

PHOTOMETRIC STUDIES OF ASTEROIDS. VII*

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Received October 29, 1957; revised November 26, 1957

ABSTRACT

Photoelectric light-curves of nine asteroids were derived with the 82-inch reflector of the McDonald Observatory. For three asteroids, light-curves were determined for the first time. The main data are collected in Tables 1 and 2 and in the figures. The sense of rotation of Eunomia being retrograde was confirmed. A possible direct rotation for Laetitia was found. The period of (321), Florentina, a member of the Coronis family, was found to be as short as 2^h52^m .

I. INTRODUCTION

This is the seventh paper of a series of publications about the light-variations of asteroids, initiated by Dr. Kuiper in 1949 and contributed by several authors, I–VI (Groeneveld and Kuiper 1954*a, b*; Shatzel 1954; Ahmad 1954; Gehrels 1956, 1957). The observations described in the present paper were made with the 82-inch reflector in December, 1955, and January, 1956. They were mostly obtained with the same photometer as in 1954, but on some nights a different photometer was used, designed and constructed by Dr. Hiltner. A high-sensitivity amplifier built by Dr. Hiltner was used also; we are greatly indebted to him for making these instruments available to us. Dr. Kuiper determined a light-curve of asteroid No. 7 on March 8, 1956, using the standard photometer and a M.I.T. amplifier; these observations are also included in this paper. The calibration of the Hiltner amplifier was made jointly by Dr. Hiltner and one of the authors. The M.I.T. amplifier was calibrated by Mr. D. Crawford.

II. THE LIGHT-CURVES

The method of observation and reduction is the same as in the previous papers. Part of the readings on the Brown recorder paper were made on a measuring machine constructed by Hiltner. All magnitudes are reduced to the B, V, U system of Johnson and Morgan. The light-curves themselves were determined in yellow only. Special color measurements were made for all asteroids observed.

The main data derived from this investigation are collected in Tables 1 and 2. Table 1 is self-explanatory. Table 2 gives, respectively, asteroid number, date of observation, and the phase coefficient α used; further, the phase angle ϕ , the logarithm of distance sun-asteroid, r ; and of the earth-asteroid, ρ , the last three taken from the Russian Ephemeris 1955 and 1956, except for 321 Florentina. These values were kindly computed by Mr. van Lüde of Heidelberg. Then follows the light-time, the observed R.A. and Decl., and their corresponding ecliptical longitude, λ , and latitude, β . The zero magnitude in Table 1 is the V brightness at maximum. These values are also inserted in one of the left-hand corners in Figures 1–26, showing the light-curves. The time co-ordinate in the figures is expressed in U.T., without correction for light-time. In single nights, open circles indicate observations of lesser weight, mostly derived from one deflection only, while the filled circles are means of two deflections. The uncertainties of the periods given in Table 1 were obtained as follows. The mean accuracy of one epoch is

* *Contributions of the McDonald Observatory*, No. 279; also *Mitteilungen der Landessternwarte Heidelberg-Königstuhl*, No. 107.

about $\pm 3^m$, and thus the uncertainty of one interval about $\pm 4^m$. The quotient of $\pm 4^m$ divided by the number of cycles used for the final determination of the period gives the tabulated uncertainties.

Iris, No. 7

Observations of asteroid No. 7 were made on December 28, 29, January 2, 5, and March 8 (Figs. 1-5). The first four nights could be used for a rediscussion of the period. This had been determined in 1950 from a 4-day interval, giving a period of 7^h7^m5 . A later determination in 1952 yielded the period 7^h3^m . From the present material a period of $7^h8^m1 = 0^d29729$ is derived, using the interval from December 28 to January 5, which contains 23 cycles. This value is uncertain by 0^m15 , or about 0^d00010 . It was used for the combination of the observations of the 4 nights into a single light-curve by cor-

TABLE 1
SUMMARY OF OBSERVATIONS

No.	Obs. Date	V of Zero	B-V	U-B	Ampl	Period	Mean V	g	Fig.
7...	Dec. 28, 1955	9 ^m 74	..	..	..	..	..	...	1, 6
	Dec. 29, 1955	9 74	+0 ^m 86	+0 ^m 48	..	..	..	...	2, 6
	Jan. 2, 1956	9 69	+ 87	+ 46	0 ^m 08	7 ^h 08 ^m 1	9 ^m 78*	6 ^m 04*	3, 6
	Jan. 5, 1956	9 64	+ 87	+ 46	..	$\pm 0\ 2$	..	..	4, 6
	Mar. 8, 1956	8 92	+ 84	..	..	..	..	..	5
8....	Dec. 20, 1955	8 93	..	..	03	$\approx 13^h6$	8 93†	6 70†	7
	Dec. 27, 1955	8 80	+ 88	..	04		8 83†	6 74†	8
	Dec. 28, 1955	8 81	+ 88	+ 48	..		8 81†	6 74†	7
	Dec. 29, 1955	8 81	+ 88	+ 49	..		8 81†	6 76†	7
	Jan. 1, 1956	8 72	+ 88	+ 47	04		8 74†	6 74†	9
11...	Jan. 3, 1956	10 56	+ 81	+ 39	07	$\approx 10^h40^m$	10 60‡	7 08‡	10
15 ...	Dec. 21, 1955	8 40	+ 83	+ 45	53	6 ^h 05 ^m 0 $\pm 0\ 2$	8 74§	5 34§	11, 14
	Dec. 24, 1955	8 49	+ 83	+ 44	53				12, 14
	Dec. 27, 1955	8 58	+ 83	+ 45	51				13, 14
16.....	Dec. 26, 1955	9 73	+ 71	+ 26	11	4 ^h 18 ^m 2 ± 1	9 78§	6 08§	15
	Jan. 2, 1956	9 87	+ 71	+ 24	12		9 92§	6 08§	16
17. ...	Dec. 23, 1955	11 38	..	..	02	..	11 40†	7 73†	17
	Dec. 24, 1955	11 42	+ 86	+ 40	13		11 50†	7 80†	17
	Dec. 25, 1955	11 44	+ 82	+ 39	09		11 49†	7 78†	18
25.. ..	Jan. 3, 1956	12 46	+ 99:	..	16	..	12 56†	8 24†	19
	Jan. 4, 1956	12.50	+ 92	+ 52	06		12 54†	8 23†	20
	Jan. 6, 1956	12 56	+ 92	+ 52	..		12 56	8 26	..
39..	Dec. 18, 1955	10 42	+ 95:	+ 46	50	5 ^h 08 ^m 3 $\pm 0\ 1$	10 66§	6 81§	21, 24
	Dec. 19, 1955	10 45	+ 91	+ 55	50				22, 24
	Dec. 28, 1955	10 60	+ 91	+ 47	53				23, 24
321.	Dec. 19, 1955	14.10	..	..	40	2 ^h 52 ^m 2 $\pm 0\ 5$	14 27§	10 45§	25
	Dec. 20, 1955	14.11:	+0 82	+0 45	0 36		14 26§	10 43§	26

* Mean over three-quarters of a period

† Mean over night's run.

‡ Mean over half-period.

§ Mean over full period.

recting for distance, phase, and light-time differentially to December 28. The result is shown in Figure 6. The agreement between the different nights is good.

If we compare the three light-curves obtained in 1950, 1952, and 1955/56, then we see a pronounced change in the amplitude: 0.29 mag. in 1950, 0.22 mag. in 1952, and 0.08 mag. in 1955/56. This means that the axis of rotation of the asteroid is considerably inclined toward the plane of the ecliptic. Consequently, the method of determining the sense of rotation, as developed in Paper I of this series, cannot be used here.

TABLE 2
ASPECT DATA

No	Obs Date	Phase Coeff *	Phase Angle	log r	log ρ	Light-Time	R A	Decl	λ	β
7	{ Dec 28	0 019M	23°7	0 3764	0 2826	15 ^m 93	11 ^h 12 ^m 5	— 1°41 ^m	170°	— 6°
	Dec 29		23 6	3769	2804	15 85	11 12 9	— 1 47	170	— 6
	{ Jan. 2		23 0	3789	2711	15 52	11 14 2	— 2 07	170	— 7
	{ Jan. 5		22 3	3802	2644	15 28	11 14 8	— 2 20	170	— 7
	{ Mar. 8		4 0	4069	1962	13 06	10 36 9	— 0 53	161	— 9
8	{ Dec. 20	028M	14 9	3084	0546	9 43	8 06 2	+19 10	119	— 1
	Dec. 27		13 9	3109	0445	9 21	8 00 6	+19 51	118	— 1
	{ Dec. 28		10 6	3113	0434	9 18	7 59 8	+19 58	117	— 1
	Dec. 29		10 1	3118	0422	9 16	7 59 0	+20 04	117	— 1
	{ Jan. 1		8 3	3128	0402	9 12	7 56 0	+20 25	117	0
11	{ Jan. 3	022M	14 6	4053	2350	14 28	4 02 2	+15 23	62	— 5
15	{ Dec. 21	042C	20 1	3412	1547	11 87	2 08 3	+28 41	40	+15
	Dec. 24		20 8	3418	1641	12 13	2 09 8	+28 20	40	+14
	{ Dec 27		21 5	3424	1728	12 37	2 11 1	+27 54	40	+14
16	{ Dec. 26	030A	9 9	4293	2499	14 78	4 22 5	+17 50	67	— 3
	{ Jan. 2		12 2	4307	2636	15 25	4 18 5	+17 01	66	— 4
17	{ Dec. 23	040C	2 6	4492	2636	15 25	5 42 0	+18 06	86	— 5
	Dec 24		3 3	4490	2641	15 27	5 41 0	+18 07	85	— 5
	{ Dec. 25		3 6	4490	2646	15 29	5 39 8	+18 08	85	— 5
25	{ Jan. 3	025M	11 0	4770	3314	17 83	7 43 9	— 9 40	120	—31
	{ Jan. 4		10 8	4770	3304	17 79	7 43 0	— 9 43	120	—31
	{ Jan. 6		10 6	4771	3290	17 72	7 41 4	— 9 48	119	—31
39	{ Dec 18	022M	19 9	4016	2771	15 73	1 51 6	— 2 16	25	—13
	Dec. 19		20 1	4017	2799	15 83	1 51 6	— 2 13	25	—13
	{ Dec 28		21 5	4029	3045	16 76	1 54 4	— 1 25	26	—12
321	{ Dec. 19	0 030A	9 7	4395	2659	15 33	3 34 3	+21 01	56	+ 2
	{ Dec. 20		10 1	0 4395	0 2675	15 39	3 33 7	+20 59	56	+ 2

* M = Müller; C = derived in 1950 or 1953; A = adopted

In 1950 and 1952 the general shape of the light-curve was the same, although the amplitude had somewhat changed. This is not true for the 1955/56 observations; the maxima and minima of the earlier observations cannot be recognized in the new light-curve. This result is compatible with the changes in aspect listed in Table 3.

The run of March 8, 1956 (Fig. 5), shows no clearly defined extremum, and the observed part of the light-curve could not be identified with any part of the light-curve of Figure 6, made 2 months earlier. If we try to determine the absolute magnitude from

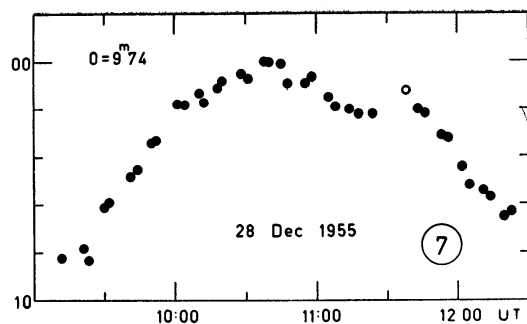


FIG. 1.—Observations of 7 Iris

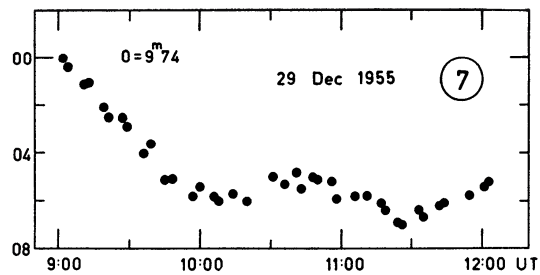


FIG. 2.—Observations of 7 Iris

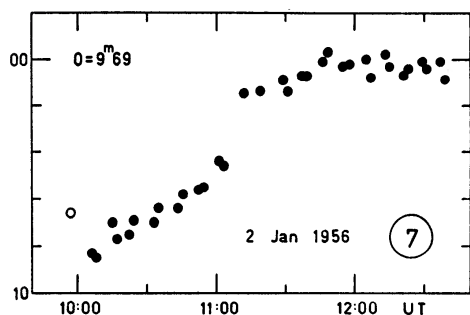


FIG. 3.—Observations of 7 Iris

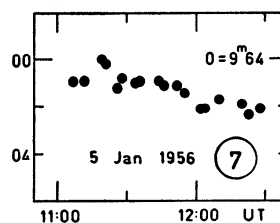


FIG. 4.—Observations of 7 Iris

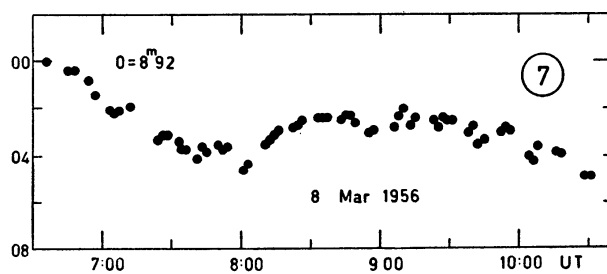


FIG. 5 —Observations of 7 Iris

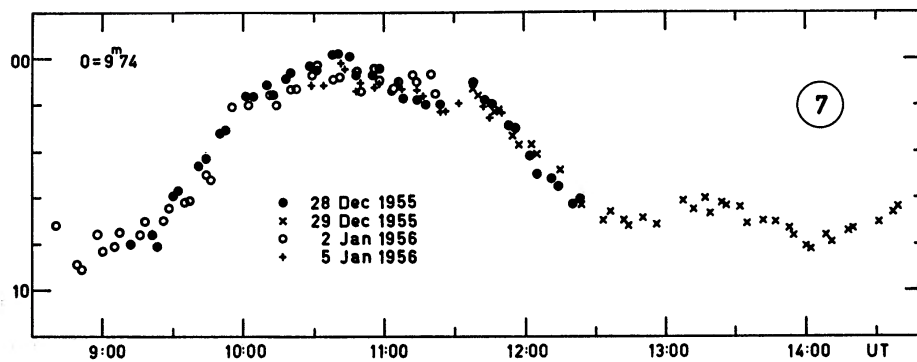


FIG. 6.—Light-curve of 7 Iris. Combination of curves 1, 2, 3, and 4, reduced to December 28, 1955

the March observations with the phase coefficient as used in December–January, we see that the *g*-value in March is about 0.2 mag. brighter. This could be explained by assuming that the phase function used was not linear. The opposition was February 29, 1956. The phase angle changed from 23.7° in December to 4° in March, so that an appreciable phase correction had to be applied. The phase coefficient used was determined by Müller (1897), which brought the observations of December–January into good mutual agreement.

Flora, No. 8

Light-curves of asteroid No. 8 were obtained in 1953 by I. I. Ahmad and I. Groeneveld and G. P. Kuiper (Paper III). No distinct variation could be detected. In 1955–56 one run of 6 hours was made (Fig. 8), with additional short runs on four other nights (Figs. 7 and 9). The long run of December 27 shows a definite change in brightness, of about

TABLE 3
ASPECTS OF IRIS

Year	λ	β	Year	λ	β
1950	271°	+4°	1955 December	170°	−6°
1952	118	−7	1956 March	161	−9

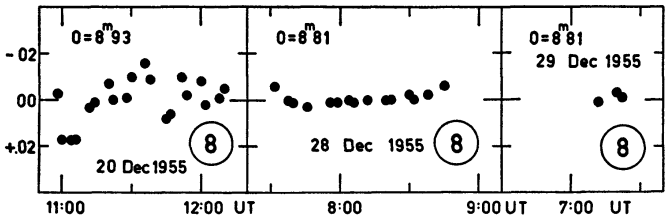


FIG. 7.—Observations of 8 Flora on three nights

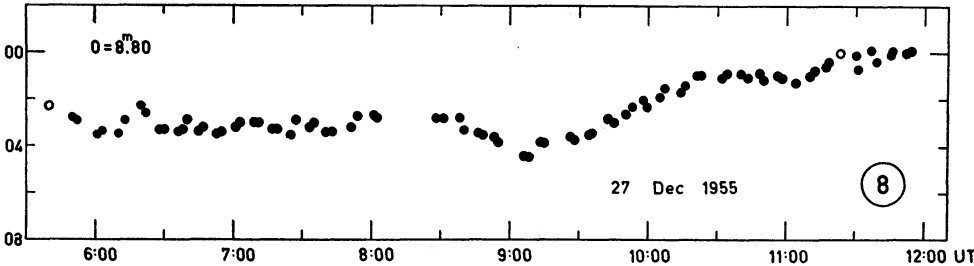


FIG. 8.—Light-curve of 8 Flora

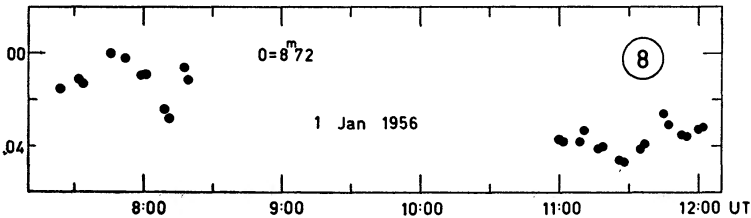


FIG. 9.—Observations of 8 Flora

0.04 mag. On January 1, 1956, two runs of about 1 hour each were made, separated by 2.5 hours. They show again a change of 0.04 mag. The observed maxima and minima brightnesses, all reduced to December 27 by correcting for distance and phase, are collected in Table 4. If we assume that the complete light-curve has two maxima and two minima, we estimate from our observations that $\frac{1}{4}$ period is about 3.5 hours, which would make the full period roughly 14 hours or 0.58 day. If the observed maxima on December 20 and January 1 are identical, then the period is found to be $11^d84/21 = 0^d564$. If the observed maximum on December 27 is the secondary maximum, the period becomes $7^d013/12.5 = 0^d561$ (from the interval December 20–27) or $4^d828/8.5 = 0^d568$ (from the interval December 27–January 1). Finally, the two observed minima on December 27 and January 1 give a period of $5^d098/9 = 0^d566$. It seems, therefore, that a period of 0.565 day satisfies our observations reasonably well; but, because of the very small amplitude, more observations are required.

TABLE 4
MAXIMA AND MINIMA OF FLORA (8)*

1955–56	U T	V(Max)	U T	V(Min)	JD(Max)†	JD(Min)†
Dec 20	11 ^h 36 ^m	8 77			461.983 I	
Dec. 27	11 54	8.80	9 ^h 07 ^m	8 85	468.996 II	468 880 I
Jan. 1	7 46	8 81	11 28	8 85	473 824 I	473 978 I

* No well-defined extrema on December 28 and 29

† Add 2435000 days

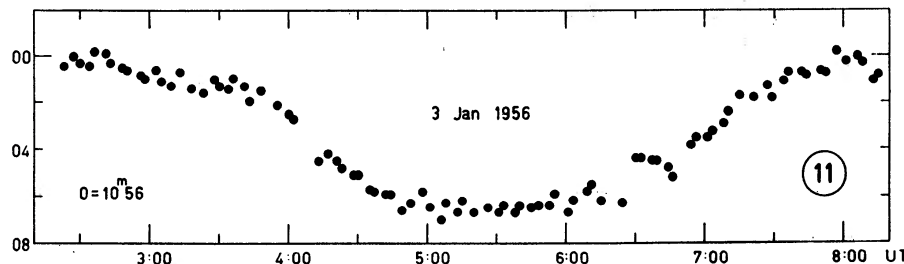


FIG. 10.—Light-curve of 11 Parthenope

Parthenope, No. 11

Parthenope was observed only on one night over 6 hours, as shown in Figure 10. The observations indicate that the half-period is 5^h20^m, the full period would accordingly be 10^h40^m. The observed minimum is very broad; in general, the light-curve resembles the 1954 light-curve of asteroids Nos. 10 and 532. Two maxima of equal brightness were observed at 2^h39^m $\pm$ 3^m and 8^h01^m $\pm$ 4^m U.T.

Eunomia, No. 15

This time Eunomia was observed on three nights, December 21, 24, and 27; the observations are given in Figures 11, 12, and 13, respectively. The data for the extrema are found in Table 5. "JD(c)" means Julian Day corrected for light-time. From these data a period of $0^d25347 = 6^h5^m0$ is found. The interval used for the determination of the period contains 23 cycles, and accordingly its accuracy becomes $\pm 0^d00013$. The

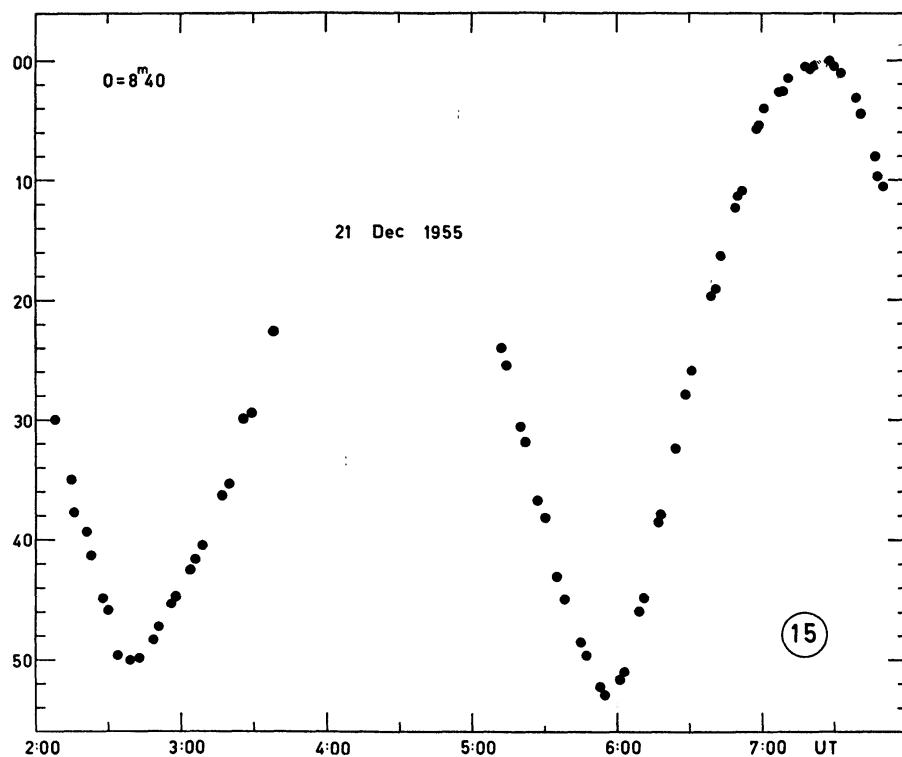


FIG. 11.—Light-curve of 15 Eunomia

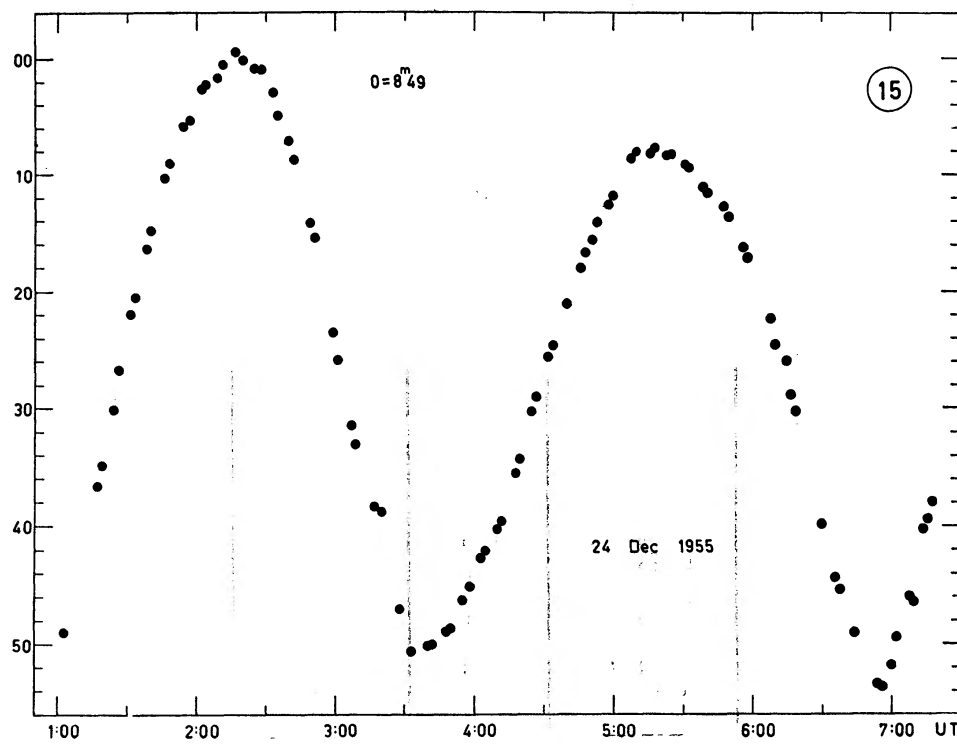


FIG. 12.—Light-curve of 15 Eunomia

three light-curves of the different nights were combined into one curve, using the period of 0^d25347. The observations of December 21 and 27 are reduced to December 24 by correcting for light-time, but the magnitudes of all three nights are reduced to unit distance to sun and earth and to zero phase angle, by applying the correction

$$5 \log r\rho + \phi a,$$

where r , ρ , and ϕ are interpolated in time for each individual observation. In Figure 14 the observations at the beginning of the light-curve are replotted at the end, and vice versa. The agreement between December 24 and 27 is perfect; but the magnitudes of

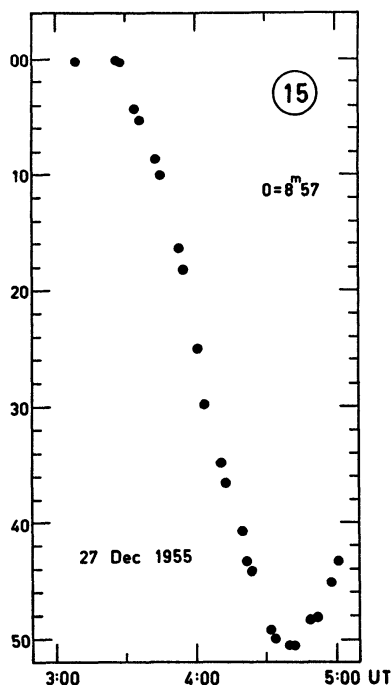


FIG. 13.—Observations of 15 Eunomia

TABLE 5
EPOCHS OF MAXIMA AND MINIMA OF EUNOMIA (15)

1955	U T.	Type	JD*	JD(c)*
Dec. 21	2 ^h 39 ^m ± 2 ^m	m II	462 6104	462 6022
	5 56 ± 2	m I	462 7472	462 7390
	7 24 ± 3	M I	462 8083	462 8001
Dec. 24	2 18 ± 3	M I	465 5958	465 5874
	3 38 ± 3	m II	465 6514	465 6430
	5 17 ± 4	M II	465 7201	465 7117
	6 56 ± 3	m I	465 7889	465 7805
Dec. 27	3 19 ± 5	M I	468 6382	468 6296
	4 40 ± 2	m II	468 6944	468 6858

* Add 2435000 days.

December 21 are systematically 0.02 mag. too high. This may have been caused by inaccuracies in the calibration constants; on this night standard stars were measured not earlier than 8^h U.T.

In Paper I the sense of rotation of Eunomia was determined. It is important to see whether the new observations are compatible with the retrograde rotation derived. Compared to the previous investigations, the amplitude was again practically the same. That means the axis of rotation is nearly at right angles to the ecliptic plane. In Table 6 the primary maxima in the different observing periods are compared, and the corrections for rotation are computed. "JD(c)" is again the Julian Day corrected for light-time; λ and β are the ecliptical co-ordinates; Δt is the time difference between consecutive

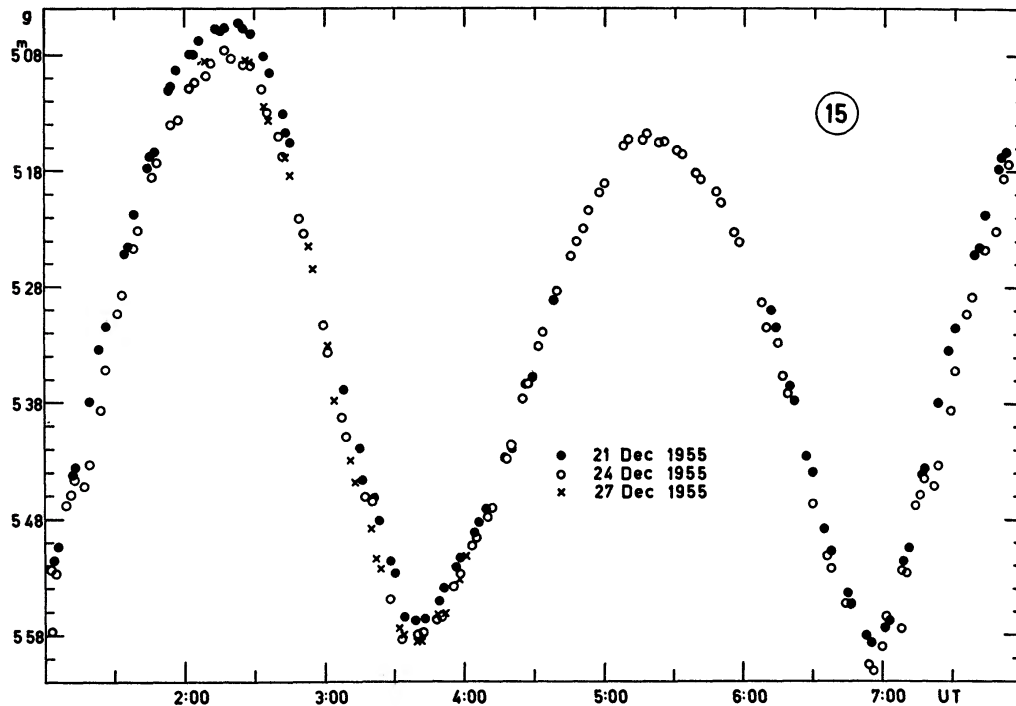


FIG. 14.—Light-curve of 15 Eunomia. Combination of curves 11, 12, and 13, reduced to December 24, 1955. Ordinates give absolute magnitudes.

TABLE 6
EPOCHS OF PRIMARY MAXIMA OF EUNOMIA

Year	JD*	JD(c)*	λ	β	Δt	$\Delta\lambda$	$\Delta\lambda^\circ \cdot p^d/360^\circ$
1950	3506 778	3506 768	288°	+ 2°	529 ^d 846	161.5	0 ^d 114
1952	4036 623	4036 614	89 5	— 3	438 145	96 5	.068
1953	4474 771	4474 759	186	—17	990 828	214	0 151
1955	5465 596	5465 587	40	+14			

* Add 2430000 days.

observed maxima; $\Delta\lambda$ is the difference in λ expressed in degrees; and the last column gives the corresponding correction of Δt . We then find with *direct* rotation:

$$\begin{aligned} n_1P &= 529^d846 - 0^d114 = 529^d732; \quad n_1 = 2090, P = 0^d2534603 \\ &\qquad\qquad\qquad \Delta P = 0^d0000565 \\ n_2P &= 438.145 - 0.068 = 438.077; \quad n_2 = 1728, P = 0.2535168 \\ &\qquad\qquad\qquad \Delta P = 0.0000171 \\ n_3P &= 990.828 - 0.151 = 990.677; \quad n_3 = 3908, P = 0.2534997 \end{aligned}$$

with *retrograde* rotation:

$$\begin{aligned} n_4P &= 529^d846 + 0^d114 = 529^d960; \quad n_4 = 2091, P = 0^d2534481 \\ &\qquad\qquad\qquad \Delta P = 0^d0000007 \\ n_5P &= 438.145 + 0.068 = 438.213; \quad n_5 = 1729, P = 0.2534488 \\ &\qquad\qquad\qquad \Delta P = 0.0000015 \\ n_6P &= 990.828 + 0.151 = 990.979; \quad n_6 = 3910, P = 0.2534473 \end{aligned}$$

Since the uncertainty of one interval is about 0.003 day, the uncertainties of the periods derived in this way are, respectively, 0.0000014, 0.0000017, and 0.0000008 day; so that

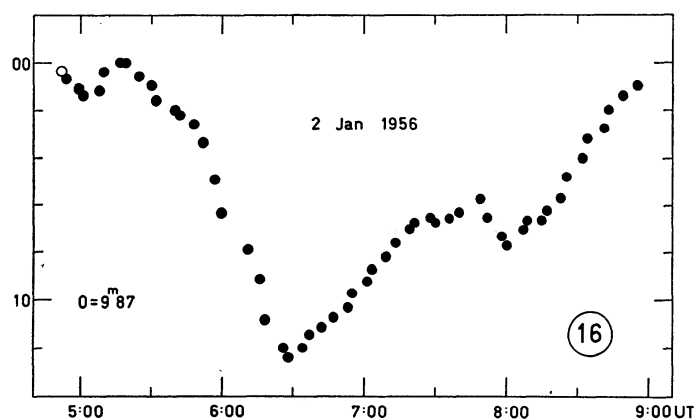


FIG. 15—Light-curve of 16 Psyche

their differences are uncertain by 0.0000022 and 0.0000019 day. It follows that the assumption of retrograde rotation gives a satisfactory period, while direct rotation does not, as found before (Paper I).

Psyche, No. 16

Asteroid No. 16 was observed on December 26, 1955 (Fig. 15) and January 2, 1956 (Fig. 16). On the first night two cycles were covered, from which a period of $4^h18^m \pm 3^m$ was found. As usual, two maxima and two minima are present. The amplitude is 0.12 mag. The secondary maximum and minimum differ by only 0.02 mag., but they are well defined. The primary maximum is relatively broad. The time interval between December 26 and January 2 corresponds to 39 cycles. From this, by superimposing the light-curves, an improved period could be derived: $0^d17929 \pm 0^d00007 = 4^h18^m2 \pm 0^m1$. The epochs of maxima and minima are given in Table 7. In order to bring the mean magnitudes of the two nights into agreement, a phase factor of 0.03 mag/degree had to be used, differing from Müller's value of 0.048 mag/degree.

Thetis, No. 17

Earlier observations of Thetis were published in Paper I. It was shown that the light-curve of the last observation day did not agree with the observations taken 3 weeks earlier. The period was determined as $12^{\text{h}}16^{\text{m}}5$. In 1955 the asteroid was observed on December 23 (Fig. 17, *a*), December 24 (Fig. 17, *b*), and December 25 (Fig. 18). When these observations were compared with the old period, difficulties arose. The observations of December 23 and 25 could be combined well enough; the observations of

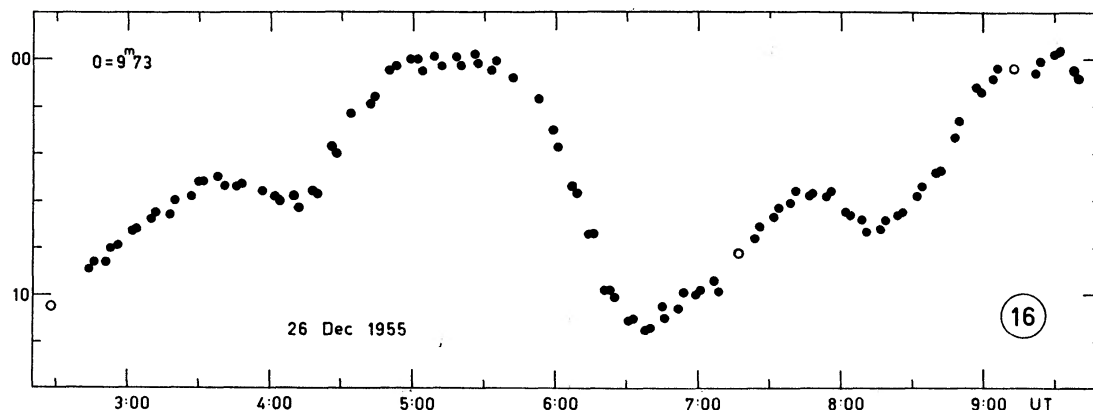


FIG 16—Light-curve of 16 Psyche

TABLE 7

EPOCHS OF EXTREMA OF PSYCHE (16)

Date	U T	Type	JD*	JD(c)*
Dec 26, 1955	$3^{\text{h}}35^{\text{m}} \pm 4^{\text{m}}$	M II	467 6493	467 6390
	$4^{\text{h}}08^{\text{m}} \pm 4$	m II	467 6722	467 6619
	$5^{\text{h}}16^{\text{m}} \pm 7$	M I	467 7194	467 7092
	$6^{\text{h}}39^{\text{m}} \pm 2$	m I	467 7771	467 7668
	$7^{\text{h}}48^{\text{m}} \pm 4$	M II	467 8250	467 8147
	$8^{\text{h}}15^{\text{m}} \pm 3$	m II	467 8437	467 8335
	$9^{\text{h}}30^{\text{m}} \pm 6$	M I	467 8958	467 8856
Jan. 2, 1956	$5^{\text{h}}17^{\text{m}} \pm 3$	M I	474 7201	474 7095
	$6^{\text{h}}29^{\text{m}} \pm 2$	m I	474 7701	474 7595
	$7^{\text{h}}41^{\text{m}} \pm 5$	M II	474 8201	474 8095
	$8^{\text{h}}02^{\text{m}} \pm 2$	m II	474 8347	474 8241

* Add 2435000 days

December 23, after differential correction for light-time, phase, and distance to December 25, are represented as crosses in Figure 18. The well-defined minimum of December 24 should have occurred on December 25 at $9^{\text{h}}37^{\text{m}}$ U.T.; however, it is not visible at all. But there still seems to be no reason to doubt the period derived in 1953. We have no explanation for this discrepancy. The amplitude in 1953 was 0.32 mag. and 0.36 mag., while in 1955 it was only 0.13 mag.

Phocaea, No. 25

From the 1950 observations of Phocaea a period of $9^{\text{h}}56^{\text{m}}7$ was found (Paper I). New observations were obtained on January 3 and 4 (Figs. 19 and 20) and a single observa-

tion on January 6 because of clouds. When we reduce the January 4 observations to January 3, by applying the usual corrections with the period found earlier, the two runs are found to be in different parts of the light-curve; the first point of January 4, reduced to January 3, follows $1^{\text{h}}32^{\text{m}}$ later than the last point of January 3. Therefore, no definite conclusions about the behavior of Phocaea during this observational period can be made except for a minimum value of the amplitude, 0.16 mag. The single observation is January 6, $7^{\text{h}}23^{\text{m}}$ U.T., and fits well into the observations of January 3.

Laetitia, No. 39

Laetitia was observed in 1949 (Paper II), 1952, and 1953 (Paper I). In 1955 observations were obtained on three nights, December 18, 19, and 28 (Figs. 21, 22, 23). The run

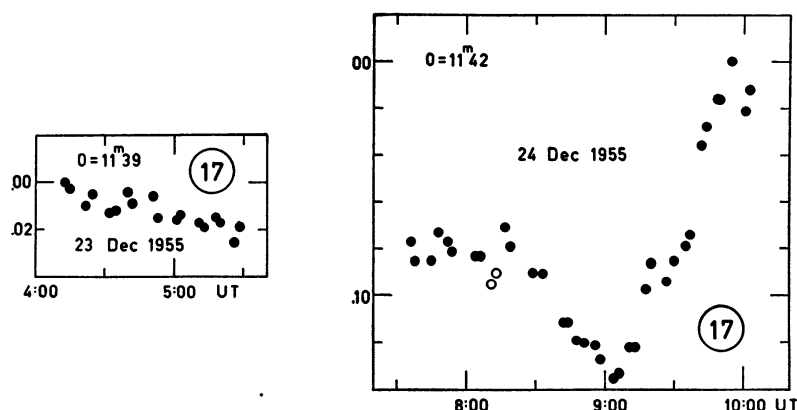


FIG. 17.—Observations of 17 Thetis on two nights

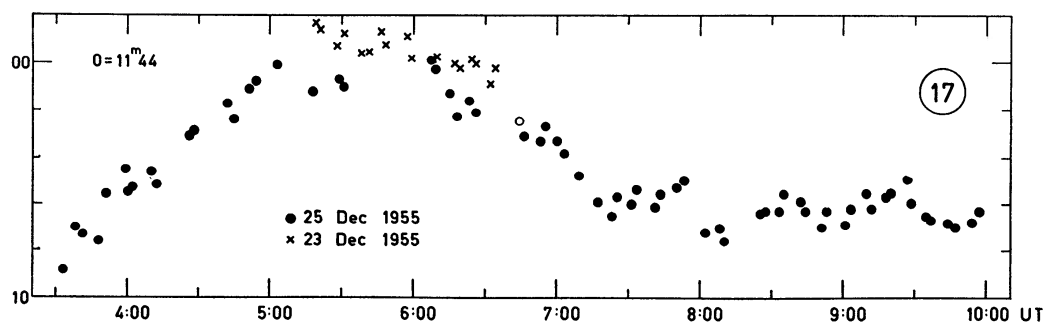


FIG. 18.—Light-curve of 17 Thetis. The observations of December 23, 1955, are reduced to December 25, 1955, and plotted as crosses.

of December 28 was interrupted by some clouds. The light-curve is very smooth. It cannot be decided which minimum is the deeper one because of a lack of observations at the time of one of the minima; we shall denote here as secondary the minimum following the primary maximum. This is in accordance with the light-curves observed in 1952 and 1953. The observed extrema are found in Table 8.

The period determined from this material is $0^{\text{d}}21408 = 5^{\text{h}}8^{\text{m}}3$. The longitude, λ , is changing from $25^{\circ}0$ on December 18 to $26^{\circ}0$ on December 28, while the latitude, β , is practically unchanged, from $-12^{\circ}9$ to $-12^{\circ}3$; hence we can neglect the motion and get nearly the true period. The interval between December 18 and 28 contains 47 cycles, so that the uncertainty of the period becomes $0^{\text{d}}00006$. With the period $0^{\text{d}}21408$, the

observations of December 18 and 28 were reduced to December 19 by correcting in the same way as for Eunomia, so that in the combined light-curve the ordinate shows the absolute magnitudes. Observations at the beginning and end of the cycle are replotted one cycle later and earlier (Fig. 24). The phase coefficient 0.022 mag/degree, used in the earlier papers, proved to be completely satisfactory.

The amplitude and the appearance of the light-curve of Laetitia are quite different in the four observed oppositions, as shown in Table 9. It follows that the equator of

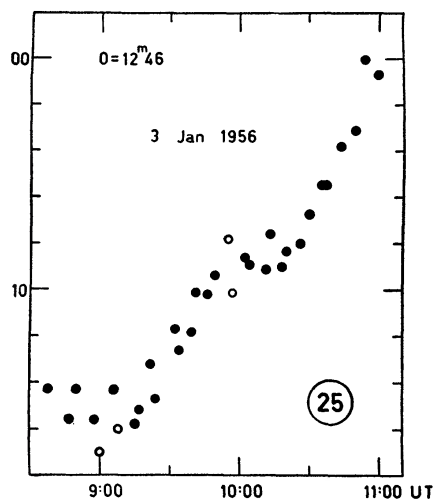


FIG. 19.—Observations of 25 Phocaea

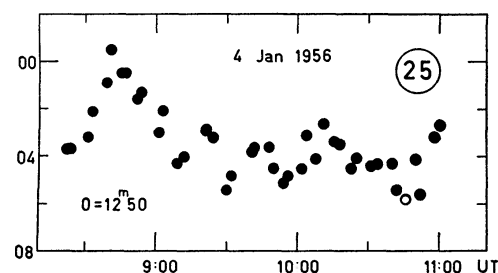


FIG. 20.—Observations of 25 Phocaea

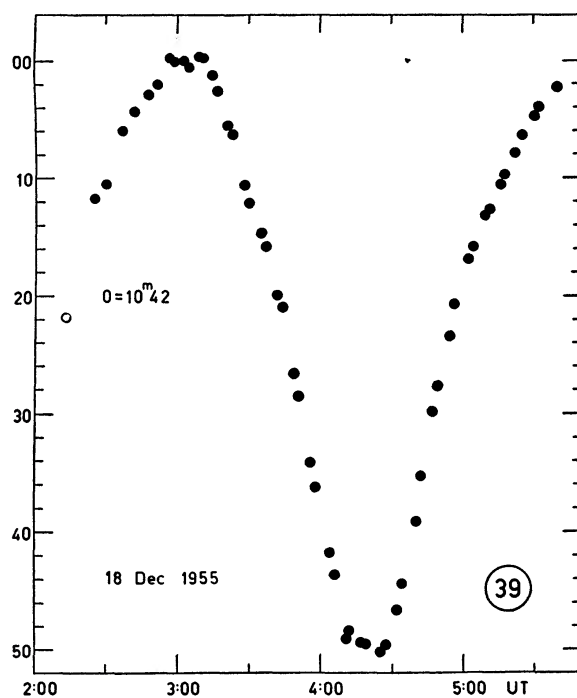


FIG. 21.—Observations of 39 Laetitia

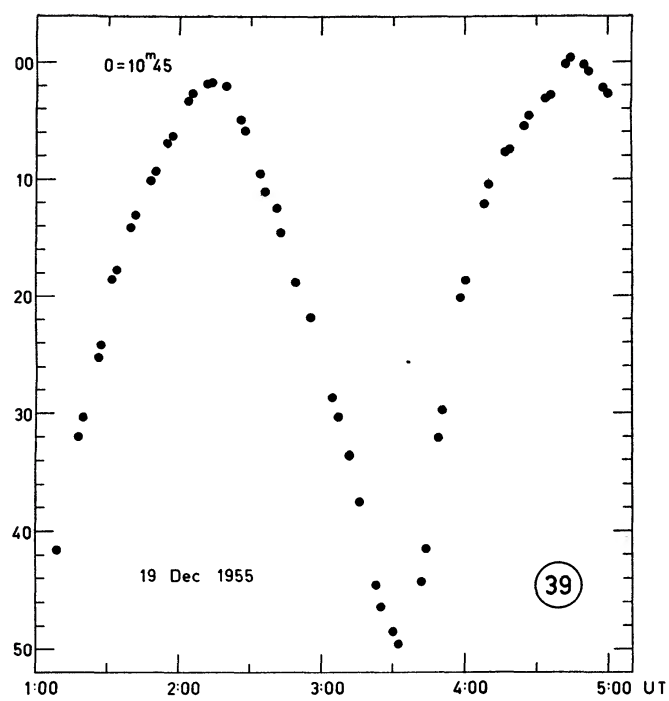


FIG. 22.—Observations of 39 Laetitia

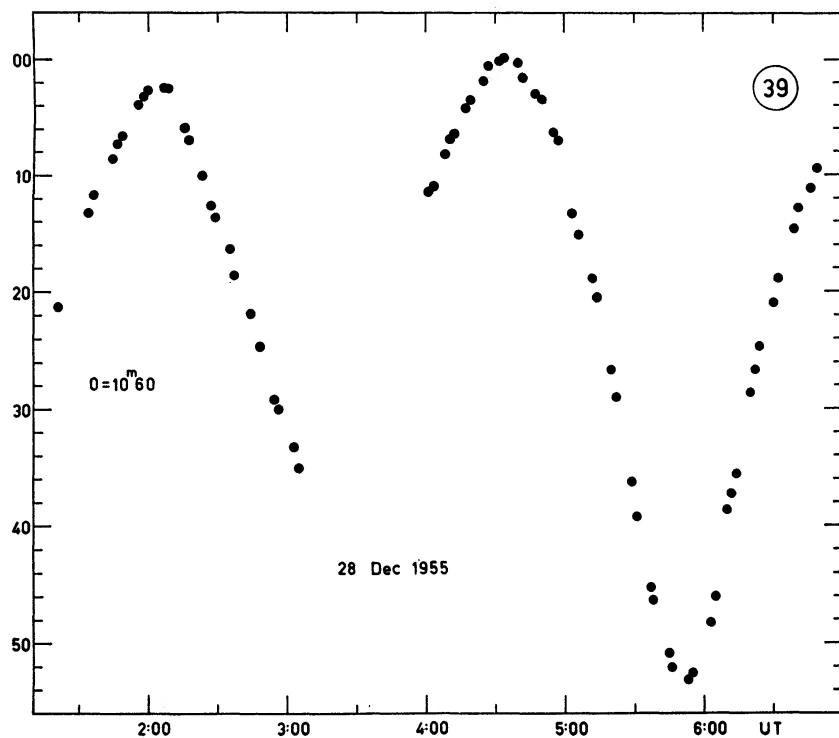


FIG. 23.—Light-curve of 39 Laetitia

Laetitia is considerably inclined on the ecliptic plane. Accordingly, if we want to determine the sense of rotation, we cannot use the simple procedure applied earlier. A correct formula will be given below. For this we must know at least an approximate position of the orientation of the axis of rotation. The method followed by Stobbe (1940) and Beyer (1953) will be applied. They used the relation between the observed amplitude of light-variation and the declination of the direction of the earth, in asteroidal co-ordinates. It was tried to obtain such a relation by scaling down the ordinate of Beyer's Figure 6 (determined for the asteroid Eros) by a factor equal to the ratio of the maximal ampli-

TABLE 8
EPOCHS OF MAXIMA AND MINIMA OF LAETITIA (39)

Date (1955)	U T	Type	JD*	JD(c)*
Dec. 18	$3^h 04^m \pm 4^m$	M I	459 6278	459 6169
	$4\ 24 \pm 2$	m II	459 6833	459 6724
Dec. 19	$2\ 14 \pm 3$	M II	460 5931	460 5821
	$3\ 32 \pm 3$	m I	460 6472	460 6362
	$4\ 45 \pm 3$	M I	460 6979	460 6869
Dec. 28	$2\ 04 \pm 3$	M II	469 5861	469.5745
	$4\ 34 \pm 3$	M I	469 6903	469.6787
	$5\ 52 \pm 2$	m II	469 7444	469 7328

* Add 2435000 days.

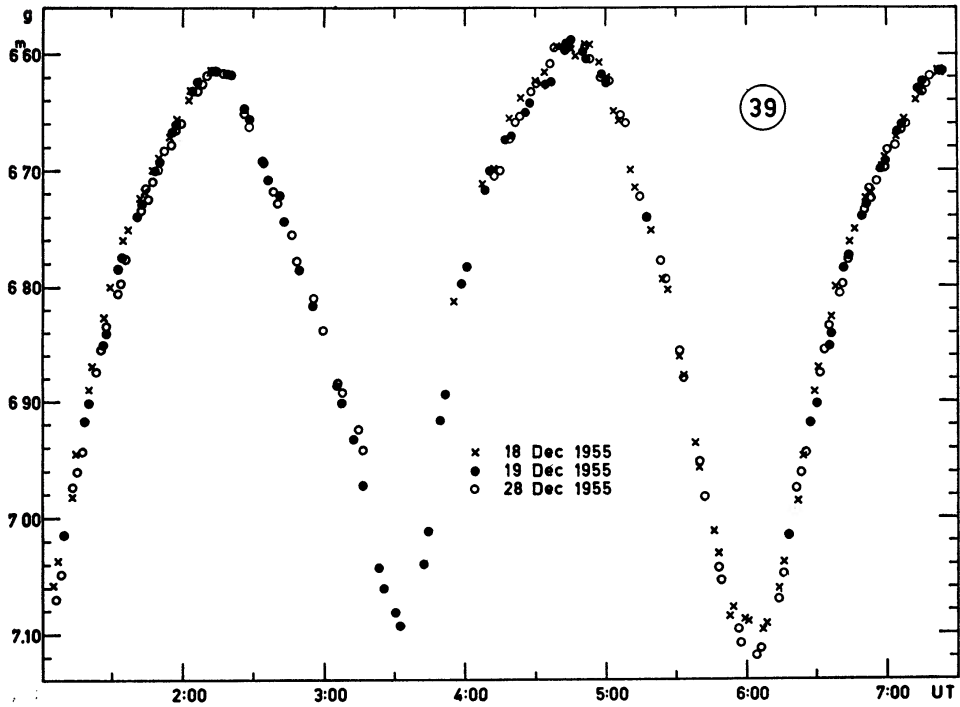


FIG. 24.—Light-curve of 39 Laetitia. Combination of curves 21, 22, and 23, reduced to December 19, 1955. Ordinates give absolute magnitudes.

tudes of the two asteroids involved (Laetitia/Eros). We realize that this may not give the correct relation; the shape of Laetitia may be quite different from that of Eros, and the scaling procedure has not been justified theoretically. But, because of the few data available, no better method was suggested. The maximum amplitude of Laetitia was determined by plotting the observed amplitudes against λ and estimating the maximum amplitude by interpolation. It appeared that the amplitude observed in 1955 was nearly maximal, so that the factor used for the conversion of the ordinates in Beyer's Figure 6 was 0.53/1.45. In this way the orientation of the axis of rotation was estimated. As expected, it was difficult to distinguish between the true axis and its mirror image with respect to the orbital plane. It is our impression that, apart from this ambiguity, the pole was ascertained with an accuracy of about 10° .

The correction of the rotational period resulting from the relative motions of earth and asteroid is based on one assumption: that the light-curve will always have a maximum when a certain meridian crosses the line of sight, whatever may be the inclination of the

TABLE 9
OBSERVED OPPOSITIONS OF LAETITIA

Year	Ampl (mag)	λ	β	Year	Ampl (mag)	λ	β
1949	0 43	232°	+15°	1953	0 22	180°	+ 7°
1952	0 18	96	-12	1955	0 53	25	-13

rotational axis. The differences in position of this meridian will then determine the corrections to be made.

If the co-ordinates of the asteroid, as seen from the earth, are λ_{ast} and β_{ast} , then an observer on the asteroid will see the earth in the position $\lambda_{\text{ast}} + 180^\circ$, $-\beta_{\text{ast}}$. The direction of the polar axis is the same for both observers (λ_{pole} and β_{pole}). We want to transform these co-ordinates into asteroidal equatorial co-ordinates, with the descending node of the ecliptic toward the asteroidal equatorial plane as the zero point of the asteroidal right ascension. If we reduce the ecliptic co-ordinates to the same zero point, we get $\lambda' = (\lambda_{\text{ast}} + 180^\circ) - (\lambda_{\text{pole}} + 90^\circ) = \lambda_{\text{ast}} - \lambda_{\text{pole}} + 90^\circ$; $\beta' = -\beta_{\text{ast}}$. The conversion in asteroidal equatorial co-ordinates is straightforward; if we denote the asteroidal right ascension with E and eliminate the other co-ordinate, the formula takes on the following form:

$$\tan E = \sin \beta_{\text{pole}} \cot (\lambda_{\text{pole}} - \lambda_{\text{ast}}) - \cos \beta_{\text{pole}} \tan \beta_{\text{ast}} / \sin (\lambda_{\text{pole}} - \lambda_{\text{ast}}).$$

With this formula the corrections to the period caused by the relative motions of earth and asteroid were computed. In our computation, two poles were used, both thought to be near the correct position of the pole and about 10° apart. They are $\lambda_{\text{pole}} = 117^\circ$, $\beta_{\text{pole}} = +24^\circ$, and $\lambda_{\text{pole}} = 112^\circ$, $\beta_{\text{pole}} = +31^\circ$.

In Table 10, E_1 and E_2 denote the E -values computed from the two poles, and JD (c) is the Julian Day corrected for light-time. The four oppositions define three intervals I, II, and III, and accordingly three values of the period. The differences between those periods should not exceed the limits set by the accuracy of the observations, roughly 0.0000015. The differences actually found are given in Table 11. From the figures in Table 11 it is apparent that the corrections are too sensitive to the assumed Laetitia pole to make this method useful. Direct rotation appears to give the smallest differences,

but the evidence for direct rotation is not conclusive. Clearly, the direction of the Laetitia pole must be known with greater precision (see also "Concluding Remarks").¹

Florentina, No. 321

Florentina is a member of Hirayama's Coronis family. It was observed on December 19 and 20 (Figs. 25 and 26); in the last night the observations were interrupted by passing clouds. Therefore, on this night the primary maximum and the following minimum are not well defined. As the object is faint and the period appeared to be extremely short, the determination of a period was somewhat more difficult than usual. The full-

TABLE 10
OPPOSITIONS OF LAETITIA

U T (Min II)	JD(c)*	λ	E_1	E_2	Δt
May 9, 1949, 9 ^h 29 ^m	3045 8837	232°	205°	209°	994 ^d 8108 437 0342 981 9437
Jan. 29, 1952, 4 56	4040 6945	96	48	68	
Apr. 10, 1953, 5 47	4477 7287	180	175	175	
Dec. 18, 1955, 4 24	5459 6724	25	10	12	

* Add 2430000 days

TABLE 11
DETERMINATION OF PERIOD OF LAETITIA

Pole	$\lambda = 117^\circ, \beta = +24^\circ$	$\lambda = 112^\circ, \beta = +31^\circ$
	Δ Period	Δ Period
With direct rotation	$\left\{ \begin{array}{l} 0^d000\ 0055 \\ \quad 000\ 0019 \\ \quad 000\ 0036 \end{array} \right.$	$\left\{ \begin{array}{l} 0^d000\ 0023 \\ \quad 000\ 0041 \\ \quad 000\ 0018 \end{array} \right.$
With retrograde rotation	$\left\{ \begin{array}{l} 000\ 0038 \\ 000\ 0100 \\ 0\ 000\ 0062 \end{array} \right.$	$\left\{ \begin{array}{l} 000\ 0041 \\ 000\ 0039 \\ 0\ 000\ 0080 \end{array} \right.$

drawn line in the figures connects the normal points, which are the mean of all observations between two measures of the comparison star. The dotted line connects uncertain observations. The observations on December 20 were interrupted from 5^h00^m to 5^h20^m U.T. The epochs of the extrema are found in Table 12. The period, determined by superimposing the light-curves of the two nights, is 0.11972 day; the intervals between the times of corresponding extrema give the periods shown in Table 13. This period, only 2^h52^m.2, is the shortest rotation period of an asteroid observed so far.

The investigation of this asteroid was undertaken to check one consequence of the collision theory: if the collision has taken place recently, then the rotation axis will not

¹ There is a correction to Paper II, p. 539. In Table 3 the April 10, 1953, II Min. should read 5^h47^m instead of 5^h37^m, and hence, on the second line from bottom: April 10, 1953, 5^h47^m $\pm$ 2^m. As a result, the computation on p. 540 needs revision.

have a fixed orientation in space but will show a rapid motion (Prendergast, unpublished). As the Coronis family is one of the youngest families, the possibility of finding this effect here was most favorable. The effect could not be demonstrated with certainty, although the varying height of the extrema might be explained in this way. A much more extended series of observations will be necessary to go beyond this general statement.

III. COLORS

Special color measurements were made during the observations for all objects studied. They are given in Table 1, under the headings $B-V$, $U-B$. The results confirm the

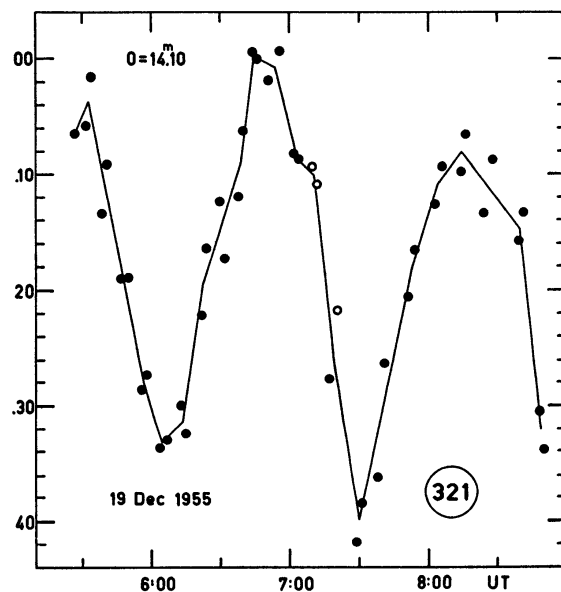


FIG. 25.—Light-curve of 321 Florentina

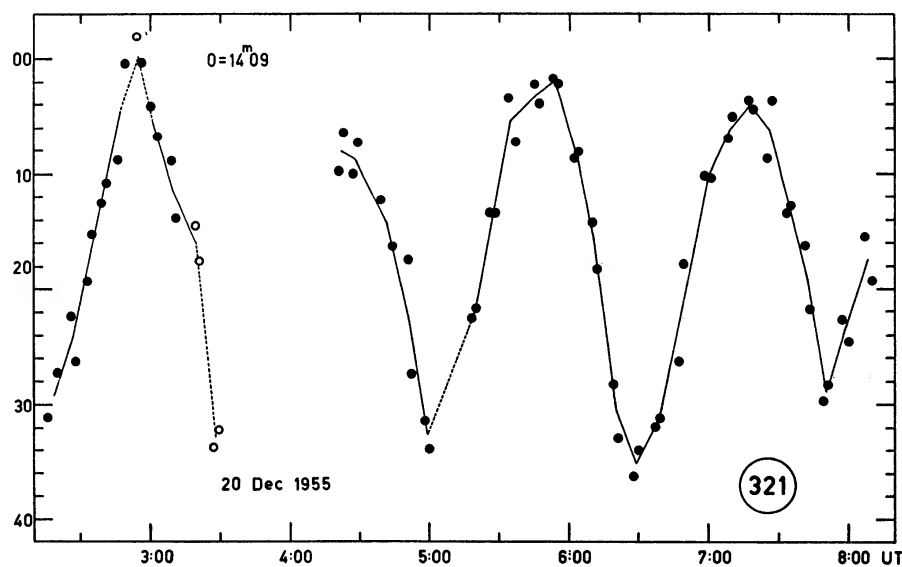


FIG. 26.—Light-curve of 321 Florentina

picture obtained earlier (see Fig. 18 of Paper II); most of the colors correspond to those of stellar spectral classes K0–K2 V. Only asteroid No. 16 has a slightly different color; it corresponds to spectral class G6 V, as did Nysa (Paper II). The colors of asteroids observed in different years agree well with each other.

IV. CONCLUDING REMARKS

The problems encountered in the study of asteroids fall into two groups: the derivation of the orbital elements; and data about the asteroids proper, such as period of rota-

TABLE 12
EPOCHS OF EXTREMA FOR NO 321

Date	U T	Type	JD*	JD(c)*
Dec. 19, 1955	$5^h 32^m \pm 3^m$	M II	460 7306	460 7199
	$6\ 05 \pm 2$	m II	460 7535	460 7428
	$6\ 49 \pm 2$	M I	460 7840	460 7734
	$7\ 30 \pm 2$	m I	460 8125	460 8018
	$8\ 17 \pm 3$	M II	460 8451	460 8345
Dec. 20, 1955	$2\ 55 \pm 3$	M I	461 6215	461 6108
	$5\ 51 \pm 3$	M I	461 7438	461 7331
	$6\ 29 \pm 2$	m I	461 7701	461 7595
	$7\ 18 \pm 2$	M II	461 8042	461 7935
	$7\ 50 \pm 2$	m II	461 8264	461 8157

* Add 2435000 days

TABLE 13
DETERMINATION OF THE PERIOD OF NO 321

Type	Period
Max II	$0^d 11932\ (9\ \text{cycles}) \pm 0^d 00042$
Min II	$11921\ (9\ \text{cycles}) \pm 00028$
Max I	$11964\ (7\ \text{cycles}) \pm 00049$
Max I	$11996\ (8\ \text{cycles}) \pm 00042$
Min I	$0\ 11980\ (8\ \text{cycles}) \pm 0\ 00028$
Average	$0\ 11957 \pm 0\ 00035$

tion, direction of the rotational axis, sense of rotation, scattering properties of the asteroid surface, shape of the asteroid. The observational data are the positions at different times, the light-curve, the changes in the light-curve with varying aspect, and the phase function.

The period of rotation is derived easily from the period of the light-curve; only a small correction is necessary, caused by the relative motion of earth and asteroid. But this relative motion may cause difficulties in determining the period; the combination of single observations made on different nights into one light-curve, as done in variable-star photometry, is usually not possible here because of varying distance and phase angle. Only when a full light-curve can be determined in a single night's run or in two successive nights, the period can be determined easily.

The orientation of the rotational axis can be determined from the changes in amplitude of the light-curve with varying aspect, as was done for Eros by Stobbe (1940) and Beyer (1953). Both authors made use of the micrometer measures of Van den Bos and Finsen (1931) in deriving the $(\Delta m, d)$ relation between the amplitude of light-curve and the inclination of axis; but for practically all other asteroids it is impossible to obtain such measures. The only way, then, is a trial-and-error method—to try different $(\Delta m, d)$ curves and choose the one that gives, within the limits of accuracy, a constant pole. In order to eliminate possible precessional motions of the rotational axis, it is advisable to make this determination from the data of one opposition only. It is clear that the observations must be extended over several months.

For the determination of the sense of rotation, the method followed in the case of Laetitia may be used. It was shown that an accurate knowledge of the direction of the rotational axis is necessary. Other factors may complicate the computation: (a) maximum light may not be reached at the times when the same meridian crosses the line of sight with varying inclinations of the rotational axis; (b) there are time shifts, as predicted by Huruhata (1940), for the case that the earth and the sun are not in the same asteroidal meridian plane. Knowledge of the scattering law of the asteroid surface would enable us to correct for this effect. An empirical determination for Eros was made by Huruhata himself, but, according to Stobbe (1941), the results are unreliable because of Huruhata's use of a wrong Eros pole. In this paper these effects are neglected.

The scattering law is needed also for a theoretical explanation of the phase function. But the latter is very incompletely known observationally. Usually a linear magnitude-phase relation is assumed, of the form $m = m_0 + \alpha\phi$ (Müller 1897). But recent results obtained by Haupt (1950) and Gehrels (1956) have shed doubt on this formula. Clearly, a thorough reinvestigation of phase functions is needed. Our experience is that a linear formula is satisfactory for small time intervals, and in most cases the values of Müller may still be used. But when large time intervals have to be bridged, difficulties arise.

The problem of the determination of the shape of the asteroid from its light-curve has not yet been solved. According to Russell (1906), no rigorous solution is possible; although this is undoubtedly correct, it may still be possible to construct a model that shows the essential features of the asteroid. But for such a computation it is necessary again to know the scattering law of the asteroid surface; moreover, one has to assume equality of albedo over the whole asteroid. Whether or not this is justified is not known.

It follows that knowledge of the scattering law is important; but this law cannot be determined from asteroid observations alone, as the light received from the asteroid gives an integrated effect over the whole asteroid surface. The best way to obtain such a relation is by observation of the moon. During a complete rotation of the moon, one surface element changes its position to the earth very little, but the direction to the sun changes widely. By taking many elements with the same albedo over the whole lunar surface, the dependence on the second parameter (angle between normal on surface element and line of sight) can be established. It is not certain, of course, that the lunar surface is optically similar to the asteroid surfaces or that the asteroids are mutually similar. Similarity of observed phase functions would strengthen this point.

It appears that determinations of the orientation of rotational axes and of reliable phase functions are at present the most urgent in asteroid physics. Both can be made during one opposition, by observing throughout the apparition. It is further important to extend our knowledge about asteroid rotational periods.

The authors wish to express their sincere thanks to Dr. Strömgren and the staff of the Yerkes observatory for their hospitality. They are particularly indebted to Dr. Kuiper for his encouraging interest and valuable discussions and for his observation of asteroid Iris. We wish to thank Dr. Hiltner for valuable aid with the various instruments. Thanks are due to Dr. Kienle, Heidelberg, for allowing a leave of absence to one of us, which made the

present investigation possible; and to Dr. Oort for his permission to finish this paper at Leiden. The observational work was carried out under grant NSF-G-1993 of the National Science Foundation.

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