

is for a three strut support made of six fine wires under tension as in Figure 13. The wires can be so thin that the scattered light is almost negligible.

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PHOTOELECTRIC PHOTOMETRY OF COMET AREND-ROLAND (1956h)

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Abstract. Photoelectric observations of comet Arend-Roland were made on eleven nights in the blue, yellow and with an interference filter centered at 4065 Å. The intensity with the interference filter decreased less rapidly than in the blue and yellow for which agreement with the results of Thiessen were found. Intensity tracings across the comet could be adequately represented by the fountain model with a Maxwell-Boltzmann distribution for the ejection velocities.

The purpose of this paper is to provide photometric information on the behavior of the coma of Comet Arend-Roland. The explanation of the appearance of a comet is complicated by variation in the behavior of individual comets as well as by the differences in behavior of the various spectroscopic species contributing to the light of the comet. Only a limited amount of accurate photometric data is available to resolve these problems. An attempt to explain the observations with the help of the fountain model is partially successful.

Photoelectric observations made on ten nights during April and May 1957 are discussed. The instrument used was the ten-inch visual refractor of the observatory. A photoelectric photometer with a 1P21 photomultiplier was attached. The diaphragm diameter of 1.5 mm combined with a focal length of 172 inches for the telescope gives an angular diameter of 1.18 for the diaphragm

projected onto the sky. The filters used were: yellow, Schott GG11; blue, Schott GG13 plus Corning 5030; an interference filter (IF) centered at 4065 Å. The transmission of the interference filter was determined with a spectrophotometer using an incandescent lamp as a light source and the resulting transmission curve is shown in Figure 1, where the peak transmission has been arbitrarily set as 100. Over the wavelength range covered by these filters appreciable errors in the quality of the image can be expected due to secondary spectrum (residual chromatic aberration) of the refractor. To minimize this effect, an intermediate focus was used which produced circles of confusion approximately 0.3, 0.4, and 0.5 in diameter as projected on the sky for the blue, yellow and interference filter respectively. The output of the photomultiplier was amplified with a DC amplifier and recorded on a Brown recorder.

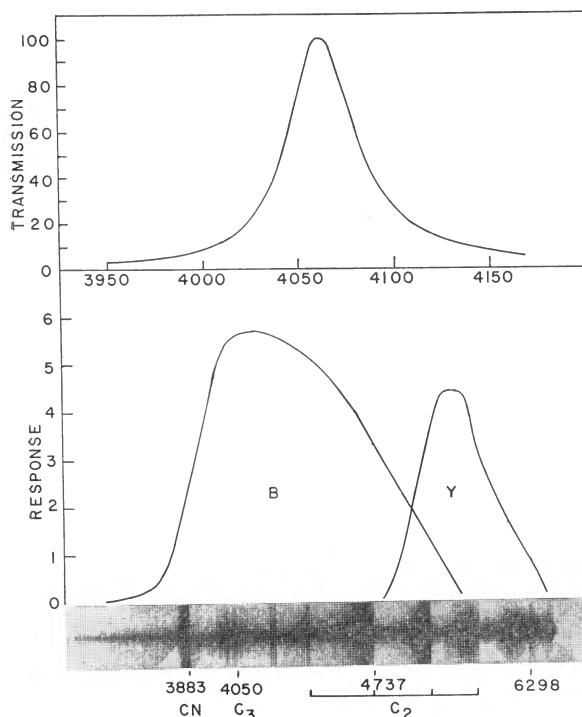


Figure 1. Top, the transmission of the interference filter relative to the peak value. Bottom, the relative response of a photometer for the blue and yellow compared with a spectrum of Comet Cunningham.

Two types of observations were obtained. First, the comet was centered in the diaphragm visually and the magnitude measured in each color. Second, the telescope was displaced in right ascension or declination and then traced over the comet in right ascension or declination to give an intensity profile of the comet in two directions. The rate of motion was $5'.92/\text{min}$ in declination and $6'.27 \cos \delta/\text{min}$ in hour angle. With the help of observations of comparison stars, the appropriate corrections for extinction were made.

The effect of extinction is large at the zenith distances for which the comet was observed and considerable uncertainty remains in the derived magnitudes. For broad band filters the correct extinction coefficient must remain quite uncertain. The distribution of intensity of the light from the comet, when it is near the sun, does not resemble that of the stars usually employed for determining the extinction coefficients. Figure 2 shows the spectral response for yellow and blue filters as given by Johnson and Morgan (1951) compared with a spectrum of Comet Cunningham as reproduced in the Atlas of Representative

Cometary Spectra. The effective wavelength for the blue magnitudes will be determined by the relative intensities of the CN , C_3 and the C_2 bands. For the yellow magnitudes the C_2 bands are mainly responsible for determining the intensity distribution. The observed colors of the comet are no simple indicator of the appropriate extinction coefficients to employ, although the presence of reflected solar light will reduce the discrepancy. An approximate indication of the effective wavelength may be obtained by using estimated intensities of the spectral features given in the Atlas of Representative Cometary Spectra with the response of the photometer to light of different wavelengths.

Of course these values will vary with the comet's distance from the sun and with the part of the comet being observed. This emphasizes the importance of using narrow pass band filters for the accurate photometry of comets. Because of these considerations, which introduce systematic errors in the magnitudes varying with

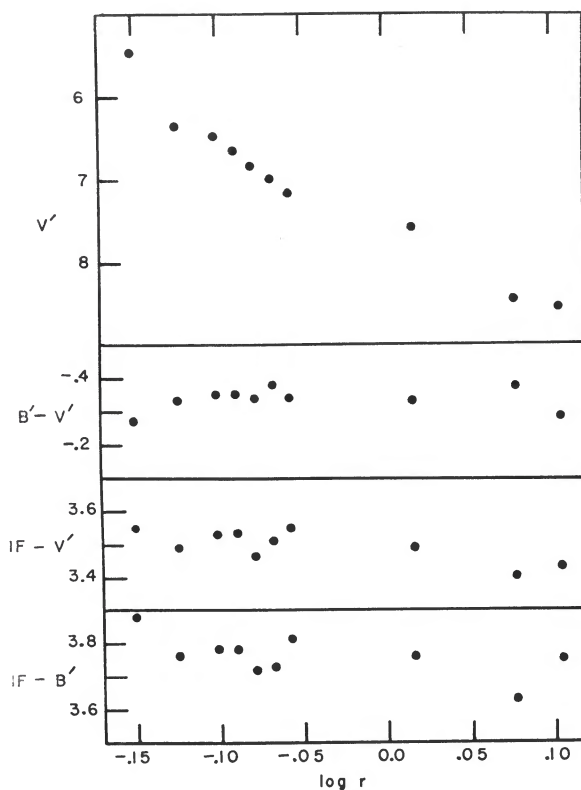


Figure 2. The colors and magnitudes of the center of the coma as a function of the distance from the sun. The observations refer to a fixed angular size, so that the area on the comet varies with the date.

the distance of the comet from the sun, and because of the limited accuracy of the data, the extinction coefficients appropriate for AO stars were adopted.

The observed magnitudes on the instrumental system for the blue, yellow and interference filter will be designated by B' , V' , and IF ; the zero point has been taken to give approximate agreement between the V and V' . For stars the following relations between the B' , V' , and IF and the B , V system of Johnson and Morgan were derived:

$$V = V' + .04(B' - V') + .03$$

$$B = B' + .32(B' - V') + 1.12$$

$$B - V = 1.28(B' - V') + 1.09$$

approximately:

$$IF - V' = 1.47(B - V) + 2.40$$

$$IF - B' = .69(B - V) + 3.25.$$

However, the transformation is a function not only of the instrumental systems but of the intensity distribution of the object being observed as well. Since the intensity distribution for the comet differs so markedly from that of the stars used in obtaining the above relations, no attempt was made to transfer the observed comet magnitudes and colors to the standard B , V system.

The magnitudes and colors observed for the comet are shown in Table I. Because of the nature of the observations no correction for the variation in distance between the earth and the comet has been applied. In general the change in size of the diaphragm as projected on the comet will tend to compensate for the change in distance; however since the intensity of the comet falls off with the distance from the center, the magnitudes obtained when the comet is more

distant will be fainter than if a constant area on the comet had been observed and corrected for the distance. Nevertheless, a plot of the observed magnitudes versus the logarithm of the distance from the sun, Figure 2, shows generally the same appearance as does that of Thiessen (1957). The values can be fitted by an approximately straight line with the value for May 1 falling somewhat high. Although this is the least reliable value here, it may be noted that Thiessen found a similar deviation at this time, which he indicated was related to solar activity.

The $B' - V'$ colors show little indication of variation over the interval covered by the observations. This is in general agreement with the result of Thiessen. The decrease in the color index observed by him is mainly determined by one point at a smaller value of r than covered by the present observations. The $IF - V'$ colors show a decrease with increasing r . This is an indication that the C_3 bands decreased in intensity less rapidly than did the C_2 bands, which are the principal contributors to the V' magnitudes. Of course, since the observations at different dates refer to different areas on the comet, the distribution of intensity over the comet should be considered; this will be discussed below.

Tracings through the head were obtained every night the comet was observed; however only four nights will be discussed here. These tracings are affected not only by the limited resolution but also by the time constant of the photometer. The response of the Brown recorder is sufficiently fast that its contribution to the shape of the tracing may be neglected. The response of the entire system was checked by observing stars and by flashing an artificial light source on the photomultiplier. A time constant of 0.8 second was determined, in agreement with the value expected from the resistance and capacitance at the input to the amplifier. With the observed

TABLE I. MAGNITUDES AND COLORS

Date	Δ	r	V'	$B' - V'$	$IF - B'$	Equivalent diaphragm diameter in 10^4 km
May 1.11	.742	.707	5.46	-.33	3.88	3.81
3.10	.798	.750	6.36	-.27	3.76	4.10
5.10	.856	.792	6.48	-.25	3.78	4.39
6.11	.886	.813	6.65	-.25	3.78	4.55
7.12	.917	.834	6.83	-.26	3.72	4.71
8.12	.947	.855	6.99	-.22	3.73	4.86
9.12	.977	.876	7.15	-.26	3.81	5.01
17.12	1.223	1.038	7.58	-.27	3.76	6.28
25.13	1.463	1.195	8.44	-.23	3.63	7.51
29.14	1.579	1.270	8.55	-.32	3.75	8.10

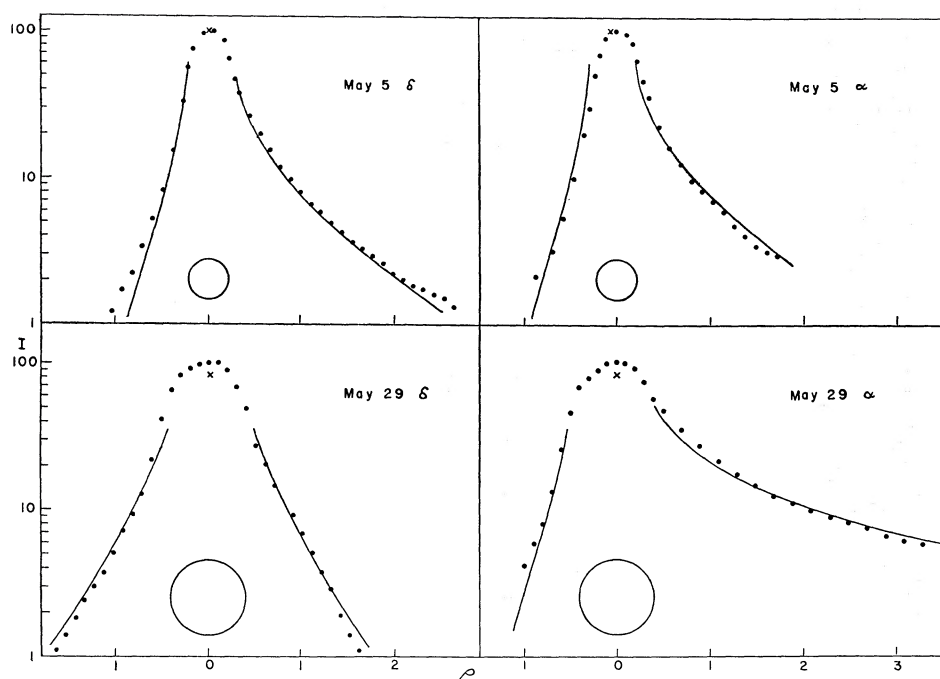


Figure 3. Intensity profiles through the nucleus of the comet; the abscissa is in units of 100,000 km. The large circles give the size of the diaphragm as projected on the comet. The dots are observed values corrected for the time constant of the photometer. The solid curves and the crosses represent the profiles predicted by the fountain model.

shapes of the curves the corrections for this effect amount to only a few percent of the observed values over most of the curves. Corrections were applied by numerically determining the rate of change of the deflection at close intervals.

Near the nucleus of the comet, the intensity changes rapidly over small distances so that the secondary spectrum and the large size of the diaphragm make it difficult to obtain the true intensity profile of the comet. However, in the outer regions this problem is much less serious. Figures 3 and 4 show several observed intensity profiles. In most cases, the variation of intensity with distance can be approximately represented on a logarithmic scale by a straight line in the outer regions of the comet, corresponding to an exponential decrease of intensity with distance.

Attempts were made to reproduce the observed intensity distributions with the fountain model employing a dispersion in the ejection velocities of the particles emitted by the nucleus. A discussion of phenomenological models of comets is given by Wallace and Miller (1958) and the relevant formulae are taken from their article. The intensity distribution was computed numer-

ically from the formula

$$N = \frac{\pi}{\rho} \int_{g_1}^{g_2} \frac{n(g)}{g} dg,$$

where $g_1 = \sqrt{a\rho} \sqrt{1 - \cos \theta}$, ρ and θ are polar coordinates in the plane of the sky with origin at the nucleus and $\theta = 0$ in the anti-solar direction, a is the acceleration in the anti-solar direction, $n(g)$ is the number of particles emitted with velocity g . Initially $n(g)$ was taken to be a Gaussian distribution. The value of the integral was determined numerically for several different values of the parameters, with g_2 taken at a convenient value for which no further significant contributions to the integral were to be expected.

In the region $45^\circ < \theta < 315^\circ$ and away from the nucleus the intensity can be represented approximately as decreasing exponentially with the distance ρ . This is in agreement with the observational results. When $1/\rho$ is small the value of $D = \frac{d \log N}{d \rho}$ is given by the derivative of the integral. Then, since the distance at which g_1 is attained is given by $\rho_1 = g_1^2/a(1 - \cos \theta)$ the ratio of the derivatives for two values of θ

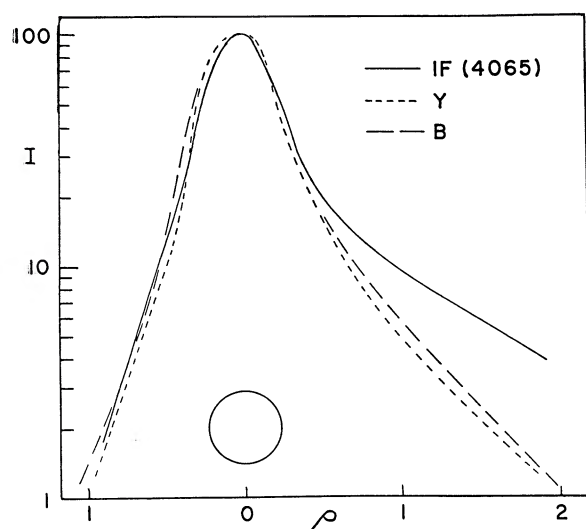


Figure 4. Intensity tracings in declination through the nucleus in three colors on May 7.

will be given by $(1 - \cos \theta_1)/(1 - \cos \theta_2)$ or for two values of θ differing by 180° as in the observations by $(1 - \cos \theta)/(1 + \cos \theta)$ for a single tracing in right ascension or declination. The observed slopes may easily be corrected for the effect of the diaphragm size if it is assumed that within the diaphragm the intensity may be represented by an exponential decrease in one dimension and is constant at right angles to that direction. When the observed and predicted ratios of slopes are compared, the agreement is poor, the observations showing more nearly equal slopes than predicted.

A simpler comparison is possible if $n(g)$ is taken as the Maxwell-Boltzmann distribution:

$$n(g) = Ag^2 e^{-\lambda g^2}.$$

Then the value of N may be computed from the integral in a closed form as

$$N = \frac{B}{\rho} e^{-\lambda a \rho (1 - \cos \theta)}.$$

While the expression for the derivative D will show the same ratio as that derived earlier, when $1/\rho$ is negligible, this formula has the advantage of simplicity in fitting the observations when $1/\rho$ is not negligible, as is the case here, and employs a physically plausible velocity distribution. When the line of sight is not perpendicular to the line joining the sun and the comet, the expressions

for g_1 must be modified. This can be accomplished by projecting onto the plane of the sky the paraboloidal envelope within which lie all trajectories for velocities of amount g_1 or less. The expressions for g_1 and N become

$$g_1 = \sqrt{a\rho \sin \varphi} \sqrt{1 - \cos \theta}$$

$$N = B \frac{1}{\rho} e^{-g_1^2},$$

where φ is the angle between the line of sight and the line joining the sun and comet.

This expression has been fitted to the observations for two nights in the blue and the resulting agreement is shown in Figure 3. The effect of integrating the light of the comet over the size of the diaphragm was introduced into the theoretical curves by numerically integrating the intensity N over the size of the diaphragm. It was assumed for this purpose that within the area covered by the diaphragm, the intensity varied only with x , the rectangular coordinate coinciding in direction with ρ at the center of the diaphragm. This assumption is poor when near the nucleus so the theoretical curves were not carried all the way in. With the diaphragm centered on the nucleus the predicted intensity can be readily obtained by expanding the exponential and integrating over the area in polar coordinates. These values, which lie on the theoretical curves, are shown by crosses in Figure 3. The infinite intensity predicted by the expression for N when $\rho = 0$, occurs because the size of the nucleus has been neglected. This does not introduce any difficulty since the discontinuity is removed on integrating over the area. The parameters of the curves are given in Table II.

TABLE II. PARAMETERS OF THE FOUNTAIN MODEL

Date	$\lambda a \sin \varphi \times 10^5 \text{ km}^{-1}$			Observed θ			Computed	
	V'	B'	IF	V'	B'	IF	θ	φ
May 5.1	2.7	1.8	1.0	49	47		38	75
7.1	2.0	2.1	1.6	55	23:		41	72
25.1	3.2	2.2		75	73		63	43
29.1	2.1	1.4		80	90		64	39

The values of θ used deviate somewhat from those predicted by the relative positions of the sun, comet and earth because of the actual procedure used in obtaining the parameters. The values used deviate in the same sense and by about the same amount as was found from photographic observations by Wenske (1958). Further