SOME THEORETICAL ASPECTS OF H AND K EMISSION IN LATE-TYPE STARS

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ABSTRACT

A velocity is defined in the subphotospheric convection zones of late-type stars that varies from star to star in the same way as the width of the emission lines H and K of Ca II The quantity is tentatively identified as the velocity of hydromagnetic waves.

I. INTRODUCTION

The aim of the present paper is to discuss certain theoretical implications of the remarkable correlation that exists between the line widths of the H and K emission in late-type stars and the absolute visual magnitudes of these stars (Wilson 1954; Wilson and Bappu 1957).

It is probably significant that all stars in which this correlation has been found possess deep convective envelopes. Below a very shallow radiative zone, extending downward from the photosphere, convection plays a decisive part in the transport of energy, until inner regions at temperatures upward of 10^6 degrees K (or even 10^7 degrees K in some cases) are reached.

Write ρ_p and T_p for the photospheric density and the effective temperature, respectively, and write ρ_a and T_a for the density and temperature at the depth at which convection first takes over the main energy transport. This may be suitably defined as the depth at which the convective transport rises to 50 per cent of the total energy flux. In giant stars of normal chemical composition (for a detailed specification of "normal composition" see Suess and Urey 1956), Hoyle and Schwarzschild (1955) found that

$$T_a \cong 10^4 \text{ degrees K} , \tag{1}$$

$$\rho_a T_a \cong 3\, \rho_p T_p . \tag{2}$$

These results were based on certain analytic approximations. Detailed numerical integrations avoiding these approximations have recently been made by one of the present authors. The integrations confirm the substantial accuracy of formulae (1) and (2). It turns out, moreover, that these formulae can be employed satisfactorily not only in giant stars but also in dwarfs of spectral class later than about G2, provided that the dwarfs also have normal chemical composition.

There seem to be two possible modes of attack on the problem of the cause of the correlation described above. One possibility is to examine in detail the conditions under which solar H and K emission takes place. Since the sun can be observed very precisely, there would seem to be a reasonable hope of arriving at a solution of the problem for this restricted case. Then, since the sun is a member of the family of stars that satisfy the correlation, a solution of the restricted solar case might be expected to suggest the solution of the general problem for all late-type stars (Goldberg 1957).

Alternatively, a broader, more empirical approach can be followed. The observed widths of the H and K lines can be regarded as defining a velocity $\frac{1}{2}W_0$ (in the notation of Wilson and Bappu). We can now search for a theoretical velocity, V, with the following properties: (i) V is given by an explicit formula in terms of quantities that can be determined from observation, and (ii) the ratio $2V/W_0$ is the same for all late-type stars. It would then seem a reasonable presumption that the velocity V has importance in the problem, and an attempt can be made to elucidate its significance.

In the following sections a quantity V satisfying both these requirements will be obtained. At a later stage we shall give a tentative discussion of the physical significance of our prescription for determining V.

To end the present introduction, we define a quantity L_K in accordance with the equation

$$\tfrac{1}{2}W_0 = 17\left[\frac{L_K}{(L_{\text{vis}})_\odot}\right]^{1/6} \text{km sec}^{-1}, \tag{3}$$

where $(L_{\text{vis}})_\odot$ is the visual luminosity of the sun. Since W_0 is determined observationally for any particular star and $(L_{\text{vis}})_\odot$ is known, L_K is numerically determined by equation (3). The correlation of Wilson and Bappu is expressed by

$$L_K = L_{\text{vis}}, \tag{4}$$

where L_{vis} represents the visual luminosity of the star in question. In general, condition (4) is satisfied to within a margin of about ± 0.5 mag., as is shown by the results given in a later section.

II. THE DEFINITION OF V FOR SUBGIANTS, GIANTS, AND SUPERGIANTS

The quantity V is to be defined by

$$\tfrac{1}{2}\rho_a V^2 = \text{Mean convective energy per unit volume at the depth at which the density is } \rho_a \text{ and the temperature is } T_a. \tag{5}$$

The right-hand side of this equation can be estimated from a result of Hoyle and Schwarzschild (1955), viz., that the convection currents move with a velocity comparable with the velocity of sound. Thus, for the convective transport to rise to 50 per cent of the total energy flux, we have

$$\left(\tfrac{10}{3}\,\Re\, T_a\right)^{1/2} (\text{mean convective energy per unit volume}) = \tfrac{1}{2}\,\pi a c\, T_p^4, \tag{6}$$

where $\Re$, a, and c are the gas constant, Stefan's constant, and the velocity of light, respectively. Eliminating the mean convective energy per unit volume at the depth denoted by the suffix a (the depth at which convection takes over the main transport of energy) and also removing ρ_a with the aid of equation (2), we obtain

$$V = \left[\frac{\pi a c T_p^3 T_a}{3\rho_p (10\Re T_a/3)^{1/2}}\right]^{1/2}. \tag{7}$$

If, further, we use the value $T_a = 10^4$ degrees K, found by Hoyle and Schwarzschild, then V can be calculated if ρ_p and T_p are specified.

The next step is to obtain a suitable formula for ρ_p. This will now be done for stars of sufficiently low T_p for ionization of hydrogen at the photosphere to be negligible. That is to say, in what follows we shall take the free electrons at the photosphere as being derived from metals. We also take the photospheric opacity, κ, per gram as arising from continuous absorption by the negative hydrogen ion, viz.,

$$\kappa \cong \frac{XP_e}{T_p^{5/2}} \exp\left[16.65 + \frac{8700}{T_p}\right], \tag{8}$$

to a sufficient approximation, where P_e is the free-electron pressure in dynes per square centimeter, given by

$$P_e = \mathfrak{R}\,\rho_p T_p \sum_i c_i x_i\,, \tag{9}$$

ρ_p being in grams per cubic centimeter, T_p in ° K, and the other symbols being defined as follows: X is the fraction by mass in the photospheric material that is hydrogen; i is an index that is summed over the metals; c_i is the number of atoms of metal i per gram of photospheric material; and x_i is the fraction of metal i that is singly ionized.

The photospheric density (Hoyle and Schwarzschild 1955) is determined by

$$\mathfrak{R}\,\kappa\,\rho_p T_p R^2 G^{-1} M^{-1} = 1\ , \tag{10}$$

where G is the gravitational constant and M and R are the total mass and photospheric radius of the star, respectively. Writing

$$c = \sum_i c_i \ \text{ and } \ \Theta = c^{-1} \sum_i c_i x_i\,, \tag{11}$$

equations (8)–(11) can be solved to give

$$\rho_p^2 = X^{-1}\mathfrak{R}^{-2} G \exp(-16.65)\left[\frac{M T_p^{1/2}}{c\Theta R^2} \exp\left(-\frac{8700}{T_p}\right)\right]. \tag{12}$$

Provided that X is taken the same from star to star, ρ_p therefore varies according to the quantity in brackets on the right-hand side of equation (12), so that V, given by equation (7), varies according to

$$\left[c^{0.5}\Theta^{0.5} R M^{-0.5} T_p^{2.75} \exp\left(\frac{4350}{T_p}\right)\right]^{1/2}. \tag{13}$$

Then, introducing the bolometric luminosity with the aid of

$$L_{\text{Bol}} = \pi a\, c R^2 T_p^4\ , \tag{14}$$

the factor (13) can be written in the form

$$\left[L_{\text{Bol}}^{1.5} c^{1.5} \Theta^{1.5} M^{-1.5} T_p^{2.25} \exp\left(\frac{13050}{T_p}\right)\right]^{1/6}, \tag{15}$$

apart from a constant factor. This latter expression describes the way that V varies from star to star. To avoid rewriting the cumbersome terms that appear in (15), define L_T by

$$L_T = T_p^{2.25} \left(\frac{L_{\text{Bol}}\, c\Theta}{M}\right)^{1.5} \exp\left(\frac{13050}{T_p}\right). \tag{16}$$

Then V varies from star to star according to $L_T^{1/6}$. A comparison with equation (3) now shows that the ratio $2V/W_0$ will be a constant from star to star, provided that L_T/L_K is constant.

It is convenient to work in terms of a "normalized" quantity, Q, given by

$$Q = \frac{L_T}{L_K}\left(\frac{L_K}{L_T}\right)_{\text{standard star}}. \tag{17}$$

Evidently, L_T/L_K is constant from star to star, provided that Q is constant. The choice of the standard star to be used in equation (17) would be quite arbitrary if very accurate data were available for all stars. This is not the case, however, so that some caution must be exercised if Q is not to be systematically distorted by an error in the estimation of L_T/L_K for the standard star. In what follows, the star HD 27697 of the Hyades, with $M_{\text{vis}} = +0.66$ and of spectral class K0 III, will be used as a standard. The value of M_{vis} is particularly well determined for this star (δ Tau), as also is the quantity W_0, used in equation (3) to determine L_K. A total of 9 spectrograms gave $\log_{10} W_0 = 1.820$, with W_0 in kilometers per second.

A magnitude, M_K, may be defined by

$$M_K = 0.66 - 2.5 \log_{10} \frac{L_K}{(L_{\text{vis}})_{\text{standard}}}, \tag{18}$$

L_K being determined by equation (3). The values given by equation (18) agree with the M_K values of Wilson and Bappu.

The present correlation between V and $\frac{1}{2}W_0$ requires Q to be close to unity for all stars. The error in the correlation can be expressed on a magnitude scale by working out the quantity $2.5 \log_{10} Q$, which should scatter around zero. For comparison with this, it may be noted that the error in the correlation of Wilson and Bappu (when also expressed on a magnitude scale) is given by $M_K - M_{\text{vis}}$.

The next step is to describe how L_T is to be evaluated. An examination of equation (16) shows that a numerical computation of L_T requires a knowledge of T_p, L_{Bol}, M, c, and Θ. We shall now consider how each of these quantities may be determined.

(i) *The Effective Surface Temperature* T_p

We shall confine our numerical estimates to stars for which Yerkes spectral types are available. The corresponding values of T_p are read off from the tables of Keenan and Morgan (Hynek 1951).

(ii) *The Bolometric Luminosity* L_{Bol}

Where a suitably large trigonometric parallax is available, L_{vis} is estimated from the parallax. Otherwise, the Yerkes magnitude scale is used to estimate L_{vis} (Keenan and Morgan, *loc. cit.*). The bolometric luminosity is then obtained by using the bolometric corrections given by Kuiper (1938) as a function of T_p.

(iii) *The Mass M*

The determination of M is much simplified by a consideration of the composite color-magnitude diagram (Sandage 1957). It appears that the evolutionary tracks of stars that are initially brighter than about +2.0 move nearly horizontally to the right in the diagram. This means that, for such stars, M is related to L_{vis} in essentially the same way as for stars that lie on or near the main sequence. For the latter stars it is a satisfactory approximation to use the relation

$$\frac{M}{M_\odot} = \left[\frac{L_{\text{vis}}}{(L_{\text{vis}})_\odot}\right]^{1/4}, \tag{19}$$

although equation (21) may somewhat underestimate M for stars of very high luminosity.

Evolving stars that were initially fainter than +2.0 have masses in the range 1.1–2$\odot$. These are stars with evolutionary tracks that rise rather steeply upward as they move to the right in the color-magnitude diagram. The subgiants and the giants of M67 are examples of this class of star. As a working hypothesis, we shall use an average mass of 1.5$\odot$ for these stars, and in our later table of results we attach an asterisk whenever M has been chosen this way. When there is no asterisk attached to our results this implies that M was estimated from equation (19).

(iv) *The Quantity c*

With an exception to be mentioned later, we shall suppose that the concentrations, c_i, are the same in all stars. Then the quantity $c^{3/2}$ cancels in the ratio $L_T/(L_T)_{\text{standard}}$.

At first sight it might seem as if this hypothesis must militate against our obtaining satisfactory numerical estimates, since the c_i's for various i's are known to vary quite appreciably between halo stars, high-velocity stars, and population I stars in the galactic disk. It must be remembered, however, that these variations concern stars with widely different kinematic behavior, whereas the stars for which H and K line data are available all belong to the solar neighborhood. For the most part, these stars may well form a rather homogeneous group so far as composition is concerned. Nevertheless, exceptions are to be expected, particularly for very old stars. Such stars can perhaps be identified by their positions in the color-magnitude diagram, where they may be expected to lie near the evolutionary sequence of M67. Where it has been thought that such stars are involved, the value of c has been reduced by a factor of 2.5 (Schwarzschild, Spitzer, and Wildt 1951), and the corresponding results in our table have been followed by a dagger (†).

(v) *The Quantity* Θ

For late-type stars in which the photospheric free electrons are derived from metals, only six elements need detailed consideration, namely, silicon, magnesium, iron, aluminium, sodium, and potassium.

Apart from a reduction of the c_i by a factor 2.5 in the exceptional cases referred to at the end of section iv, the concentrations have been taken from the relative abundances given by Suess and Urey (1956). According to these authors, the concentrations of the six metals relative to hydrogen are

$$\begin{gathered} \text{Si} = 2.5 \times 10^{-5}\,, \ \text{Mg} = 2.3 \times 10^{-5}\,, \ \text{Fe} = 1.5 \times 10^{-5}\,, \\ \text{Al} = 2.4 \times 10^{-6}\,, \ \text{Na} = 1.1 \times 10^{-6}\,, \ \text{K} = 7.7 \times 10^{-8}\,. \end{gathered} \tag{20}$$

Since the number of hydrogen atoms is $6 \times 10^{23}X$ per gram of photospheric material, it follows that the appropriate values of c_i for the metals are given immediately on multiplying (20) by $6 \times 10^{23}X$.

Now write A_i for abundances taken relative to silicon, the appropriate values of A_i being

$$\begin{gathered} 1 \text{ for Si}\,, \ 0.91 \text{ for Mg}\,, \ \ 0.6 \text{ for Fe}\,, \\ 0.095 \text{ for Al}\,, \ 0.044 \text{ for Na}\,, \ 0.0031 \text{ for K}\,. \end{gathered} \tag{21}$$

The quantity Θ is given by

$$\Theta = \frac{\sum_i c_i x_i}{\sum_i c_i} = \frac{\sum_i A_i x_i}{\sum_i A_i}\,, \tag{22}$$

and the values of the x_i's are calculated from the well-known ionization equations,

$$\log_{10} \frac{x_i}{1 - x_i} = -\frac{5040}{T_p} I_i + \tfrac{5}{2} \log_{10} T_p + w_i - \log_{10} P_e\,, \tag{23}$$

where I_i is the ionization potential of element i measured in electron-volts and w_i is the appropriate statistical weight factor. Equations (9), (12), (22), and (23), together with the above values of c_i and A_i, yield Θ when T_p is specified.

A satisfactory degree of accuracy can be obtained with a somewhat less involved procedure, however. It is known from a study of stellar models that $\log_{10} \rho_p \cong -7.5$ for subgiants of luminosity class IV; that $\log_{10} \rho_p \cong -8.0$ for giants of luminosity class III; that $\log_{10} \rho_p \cong -8.25$ for giants of luminosity class II; and that $\log_{10} \rho_p \cong -8.5$ for supergiants of luminosity class I*b* (except for the last of these, which depends on subsequent integrations, cf. Hoyle and Schwarzschild 1955). If these values of ρ_p are used in place of equation (12), the calculation of Θ is much simplified without substantial loss of accuracy. Values of Θ, together with other information, are given in Table 1 for the various luminosity classes. The visual magnitudes are on the Yerkes scale, and the bolometric corrections (B.C.) are in magnitudes.

III THE DEFINITION OF V FOR LATE-TYPE DWARFS

We shall use essentially the same method of defining V as was used above. A difference in detail arises, however, because there is an important difference in the nature of the subphotospheric convection. In giants the energy flux is not necessarily carried by convection simply because the material is in convective motion. An additional requirement is that the heat capacity of the material be adequate to bear the energy flow. This condition is represented by equation (6). In order to promote the maximum rate of convective transport, the moving material attains a speed comparable with that of sound This is expressed in equation (6) by the factor $(10\mathfrak{R}T_a/3)^{1/2}$.

The situation is different in late-type dwarfs. Here the heat capacity of the material is far greater because of much higher values of ρ_p (-6.4 may be used as a general average lor $\log_{10} \rho_p$ in dwarfs, as compared with values ranging from -7.5 to $-8\ 5$ for giants and supergiants). With the larger heat capacity, the convectively unstable material carries the energy flux while moving at subsonic speeds. Hence it is clear that the factor $(10\mathfrak{R}T_a/3)^{1/2}$ cannot be retained on the left-hand side of equation (6). We shall replace this factor by the quantity V itself, writing

$$V \text{ (mean convective energy per unit volume)} = \tfrac{1}{2}\, \pi ac\, T_p^4\,, \tag{24}$$

in place of equation (6). Instead of equation (7) we now obtain

$$V = \left(\frac{\pi\, a\, cT_a T_p^3}{3\,\rho_p}\right)^{1/3}. \tag{25}$$

The discussion proceeds exactly as before, leading to the requirement that

$$Q = \frac{L_T}{L_K}\left(\frac{L_K}{L_T}\right)_{\text{standard star}}$$

be constant from star to star, where L_T is now defined by

$$L_T = c\Theta L_{\text{Bol}} M^{-1} T_p^{1\ 5} \exp\left(\frac{8700}{T_p}\right). \tag{26}$$

The quantities T_p, L_{Bol}, and c_i are obtained in exactly the same way as before, M is obtained from equation (19), and $-6\ 4$ is used for $\log_{10} \rho_p$ in the computation of the values of Θ in Table 2.

A point remains, concerning the choice of standard star. It would be possible to retain the same standard as for the giants, namely, HD 27697. But this would have the inconvenience that L_T and $(L_T)_{\text{standard}}$ would have to be computed from different formulae (16) and (26) and that various quantities that cancel in the ratio $L_T/(L_T)_{\text{standard}}$, when the same formula is used, would no longer cancel. This suggests that a different standard be chosen for the dwarfs, one that allows equation (26) to be used for both L_T and

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TABLE 1

NUMERICAL DATA FOR LUMINOSITY CLASSES I*b*, II, III, AND IV

Spectral Class	T_p	B C	M_{vis}	Θ
		Luminosity Class IV		
G0	5750	−0 16	+3 2	1 00
G2.	5350	− 22	+3 3	0 98
G5	5080	− 29	+3 4	0 96
G8	4870	− 34	+3 4	0 91
K0	4650	− 39	+3 4	0 77
K1.	4450	− 45	+3 5	0 64
K2 ..	4280	−0 51	+3 5	0 53
		Luminosity Class III		
G0....	5300	−0 23	+0 7	1 00
G2. .	4990	−0 30	+ 4	0 98
G5...	4650	−0 39	+ 4	0 91
G8 ..	4440	−0 45	+ 2	0 81
K0...	4200	−0 54	+ 2	0 64
K1..	4000	−0 77	+ 1	0 51
K2..	3810	−1 0	+ 0	0 38
K3...	3660	−1 2	− 1	0 31
K5..	3550	−1 4	− 3	0 24
M0.	3340	−1 7	− 4	0 16
M1.	3200	−1 9	− 4	0 090
M2..	3090	−2 2	− 4	0 075
M3..	2980	−2 5	−0 5	0 065
		Luminosity Class II		
G0.... ...	5150	−0 26	−2 0	1 00
G2..	4770	−0 36	−2 0	0 97
G5..	4470	−0 43	−2 0	0 86
G8..	4220	−0 54	−2 1	0 68
K0	4010	−0 77	−2 1	0 48
K1..	3850	−0 93	−2 2	0 37
K2..	3700	−1 1	−2 2	0 27
K3..	3540	−1 4	−2 3	0 19
K5	3430	−1 6	−2 4	0 16
M0.	3270	−1 8	−2 4	0 11
M1.	3150	−2 1	−2 4	0 085
M2.	3070	−2 3	−2 4	0 075
		Luminosity Class I*b*		
G0...	5000	−0 30	−4 5	1 00
G2...	4600	−0 40	−4 5	0 97
G5...	4290	−0 50	−4 5	0 85
G8...	4000	−0 77	−4 5	0 59
K0...	3820	−1 0	−4 5	0 43
K1...	3700	−1 1	−4 5	0 34
K2...	3590	−1 4	−4 5	0 27
K3...	3430	−1 6	−4 5	0 19
K5 .	3320	−1 7	−4 5	0 15
M0..	3210	−1 9	−4 5	0 12
M1..	3100	−2 2	−4 5	0 093
M2..	3050	−2 3	−4.5	0 081

$(L_T)_{\text{standard}}$. To this end, we standardize with respect to the brighter component of 70 Oph (HD 165341). The visual magnitude of this star, as determined by the large trigonometric parallax of 0.188″, is +5.4.

IV. RESULTS

Where the visual magnitudes in Table 3 are obtained from trigonometric parallaxes, the letter "P" is used. Where the visual magnitudes are taken from the Yerkes scale the letter "Y" is attached. For the meaning of the asterisks and daggers attached to the results in the 2.5 $\log_{10} Q$ column, see sections iii and iv of Section II.

V. DISCUSSION

The mean error in the present correlation is 0.59 mag. This corresponds to an average error of no more than 9 per cent in the estimation of the H and K line width. For comparison, the mean error in the correlation of Wilson and Bappu is 0.62 mag., corresponding to an average discrepancy of 10 per cent in the line width. Hence the present correlation between V and $\frac{1}{2}W_0$ has an accuracy closely comparable with that of the correlation of Wilson and Bappu. Two considerations may be mentioned that, if taken into account, would increase the strength of the present correlation still further.

TABLE 2

NUMERICAL DATA FOR LUMINOSITY CLASS V

Spectral Class	T_p	B C	M_{vis}	Θ
G2	5730	−0 07	+4 7	1 00
G5	5520	− 08	+5.1	0 88
G8	5320	− 09	+5 6	0 79
K0	5120	− 10	+6 0	0 66
K1	4920	− 11	+6 2	0 54
K2	4760	− 12	+6 4	0 42
K3	4610	− 17	+6 9	0 31
K5	4400	− 30	+7 8	0 23
K6	4000	−0 72	+8 5	0 12

1. Equation (21) for M undoubtedly underestimates the masses of the supergiants. The use of a too small value of M for these stars causes L_T to be too large, thereby making log Q systematically positive for these stars, as, indeed, the entries in the table show it to be (i.e., the *average* value of log Q is pronouncedly positive).
2. The use of the Yerkes magnitudes at luminosity classes I and II introduces quite large errors, which are obvious in many cases, e.g., for HD 27022, HD 173764, and HD 223719. It is interesting to note that in such cases the present calculations and the measures of Wilson and Bappu show almost identical divergencies from the line-width data. The implication is that much of the error in our estimation of the line widths has been artificially introduced by the use of incorrect visual magnitudes.

It probably will be wondered how such an apparently unpromising formula as (16) comes to give such satisfactory results. The general reason for this will now be explained. Write

$$L_{\text{Bol}} = \theta L_{\text{vis}} , \tag{27}$$

so that $-2.5 \log_{10} \theta$ is the bolometric correction, and write equation (16) in the form

$$L_T = L_{\text{vis}} (\theta\Theta)^{3/2} c^{3/2} (L_{\text{vis}} M^{-3})^{1/2} \left(T_p^{2\ 25} \exp \frac{13050}{T_p}\right). \tag{28}$$

TABLE 3

RESULTS OF COMPUTATION

Star (HD No)	Spectral Class	M_K	M_{vis}	M_{Bol}	M_K-M_{vis}	$2 5 \log_{10} Q$
1013	M2 III	-0 5	-0 4Y	-2 6	-0 1	-0 4
3627	K3 III	+0 1	-0 1Y	-1 3	+0 2	+0 4
3651	K IV	+5 7	+5 8P	+5 7	-0 1	-0 3
4128	K0 III	+0 9	+0 8P	+0 3	+0 1	+0 2
4628	K4 V	+7 3	+6 5P	+6 3	+0 8	+0 0
6186	K0 III	+1 2	+1 5P	+1 0	-0 3	-0 3
6860	M0 III	-0 9	-0 4Y	-2 1	-0 5	-0 5
9270	G8 III	-0 2	+0 4Y	0 0	-0 6	-0 3
9927	K3 III	+0 4	-0 1Y	-1 3	+0 5	+0 6
10476	K IV	+6 3	+5 8P	+5 7	+0 5	+0 2
12929	K2 III	+1 3	+0 2P	-0 8	+1 1	+0 8*†
16160	K4 V	+6 8	+6 6P	+6 3	+0 2	-0 6
17506	K3 Ib	-4 7	-4 5Y	-6 1	-0 2	+0 4
17925	K0 V	+5 4	+6 4P	+6 3	-1 0	-1 1
18322	K2 III	+1 6	+1 0P	0 0	+0 6	-0 1*†
18884	M2 III	-1 5	-0 4Y	-2 6	-1 1	-1 4
20630	G5V	+4 6	+4 9P	+4 8	-0 3	+0 1
20797	M0 II	-3 6	-2 4Y	-4 2	-1 2	-1 3
21120	G8 III	+0 4	+0 4Y	0 0	0 0	+0 3
22049	K2 V	+6 3	+6 1P	+6 0	+0 2	-0 3
23249	K0 IV	+4 8	+3 7P	+3 3	+1 1	+0 5*
26630	G0 Ib	-4 6	-4 5Y	-4 8	-0 1	+0 8
26965	K0 V	+5 4	+5 9P	+5 8	-0 5	-0 6
27022	G5 III	+1 7	+0 2Y	-0 2	+1 5	+1 8
29139	K5 III	+0 1	-0 08P	-2 2	+0 9	+1 0
31398	K3 II	-1 9	-2 3Y	-3 7	+0 4	+0 2
31767	K2 II	-1 9	-2 2Y	-3 3	+0 3	+0 3
32068	K4 II	-2 1	-2 4Y	-3 9	+0 3	+0 2
36389	M2 Ib	-5 0	-4 5Y	-6 8	-0 5	0 0
39400	K2 II	-2 4	-2 2Y	-3 3	-0 2	-0 2
39801	M2 Ib	-5 7	-4 5Y	-6 8	-1 2	-0 7
40239	M3 II	-3 0	-2 4Y	-4 9	-0 6	-0 3
44478	M3 III	-1 3	-0 4P	-2 9	-0 9	+0 6*
44537	M0 Iab	-5 5	-6:Y	-7 9	+0 5	+1 1
45416	K1 II	-1 8	-2 2Y	-3 1	+0 4	+0 6
47731	G5 Ib	-4 0	-4 5Y	-5 0	+0 5	+1 6
48329	G8 Ib	-5 0	-4 5Y	-5 3	-0 5	+0 4
50877	K3 Ib	-5 2	-4 5Y	-6 1	-0 7	-0 1
67594	G2 Ib	-4 9	-4 5Y	-4 9	-0 4	+0 7
74395	G2 Ib	-4 1	-4 5Y	-4 9	+0 4	+1 5
81797	K3 II-III	-1 0	-1 2Y	-2 3	+0 2	+0 2
82885	G8 V	+4 0	+5 6P	+5 5	-1 6	-1 4
84441	G0 II	-2 2	-2 0Y	-2 3	-0 2	+0 3
86663	M2 III	-0 9	-0 4Y	-2 6	-0 5	-0 8
89484	K1 III	+0 6	+0 3Y	-0 5	+0 3	+0 4
89485	G7 III	+1 7	+1 1Y	+0 7	+0 6	-0 2*†
89758	M0 III	+0 6	+0 7P	-1 0	-0 1	-0 2
92125	G3 II	-3 8	-2 0:Y	-2 4	-1 8	-1 1
94264	K0 III-IV	+2 1	+1 8Y	+1 4	+0 3	+0 4
95689	K0 III	+0 1	+0 2Y	-0 3	-0 1	0 0
98839	G8 II+	-0 2	-1 6Y	-2 2	+1 4	+1 8
100029	M0 III	-0 1	-0 4Y	-2 1	+0 3	+0 3
101501	G8 V	+5 1	+5 5P	+5 4	-0 4	-0 2
102212	M1 III	+0 1	-0 4Y	-2 1	+0 5	0 0
107328	K1 III	+0 6	+0 1Y	-0 7	+0 5	+0 6
111028	K1 III-IV	+3 7	+2 2P	+1 6	+1 5	+0 2*†
112300	M3 III	-0 2	-1 1Y	-3 6	+0 9	+1 0
113226	G8 III	+1 5	+0 4P	0 0	+1 1	+0 6*†
114710	G0 V	+5 1	+4 6P	+4 5	+0 5	+1 1
115043	G1 V	+3 7	+4 6Y	+4 6	-0 9	-0 3
119228	M2 III	-1 0	-0 4Y	-2 6	-0 6	-0 9
124294	K3 III	+0 9	-0 1Y	-1 3	+1 1	+0 8*†
124897	K1 III	+0 9	-0 3P	-1 2	+1 2	+1 1 †
127665	K3 III	-0 4	+0 6P	-1 3	-0 3	-0 1
128902	K4 III	+0 6	-0 2Y	-1 5	+0 8	+0 9
129989	K0 II-III	-0 9	-1 0Y	-2 0	+0 1	+0 3
131156Br	G8 V	+5 9	+5 4P	+5 3	+0 5	+0 7
131156F	K4 V	+7 9	+7 4P	+7 2	+0 5	-0 4
131511	K1 V	+4 5	+5 6P	+5 5	-1 1	-1 3
131873	K4 III	+0 1	-0 2Y	-1 5	+0 3	+0 4
131977	K5 V	+6 4	+6 8P	+6 5	-0 4	-1 4
135722	G 8 III	+1 7	+0 7P	+0 2	+1 0	+0.6*†
137759	K2 III	+1 3	+0 8P	-0 2	+0 5	+0 0*†
138716	K1 III	+3 1	+1 5P	+0 7	+1 6	+0.6*†
143107	K3 III	+0 2	-0 1Y	-1 3	+0 3	+0 1 †
145001	G8 III	-0 2	+0 4Y	0 0	-0 6	-0 3
145148	K0 IV	+4 5	+3 1P	+2.7	+1 4	-0 4*†
145328	K0 III-IV	+2 7	+1 8Y	+1 4	+0 9	-0 2*†
146051	M0 III	-0 4	0 0P	-1 7	-0 4	-0 4
148478	M1 Ib	-4 9	-4 5Y	-6 7	-0 4	+0 1
148856	G8 III	+0 1	+0 4Y	0 0	-0 3	0 0
149661	K0 V	+4 8	+5 5P	+5 4	-0 7	-0 7
153210	K2 III	+0 6	+0 2P	-0 8	+0 4	+0 2*†
155885Br	K1 V	+6 5	+6 4P	+6 3	+0 1	-0 2
155886F	K2 V	+6 4	+6 4P	+6 3	0 0	-0 5
156283	K3 II	-1 2	-2 3Y	-3 7	+1 1	+0 9
157999	K5 II	-2 1	-2 4Y	-4 0	+0 3	+0 3
159181	G2 Ib	-5 5	-4 5Y	-4 9	-1 0	+0 1
160269	G1 V	+3 4	+4 2P	+4 1	-0 8	-0 2
163588	K2 III	+1 3	+1 1P	+0 1	+0 2	+0 2
163770	K1 II	-2 4	-2 2Y	-3 1	-0 2	+0 0
163993	G9 III	+1 3	+0 3Y	-0 2	+1 0	+0 6*†
164341Br	K0 V	+5 4	+5 4P	+5 3	0 0	standard
164341F	K6 V	+6 8	+7 1P	+6 4	-0 3	-1 5
168656	G8 III	+1 7	+0 4Y	0 0	+1 3	+1 1*†
169916	K2 III	+1 7	+1 1P	+0 1	+0 6	-0 2*†
173764	G5 II	-4 4	-2 0Y	-2 4	-2 4	-1 8
180809	K0 II	-2 4	-2 1Y	-2 9	-0 3	-0 1
181391	K0 III	+1 3	+2 4P	+1 9	-1 1	-1 0*
185144	K0 V	+5 9	+5 9P	+5 8	0 0	-0 1
185758	G0 II	-2 0	-2 0Y	-2 3	0 0	+0 5
186791	K3 II	-1 8	-2 3Y	-3 7	+0 5	+0 3
189276	K5 II-III	-0 7	-1 0Y	-2 5	+0 3	+0 3
190406	G1 V	+4 6	+4 7P	+4 6	-0 1	+0 5
191026	K0 IV	+2 8	+2 7P	+2 3	+0 1	-0 1
192713	G2 Ib	-5 5	-4 5Y	-4 9	-1 0	+0 1
192947	G9 III	+1 2	+1 5P	+1 0	-0 3	-0 2
196755	G5 IV	+2 4	+1 7P	+1 4	+0 7	+0 7
197989	K0 III	+1 3	+0 6P	+0 1	+0 7	+0.7
198149	K0 IV	+2 9	+2 6P	+2 2	+0 3	+0 3*
201091Br	K5 V	+6 4	+7 6	+7 3	-1 2	-2 3
201092F	K7 V	+8 2	+8 4P	+7 4	-0 2	-1 6
202109	G8 II	+1 2	-2 1Y	-2 6	+3 3	+3 8
204075	G4 Ib	-3 5	-2 0:Y	-2 4	-1 5	-0 7
204867	G0 Ib	-5 0	-4 5Y	-4 8	-0 5	+0 4
206778	K2 IB	-4 2	-4 5Y	-5 9	+0 3	+1 1
206859	G5 Ib	-3 7	-4 5Y	-5 0	+0 8	+1 9
207089	K1 Ib	-6 0	-4 5Y	-5 6	-1 5	-0 8
208606	G8 Ib	-4 9	-4 5Y	-5 3	-0 4	+0 5
209747	K4 III	-0 4	-0 2Y	-1 5	-0 2	0 0
209750	G2 Ib	-5 5	-4 5Y	-4 9	-1 0	+0 1
210745	K1 Ib	-4 7	-4 5Y	-5 6	-0 2	+0 5
212943	K0 IV	+3 1	+1 4P	+1 0	+1 7	+0 6*†
216386	M2 III	-0 5	-0 4Y	-2 6	-0.1	-0 4
216946	K5 Ib	-4 7	-4 5Y	-6 2	-0 2	+0 2
217906	M2 II-III	-1 6	-1 4Y	-3 7	-0 2	-0 3
218329	M2 III	-0 7	-0 4Y	-2 6	-0 3	-0 6
218356	K0 Ibp	-2 2	-2 2Y	-3 2	0.0	+0 4
219134	K3 V	+6 5	+6 5P	+6 3	0 0	-0 7
219615	G8 III	+1 8	+0 7P	+0 3	+1 3	+0 4*†
220954	K1 III	-0 9	+0 1Y	-0 7	-1 0	-0 9
222404	K1 IV	+2 7	+2 2P	+1 8	+0 5	+0 2
223719	K4 II	-0 4	-2 4Y	-3 8	+2 0	+1 9
Hyades**						
27371	K0 III	+0 6	+0 68	+0 14	-0 08	-0 02
27697	K0 III	+0 6	+0 66	+0 12	-0 06	standard
28305	K0 III	+0 4	+0 54	+0 0	-0 14	-0 06
28307	K0 III	+1 0	+0 80	+0 26	+0 20	+0 25

** For the Hyades stars the results depend upon nine spectrograms of each star They are thus of considerably greater weight than nearly all of the others Moreover, the absolute magnitudes of the Hyades are known accurately by other means than trigonometric or spectroscopic parallaxes

Working from the right-hand end of this formula, (i) $T_p^{2.25} \exp 13050/T_p$ varies only very slowly with T_p, for the values of T_p that are of interest. (ii) $(L_{\text{vis}} M^{-3})^{1/2}$ is also almost constant from star to star. Indeed, if we had taken $L_{\text{vis}} \propto M^3$ for the mass-luminosity relation, this term would have been exactly constant. (iii) With the exception of the few stars marked by a dagger in the tables, c has been taken constant from star to star. (iv) By what can only be described as freak behavior, the product of $\theta\Theta$ turns out to be almost constant over the whole range of spectral types from G0 to M3. Moreover, although $\theta\Theta$ does decrease slightly over this range, the decrease tends to be compensated by a corresponding increase in the factor $T_p^{2.25} \exp 13050/T_p$. The net effect is to give

$$L_T = (\text{almost constant factor})\, L_{\text{vis}}\,, \tag{29}$$

thereby yielding the correlation of Wilson and Bappu.

This explains why the correlation of the present paper bears such a close similarity to that of Wilson and Bappu. We also see how it comes about that the correlation must be with the visual luminosity, not with the bolometric luminosity, for the bolometric correction θ is required to cancel against the ionization factor Θ. This insight into the significance of L_{vis} rather than L_{Bol} provides a strong reason for believing that the correlation established by our definition of V is not accidental.

It is emphasized that the derivation of equation (29) requires c (the total concentration of metals) to be constant from star to star. According to the above investigation, the H and K line widths should not agree with the correlation of Wilson and Bappu if c were variable. In particular, our arguments require the line width to be abnormally small in stars of low metal content. This is a definite prediction of the theory, one that should be capable of a direct observational test.

VI. INTERPRETATION

Even if the significance of our definition of V be granted, a large part—perhaps the larger part—of the problem still remains. It is necessary to explain how the velocity V, defined in the subphotospheric convective material, comes to be translated into the H and K emission line widths, the emission cores of the H and K lines being presumably formed in material that lies appreciably *above* the photosphere.

It is important to realize in this connection that V is not a simple material velocity. Definition (5) includes the whole convective energy per unit volume. In the giants the main contribution to the convective energy comes from hydrogen ionization. Without this contribution, there would be no correlation such as we have obtained.

It is tempting to argue that the rising convective elements explode like bombs, drawing on the ionization energy of the hydrogen as they expand. In this way the quantity V might be related to the velocity of the shock waves generated in such explosions. There is a serious difficulty, however, in explaining how the shock waves travel upward to the photosphere and beyond into the chromosphere, always maintaining the velocity V. Perhaps it might be argued that the velocity of the shock waves increases by some factor, γ say, as the waves travel into the more rarified upper regions and that γ is a constant from star to star. The latter hypothesis scarcely seems very plausible, however, in view of the wide range of density from one star to another. Moreover, there is very little room for any increase in V to occur in this way. So far, we have only considered relative values of V. We now make an absolute determination of V for our standard giant star HD 27697.

The relevant observational data are: spectral class, K0 III; effective temperature, $T_p = 4200°$ K; and $\frac{1}{2}W_0 = 34$ km/sec. Referring back to equations (1) and (7), we can calculate V by using this value of T_p, together with the value $\log_{10} \rho_p = -8.0$ used above for luminosity class III. The result is $V = 32.5$ km/sec, in very satisfactory agreement with $\frac{1}{2}W_0$. The implication is that the ratio $2V/W_0$ is not merely a constant but that

$$2\,V = W_0\,. \tag{30}$$

If this is the case, the subphotospheric V must be transmitted upward without change into regions above the photosphere. Such a requirement would militate against the shock-wave hypothesis.

An alternative suggestion is that a strong magnetic field exists in the subphotospheric convection region, the magnetic energy being in equipartition with the convective energy, viz.,

$$\frac{H^2}{8\pi} = \text{Convective energy per unit volume at the depth at which the density is } \rho_a \text{ and the temperature is } T_a . \tag{31}$$

Equations (5) and (31) give

$$V^2 = \frac{H^2}{4\pi \rho_a}; \tag{32}$$

so that V is just the hydromagnetic wave velocity. Although this interpretation seems attractive, the problem remains to be solved, however, as to the connection of such a hydromagnetic wave velocity with the material motions occurring above the photosphere. A general theorem of the following form would be implied: that a source of hydromagnetic waves can induce material motions of the same speed as thatof the waves at the source, but greater speeds cannot be induced. The investigation of the correctness or otherwise of this conjecture is a matter of such considerable difficulty that it scarcely lies within the scope of the present paper.

VII. THE SUN

It is of interest to apply these latter considerations to the sun. With $W_0 = 34$ km/sec, equations (30) and (32) require the sun to possess a photospheric magnetic field of the order of 10^3 gauss. At first sight, it might appear that this inference is so markedly at variance with observation that the magnetic hypothesis must be discarded. Yet a closer examination of the problem shows that this issue is not so clear-cut.

The surface structure of the sun determined by recent observations (Leighton, private communication; Blackwell, Dewhirst, and Dollfuss, private communication; Schwarzschild, Rogerson, and Evans, private communication) shows that the typical granule diameter is about 750 km, about five times greater than the photospheric scale height. This suggests that the granules are highly flattened and that any magnetic field local to them is likely to be largely horizontal in direction (as required by condition i of Sec. VI). Consequently, such a magnetic field, fluctuating over regions with dimensions of the order of the granule diameter, would not be readily observed by polarization measures with existing equipment. A radial field component of order 10^2 gauss might be expected to be found, if observation could be confined to a single granule.

There is a feature that actually supports the existence of a field of the present order over regions of granular dimensions, namely, the occurrence of the small dark pores that precede the formation of spots. The darkening would seem to require the presence of a strong magnetic field, otherwise the pores would immediately be compressed and heated by the surrounding gas. It is natural to associate a pore with a region where the direction of the magnetic field has become vertical rather than horizontal.

It may be added that a magnetic field of 10^3 gauss produces a Zeeman broadening on a line of unit Landé factor that is approximately equivalent to a turbulent motion of 0.75 km/sec. This degree of broadening is too small to serve as a criterion of the presence or absence of such a field, but the case may be otherwise for lines of larger Landé factor.

VIII. THE FORMATION OF THE H AND K LINES

As has already been indicated, the present paper is seriously incomplete, in that it does not offer a mechanism for the formation of the H and K lines. Our point is that we are content to derive a velocity V that varies from star to star in the same way as the empirical velocity $\frac{1}{2}W_0$ derived from the line widths. The results set out in Table 3 are given relative to a standard star and hence do not by themselves indicate more than $V \propto \frac{1}{2}W_0$. The absolute value of V determined for the standard star, δ Tau, agrees so well with $\frac{1}{2}W_0$, however, as to suggest the stronger result, $V = \frac{1}{2}W_0$. If we accept the latter equality instead of the weaker proportionality, we must suppose that V determines the total width of the H and K lines and hence that these lines are not subject to appreciable radiative broadening. This would imply that the lines are formed in a region of comparatively small optical depth, τ, of order unity.

This point of view is open to observational test. If the lines are formed at τ of order unity, we should expect the total intensity of the K emission to exceed the intensity of H emission, because of the greater f-value of the K line. If, on the other hand, the emission takes place at large optical depth, say $\tau \cong 10^3$, the total intensities of the two lines should be effectively the same. Preliminary results do suggest a difference in intensity, favoring the present point of view, namely, that the lines are formed at an optical depth not much greater than unity.

It is emphasized that if the lines are formed at great optical depth, two different factors contribute to their width—a basic "Doppler width" and the radiative broadening. Conceivably, our V might still be associated with the Doppler width, but this would require the radiative broadening to be proportionately the same in all stars. How this could come about is not clear (for a discussion of this problem, see Wilson 1957).

Note added in proof: Since this paper was written, other suggested explanations of the width-luminosity correlation have appeared (de Jager 1958; Schatzman 1958). A third paper, expressing essentially the same viewpoint as that of the present authors, is in the process of publication (Kraft 1958).

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