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REDISCUSSION OF ECLIPSING BINARIES. IV
RX HERCULIS AND OTHER A STARS*

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ABSTRACT

A new set of spectrograms forms the basis for determination of the masses and radii of the components of RX Her, appreciable corrections being required to the older values. The four A-type eclipsing systems with best-determined properties are RX Her, AR Aur, WW Aur, and β Aur. These properties are re-examined and compared with those of nearby single stars. In the relations between color and spectral type, between mass and luminosity, and between color and luminosity the eclipsing binaries show excellent agreement with nearby stars of the main sequence. The greatest uncertainty in the comparisons for stars near type A0 arises from the uncertainty in the temperature scale.

INTRODUCTION

The numerous investigations by Struve and his collaborators, revealing instabilities of various kinds in the atmospheres of close binaries, raise questions as to the validity of inferring the fundamental properties of single stars from those of eclipsing binaries. The most comprehensive discussion of the fundamental properties of eclipsing binary stars that has been published in recent years appears to be that of Kopal (1955). As discussed at the National Science Foundation Conference on Binary Stars (Popper 1957*a*), many of the data used by Kopal are of questionable reliability. The purpose of the present paper is to carry out a comparison of the fundamental properties of eclipsing binaries of spectral type A with the properties of nearby stars in the same region of the H-R diagram. Only those binaries for which the properties are well determined will be considered.

Among the brighter A-type eclipsing binary systems, there are four—β Aur, AR Aur, WW Aur, and RX Her—in which the two components are similar in mass, radius, and surface brightness and for which the spectrograms and light-curves show that complications arising from ellipticity, reflection, or other interaction between components are slight. For all except β Aur, the minima are deep enough that modern photoelectric light-curves yield geometric elements of considerable reliability. A new light-curve of high precision is needed for β Aur. From the spectrographic standpoint, the values of $a \sin i$ and $\mathfrak{M} \sin^3 i$ have been well determined for all systems but RX Her, for which a discussion based upon new observations is given here. It may be pointed out that Wood (1948) came to the conclusion that the components of RX Her are about 2 mag. more luminous for their masses than expected from the mass-luminosity relation.

RX HERCULIS

Photometric Analysis

The independent photoelectric observations and analyses of the light-curve of RX Her[1] by Magalashvili (1953) and Wood (1948) lead to results in good agreement, as shown in Table 1. Both sets of observations were obtained with Sb-Cs photocells without filter. The two authors agree that primary minimum is a transit. In Table 1, u is the coefficient of limb darkening; a_b and a_f are the semimajor axes of the equatorial

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* The observations of RX Her reported in this paper were obtained under a co-operative agreement with the Mount Wilson and Palomar Observatories.

[1] HD 170757, var; A0; $\alpha_{1900} 18^{h}26^{m}.0$; $\delta_{1900} + 12°33'$.

sections of the brighter and fainter star, respectively, in units of the radius of the relative orbit; i is the orbital inclination; L is the light of each star; r is the mean radius; and L (corrected) is the light corrected for reflection. Miczaika's (1957) two-color photoelectric observations, which he did not analyze, do not differ significantly from the others. The observations of all three authors show considerable scatter, perhaps owing in part to the variability of a comparison star and perhaps in part to the intrinsic variability of RX Her (see also Günther 1955). Because of the scatter, it has not appeared worthwhile to try to obtain a "best" analysis of the light-curve by combining the three sets of observations. For the purposes of this discussion, the agreement between the two sets of quantities in Table 1 is sufficiently good. We obtain the "adopted" elements of Table 1 from Wood's analysis, taking for the average radius of each star, r, the mean of the three mutually perpendicular axes computed dynamically, and for the light of each star (blue-violet region of the spectrum) Wood's values corrected for the computed reflected light. These values of r and L lead to a ratio of surface brightnesses, $J_b/J_f = 1.18$.

TABLE 1

PHOTOMETRIC ELEMENTS OF RX HERCULIS

	Wood	Magalashvili		Adopted
a_b	0.232	0.223	r_b	0.229
a_f	0.188	0.190	r_f	0.186
i	85°.6	86°.2	i	86°.0
L_b	0.636	0.615	L_b (corrected)	0.643
L_f	0.364	0.385	L_f (corrected)	0.357
u (assumed)	0.6	0.4		

It may be noted that Petrie's (1950) value of the luminosity ratio, 0.59/0.41 (Δm = 0.39), lies outside the range of solutions given by Wood. Because of the scatter of Petrie's individual observed values (kindly supplied to me by Dr. Petrie) and because of the additional uncertainty in evaluating the luminosity ratio caused by a small difference in spectral type between the components, it is not felt that the discrepancy is a serious one.

The magnitude and colors of RX Her were measured on three nights in June, 1956, with the 20-inch reflector on Palomar Mountain.[2] The results, corrected to the U, B, V system, are $V = 7.28$ at maximum light, $B - V = +0.06$, $U - B = -0.02$. Use of the Q method of Johnson and Morgan (1953; Johnson 1958) leads to a color excess, $E_y = +0.08$, and unreddened colors, $B - V = -0.02$, $U - B = -0.08$. These colors are those of a main-sequenced star of spectral type B9.5. The type given by Miss Roman (1956) is A0. The small amount of reddening is not inconsistent with the weak interstellar K line observed in the spectrum.

From relations (Popper 1959) between $B - V$ and effective temperatures and between effective temperatures and bolometric corrections, one may obtain the relation between $B - V$ and the surface brightness, J. The ratio $J_b/J_f = 1.18$ corresponds, in the vicinity of spectral type A0, to a difference in $B - V$ of 0.06 mag. between the two components. We may adopt the colors $B - V = -0.04$ for the hotter, brighter component, and $B - V = +0.02$ for the fainter component.

Spectrographic Analysis

Sanford (1928) has published spectrographic orbits of the components of RX Her. Examination of his spectrograms reveals that the hydrogen lines of the two components,

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[2] The U filter used for all observations in June, 1956, was Corning 5840 instead of the Corning 9863 as reported (Popper 1957).

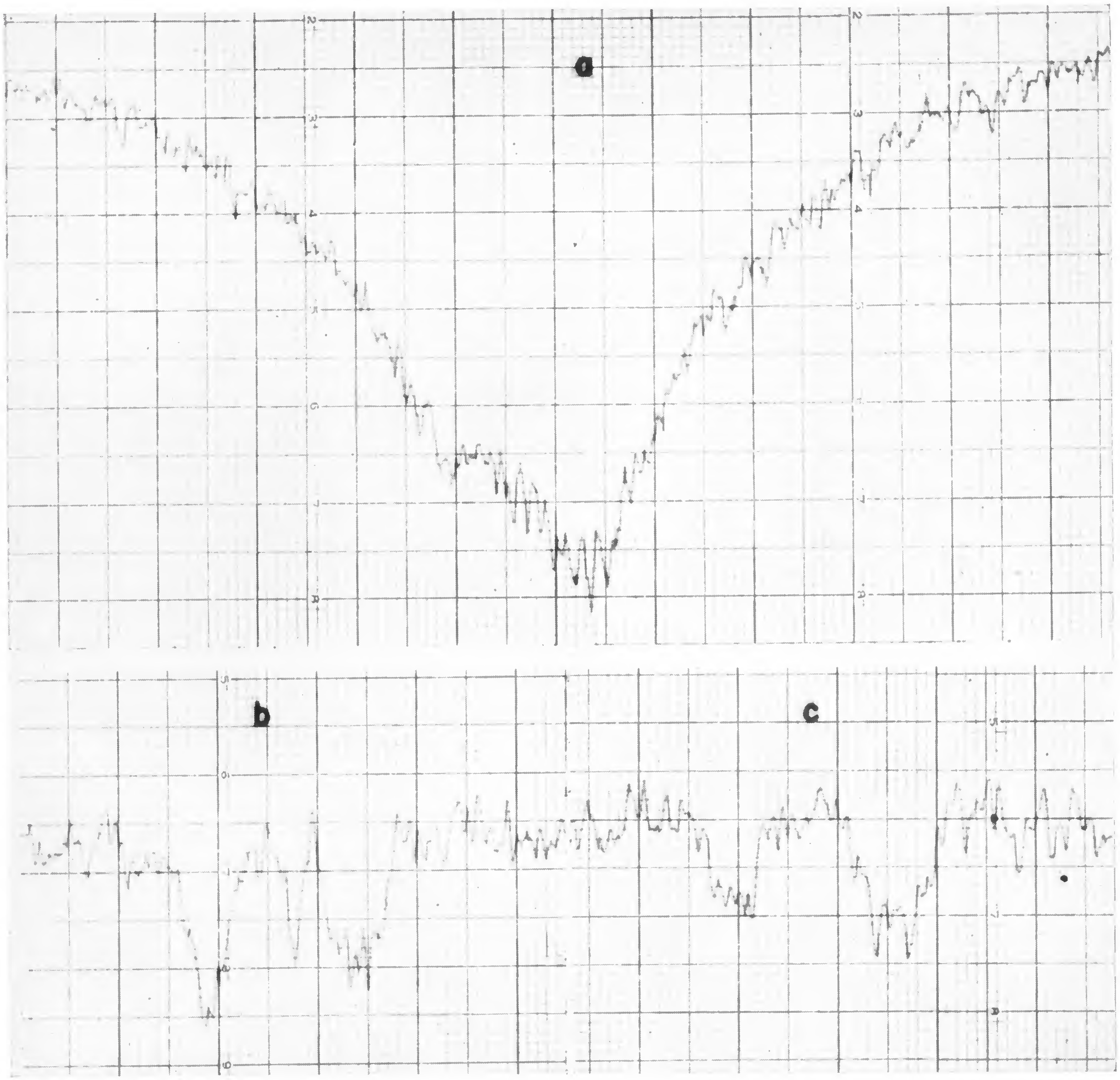

FIG. 1.—Tracings of spectrogram Ce 11205 (0.82 phase). *a:* Hδ; *b:* K line; *c:* λ 4481. Wave length increases toward the right. In Hδ, the line of the cooler component is seen as a plateau shortward of the minimum of intensity due to the hotter component. In the K line, the sharp interstellar line is seen between the two stellar components.

although both clearly visible, are badly blended; that λ 4481 was often difficult to see; and that the K line is the only other line that can be measured. Consequently, a new series of spectrograms, dispersion 10 A/mm, was obtained with the 100-inch reflector. Inspection of these shows that even the K line could not be expected to yield reliable velocities with dispersion appreciably less than 10 A/mm because of blending with an interstellar K line. Although no additional lines other than those mentioned are measurable on the new spectrograms, the clear resolution of the K line, the greater dispersion, and the easier visibility of λ 4481 on all spectrograms may be expected to yield improved precision. The appearance of the K line, of Hδ, and of λ 4481 on a 100-inch spectrogram are shown in Figure 1.

TABLE 2

RADIAL VELOCITIES OF RX HERCULIS

PLATE CE	U.T.	JD_{helioc} 2435000+	PHASE	HOTTER STAR		COOLER STAR	
				Velocity	Residual	Velocity	Residual
	1956						
10423......	Apr. 21.489	584.991	$0^{\mathrm{P}}601$	+ 55	− 1	−137	−14
10427......	22.385	585.887	.105	−121	−11	+ 84	+10
10434......	23.507	587.009	.736	+113	+ 1	−195	− 6
10744......	Sept. 24.117	740.617	.102	−110	− 2	+ 71	0
10763......	Oct. 13.122	759.621	.786	+109	+ 1	−181	+ 4
10766......	14.110	760.609	.342	−140	+ 2	+103	− 8
	1957						
11196......	June 15.383	1004.887	.687	+107	+ 6	−173	+ 4
11197......	15.463	1004.967	.732	+107	− 4	−184	+ 4
11199......	16.312	1005.816	.209	−156	+ 4	+135	+ 3
11200......	16.378	1005.882	.246	−161	+ 3	+137	0
11205......	17.396	1006.900	0.819	+ 97	− 2	−167	+ 7

TABLE 3

ELEMENTS OF VELOCITY VARIATION

	THIS PAPER	SANFORD		SOLAR UNITS	
				This Paper	Sanford
K_1	138±1	131±4	$a \sin i$........	10.6±0.1	9.7
K_2...........	163±2	146±5	$\mathfrak{M}_1 \sin^3 i$.....	2.7±0.1	2.1±0.3
γ............	− 26±1	− 25	$\mathfrak{M}_2 \sin^3 i$.....	2.3±0.1	1.8±0.3

The new spectrograms and measured velocities, based upon the measures of the K line and λ 4481, are given in Table 2. Phases computed from Wood's elements,

$$JD_{\mathrm{min}} = 2432380.174 + 1.7785720E ,$$

appear to require no appreciable correction. The circular elements computed from these velocities are shown in Table 3, along with Sanford's elements. The measures and computed curves are shown in Figure 2. Since Sanford must have measured blended lines for the most part, the new velocity amplitudes are larger than the old. The change in amplitudes gives an increase in the orbital radius of 9 per cent and an increase in the masses of 28 per cent. The importance of using adequate dispersion is evident. The velocity from the interstellar K line is -13 ± 1 km/sec.

PROPERTIES OF A-TYPE ECLIPSING BINARIES

In the discussion to follow, the spectrographic data have been taken from Slocum (1942) for WW Aur, from Harper (1937) for AR Aur, from Smith (1948) for β Aur, and from the present paper for RX Her. Except for the magnitudes and colors, for which my own observations obtained at the Lowell and Palomar Observatories are used, the photometric data have been taken from Huffer and Kopal (1951) for WW Aur, from Huffer and Eggen (1947) for AR Aur, and from Piotrowski (1948; Stebbins' observations) for β Aur. The photometric observations of RX Her are discussed earlier in this paper.

Table 4 gives the observed magnitude and color of the combined light for each system, the color excess, E_y, obtained by the Q method (Johnson and Morgan 1953),

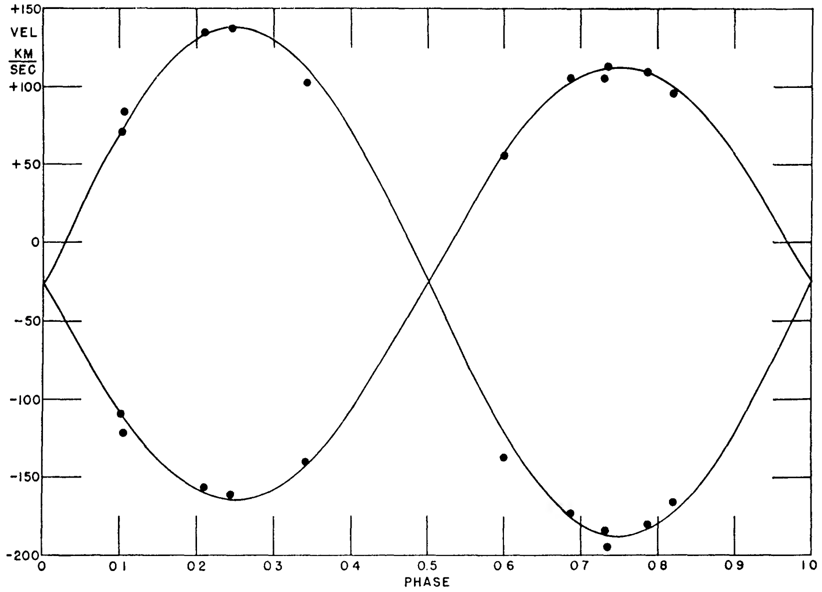

FIG. 2.—Radial-velocity observations and computed curves for RX Herculis

TABLE 4

MAGNITUDES, COLORS, AND SPECTRA

STAR	OBSERVED				$B-V$ (CORRECTED)	SPECTRUM	
	V	$B-V$	$U-B$	E_y		From $B-V$	Observed
AR Aur b......	6.1	−0.02	−0.16	+0.04	−0.09	B8	B9 V
f......					− .03	B9.5	
RX Her b......	7.3	+ .06	− .02	+0.08	− .04	B9.5	A0 V
f......					+ .02	A1	
β Aur	1.9	+ .04	+ .03	0	+ .04	A1	A2 IV
WW Aur b.....	5.8	+0.19	+0.13	0	+ .17	A5–7	A7 V
f.....					+0.22	A7	

and the corrected colors of the two components. In computing the latter, the color differences between components have been obtained from the surface-brightness ratios as described in the discussion of RX Her. No color difference was obtained for β Aur because the minima are not deep enough for a good determination of the surface-brightness ratio. Also given in Table 4 are the observed spectral types of the combined light and the types corresponding to the deduced values of $B - V$ (Johnson and Morgan 1953). The spectral types of RX Her and WW Aur are by Miss Roman (1956), that of AR Aur is by Petrie (1948), and that of β Aur is by Miss Roman (1949). Morgan, Harris, and Johnson (1953) classified β Aur as A2 V.

In the solution of light-curves with two nearly equal partial eclipses, as is well known, the ratio of the radii of the components, k, cannot be determined with precision from the light-curve alone. In the solutions of the light-curves of WW Aur, AR Aur, and β Aur, the authors cited above have obtained k by adopting Petrie's value (1939) of the luminosity ratio of the components in each case. In none of the cases are the components greatly different in luminosity; so there is little doubt that the value of k obtained is approximately correct. Nevertheless, for reasons given in the discussion of RX Her and elsewhere (Popper 1957), I feel that the exact value for the magnitude difference given by Petrie in each case cannot be assumed, when combined with the observed surface-brightness ratio, to yield a value of k with an uncertainty as small as that implied by Petrie's internal probable errors of the luminosity ratios.

In addition to estimates from the light-curve and the use of the spectrographic light-ratio, two other methods of determining the ratio of the radii are available. The first is from the color changes in the two minima. This method was first published by Danjon (1928) and has been discussed by me (Popper 1956). For stars with components as nearly equal in color as those in the systems discussed here, application of this method requires two-color observations of great precision. Those of Miczaika (1957) for RX Her and of Piotrowski and Serkowski (1956) for WW Aur, while showing the effect of the color differences of the components within primary minima, are not adequate for determinations of k. The second method is to obtain the luminosity ratio of the components by assuming that the ratio is the same as that for main-sequence stars of the observed surface-brightness ratio and mean spectral type or, what is equivalent, of the same difference in $B - V$. Since for all four systems the more massive star has both the larger radius and the greater surface brightness, this assumption that the pair is parallel to the main sequence in the H-R diagram appears a reasonable one. For all cases except β Aur, the surface-brightness ratio is well obtained from the light-curve. Neither this method nor that of Petrie for determining the luminosity ratio is entirely satisfactory. But, for the stars under consideration, these determinations, together with the analyses of the light-curves, do serve to set narrow limits on the values of the radii (again with the exception of β Aur).

In the compilation of the masses and radii contained in Table 5, the periods, velocity amplitudes, orbital inclinations, and sum of the radii are taken from the papers cited earlier, except for the velocity amplitudes of RX Her, which are from this paper. The masses are known for all the systems with a precision of a few per cent. The uncertainty of the orbital inclination of β Aur, though greater than for the other systems, is not sufficiently great to cause an appreciable uncertainty in the masses. Two sets of values are given for the radii in Table 5 (except for β Aur). They are, in the column labeled "(1)," values obtained with the use of k determined from the spectrographic evaluations of the luminosity ratios, and in the column labeled "(2)," values obtained with k determined by evaluating the luminosity ratio from the slopes of the main sequence corresponding to the observed surface-brightness ratios. In the subsequent discussion I have adopted the mean of the two determinations of the radius for each component. My principal purpose in discussing two values of the radii is to indicate what I consider to be more realistic estimates of the uncertainties than those obtained by a literal adoption of the internal probable errors of the luminosity ratios given by Petrie and adopted by the photometric investigators. In no case, except that of β Aur, is the uncertainty of the radius greater than about 10 per cent of its value. The much greater uncertainty in the case of β Aur is discussed at length elsewhere (Popper 1959), where the urgent need for a new light-curve for this important system is emphasized.

In order to compute the bolometric absolute magnitudes of the stars, in addition to knowing the radus, one must adopt an effective temperature, T_e, for each component. The calibration of T_e with respect to $B - V$ as argument is discussed in another paper (Popper 1959), where an uncertainty of about 1000° for A0 stars is demonstrated. Because of this uncertainty, two values are given for M_{bol} for each star in Table 5, one based on the higher scale of temperatures and one on the lower. In computing the absolute magnitudes, the effective temperature, 5750°, and the absolute bolometric magnitude of the sun are required. For the latter the value +4.77 is used, obtained from the value of V, −26.73, by Stebbins and Kron (1957) combined with the bolometric correction of the sun, −0.07 (Popper 1959). The resulting values of M_{bol} for the eclipsing binaries are contained in Table 5, as are the values of M_V from the two temperature scales. The two values of the bolometric corrections used in computing M_V for each star are those corresponding to the two temperatures obtained from $B - V$. Only one value of M_V is given for β Aur, that obtained from its parallax, for which the value 0".037 is used. The uncertainty of this quantity is also discussed in the paper last referred to. The two values of M_{bol} for this star are derived from M_V and the bolometric corrections corresponding to the two temperature scales for $B - V = +0.04$.

TABLE 5

COLORS, MASSES, RADII, AND ABSOLUTE MAGNITUDES

STAR	$B-V$	MASS	RADIUS		M_{bol}		M_V	
			(1)	(2)	High T_e	Low T_e	High T_e	Low T_e
AR Aur b.......	−0.09	2.55	1.8	2.0	+0.2	0.9	0.9	1.3
f.......	− .03	2.30	1.8	1.6	+0.5	1.2	1.1	1.5
RX Her b.......	− .04	2.75	2.4	2.5	−0.2	0.5	0.4	0.8
f.......	+ .02	2.33	2.0	1.9	+0.6	1.2	1.0	1.5
β Aur b.......	+ .04	2.35	2.5:	....	0.0	0.2	0.5	
f.......		2.25	2.3:	....				
WW Aur b.......	+ .17	1.81	1.9	2.0	+1.7	1.9	1.8	2.0
f.......	+0.22	1.75	1.9	1.9	+1.9	2.1	2.0	2.2

COMPARISON WITH NEARBY STARS

The principal aim of this paper is to compare the properties of close (but non-interacting) binary stars on or near the main sequence with those of similar single or wide double stars. The early evolutionary histories of the stars in one group must have been quite different from those of stars in the other. Although the close binary stars under consideration are among those for which the most reliable data are available, it is clear that uncertainty in the temperature scale sets a limit to the precision with which the comparisons can be made.

The first comparison, Figure 3, is in the mass-luminosity relation. The only visual binaries with reliable masses in the same range of masses as the eclipsing binaries are Sirius A and Procyon A; γ Vir is included for good measure. Data for the visual binaries are from Strand (1957), except that the new scale of bolometric corrections (Popper

1959) has been used. The two points for each component of the eclipsing binaries, taken from Table 5, correspond to the two temperature scales mentioned in the preceding section. The two for Sirius are from the two bolometric corrections corresponding to the two temperature scales and the color of the star.

The second comparison, Figure 4, is in the H-R diagram for stars closer to the sun than 20 pc. Data for the latter are from a paper by Eggen (1955). Also shown in Figure 4 is the "zero-age" main sequence of Johnson (1957). A third comparison of interest is contained in the last two columns of Table 4, where the spectral types are seen to be normal for the colors of the stars.

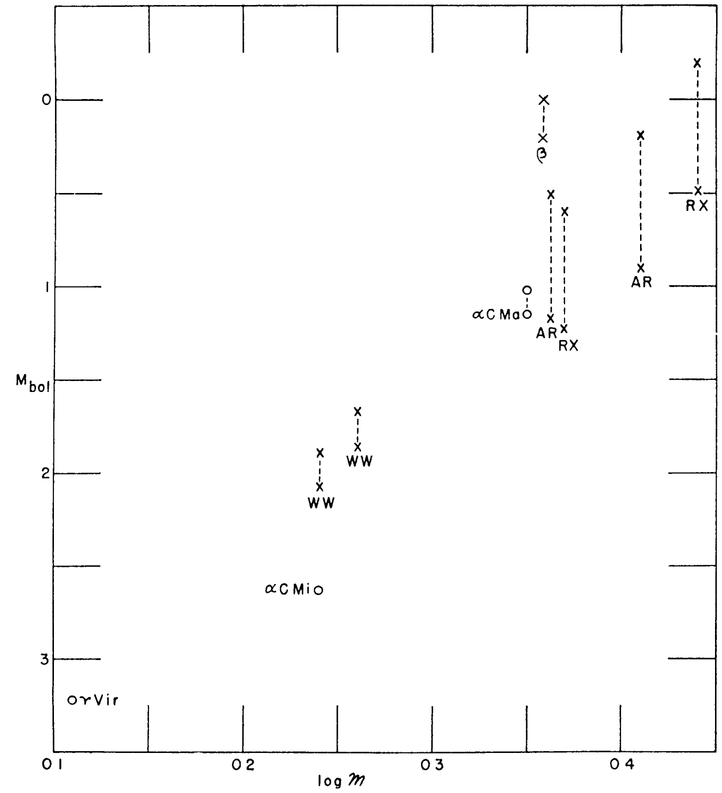

FIG. 3.—Mass-luminosity relation for visual and eclipsing binaries. The two points for a given star correspond to the two temperature scales.

The principal conclusion to be drawn from this discussion is that, when only reliable data are used, eclipsing binaries—at least those not strongly interacting—are similar to single stars. In addition, they form a homogeneous group with respect to physical properties. These conclusions should give us some confidence that fundamental data, obtained from eclipsing binaries in regions of the H-R diagram for which such comparisons cannot be readily made, may be applied to single stars. As pointed out in the introduction, doubts on occasion have been expressed in this matter.

Despite the apparent close correspondences demonstrated here between the properties of eclipsing binaries and those of single stars, one should be careful not to assume that the eclipsing binaries are "normal" stars in all respects. It is axiomatic that no eclipsing binary is completely "normal," and painstaking investigations of these systems might be repaid by revealing some peculiarities.

It would be desirable to extend comparisons between close binaries and single stars to other spectral types. The best eclipsing systems of early type for which both spectrographic and photometric analyses appear to lead to reliable results are Y Cyg, O9.5, and U Oph, B5. In addition, CW Cep, B3, is a promising system from this standpoint. For O and B stars there are, however, no reliable mass determinations of visual binaries for comparison. Moreover, at the present time the temperature scale for hot stars is very much in doubt, so that absolute magnitudes of eclipsing stars are uncertain. Cooler main-sequence eclipsing binaries for which the comparisons could be made are ZZ Boo, F0; CD Tau, F2; VZ Hya, F5; WZ Oph, G0; UV Leo, G0; and YY Gem, M1. The variable CD Tau is a member of a close visual double, so that photometry will be difficult. Adequate radial-velocity observations are available for ZZ Boo, but the period is so close to 4 days that complete coverage of the light-curve would be an extremely laborious, though very rewarding, observational task. Spectrograms of higher dispersion than hitherto employed are needed for CD Tau, VZ Hya, WZ Oph, and UV Leo, in order to make certain of the amplitudes of velocity variation. Photoelectric light-curves of high quality are needed for ZZ Boo, CD Tau, and VZ Hya. I have obtained a light-curve (unpublished) of WZ Oph, and two sets of published photometric results for UV Leo (Perek 1952; Wellmann 1954) are in good agreement. The one important datum required for YY Gem is its bolometric correction.

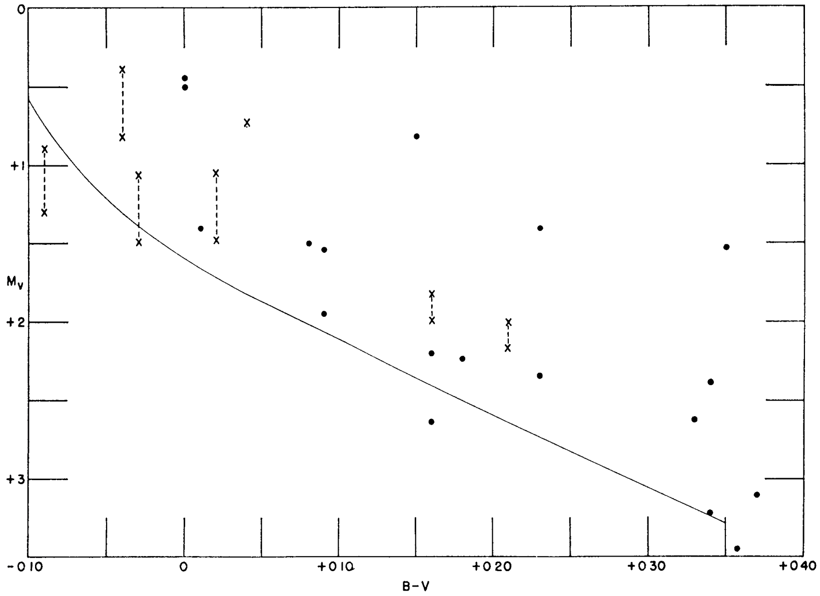

FIG. 4.—H-R diagram for nearby stars and eclipsing binaries. *Dots:* stars closer than 20 pc; *crosses:* eclipsing binaries. The two points for each component of the eclipsing binaries (except β Aur) correspond to the two temperature scales. The curve is the "zero-age" main sequence of Johnson.

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