Astrometric Analysis of Lalande 21185

SARAH LEE LIPPINCOTT
Sproul Observatory, Swarthmore College
(Received May 2, 1960)

The variable proper motion of Lalande 21185 was discovered in 1941 from plates taken with the 24-inch Sproul telescope. Measurements and reductions of the plates over the interval 1912–1959 yield $+0\overset{''}{.}399 \pm 0\overset{''}{.}002$ for the relative parallax and indicate a period of $8\overset{y}{.}0 \pm 0\overset{y}{.}3$ with $0\overset{''}{.}0336 \pm 0\overset{''}{.}0024$ for the value of the semi-axis major of the photocentric orbit. Assuming a mass for the primary, appropriate for a dM2 star, a mass of the order of 0.01 ⊙ is found for the unseen companion.

INTRODUCTION

LALANDE 21185, BD+36°2147, ($10^h57\overset{m}{.}9$ +36°38′, 1900) vis. mag. 7.46 spectrum M2V, at a distance of 8.1 light-years, is an astrometric binary with a period of 8 years. The system has a space velocity of 103 km/sec. From the plates taken with the Sproul 24-inch refractor, variable proper motion was discovered by Land in 1941 and announced by van de Kamp (1944). Due to the small amplitude of only $0\overset{''}{.}03$, a long time elapsed before the proper period could be assigned to the perturbation. The current investigation includes the material from 1912–1914 (Miller 1919), 1921–1926 (Pitman 1928), and 1912–1940 (Land 1942), and supersedes the former results for parallax and proper motion.

MATERIAL AND MEASURES

The procedure described in previous Sproul papers has been followed (van de Kamp and Lippincott 1949). The material includes 3591 exposures on 955 plates taken on 315 nights, resulting in a total weight 783. The material is summarized by two early normal points and yearly means from 1938 on, given in Table I. The regular exposure time from 1912–1926 was 15 min, 1937–38: 9 min, 1939–41: 2 min, 1942–45: $1\frac{3}{4}$ min, 1946 on $\frac{3}{4}$ min. The dependences and their yearly changes are given in Table II, which also includes the standard frame and the position for Lal 21185 in 1940.00. The spectra of the reference stars were kindly furnished by Vyssotsky. The image of Lal 21185 was reduced to magnitude 10.9 by the use of a rotating sector with 4.4% opening, thus providing close magnitude compensation with the three reference stars.

The measurements used in the current investigation have been made over the time interval 1937–1959 by the following persons on the Gaertner machine: Dorothy Allen, Julie E. Damkoehler, Marion Heaney, Candida Kranold, Gustav Land, Sarah L. Lippincott, Betty A. Mateer, John Merrill, Jr., Jane Piper, E. Günther Reuning, Nancy G. Roman, Sara M. Smith, Kaj Aa. Strand, Marion E. Wolff, and Richard D. Woltman. Two thirds of the plates have been measured by the author. Approximately half of the plates have been measured twice, some three times. When additional years were measured, an earlier selection of plates consisting of four or five consecutive nights were remeasured in order to check the constancy of the machine-measurer combination. In all cases averages have been used with no increase in plate weight.

PRELIMINARY REDUCTIONS

The solution for parallax and proper motion in x and y from 1912–1940 (Land 1942) provided the basis for the ephemerides until the current solution was made. The color equation amounts to $\gamma_x = +0\overset{mm}{.}0013$, $\gamma_{x2} = +0\overset{mm}{.}0003$ and $\gamma_y = +0\overset{mm}{.}0018$ (Lippincott 1957). Graphical attempts at finding an orbit which satisfied Kepler motion and represented the perturbed proper motion led to a period of $1\overset{y}{.}14$ (van de Kamp and Lippincott 1951). The true value of the period was obscured for a long time due to the inherent errors of the same magnitude as the amplitude of the perturba-

TABLE I. Normal points of residuals.

Epoch	Σn	Σp	$q_x t^2$ (unit 0.0001 mm)	$q_y t^2$	orbital motion in x (unit 0.0001 mm)	y	v_x (unit .0001 mm)	v_y
1913.20	16	16	+55	−41	−18	0	−3	−2
23.27	17	17	22	16	+3	+2	+10	+3
38.20	7	13	0	0	−13	+2	0	−2
39.15	7	13	0	0	+3	+2	+3	−7
40.06	11	25	0	0	+14	0	−3	−1
41.18	13	35	0	0	+14	−3	+6	+3
42.30	11	32	0	0	+4	−4	−13	−5
43.26	5	13	+1	−1	−6	−4	−18	+10
44.15	25	80	1	1	−14	−2	−3	+4
45.25	14	40	2	1	−18	0	−2	−3
46.18	9	26	+3	−2	−14	+2	+3	+5
47.23	18	43	4	3	+3	+2	−14	+3
48.09	34	84	5	4	+14	0	+6	−7
49.12	15	41	6	5	+14	−3	0	+5
50.19	19	51	8	6	+4	−4	+9	0
51.12	14	40	+10	−7	−4	−4	+13	+3
52.18	11	28	11	8	−13	−2	+6	−3
53.14	16	46	13	10	−18	0	−2	−13
54.13	11	30	15	11	−15	+2	+8	+12
55.20	11	32	18	13	+2	+2	−2	+8
56.16	8	22	+20	−15	+14	0	0	−4
57.19	6	16	23	17	+14	−3	−15	+7
58.27	9	19	26	19	+4	−4	−3	−10
59.17	8	21	28	21	−6	−4	−12	+6

TABLE II. Reference system.

No.	m_{pv}	Sp.	Standard frame x_s	y_s	Dep. 1940	ΔD/yr	relative μ_x	μ_y
			mm	mm				
1	11.3	K0	−49.69	−35.30	+0.2149	+0.00287	+0″.015	−0″.008
2	10.9	G0	−30.93	+47.43	0.4011	−0.00308	+0.017	+0.019
3	10.5	F8	+80.62	−12.13	0.3840	+0.00021	−0.032	−0.009
Lal 21185	(10.9)	M2	+ 7.87	+ 6.76				

tion. It now seems clear that the period is close to eight years. The plane of the orbit of the unseen companion is inclined close to 90° and is nearly parallel to the right ascension coordinate. The perturbation, therefore, exhibits itself primarily in right ascension which contributed to the delay in the proper recognition of the period. Normal point remainders provide no distinction between $P=7^y.8$ and $8^y.5$ as to fitness after adjustments to the proper motion and the introduction of a term for secular acceleration.

There is no doubt as to the necessity for a quadratic term over the interval 1912–1959. Although Lal 21185 has little proper motion in right ascension, the proper motions of the reference stars cause a spurious acceleration (van de Kamp 1951). Accurate proper motions for the reference stars are not available; the field is not rich in stars, and the system of three reference stars used represents one of the largest configurations on the Sproul program. An attempt was made to determine the relative proper motions of the reference stars from a long-exposure plate taken with the Sproul 24-inch refractor in 1912 and one in 1959 where 14 field stars were measured. Reference stars 1 and 3 nevertheless are so near the edge of the plate that the determinations of their positions and hence motions are extrapolations. The resulting relative proper motions in Table II are given with reservation. The expected effect on the observations is summarized in Table III.

TABLE III. Secular acceleration.

	x	y
perspective acceleration $-1.024\times10^{-6}\,\mu V\pi t^2$	$-0''.00002\,t^2$	$-0''.00017\,t^2$
spurious acceleration $-[\Delta D]t^2$	+0.00002	+0.00008
Total expected effect on observations	0.00000	−0.00009
q from least-squares solution	+0.00014 ± 1 (p.e.)	−0.00011 ± 1 (p.e.)

TABLE IV. Results of solutions in x and in y.

	Orbital elements I	II	p.e.
	mm	mm	mm
c_x	+7.848845	+7.848728	
μ_x	−0.030190	−0.030182	±0.000011
q_x	+0.0000077	+0.0000072	±0.0000006
π_x	+0.02099	+0.02100	±0.00015
c_y	+6.766978	+6.766938	
μ_y	−0.250545	−0.250544	±0.000008
q_y	−0.0000057	−0.0000058	±0.0000005
π_y	+0.02176	+0.02176	±0.00022
(B)	+0.000468	+0.000362	±0.000123
(G)	+0.001671	+0.001636	±0.000129
(A)	+0.000332	+0.000323	±0.000090
(F)	−0.000163	−0.000140	±0.000096
p.e. 1_x	±0.00235		
p.e. 1_y	±0.00169		
$[pv_x^2]$	0.00375565	0.00375680	
$[pv_y^2]$	0.00195265	0.00199617	

LEAST-SQUARES SOLUTION

The conditional equations for the least-squares solution are

$$R_x=\Delta c_x+\Delta\mu_x t+q_x t^2+(B)x+(G)y+\Delta\pi P_\alpha,$$
$$R_y=\Delta c_y+\Delta\mu_y t+q_y t^2+(A)x+(F)y+\Delta\pi P_\delta,$$

where t is counted from 1940.0000; R_x, R_y are the remainders from the ephemeris computed from the 1940 solution; q is the value for the acceleration. The geometric elements, (B) (A) (G) and (F) are the Thiele-Innes constants multiplied by $-\alpha/a$. They yield the scale α of the photocentric orbit. Two sets of elements were considered:

$$\text{I}\quad P=8^y.0,\quad T=1939.3,\quad e=0.30$$
$$\text{II}\quad P=8^y.2,\quad T=1939.0,\quad e=0.20,$$

resulting in two sets of elliptic rectangular coordinates x, y, for use in the conditional equations. Solutions were made on an IBM 650 electronic computer from the 315 weighted nightly means in x and in y, using both sets of orbital elements. The values in Table IV were found; the scale factor for the 24-inch Sproul refractor is 1 mm = 18″.87. The solution using orbital elements I is adopted, keeping the other in reserve for possible use in the future. Normal points of the night residuals are given in Table I. Figures 1 and 2 show the normal points plotted with respect to the orbital motion and to the quadratic term effect.

A combined solution in x and y for parallax was made with the results shown in Table V.

PARALLAX

The weight of the combined solution in x and in y for parallax is 322.40 yielding ±0″.0020 for the formal probable error.

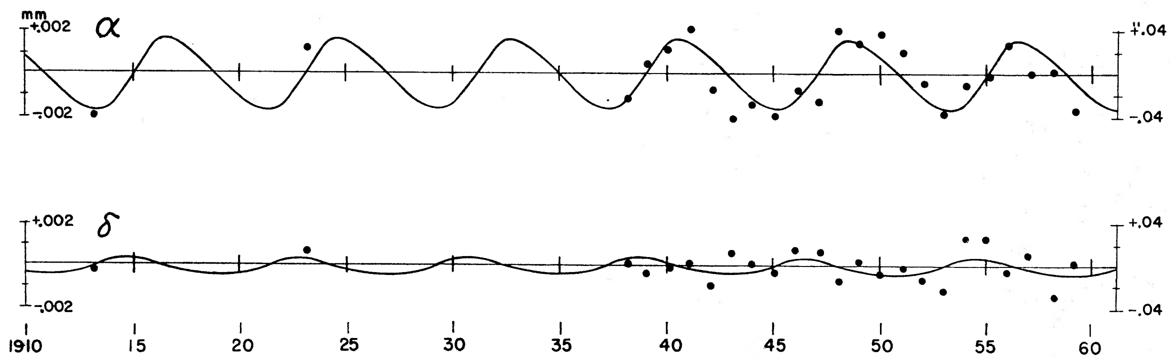

FIG. 1. Lalande 21185. Mean residuals from solution for μ, q, π represented by Kepler motion, $P=8^{y}\!.0$, $T=1939.3$, $e=.30$.

Lalande 21185 is well placed with respect to the ecliptic to give an accurate parallax in declination from the present accumulation of data: $\pi_\delta=+0''\!.4106 \pm 0''\!.0042$ (p.e.); $\pi_\alpha-\pi_\delta=-0^{\text{mm}}\!.00077=-0''\!.014\pm0''\!.005$ (p.e.). This difference is disappointingly large in view of the probable errors for π from the x and y solutions. A number of sources of error play a role in the parallax determination in x from which the y coordinate is free. The linear correlation of the parallax factors in right ascension with the time of the night has long been a concern of parallax observers. The spread of the reference configuration in x is very large shown by $[x^2]=99$ cm², $[y^2]=36$ cm² (Lippincott 1957) which undoubtedly contributes to the large size of the probable error of unit weight in x and may well have introduced systematic errors into the parallax determination in right ascension.

Recently we have become aware that the parallax determinations in right ascension at the Sproul Observatory are subject to systematic errors depending on the particular years of the observations (Lippincott 1957). Differential parallax solutions grouped biennially were made from the night residuals from the above full solution in x and in y. The resulting differential parallax corrections are given in Table VI with the time intervals and sum of the night weights. The probable errors are computed from the weights of the parallax given for each solution combined with the probable errors of unit weight obtained from the separate x and y solutions for the six variables. The formal probable error from the parallax solution in x must be regarded as spuriously low. No satisfactory explanation has been found for the differences in the parallax determinations in x; the y coordinate appears free from systematic errors.

The relative parallax is reduced to absolute in the usual Sproul manner taking into account the magnitudes and the spectra of the reference stars at galactic latitude +67°. The resulting absolute parallax for the 1912–1959 Sproul material is $+0''\!.4039\pm0''\!.0021$. This value is compared with $+0''\!.384\pm0''\!.007$ (p.e.) from the three determinations: McCormick, Yerkes and Van Vleck Observatories adjusted for the Yale Catalogue precepts. Combining this value with the current Sproul value we find $+0''\!.402\pm0''\!.002$ for the weighted mean value and its probable error.

TABLE V. Results of combined solution in x and y for parallax.

	mm		p.e.
c_x	+7.848842		
μ_x	−0.030190	$=-0''\!.56968$	$\pm0''\!.00020$
q_x	+0.0000077	=+0.00014	±0.00001
c_y	+6.767171		
μ_y	−0.250542	=−4.72773	±0.00015
q_y	−0.0000057	=−0.00011	±0.00001
π	+0.02114	=+0.3989	±0.0021
(B)	+0.000444	=+0.0084	±0.0023
(G)	+0.001670	=+0.0315	±0.0024
(A)	+0.000293	=+0.0055	±0.0017
(F)	−0.000176	=+0.0033	±0.0018
p.e. 1	±0.0020	=±0.0377	

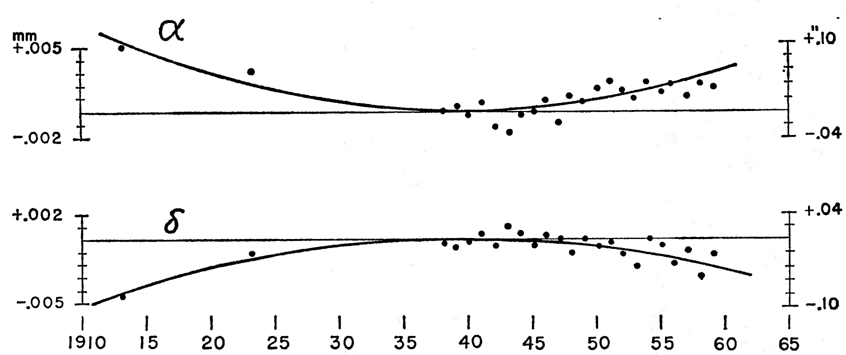

FIG. 2. Lalande 21185. Observed acceleration after allowing for μ, π, and orbital motion.

TABLE VI. Differential parallax with respect to interval of time.

Interval	Σp	$\Delta\pi_x$	p.e.	$\Delta\pi_y$	p.e.
		unit 0.0001 mm			
1912.3–1941.4	119	− 1.0	±3.4	− 0.3	±3.6
42.1– 44.4	125	+ 5.9	3.5	+ 3.5	3.2
44.9– 46.4	66	+13.2	4.8	+ 1.2	3.9
46.9– 48.4	127	+17.4	3.4	+ 0.4	3.4
48.9– 50.4	92	+ 3.7	4.1	+ 2.3	3.6
51.0– 52.4	68	−46.6	5.1	− 1.8	4.3
53.0– 55.4	108	− 6.1	5.2	+ 3.7	3.8
56.1– 59.3	78	−17.9	5.3	−11.1	3.8

ORBITAL ANALYSIS

The sums of the squares of the residuals from the two solutions with different orbital elements show no significant differences. The interpretation is not dependent on a choice of one or the other orbit. In this investigation we have chosen to work with $P=8^{y}.0$, $T=1939.3$ and $e=0.30$. The geometric elements yield

$$i=79°\pm4°$$

$$\alpha=+0^{mm}.0017.8=0''.0336\pm0''.0024=0.083 \text{ a.u.}$$

The lower limit of the mass of the companion can be derived from

$$(B-\beta)(M_A+M_B)=0.021(M_A+M_B)^{\frac{1}{3}}$$

assuming several appropriate values for (M_A+M_B) and the luminosity function, β. Corresponding values for M_B are given in Table VII.

The absolute visual magnitude of the visible component is 10.49; using Limber's bolometric corrections for this star $M_{bol}=8.56$; its effective temperature is 3300°K (Limber 1958). The mass-luminosity relation indicates 0.33 ⊙ as a likely value for the mass of Lal 21185 A. It seems unlikely that the B component could be as bright as $10.5=m_{pv}$, $\Delta m=3$, and have gone unseen at a distance of 1″. Also it seems unlikely that for $M_{pv}=13.5$ the mass could be as small as 0.03 ⊙ in view of the value 0.04 ⊙ determined for each component of L726–8 where $m_{pv}=15.4$, 15.8.

TABLE VII. Values for M_B.

M_A+M_B	M_B $\Delta m=\infty$	M_B $\Delta m=5$	M_B $\Delta m=3$	max. dist. 1961
0.250 ⊙	0.008 ⊙	0.011 ⊙	0.023 ⊙	1″.0
0.300	0.009	0.012	0.027	1.1
0.400	0.011	0.015	0.035	1.2
0.500	0.013	0.018	0.043	1.3

It is concluded that $\Delta m>3$ which leads to a mass of 0.01 ⊙ for the fainter component assuming the mass for A to be between 0.3 ⊙ and 0.4 ⊙.

The companion is probably extremely red; therefore, observations in the infrared would decrease the Δm, thereby facilitating its detection. Some image scanning photoelectric device in the infrared, taking advantage of the time of greatest separation and the position angle might yield the positive results needed for a rigorous mass determination.

ACKNOWLEDGMENTS

I wish to thank Dr. Edgar Mullins, Jr., of the Mathematics Department of Swarthmore College for adapting a program to accommodate the above least-squares solutions and for carrying them out on an IBM 650 electronic computer. I am indebted to all the measurers whose names appear in the text and to all the observers who contributed to the accumulation of the 955 plates. I am deeply indebted to Dr. van de Kamp for his all inclusive role in making possible this investigation.

This work was partially supported by the National Science Foundation.

REFERENCES

Land, G., 1942, *Astron. J.* **49**, 153.
Limber, D. N., 1958, *Astrophys. J.* **127**, 363.
Lippincott, S. L., 1957, *Astron. J.* **62**, 55.
Miller, J. A., 1919, *Sproul Publication*, **4**, 63.
Pitman, J. H., 1928, *Sproul Publication*, **10**, 66.
van de Kamp, P., 1944, *Proc. Amer. Phil. Soc.* **88**, 5.
van de Kamp, P., 1951, *Pop. Astron.* **LIX**, 251.
van de Kamp, P., and Lippincott, S. L., 1949, *Astron. J.* **55**, 16.
van de Kamp, P., and Lippincott, S. L., 1951, *Astron. J.* **56**, 49.