THE SOLAR LIMB INTENSITY PROFILE*

JOHN B. ROGERSON, JR.
Princeton University Observatory
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ABSTRACT

A selected photograph of the solar limb, taken by a balloon-borne telescope, has been analyzed to give the intensity distribution of the solar limb at λ 5400 A. This work extends existing limb-darkening measurements to within about 1 second of arc of the limb.

INTRODUCTION

In the early fall of 1957, a pre-programmed, self-orienting solar telescope was sent into the stratosphere by balloon, equipped with an automatic 35-mm camera for the purpose of securing a photograph of the solar granulation unblurred by the poor seeing qualities of the daytime sky. This entire experiment has been described in detail by Martin Schwarzschild (1959).

Inasmuch as the primary goal of the experiment was attained in the first attempt, it was decided to use the second flight for securing a high-definition photograph of the solar limb. It was felt that, in addition to obtaining for the first time the intensity distribution in the region of the extreme solar limb, this information could subsequently be used to determine more securely the distribution of temperature in the outermost layers of the photosphere. The modifications to the instrument for this experiment are described by Schwarzschild (1959).

The effective wave length for the limb photograph is 5400 A, with the half-maximum response points roughly at 5100 A and 5800 A. This figure has been arrived at by taking into account the spectral energy distribution of solar radiation, the spectral transmission properties of the glass filter used, and the spectral sensitivity of the type V-G photographic emulsion. No account was taken of the spectral transmission of the earth's atmosphere or the telescope, inasmuch as these were quite constant over the relevant spectral range.

MICRODENSITOMETER TRACINGS

Out of nearly 50 exposures containing the solar limb, 1 was selected for analysis, primarily on the basis of high definition and good focus but also with consideration for freedom from photographic defects. This exposure, which bears the Princeton designation 569-5, was taken on October 17, 1957, at 11:12 A.M. Central Standard Time, at a point on the solar limb approximately 34° west of the north end of the diameter containing the solar rotation axis.

Inspection of the selected exposure under magnification indicates that the granulation pattern, greatly foreshortened, rapidly fades out toward the limb, becoming essentially undetectable in the outermost 2 seconds of arc. Any granulation-caused waviness of the limb itself must, if present at all, be of very small amplitude, estimated at less than 0".2. These considerations make well-defined limb tracing possible.

Three microdensitometer tracings were made along different radii of the solar image. These tracings were separated along the limb by intervals of roughly 20 seconds of arc

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so that they are quite independent. The scanning window was a 10 μ by 500 μ rectangle with the long dimension oriented parallel to the limb. The scale of the negative is such that 1 mm corresponds to 3".4 or 2440 km on the sun's disk. Over the 500 μ the solar limb curves by only 0".0015 so that no averaging of different limb distances by the slit is to be expected. The window width directly integrates over a range of limb distances, and a smearing of the tracing is unavoidable. However, by using only a 10 μ width, which corresponds to only 24 km on the disk, the smearing can be made quite small. In fact, the corrections to remove the effect of the slit width are negligible, being in all cases smaller than one-third of a unit in the last place to which the data were read from the tracings.

The individual tracings were read at equidistant points from a region of zero density outside the limb to a point roughly 10 seconds of arc inside the limb. The reading points were separated by an interval that was convenient on the tracing and which corresponds to 0".169 or 122 km on the sun.

CALIBRATION

The photographic film exposed during this experiment was developed in 100-ft. lengths. Among the approximately 800 frames developed simultaneously with the chosen limb exposure were a large number of exposures of the solar disk. One of these exposures, with Princeton designation 575-4, was chosen to provide the calibration.

The ends of each frame are exposed less than the frame center due to neutral density filters mounted for calibration purposes near the film plane of the balloon telescope. The one filter had a measured transmission of 78.2 per cent and the other, 48.4 per cent. Thus by measuring the film density on both sides of a filter edge, one has a pair of points on the calibration curve. Further, due to a linear exposure gradient along the filter edge caused by a construction error in the shape of the rotating shutter, it was possible to obtain different pairs of points on the calibration curve from the same filter edge. Altogether, 3 pairs of points were measured across each of the two filter edges, making a total of 6 pairs of points. These pairs were then individually slid along the intensity axis until all 12 points delineated a smooth curve which has an uncertainty of the order of 5 per cent.

The calibration curve points up the fact that while the slow, fine-grained, high-contrast V-G film is fine for exhibiting the small intensity contrasts in the granulation pattern, its large threshold value forbids its use at the low intensities of the extreme solar limb. This unfortunate circumstance has as its consequence that this investigation can say nothing about the extreme limb, and, indeed, the outermost second of arc is somewhat uncertain.

OBSERVED LIMB

Each of the three limb tracings was reduced to an intensity scale by means of the calibration curve. Because of the accidental linear exposure gradient mentioned earlier, the three limb intensity contours differed from one another by various scale factors. This difficulty was rectified by making the areas under each of the three contours equal, that is, multiplicative constants were applied to two of the three to make their areas equal to that of the third.

The three limb contours, being at this stage on the same intensity scale, were averaged point by point, with equal weight given to each. The position of the limb is arbitrarily chosen to coincide with the outermost point for which the calibration is trusted. The shape of the observed limb indicates that the intensity is falling rapidly at this point, and it is estimated that the true limb must be within $\frac{1}{3}$ second of arc of this point. By true limb is meant the inflection point in the true-intensity profile. The mean observed-limb profile is illustrated in Figure 1 and tabulated in Table 2, where it appears under the heading J_{obs}. In both Figure 1 and Table 2 the intensity has been expressed in terms of the disk's central intensity with the help of the Michigan limb-darkening data as described below. Also in Table 2, the column headed σ gives the mean errors of J_{obs} computed from the internal consistency of the three tracings. The internal agreement of the three tracings is quite satisfactory with the mean error of the mean values averaging only 0.64 per cent and being in the worst case 2.5 per cent.

APPARATUS FUNCTION

Originally it was hoped that the observed-limb profile combined with one of the published eclipse-derived true-limb profiles (Kristenson 1951) might provide an estimate of the apparatus function. These hopes were abandoned when it became apparent that the extreme solar limb (which is most sensitive to the smearing effect of the apparatus) could not be determined because of the film that was used.

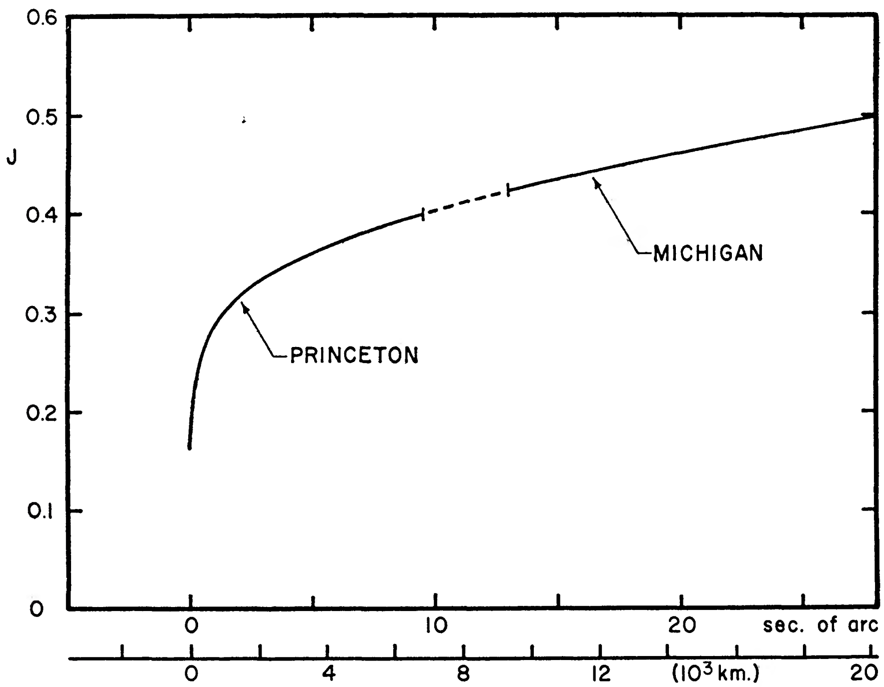

FIG. 1.—Illustration showing how the Princeton solar limb and the Michigan limb-darkening data fit together. The dashed section shows the region not covered by either data. J is the intensity expressed as a fraction of the intensity at the center of the disk.

Inasmuch as no objective method of determining the apparatus function was available, it became necessary to assume one in order to retrieve the true-limb intensity profile from the observed. It was therefore decided to use the diffraction pattern of the 12-inch mirror as the apparatus function. Considerations leading to this decision are (1) atmospheric seeing disturbances are likely to be eliminated by the high altitude at which the exposure was made, (2) the telescope optics are extremely good and were in fine adjustment at launch, and (3) the wealth of fine detail visible in the best exposures suggested that the full resolving power must be nearly realized. This choice does ignore the possibility of scattered light in the telescope which is believed to be small.

The proper diffraction pattern to use can be pictured as follows. Imagine an infinitely long luminous line of constant intensity, e.g., a line on the solar disk parallel to the limb. Produce an image of this with a 12-inch diameter optical system, then measure the distribution of intensity across the line image. This profile, normalized to unit area, is the apparatus function. The integral involved in the line-diffraction pattern can then be evaluated in terms of Struve functions which exist in tabular form (Watson 1945). This diffraction pattern is given in Table 1, where it has been arbitrarily terminated at the point that it falls below 0.5 per cent of its central value. The ordinates tabulated are adjusted so that the area under the apparatus function is unity.

TRUE LIMB

The observed-limb function, $O(x)$, is the result of folding together the true-limb function, $T(x)$, and the apparatus function, $A(x)$. These three functions are related by the convolution equation

$$O(x)=\int_{-\infty}^{+\infty} T(x')\, A(x-x')\, dx' , \tag{1}$$

TABLE 1

LINE-DIFFRACTION PATTERN FOR 12-INCH APERTURE

x (″)	A*	x (″)	A*	x (″)	A*	x (″)	A*
0.000	0.2008	0.423	0.0067	0.846	0.00[illegible]4	1.269	0.0014
.085	.1742	.508	.0083	0.930	.0030	1.353	.0014
.169	.1134	.592	.0081	1.015	.0022	1.438	0.0010
.254	.0518	. 677	.0047	1.100	.0014		
0.338	0.0160	0.761	0.0024	1.184	0.0012		

* Ordinates normalized for use in eq. (2).

which in the present case must be solved for $T(x)$. For purposes of computation, the integral in equation (1) is replaced by a sum, i.e.,

$$O_n=\sum_{m=-17}^{m=+17} T_n A_{n-m} , \tag{2}$$

where the limits on m in the summation reflect the number of ordinates in the apparatus function. The error involved in replacing equation (1) by equation (2) is satisfactorily small if, over the range of one step in the index n, both $T(x)$ and $A(x)$ vary essentially linearly. To assure this, the computations were carried out with limb-distance steps of $0\rlap{.}''08459$, i.e., one-half the interval at which the observed-limb tracings were read.

The iteration scheme used in solving equation (2) is not new (Lindsay 1931) but will perhaps bear a brief description. Use $O(x)$ as an approximation to $T(x)$ and carry out equation (2). The residuals between $O(x)$ and $C(x)$, the computed function, are then added to $O(x)$ to give the next approximation to $T(x)$. In each step of the iteration the residuals, $(O - C)$, are added to the current approximation to $T(x)$ to obtain the next approximation. The iteration scheme is performed until the $(O - C)$'s become satisfactorily small, which in this case means smaller than the mean errors of the observed function itself.

It can be seen from equation (2) that the determination of the true-limb function depends on the nearby observed-limb values. The true-limb values very near the limb thus depend partially on the unknown extreme limb and are therefore uncertain. For these calculations the extreme limb was approximated by extrapolating linearly to zero intensity from the outermost observed-limb values, a procedure which is likely to affect only the two or three outermost values of the true limb. The outermost six values in

Table 1 are marked with colons to indicate uncertainty due to this cause, the uncertainty increasing toward the limb.

The computations were carried out on a model 650 IBM electronic computer. Three iterations sufficed to give the adopted true-limb contour which is illustrated in Figure 1 and tabulated in Table 2 under the column headed J_{true}. It should be noted that each trial $T(x)$ has to be graphically smoothed, since random errors grow in the solution of equation (2) (van de Hulst 1941).

CONNECTION OF INTENSITY SCALE TO LIMB-DARKENING DATA

Unfortunately the present measures do not extend far enough inside the limb to overlap any limb-darkening measures known to the author which themselves are securely related to the intensity at the center of the disk and which are in the appropriate wavelength region. It was decided therefore to tie in the present data with the Michigan photoelectric limb-darkening work published by Pierce, McMath, Goldberg, and

TABLE 2

OBSERVED- AND TRUE-LIMB CONTOURS EXPRESSED IN TERMS OF INTENSITY AT DISK CENTER

Limb Distance		$\cos\theta$	J_{obs}	σ	J_{true}*
″	km				
0.00	000	0.0000	0.160	±0 0041	0.162::
0.17	122	.0187	.193	.0034	.197:
0.34	244	.0265	.226	.0026	.233:
0.51	366	.0324	.247	.0020	.255:
0.68	488	.0374	.262	.0018	.268:
0.85	610	.0419	.275	.0019	.277:
1.02	732	.0459	.284	.0018	.285
1.18	854	.0495	.291	.0019	.292
1.35	976	.0529	.297	.0017	.298
1.52	1099	.0562	.303	.0013	.304
1.69	1220	.0592	.308	.0009	.309
1.86	1342	.0621	.313	.0007	.313
2.03	1464	.0650	.317	.0006	.317
2.20	1586	.0675	.321	.0011	.321
2.37	1709	.0700	.324	.0008	.324
2.54	1831	.0725	.327	.0008	327
2.88	2074	.0772	.333	.0009	.333
3.21	2318	.0816	.339	.0011	339
3.55	2563	.0857	.343	.0010	.343
3.89	2807	.0897	.348	.0012	.348
4.23	3051	.0935	.352	.0015	352
4.57	3295	.0972	.356	.0016	.356
4.91	3539	.1007	.360	.0023	.360
5.25	3783	.1041	.363	.00 4	.363
5.86	4225	.1100	.369	.0017	.369
6.98	5032	.1200	.379	.0027	.379
8.19	5909	.1300	.389	.0037	.389
9.50	6849	.1400	0.399	0 0044	.399
12.92	9319	.1631	....	....	.424
20.46	14760	.2048	....	....	.466
32.49	23440	.2573	....	.....	.516
51.49	37140	0.3223	...	. ....	0.572

* The last four entries are from the Michigan data.

Mohler (1950). These data require only a short extrapolation from our innermost point and are given for λ 5485.

The Michigan limb-darkening measures were reduced to λ 5400 in the following way. The limb-darkening measures of Peyturaux (1955) are given for λ 5768 and λ 5355. For each value of cos θ, one can form the difference $J(\lambda\ 5768) - J(\lambda\ 5355)$, where J is the intensity expressed in terms of the central intensity. Assuming that J varies linearly with wave length over this range, one immediately deduces $J(\lambda\ 5485) - J(\lambda\ 5400)$ which permits the reduction of the Michigan measures to λ 5400. These corrections are not large, amounting to a maximum of 1.6 per cent.

For the adjustment of the Princeton intensity scale to the Michigan intensity scale, one plots the Michigan data in the form log J versus cos θ on translucent graph paper. Similarly, one plots on a separate sheet of translucent graph paper log $T(x)$ versus cos θ. Smooth curves are drawn through both plots with slight extrapolations toward each other. Then by superimposing the graphs, keeping the cos θ scales in registration, one can slide one graph relative to the other along the log J axis until the two curves fuse into one smooth curve. The amount of relative shift between the two graphs is then the logarithm of the factor needed to bring the two intensity scales into coincidence. Figure 1 shows the Princeton true-limb profile and how it fits with the Michigan limb-darkening measures. The dashed portion indicates the region where extrapolation was necessary.

SUMMARY AND DISCUSSION

The true solar limb has been obtained from a limb photograph taken by the Princeton solar balloon telescope. The extreme outer limb is not determined because of the poor characteristics at low intensities of the film used. This lack of information on the extreme limb reflects through the convolution equation an uncertainty in the outermost second of arc of the derived true-limb contour. The apparatus function used in unfolding the true limb from the observed limb is assumed to be the diffraction pattern of the 12-inch telescope aperture. If this assumption is in error, for example if the effective telescope aperture is only 10 inches, then the uncertainty in the outermost second of arc would be increased, but the remaining limb values would not be much affected. The present corrections from the observed to the true limb are of the order of $\frac{1}{3}$ of 1 per cent at limb distances of 1 second of arc. Changing the effective telescope aperture to 10 inches would probably increase these corrections to not more than 1 per cent. It is difficult to believe that the effective telescope aperture can be much less than 10 inches, in view of the excellent definition obtained. It would appear that the present limb data are likely to be trustworthy to within about 5 per cent out to a limb distance of 1 second of arc, with the major source of uncertainty attributable to the photographic calibration.

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