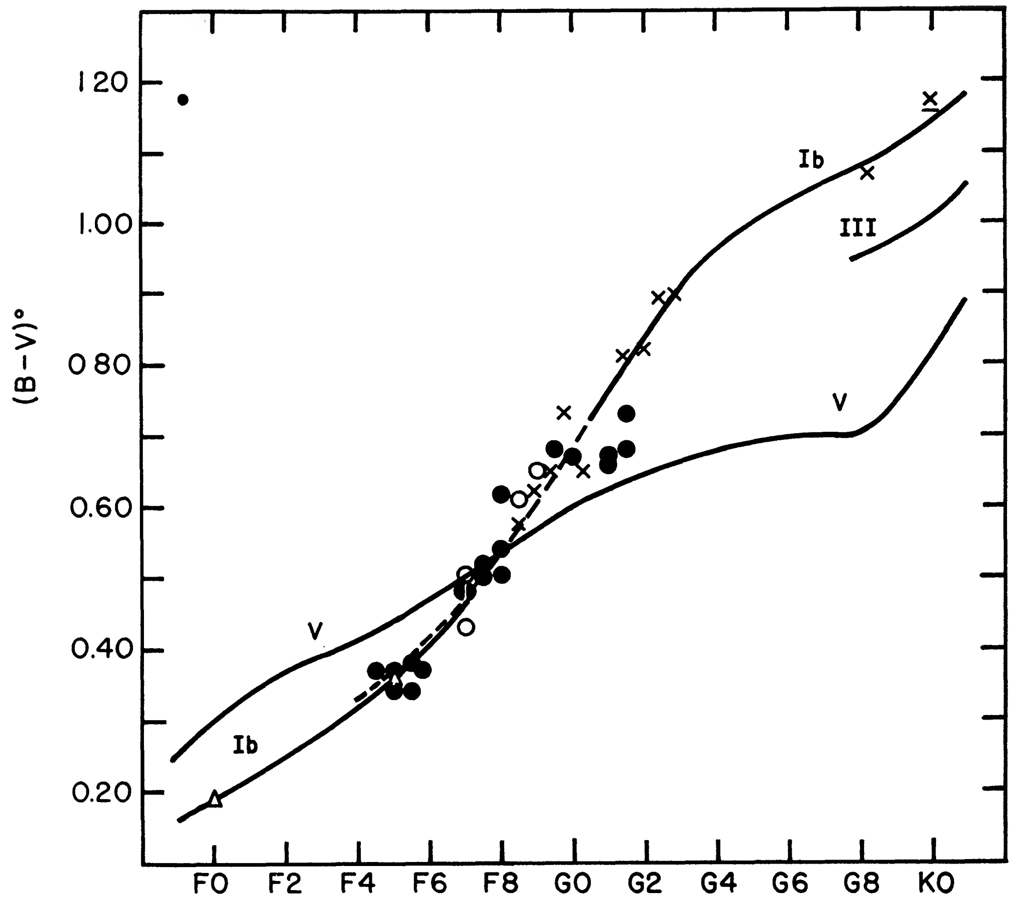

FIG. 7 —Spectral-type–intrinsic-color diagram. Triangles are nonvariable supergiants; filled circles correspond to individual spectrograms of U Sgr, δ Cep, EV Sct, Polaris, and DL Cas; open circles indicate low-weight spectrograms of these stars; and the crosses correspond to spectral types of cepheids at minimum light, as explained in the text. Underscored crosses have low weight. The dashed line represents the relation between $(B - V)^{\circ}$ and spectral types derived from the Γ-photometry.

TABLE 4

ADOPTED INTRINSIC COLORS FOR CLASS *Ib* STARS

Sp	$(B-V)$	Sp	$(B-V)$
F0	0 19:*	G1	0 76
F2	25:	G2	0 84
F5	36	G3	0 91
F6	41	G4	0 96
F7	46	G5	1 00
F8.	53	G6	1 03:
F9 .	61	G8	1 08:
G0	0 68	K0	1 14:

* Colons indicate uncertainty

VI. DISCUSSION

Three lines of discussion may be pursued. First, Figure 10 illustrates that our excesses are systematically slightly different from those of all other investigators. Is it possible to explain why these differences exist? Second, we may inquire how the relation between color and period for galactic cepheids compares with that of the cepheids in the Magellanic Clouds. Third, we may ask if any systematic color effect emerges when cepheids are grouped according to types A, B, and C (Gascoigne and Eggen 1957; Sandage 1958*a*).

Considering the first point, for $E(B-V) \gtrsim 0.25$, we find that the excesses of Kron and Svolopoulos (1959) are systematically smaller than ours. We recall that these investigators based their excess determinations on a group of bright non-variable supergiants which they assumed to be unreddened. We have already expressed doubt as to the validity of this view. However, the possibility remains that the error is in our system, since the two largest excesses in Figure 10 belong to cepheids of period 22 and 17 days. We have pointed out that stars with long periods tend to have I*ab* characteristics; if these two stars were actually redder than I*b*'s for a given spectral type, we would measure too large an excess for them by assigning colors from the correlation of Figure 7. Application of the Γ-photometry to I*ab* stars (see Fig. 3) might tend to produce a similar error. However, U Sgr ($P = 6.7$ days) is a member of M25, and it has $E(B-V) = 0.50$ from the photometry of the B stars. Kron and Svolopoulos, however, give $E(B-V) = 0.40$. The fact that, among the stars of large excess in Figure 10, this star shows an average sort of deviation from the 45° line strongly suggests that the major part of the error cannot be in our system.

TABLE 5

COMPARISON OF EXCESSES FOR CEPHEIDS WITH Γ-MEASUREMENTS AT MORE THAN ONE PHASE

Star	Sp (from Γ)	$E(B-V)$	Wt	$E(B-V)$ (Mean)
SZ Aql .	$\mathfrak{F}$8 6 $\mathfrak{G}$1 8	0 58 63	1 1	0 60
Y Oph	$\mathfrak{F}$5 7 $\mathfrak{F}$8 6	79 81	½ 1	80
S Sge	$\mathfrak{G}$2 4 $\mathfrak{G}$2 2	14 .10	½ 1	11
WZ Sgr	$\mathfrak{F}$8 8 $\mathfrak{F}$9 5	77 57	1 1	67
V 482 Sco	$\mathfrak{F}$7 3 $\mathfrak{F}$8 3	50 45	½ 1	47
SS Sct	$\mathfrak{G}$0 4 $\mathfrak{F}$9 3 $\mathfrak{F}$9 2	29 31 33	½ ½ 1	32
X Vul .	$\mathfrak{G}$0 4 $\mathfrak{F}$9 1 $\mathfrak{G}$0 0	82 80 80	½ 1 1	80
SV Vul	$\mathfrak{G}$8 6* $\mathfrak{G}$6 8*	52 0 57	½ ½	0 54

* From spectral types, it is known that, at the phases in question, this star lies on the declining branch of the calibration-curve of Fig 3

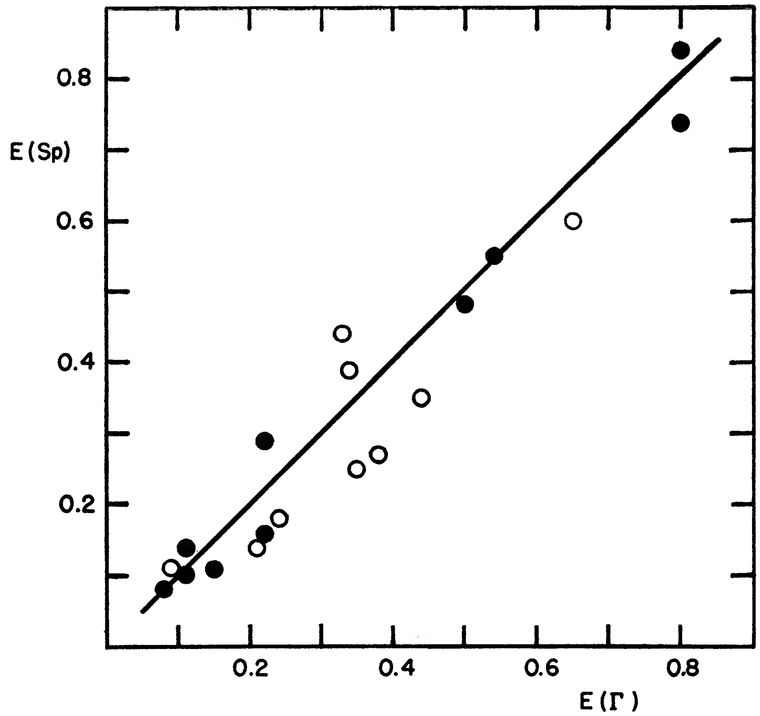

FIG. 8.—A comparison of excesses derived from Γ-photometry and from ordinary spectral classification. Each point corresponds to an individual cepheid Filled circles have the highest weight and correspond to cepheids having more than one spectrogram and more than one Γ-measurement.

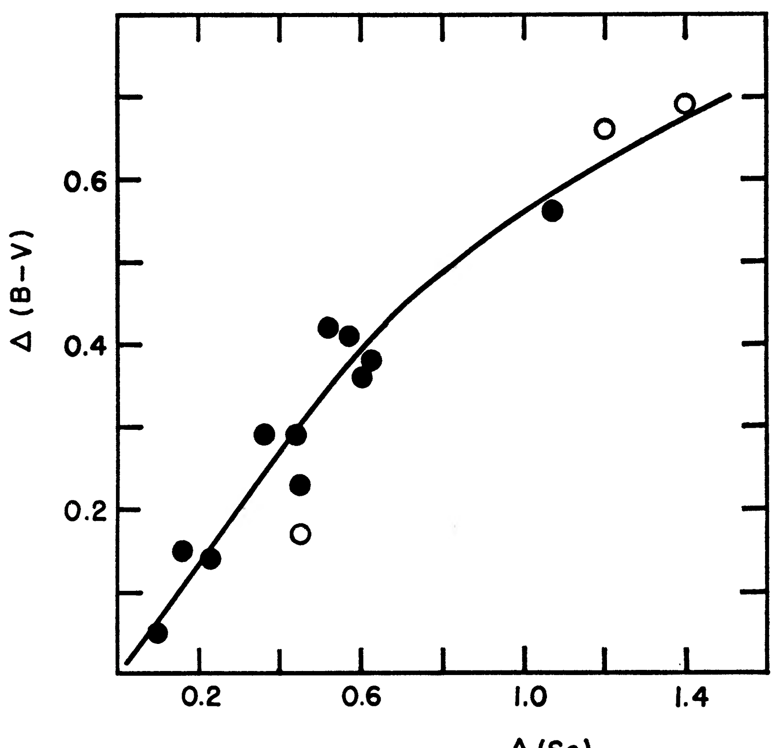

FIG. 9 —The range in MK spectral type versus the $(B - V)$ color amplitude for individual cepheids. Open circles have low weight. Notice that $\Delta(B - V)$ is a directly observable quantity and does not depend on the color excess.

The other two investigations (Gascoigne and Eggen 1957; Walraven, Muller, and Oosterhoff 1958) both assume that, at a given period, the colors at maximum light of the Magellanic Cloud cepheids must agree with those of the galactic cepheids. In the former investigation, 10 Magellanic Cloud cepheids were observed, all having periods longer than 16 days. All galactic cepheids were assumed to have the same color at maximum light as the average for these stars, viz., $(B-V) = 0.36$, with no period dependence. The Dutch observers made essentially the same assumption but also forced agreement

TABLE 6

COLOR EXCESSES $E\ (B-V)$ FOR CEPHEIDS*

Star	log P	E(K and S)†	E(G and E)†	E(W, M, and O)†	E (Present)	Wt (Present)
SU Cas	0 29	0 25	0 23		0 25	1
DT Cyg	0 40	09	11	. .	095	1
SZ Tau	0 50	33	36	0 43	30	½
SS Sct	0 56	.	40	52	32	1
RT Aur	0 57	14	05		07	1
SU Cyg	0 58	20	05	10	10	1
α UMi	0 60	07	08		08	1
T Vul	0 65	15	11	. .	11	1
FF Aql	0 65	27	31	35	30	1
V 482 Sco .	0 66			49	46	½
AP Sgr.	0 70		. .	27	27	½
V 350 Sgr .	0 71	. .	34	39	32	½
δ Cep .	0 73	14	11	. .	11	1
Y Sgr	0 76	23	27	32	19	½
RV Sco .	0 78			41	37	½
X Vul	0 80		75		81	1
RR Lac	0 81		29		18	½
XX Sgr	0 81		52	.	53	½
BB Sgr.	0 82	. .	. .	49	40	½
U Sgr	0 83	40	48	55	50	1
V 496 Aql	0 83	. .	70	72	55	½
U Aql	0 85	35	41	51	42	½
X Sgr	0 85	. .	19	26	22	½
η Aql	0 86	20	22	24	16	1
W Sgr	0 88		19	19	24	½
U Vul	0 90	. .	. .	77	68	½
S Sge .	0 92	14	25	24	14	1
FN Aql	0 98		64	80	55	½
YZ Sgr	0 98	. .	47	52	35	½
ζ Gem	1.01	12	33		18	1
Z Lac	1 04	. . .	47	. .	44	1
TT Aql	1 14	43	54	59	42	½
X Cyg	1 22	28	43	. .	29	1
Y Oph	1 23	58	79	87	77	1
SZ Aql	1 23		60	66	60	1
WZ Sgr	1 34	56	59	68	67	½
T Mon	1 43	0 27	52	0 50	37	1
SV Vul	1 65		0 67		0 55	1

* Stars for which either spectrograms around the entire cycle or more than one Γ-measurement were available are given weight unity Stars with single Γ-observations or spectrograms are assigned weight ½

† K and S = Kron and Svolopoulos (1959); G and E = Gascoigne and Eggen (1957); W, M, and O = Walraven, Muller, and Oosterhoff (1958)

with δ Cep ($P = 5.4$ days) at maximum, i.e., $(B-V) = 0.34$; the period dependence was therefore quite small. Our excesses run, on the average, 0.05–0.1 mag. smaller than the excesses derived in both these investigations.

The explanation for this anomaly is probably to be found in the assignment of the colors to the Magellanic Cloud cepheids. Owing to the apparent faintness of these stars, it is natural to pick for observation the brightest examples at a given period. Sandage (1958*a*) has shown, however, that the brightest cepheids of a given period are also the bluest, intrinsically, because of the width of the cepheid strip in the HR diagram. Furthermore, observations by Arp (1959) of 15 additional Small Magellanic Cloud cepheids with periods over 16 days indicate that there are a number of cepheids redder at maximum light than the stars observed by Gascoigne and Eggen. The possibility cannot be

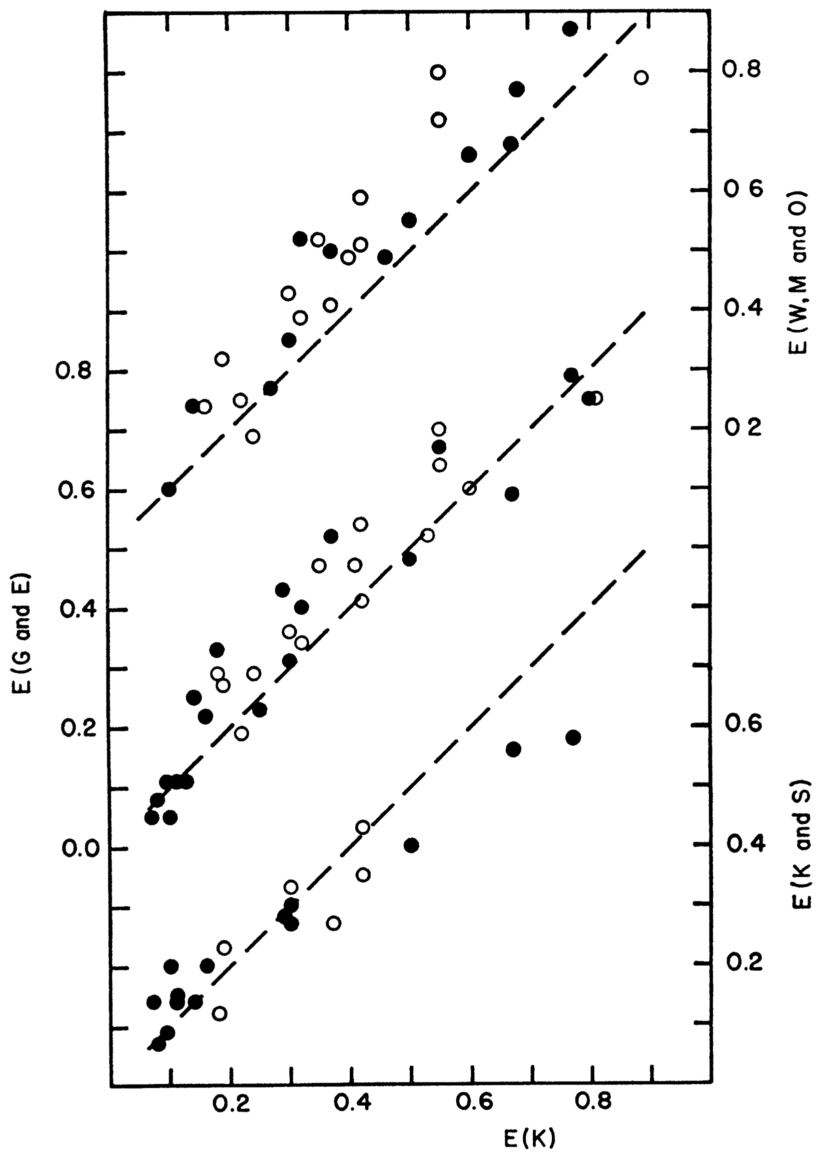

FIG. 10 —Color excesses for cepheids from the present investigation compared with those from other investigations. The initials of the observers are given and are explained in the text Open circles are low-weight points of the *present* investigation.

ruled out, of course, that some of these stars are actually reddened, but Sandage's result that the cepheid strip in the HR diagram is 0.2 mag. $(B-V)$ wide makes it seem likely that at least a few of the Arp stars are intrinsically as red as observed. Furthermore, in extending the intrinsic color line to shorter periods, the Dutch observers chose an unfortunate example in δ Cep. The star is F5 I*b* at maximum light and therefore is intrinsically one of the bluest cepheids. Indeed, the mean intrinsic color at maximum light for the four cepheids in clusters and the two with companions, mentioned in Section I, is $(B-V) = 0.45 \pm 0.03$ (m.e.). The difference between this and the color of δ Cep at maximum light is just about the amount by which the present excesses deviate from those of the Dutch observers on the average. It therefore appears that our excesses are about midway between those of Gascoigne and Eggen and the Dutch observers, on the one hand, and Kron and Svolopoulos, on the other.

The comparison of the colors of the galactic and Magellanic Cloud cepheids is contained in Figures 11 and 12. In the former, we have plotted the intrinsic colors of the galactic cepheids at maximum light as a function of log P; only cepheids having excess

TABLE 7

COLOR EXCESSES $E(B-V)$ FOR SOME BRIGHT SUPERGIANTS

Star	$(B-V)$ (Feinstein 1959; Kron 1958)	Sp	$E(B-V)$ (Kron 1958)	$E(B-V)$ (Present)
α Per	0 48	F5 I*b*	0 11	0 12
γ Cyg	0 62	F8 I*b*	05	09
β Aqr	0 83	G0 I*b*	07	15
α Aqr	1 00	G2 I*b*	11	16
β Dra	0 97	G2 I*b*	12	13
9 Peg	1 14	G5 I*b*	0 15	14
ϵ Gem	1 37	G8 I*b*		0 29:

determinations of high weight have been included, 19 in all. Half the color amplitude (Gascoigne and Eggen 1957) has been applied to each star to obtain also the median color. A rough envelope of the points at maximum light has been transcribed to the corresponding Arp (1959) diagram for the Magellanic Cloud cepheids and is shown in Figure 12. Since the galactic cepheids have periods between 2.5 and 45 days, we confine our attention to that subgroup of the Magellanic Cloud cepheids as well. It must, first of all, be admitted that the Magellanic Cloud subgroup contains about three times as many cepheids as our galactic group, and any comparison of the statistics is bound to be somewhat distorted. However, the following tentative points can be made: (1) Within the period interval specified, very few of the Cloud cepheids are bluer than the galactic cepheids. (2) The tendency for galactic cepheids with periods around 10 days to be redder than average is mimicked by the Cloud cepheids. (3) The "dispersion" in intrinsic color of the Cloud cepheids at a given period appears to be greater than that of the galactic cepheids. (4) The Cloud cepheids on the average are about 0.15 $(B-V)$ redder at maximum light than the galactic cepheids.

Points 1, 3, and 4 strongly suggest that agreement in the period-color relation in the two stellar systems is possible, provided that the Cloud cepheids are reddened by interstellar material within the Clouds themselves, which amounts, on the average, to about 0.15 mag. Observational tests of this hypothesis are immediate and twofold. First, slit spectrograms and/or Γ-photometry at maximum light of the redder stars might be obtained. For example, the Cloud cepheid at log $P = 0.90$ and $(B-V)_{\max} = +0.90$

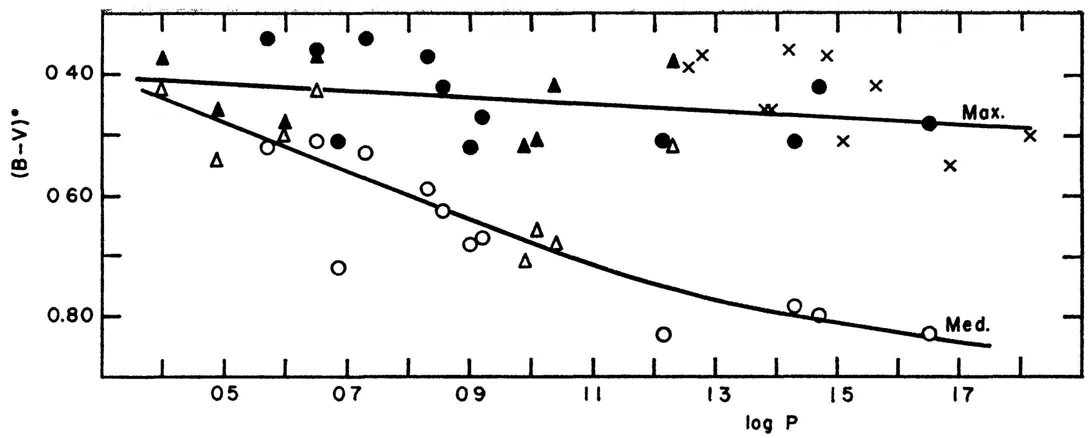

FIG. 11.—The maximum and median unreddened colors of cepheids as a function of log P. Triangles are C-type cepheids (Gascoigne and Eggen 1957) Crosses are the 10 Magellanic Cloud cepheids reported by Gascoigne and Eggen.

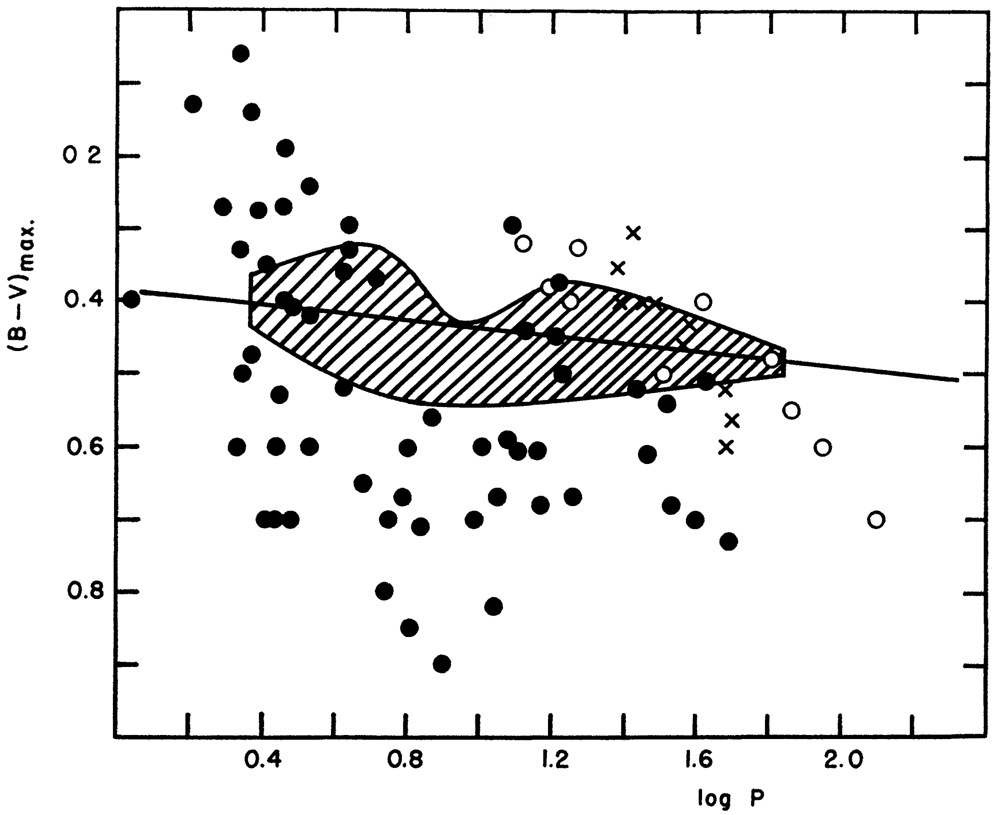

FIG. 12.—The colors at maximum light of galactic and Magellanic Cloud cepheids. Filled circles are the SMC stars observed by Arp (1959); crosses are the 10 Gascoigne-Eggen (1957) cepheids; and open circles are some additional stars of the SMC observed by Gascoigne (reported by Arp 1959). The shaded area is a rough envelope of the region corresponding to maximum light in Fig. 11.

would have to be a G3 I*b* star (from Fig. 7) if there is no reddening. This is totally unlike the behavior of any galactic cepheid at maximum light. On the other hand, it might, for example, be an F7 I*b* star with an excess of 0.43 mag. Second, polarization measurements could be made to see whether there is an average increase in polarization with increased color. In view of the fact that one is dealing with fifteenth-magnitude stars, the latter suggestion is probably more practicable than the former.

Finally, we may remark on the period-color relation in connection with the assignment of the A, B, C types. In Figure 11, triangles denote stars of type C; the others are of type A or B. Most of our C types are the 10-day-period variety. For these, at least, there is no evidence that, at median light, they are appreciably different in color from the A and B types (cf. Sandage 1958*a*), though at maximum light they appear to be a little redder than average. The only possible exception is the 17-day star Y Oph, which is very blue at median light. With the addition of more cepheids in subsequent papers of this series, we hope to give a more definitive answer to this question.

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