THE VARIABILITY OF SUPERGIANTS*

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Received February 4, 1957; revised March 21, 1957

ABSTRACT

Eight A- and F-type I*a* supergiants, selected effectively at random, were all found to be variable in radial valocity. The variability is semiregular in the manner of the M-type semiregular stars. Periods derived from these eight stars and from published observations of similar stars show that they are functions of luminosity and spectral type. Several pieces of evidence suggest that the variability is due to pulsation. Values of the pulsation quantity, $P(\rho/\rho_{\odot})^{1/2}$, where $\rho_{\odot}$ is the mean solar density, are approximately constant for both early- and late-type semiregular supergiants and are equal within a factor of 2 to the value for the classical cepheids. A survey of variables in the upper part of the H-R diagram suggests that probably all stars brighter than $M_v = +1$ and to the right of the main sequence are variable. The values of the pulsation quantity for all these classes of variables differ from those for the classical cepheids by factors of only 2 or 3, except possibly in the case of the M giants.

I. INTRODUCTION

From the appearance of H-R diagrams of associations containing supergiants and from current theories of evolution, we may surmise that the evolutionary tracks of the stars at the bright end of the main sequence are roughly horizontal and toward the right. These tracks are bisected vertically by a region of instability, namely, that of the classical cepheids. Of the adjacent regions we know only that that of the late M stars is also one of instability (Stebbins and Huffer 1930). However, from the frequency of suspected variability among high-luminosity stars of all types in the Lick and Mount Wilson radial-velocity catalogues, one might suspect that, upon closer inspection, all these stars would be found to be unstable. In this paper we ask whether this is true, with the expectation from available data that the instability is less pronounced among the early- and late-type supergiants than among cepheids.

A sample of early- and intermediate-type supergiants has been investigated for variability in radial velocity, with the result that all were found to be variable in a semiregular manner. The cause of the variation has been considered, and it is concluded that it could well be due to pulsation of a complex character. A survey of variability in the upper part of the H-R diagram leads to several general conclusions.

II. RADIAL VELOCITIES

Nine A- and F-type supergiants were selected for observation with the coudé spectrograph of the McDonald 82-inch reflector. These included nearly all such stars of luminosity class I*a* as could be photographed at a dispersion of $8\frac{1}{2}$ A/mm with mean exposure times of less than $1\frac{1}{2}$ hours. α Persei, F5 I*b*, was to serve as a control, but its radial velocity was also found to be variable. Observations were made, whenever possible, at nightly intervals during a 1-month observing session.

Measurements of the spectra quickly showed that one obtains, in general, different radial velocities for lines of different ions and even for lines of different multiplets of the same ion. For HR 1040 the radial velocity-curves as obtained from λ 4028 and λ 4030 of Si II and from $\lambda\lambda$ 4508, 4515, 4520, and 4522 of Fe II are shown in Figure 1. In general, velocity-curves obtained from different metallic lines are similar in shape but may be displaced in velocity, This has been found to be true also in the cases of ϵ Aurigae (Struve

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[*] *Contributions from the McDonald Observatory,* No. 275.

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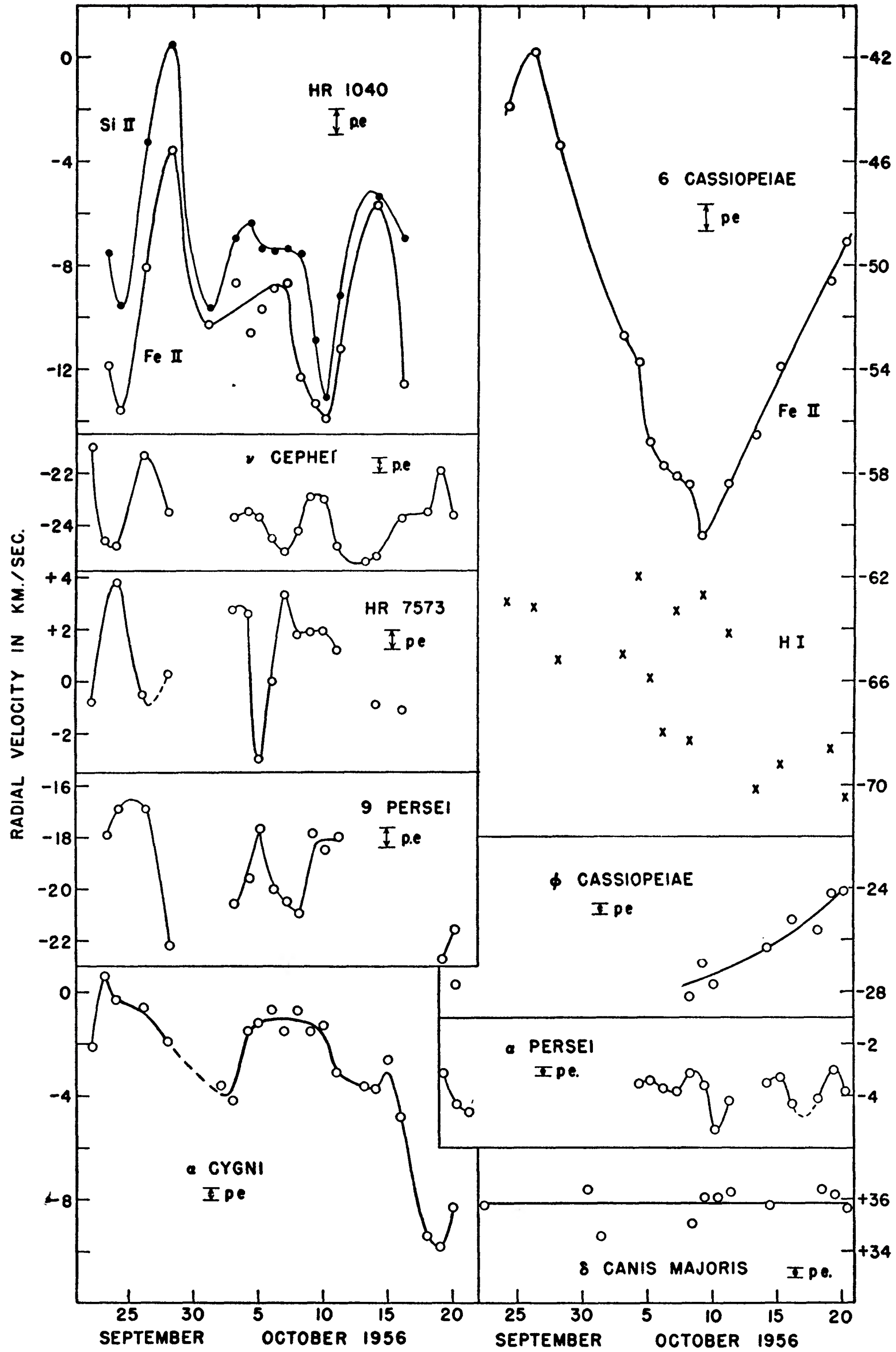

FIG. 1.—Radial-velocity measures for nine A and F supergiants. The times are in Universal Time, and the velocities refer to the mean of measures of λλ 4508, 4515, 4520, and 4522 of Fe II, unless otherwise indicated. The probable errors show the mean internal consistency per plate for each star.

and Elvey 1930) and β Orionis (Sanford 1947). The observed differences, which are of the order of 0.05–0.1 A, probably cannot be explained by uncertainties in the wavelength system used (namely, Moore 1945), since the amount and sometimes the sense seem to vary from star to star. For instance, for the Si II and Fe II lines mentioned above, the differences of the means, $\bar{V}$ (Si II) $-$ $\bar{V}$ (Fe II), for the stars HR 1040, 9 Persei, 6 Cassiopeiae, HR 7573, and α Cygni are +2.8, +2.0, +0.7, +0.6, and 0.0 km/sec, respectively. The lines appear to be symmetrical at all times.

However, the hydrogen lines sometimes show a different variation with time from that of the metallic lines. For two stars (6 Cassiopeiae and ν Cephei) velocities were obtained for $H\gamma$ from measures of both the center of the line and the mid-point between the edges of the core. These measures are consistent in showing that in these two stars the core of $H\gamma$ is always displaced shortward of the metallic lines and shows little or no variation with time (see 6 Cassiopeiae in Fig. 1). Unless otherwise indicated, the velocity-curves in Figure 1 were derived from lines of Fe II only.

The estimated probable errors shown for each star in Figure 1 were obtained from the mean internal error per plate. For one star, δ Canis Majoris, for which the measures in Figure 1 show no significant variation during the interval of 1 month, we may compare the estimated probable error, namely, 0.39 km/sec, with the scatter of the velocities computed as a probable error, which is 0.37 km/sec. The conclusion is that the estimates of probable error are realistic.

The velocity variations shown in Figure 1 are in excess of their probable errors in all cases except for α Persei and δ Canis Majoris. For the former, the approximate period derived later is in agreement with that obtained by two other observers. For δ Canis Majoris, Wright (1910) found a variable velocity with a range of 3 km/sec and a period of about 9 months, so it is not surprising that the present observations taken during the course of 1 month failed to show a significant variation.

The conclusion is that all the A- and F-type supergiants investigated are variable in radial velocity. To this we shall add published results on two late B-type supergiants. Therefore, it seems likely that all early- and intermediate-type supergiants of luminosity class I*a* and perhaps also those of class I*b* are velocity variables. The variations are semi-regular in manner, and α Cygni seems to be a fairly typical example. The variability of this star has been known for many years, primarily from the extensive work of Paddock (1935). He has shown that α Cygni has a dominant variability with a period of 11.7 days plus probably smaller fluctuations with different periods. From his observations of twenty cycles we conclude that the dominant period is moderately constant in duration and can be determined from a single cycle with a probable error of only ±13 per cent, whereas the amplitude is much more variable, giving a probable error of ±50 per cent per cycle. Therefore, in the following discussion of the velocity-curves of Figure 1 an attempt will be made to obtain periods of the dominant variation. We expect that for half the stars we shall obtain values that are in error by more than about ±13 per cent, since generally only one or a few cycles have been observed for each star. However, this may not be our largest source of error. In the succeeding analysis we shall need absolute luminosities which are not likely to be within 20 per cent (0.2 mag.) of their actual values.

The results of the next section are contained in Table 1 and the last two paragraphs of the section.

III. PERIODS

HR 1040, A0 Ia.—There are maxima (Fig. 1, Fe II) on September 28.4 and October 13.8, with probably one in between. Assuming two cycles to have elapsed in 15.4 days, the period is 7.7 days. The velocity was first found to be variable by Young (Moore 1911).

ν Cephei, A2 Ia.—Maxima were observed on September 26.3 and October 4.2, 9.6, and 19.2. Since three cycles occurred in 22.9 days, P = 7.6 days.

HR 7573, A2 Ia.—Times of $V = +1$ km/sec on descending branches of the velocity-curve occurred on September 25.7 and October 4.5 and 11.2. Two cycles in 15.5 days yield $P = 7.7$ days.

9 Persei, A2 Ia.—The observations are insufficient for a determination of the period. Maxima are observed on September 25.4 and October 5.3, but it is not clear whether or not a maximum occurred between these times. $P \leq 9.9$ days.

α Cygni, A2 Ia.—Prominent minima were observed on October 3.1 and 18.8, an interval of 15.7 days. For the first and incompletely observed cycle, we observed times of $V = -2$ km/sec on rising branches on September 22.3 and October 4.2. We would have decided upon a mean period of 13.8 days from the present observations, but we shall use Paddock's value of 11.7 days.

6 Cassiopeiae, A3 Ia+.—Less than one cycle was observed, but a slight extrapolation yields $P = 30$ days.

φ Cassiopeiae, F0 Ia.—The observations are insufficient to determine a period. The velocity was announced as variable by Adams, Joy, and Sanford (1924).

α Persei, F5 Ib.—Successive minima were observed on October 5.1 and 8.7, an interval of 3.6 days, and on October 15.0 and 19.4, an interval of 4.4 days. These suggest a mean period of about 4.0 days. Many other observers have studied α Persei. Most of these, e.g., Pitman (1912), were looking for a regular variation and tried to find a period by a superposition of different cycles. When this proved to be unsuccessful, as we might expect from an inspection of Figure 1, they concluded that the velocity was constant. Hnatek (1912) concluded that $P = 4.09$ days. From scattered observations, Newall (1900) suggested $P = 4.20$ or 16.8 days but later was unable to superimpose additional observations onto his curve. We tentatively conclude that $P = 4.0$ days.

δ Canis Majoris, F8 Ia.—The few observations in Figure 1 show no significant deviation from constancy during 1 month. Wright's (1910) suggestion for a period of 9 months should be checked.

A survey was made for high-luminosity stars with variable radial velocity in the catalogues of the Lick Observatory (Campbell and Moore 1928; Moore 1932) and by Wilson (1953). Many such stars were found, but only for the following cases have periods been published. These stars are distinguishable from binaries primarily by the irregularities in their velocity-curves. Furthermore, in some cases the published period is too short to be compatible with even a contact binary system.

σ Cygni, B9 Iab.—The spectrographic observations of Henroteau (1917) show a mean period of about 11.1 days. This has been confirmed at the Lick Observatory (Herbig 1956), although there is considerable scatter and variations of a different cycle length may well be superimposed.

β Orionis, B8 Ia.—The period by Plaskett (1909) is 21.9 days, and, although he did not publish his individual velocities, the shortness of this period seems to rule out binary motion. Scattered observations by Sanford (1947) failed to fit Plaskett's orbit but do confirm the existence of a semiregular variability of about Plaskett's period.

ε Aurigae, F0 Iap.—Ludendorff (1924), Struve and Elvey (1930), and McLaughlin (1934) have shown that the velocity-curve consists of a semiregular fluctuation with a mean period of about 110 days superimposed on the 27-year binary motion.

89 Herculis, F2 Ia.—Böhm-Vitense (1956) has found a semiregular velocity fluctuation with a period of about 70 days.

A few of these stars have been observed to vary in light. Brodskaya (1951) reports both β Orionis and α Cygni to be light-variables. In the observations of α Cygni by Fath (1935), of ε Aurigae by Huffer (1932), and of 89 Herculis by Worley (1956) the light-fluctuations are semiperiodic, with mean periods that are in agreement with those determined spectrographically. For these three stars the light-fluctuations have mean ranges from 0.05 to 0.10 mag., whereas the corresponding ranges in radial velocity are 8–15 km/sec. The ratio of these, namely, 150 km/sec per mag., is intermediate between the corresponding ratio of 600 for β Canis Majoris stars (Struve 1955), which are of earlier spectral type, and 35 for cepheids (Eggen 1951), which are later.

From an inspection of the periods in Table 1 we see that the periods within a given box are rather similar but that there tend to be large differences from box to box. We conclude that the period of a star is determined largely or entirely by its luminosity and spectral type and that the period increases with both luminosity and spectral type.

IV. LUMINOSITIES AND MASSES

For further discussion we shall wish to have accurate luminosities. Since the luminosity class among supergiants indicates the absolute magnitude to within only about 1 mag., absolute magnitudes were obtained from membership in associations and from hydrogen-line strengths calibrated from stars in associations.

The first six stars in Table 3 are probably members of the associations (Morgan, Whitford, and Code 1953) listed in the fifth column. The spectral classification is by Morgan, Code, and Whitford (1955). The distance moduli of the associations were determined from the remaining members (Morgan, Whitford, and Code 1953), using the photometry of Hiltner (1956) and the luminosity calibration by Johnson and Hiltner (1956). 9 Persei, although probably a member of h and χ Persei, has a velocity 20 km/sec greater than that of the cluster. Its luminosity was determined from hydrogen-line strengths.

TABLE 1

PERIOD IN DAYS OF SEMIREGULAR EARLY-TYPE SUPERGIANTS

LUM CLASS	SPECTRAL-TYPE GROUP			
	B8–B9	A0–A3	F0–F2	F5–F8
I*a*+		6 Cas 30	.	.. .
I*a*	β Ori 21 9 σ Cyg 11 1	HR 1040 7 7 ν Cep 7 6 HR 7573 7 7 9 Per 9 9 α Cyg 11 7	ϕ Cas ... ϵ Aur 110 89 Her 70 . . .	δ CMa 270: .. . .
I*b*		.. ..	. . . . .	α Per 4 0

It was found that for A-type supergiants the hydrogen-line strengths are extremely sensitive to absolute magnitude. Hδ was selected for measurement because it is completely free of blends and at a wave length where the continuum is very level. Spectrograms of $8\frac{1}{2}$-A/mm dispersion of stars in h and χ Persei and other associations were obtained for calibration purposes. The stars are listed in Table 2 and the calibration is shown in Figure 2. We assumed a true distance modulus of 11.8 for h and χ Persei (Johnson and Hiltner 1956). For spectral types, photometry, and luminosity calibration we used the sources referred to in the previous paragraph. The calibration then yielded the luminosities of the last three stars in Table 3. The early A stars in Table 3 yield the period-luminosity relation shown in Figure 3.

The masses of supergiants and giants are not reliably known. Furthermore, the mass-luminosity relation (Kuiper 1938*b*) is defined only for stars on the main sequence. It will be necessary to invoke some evolutionary scheme to obtain masses of supergiants and giants from those of main-sequence stars. The current theories of stellar evolution state that stars at the bright end of the main sequence evolve into the region of the supergiants

TABLE 2

STARS USED FOR A LUMINOSITY CALIBRATION

Assoc	Distance (parsecs)	Star	Spectral Type	M_v	Hδ E W (A)
I Per	2290	HD 12953	A1 I*a*	−7 9	1 7
		14433	A1 I*a*	−7 1	2 5
		13476	A3 I*ab*	−6 8	3 2
		15316	A3 I*ab*	−6 6	3 7
		17378	A5 I*a*	−7 8	3 6
I Cam	770	HR 1040	A0 I*a*	−6 6	1 6
I Cep	680	ν Cep	A2 I*a*	−6 3	3 2
I Cas	2510	6 Cas	A3 I*a*+	−8 3	2 0
II Cep	3450	HD 213470	A3 I*a*	−7 4	2 9

TABLE 3

EARLY- AND INTERMEDIATE-TYPE SUPERGIANTS WITH INDIVIDUAL LUMINOSITIES

Star	Spectral Type	Period (days)	M_v	Source	Q (days)
β Ori	B8 I*a*	21 9	−8 6	Mem. in I Ori	0 073
σ Cyg	B9 I*ab*	11 1	−6 4	Mem. in IV Cyg	110
HR 1040	A0 I*a*	7 7	−6 6	Mem. in I Cam	060
ν Cep	A2 I*a*	7 6	−6 3	Mem. in I Cep	054
6 Cas	A3 I*a*+	30	−8 3	Mem. in I Cas	060
α Per	F5 I*b*	4 0	−4 5	Mem. in α Per cluster	028
HR 7573	A2 I*a*	7 7	−6 8	Hδ E.W.=3 1 A	040
9 Per	A2 I*a*	≤9 9	−7 4	Hδ E.W.=2 6 A	037
α Cyg	A2 I*a*	11 7	−7 5	Hδ E.W.=2 4 A	0 042
Mean					0 062 ±0 017 p.e.

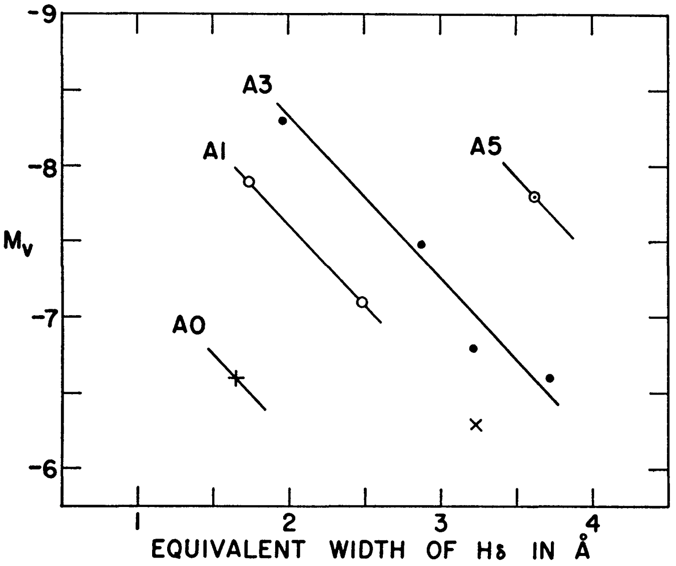

FIG. 2 —Luminosity calibration from hydrogen-line strengths. Plotted vertically are absolute visual magnitudes. The various symbols differentiate stars of different spectral types; the isolated *x* refers to a star of spectral type A2.

with only small changes in bolometric magnitude. To find out where individual supergiants originated on the main sequence, it will be necessary to trace the evolutionary tracks. These are not at present well determined from theory, particularly in the case of the more massive stars ($m > 10\ m_{\odot}$), so we shall resort to cluster color-magnitude diagrams. Such diagrams are instantaneous loci of stars on numerous evolutionary tracks and hence may not coincide with individual evolutionary tracks. However, in view of the finding by Sandage and Schwarzschild (1952) that the time required for a star to move across the color-magnitude diagram is only about one-twentieth of its lifetime near the main sequence, we conclude that supergiants in clusters originated from very nearly the break point of the main sequence. Two clusters with supergiants have

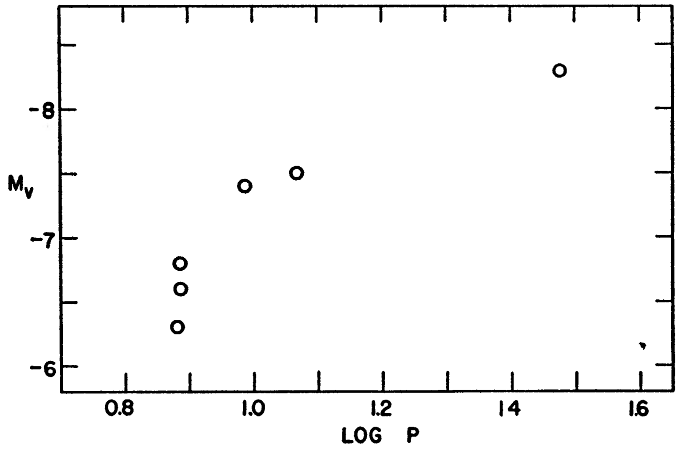

FIG. 3.—The period-luminosity relation for A0–A3 supergiants

TABLE 4

EVOLUTIONARY LUMINOSITY CHANGES FOR SUPERGIANTS

Spectral Class	M_{bol} (Present) − M_{bol} (Initial)	Spectral Class	M_{bol} (Present) − M_{bol} (Initial)	Spectral Class	M_{bol} (Present) − M_{bol} (Initial)
Late B, A	−0 8	F . . .	−1 4	M ..	−1.6

published photoelectric photometry: h and χ Persei (Johnson and Morgan 1955; Johnson and Hiltner 1956) and the α Persei cluster (Harris 1956*a*). By fitting the zero-age main sequence (Sandage 1957) to the B stars in the α Persei cluster and comparing with Taylor's (1954) evolutionary tracks, it is found that the brightest B star whose membership in the cluster is generally conceded originated at $M_v = -1.2$ and $B-V = -0.20$. With this as the original position of α Persei, F5 I*b*, its increase in M_{bol} has been 1.4 mag.

For h and χ Persei this procedure is much less certain, since the location of the zero-age main sequence is not known above $M_v = -2$ and evolutionary tracks are not available for $m > 10\ m_{\odot}$. By extrapolation, it will be assumed that these supergiants originated from a region at $M_v = -4$ and $B-V = -0.3$. The evolutionary luminosity changes so derived are listed in Table 4 and will be adopted. These changes will be subtracted from the present M_{bol} of supergiants before applying Kuiper's mass-luminosity relation (1938*b*) in the form

$$\log\left(\frac{m}{m_{\odot}}\right) = 0.51 - 0.113 M_{\text{bol}}\,(\text{initial})\,. \tag{1}$$

The adoption of the luminosity increases in Table 4 causes a decrease in log m by 0.18 at the most, or a decrease of ≤ 23 per cent in $\rho^{1/2}$, where ρ is the mean stellar density.

The question as to whether stars exist with masses much greater than, say, 10 $m_{\odot}$ cannot be settled with the present data. Although it is true that such a limit would produce better agreement in the Q-values (see below) of the most luminous supergiants of this group, namely, β Orionis (32 $m_{\odot}$) and 6 Cassiopeiae (26 $m_{\odot}$), with the others (minus σ Cygni), these data are probably not accurate enough to draw any convincing conclusions in this regard.

V. PERIOD-DENSITY RELATION

If these stars are a homogeneous group of pulsating stars, then the pulsation quantity $Q = P(\rho/\rho_{\odot})^{1/2}$, where $\rho_{\odot}$ is the mean solar density, should be a constant for the group and roughly equal to the theoretical value. Mean densities were obtained for the stars in Table 3 from their absolute magnitudes through the use of (1) the bolometric corrections of Kuiper (1938*a*), (2) the temperature calibration of Keenan and Morgan (1951), and (3) the mass calibration of the previous section. We are not able to estimate the possible error involved in the application of Kuiper's bolometric corrections to stars as luminous as some of these. In the case of α Persei the temperature calibration by Keenan and Morgan for supergiants was not used but rather that for main-sequence stars, in view of the discovery of 0.12-mag. reddening for this star by Harris (1956*b*). The results tabulated in Table 3 are plotted in Figure 4. For comparison, the theoretical and observed values for the classical cepheids have been drawn in. The former is for a model of polytropic index 3, pulsating in the fundamental mode. It was computed by Epstein (1950), who found it to be insensitive to a wide range of central concentrations of the assumed models. The latter comes from a determination by Kraft (1953) after the change in zero point of the cepheid period-luminosity relation. Kraft used Kuiper's mass-luminosity relation directly without allowing for possible evolutionary changes in M_{bol}. For consistency with our previous treatment of other supergiants and in accordance with Table 4, we shall decrease Kraft's Q by 17 per cent to 0.027 day.

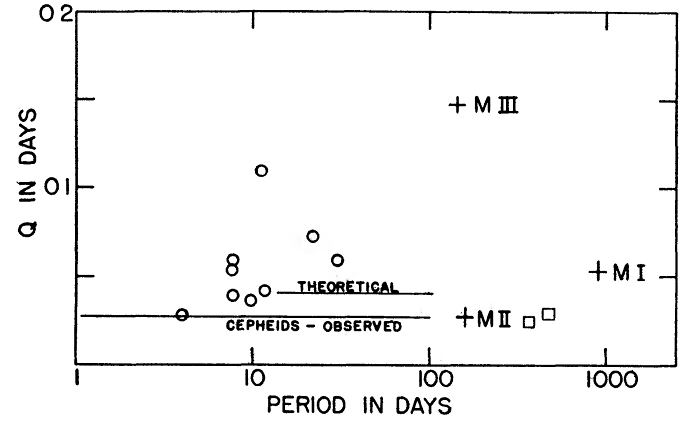

FIG 4.—The pulsation quantity $Q = P(\rho/\rho_{\odot})^{1/2}$ as computed for individual B-, A-, and F-type supergiants (*circles*). The squares represent two semiregular M stars that are members of h and χ Persei. The three crosses represent semiregular M stars, grouped according to luminosity class, whose individual absolute magnitudes are not known. The lines represent the theoretical and observed values for the classical cepheids.

The values of Q for the early-type supergiants all lie close to the theoretical value, except possibly that for σ Cygni. The absolute magnitude of this star is not well determined, since its association has only four known members and three-color photometry is

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not available for them. However, the derived absolute magnitude is consistent with its luminosity classification, and it is unlikely that it is too faint by the 1.4 mag. required to produce agreement with the other stars.

The semiregularity of these early-type supergiants leads us to wonder whether they could be early-type counterparts of the semiregular M stars. The light-curves of the two groups are similar in character. We shall now consider their values of Q. Table 5 gives data on two semiregular M stars with known periods and with individual instantaneous absolute magnitudes from membership in h and χ Persei (Blanco 1955). Table 6 gives mean data for stars for which only luminosity classes by Keenan (1942) are known. All the stars in Keenan's list with known periods have been included. The assumed luminosities were taken from the calibration by Keenan and Morgan (1951) and Johnson and Hiltner (1956). We used the temperature calibration of the former reference and the mass calibration of the previous section for the supergiants and bright giants (lum. class II). The bolometric corrections by Kuiper (1938*a*) had to be extrapolated in spectral type and luminosity, causing possible errors of perhaps 1 mag. The resultant values of Q are shown in Figure 4, where the two squares represent the stars of Table 5 and the three crosses those of Table 6. The values of Q for the M supergiants and bright giants agree, well within our accuracy, with the values derived from the early- and intermediate-type semiregular stars. We conclude that the variability of these two groups of stars is similar.

TABLE 5

SEMIREGULAR M STARS WITH INDIVIDUAL LUMINOSITIES

Star	Spectral Type	M_v	Period (days)	Q (days)
BU Per	M4 Ib	−4 3	365:	0 023
SU Per	M3 Iab	−5 4	477	0.028
Mean		..	. .	0 025

TABLE 6

SEMIREGULAR M STARS WITHOUT INDIVIDUAL LUMINOSITIES

Luminosity Classes	Spectral Types	No of Stars	Mean Period (days)	Mean Q (days)
I*a*, I*b*	K3–M4	4	882	0 053:
I*b*-II, II	M4–M6	5	159	.027
II-III, III, III-IV	M0–M6	27	193	0 147:

The masses of M III stars are not known from binaries, and these stars have not been observed in galactic clusters, except perhaps in M11 (Johnson, Sandage, and Walquist 1956). If they evolved from a Hyades-Praesepe–type cluster, then they would have experienced very little change in luminosity and their masses are about 7 solar masses. On the other hand, if they evolved according to an M67-type cluster, their luminosity has increased greatly, and their masses are about 1 $m_{\odot}$. Sandage (1957) has recently investigated the origins of early K giants and concluded that, whereas stars from a long section of the main sequence (late B to late F) have been funneled into the region of the giants, the bulk of the K giants came from early F dwarfs. Considering the relative contributions of the various original dwarfs to the present K giants, one obtains an average mass for the latter of 1.6 $\mathfrak{m}_\odot$. For want of better data, we shall assume that M giants originate from K giants, so that this is also the mean mass of M giants.

The mean value of Q for the 27 M giants in Table 6 was computed as in the case of the supergiants, except that it was assumed that all giants have a mass of 1.6 $\mathfrak{m}_\odot$. This Q is about three to five times larger than for other groups of stars. To explain this difference as an error in the observations would require that the M_v's or the bolometric corrections be too faint by 1.6 mag. The mean mass of 1.6 $\mathfrak{m}_\odot$ cannot be too high by very much, certainly not by the factor of 9 required to produce agreement in the Q's. So either the mean M_{bol} of M giants is about $-4\frac{1}{2}$, or there is a real difference in Q between M giants and M supergiants. It should be pointed out in passing that, if we had assumed constancy of M_{bol} during evolution, the resultant mean mass of 7.2 $\mathfrak{m}_\odot$ would yield a Q that was about six to eleven times larger than for other stars.

VI. NATURE OF THE VARIABILITY

Is the variability of the semiregular stars due to pulsation or to random atmospheric inhomogeneities like turbulence, prominences, or other types of mass motion? In the latter case the chance occurrence of an eddy or streamer with an unusually large Doppler displacement would produce an asymmetrical broadening of the spectral lines and hence a variable velocity. The line widths would generally be variable, whereas for pulsation they would be constant within our accuracy.

To help answer this question, observations of α Cygni at a dispersion of $4\frac{1}{2}$ A/mm and a resolution of 0.07 A were obtained. Figure 5 shows the profiles of λ 4233 Fe II obtained at two times when the radial velocities differed by 9.2 km/sec. The profiles fit best the notion of constant line width during velocity variation. A similar conclusion was reached by Adams (1956) for the semiregular M supergiant α Orionis. However, since the cause of the large breadths of lines in supergiants is obscure at the present time, we cannot be very positive when discussing the constancy of their breadths.

There are other conditions which must be met by a pulsating star. Whether or not they are satisfied by stars in which turbulence or atmospheric mass motion is the dominant cause of the variability cannot be stated without specific models. One such condition is the constancy of Q, which we have seen to be obeyed by the semiregular supergiants from late B to late M. Another condition is the rough equivalence of Q with expected values. Epstein (1950) has shown that the pulsation properties of variables depend on the effective polytropic index and the mode of oscillation, with resulting values of Q that vary by factors of 2 or 3 from the theoretical value given in Figure 4. The values of Q for the semiregular supergiants are all within this range. It should be remarked, however, that other situations can lead to a relation of the type $P(\rho)^{1/2} =$ Constant. Consider, for example, stars in which particles are ejected at a given fraction of their escape velocity and travel a given fraction of the star's radius. Such situations, though, are likely to lead to values of Q that differ from these by several orders of magnitude.

A fourth condition for pulsation is a correlation in phase of light- and velocity-curves. Such a correlation seems to exist in simultaneous photoelectric and spectrographic observations (Fath 1935). McLaughlin (1930) states that the phase shift between light- and velocity-curves for ϵ Aurigae are those to be expected for pulsating stars of that period. The evidence of this section favors a pulsation explanation for the velocity and light-variations of semiregular stars.

VII. A SURVEY OF VARIABILITY IN THE UPPER PART OF THE H-R DIAGRAM

The known classes of variable stars brighter than $M_v = +1$ have been plotted in Figure 6. The horizontal scale is one of reciprocal temperature. The main sequence has been drawn according to the luminosity calibration of Johnson and Hiltner (1956). The rectangles indicate the limits of individual classes, based on limited material. The numbers near the names give the ratio of Q for that class to Q ($=0.027$ days) for the classical cepheids. The theoretical value of Q for the cepheids is 1.5 times the observed value. For the population II cepheids and the RR Lyrae stars we used a mass of 1.25 $m_{\odot}$ (Sandage 1956).

The periods, luminosities, and spectral types of the known β Canis Majoris stars have been given by Struve (1955). Although these stars lie to the right of the bright end of the main sequence, the position of the latter at age zero is not known at this time. Therefore, we are not able to determine with accuracy the evolutionary paths and hence their masses. We shall simply assume that the luminosities of β Canis Majoris stars have increased by 0.5 mag. and then compute their masses from equation (1). Employing, as in Section V, the bolometric corrections by Kuiper (1938*a*) and the temperature-spectrum relation by Keenan and Morgan (1951), a mean Q-ratio of 0.9 was obtained for this group.

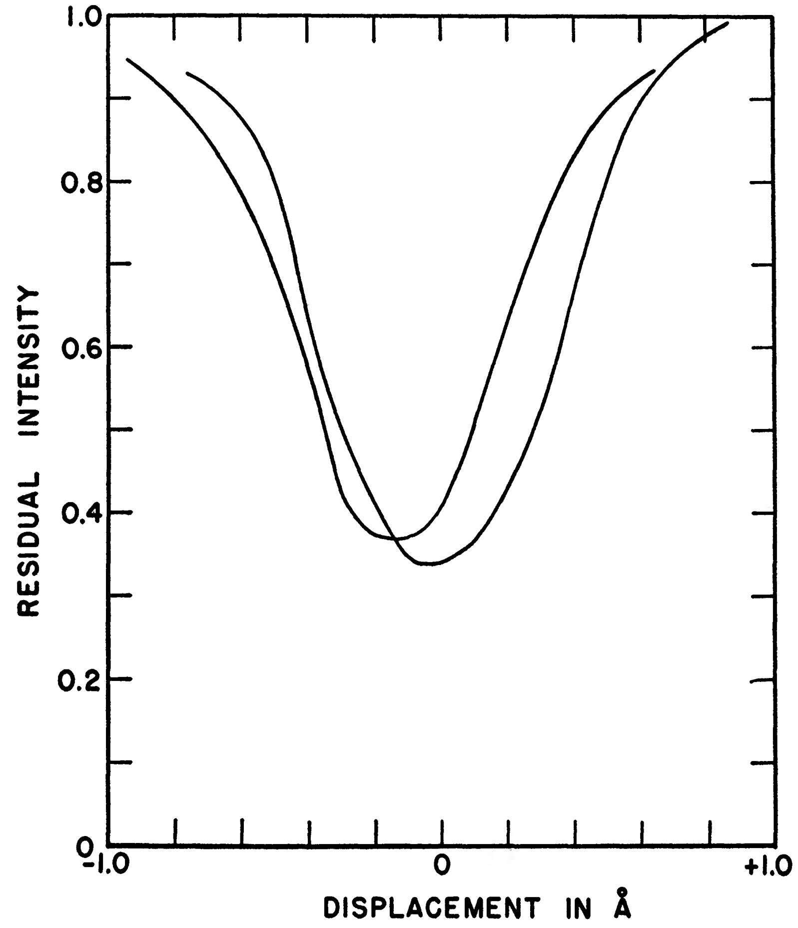

FIG. 5.—Profiles of λ 4233 Fe II on October 6.14 (*right*) and 19.24 (*left*), 1956. The abscissae are Doppler shifts from the laboratory position after correction for the earth's annual motion along the line of sight at the times of observation.

The Q-ratios for the M semiregular variables come from Tables 5 and 6. The period-spectrum-luminosity relation for the long-period variables (LPV) has been tabulated by Payne-Gaposchkin (1954). In this case we are even more uncertain about the masses because the sequence extends from the region of M II stars, which have masses of about 11 $m_{\odot}$, to that of the M III stars, whose mean mass is assumed to be 1.6 $m_{\odot}$. The Q-ratio varies from 0.6 $(m/m_{\odot})^{1/2}$ for the brighter members of the sequence to 1.2 $(m/m_{\odot})^{1/2}$ for the fainter ones. If we accept these masses, the Q-ratio varies from 2.2 for the brightest LPV to 1.5 for the faintest. Here again we should point out that if we had used the mass-luminosity relation directly without assuming evolutionary changes in luminosity, the LPV would have the rather large Q-ratios of 2.5–4.4.

Arp's (1955) data on the variables in globular clusters have been used for the population II cepheids. In keeping with out treatment of other variables, we have used the short periods for the four stars in question in Arp's paper. If we had used the long periods, the Q-ratio for the population II cepheids would have taken on two values, 2.6 and 5.2. Sandage (1956) has obtained good agreement between the observed and theoretical values of Q for the RR Lyrae stars of Bailey types *a* and *b*; for Bailey type *c* the value of Q is 1/1.5 of this (Roberts and Sandage 1955).

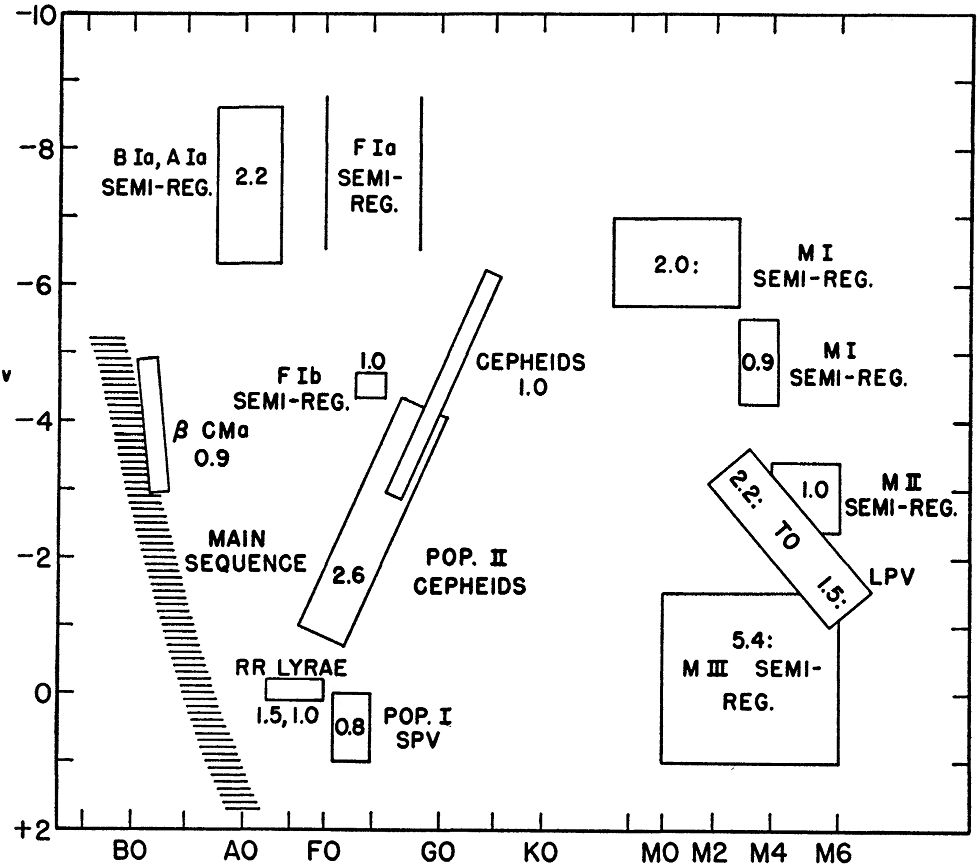

FIG. 6.—The upper part of an H-R diagram, showing classes of variable stars. The horizontal scale is one of reciprocal temperature. The numbers near the class names are the ratio of Q for the class to that for the classical cepheids.

The data on the population I short-period variables (SPV) is by Eggen (1957); we assumed an absolute magnitdue of $M_v = +0.5$ for the class. Kushwaha (1957) has shown that stars of this mass (about 2.5 $m_{\odot}$) increase their luminosity by only about 0.2 mag. while moving from the main sequence to late A; we shall assume a change of 0.3 mag. during evolution to a mean spectral type F2 III.

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It appears possible from Figure 6 that all supergiants and giants are variable. Some additional regions should be investigated. It is known (see, e.g., Herbig and Spalding 1955) that at least a large percentage of the G and K giants and bright giants are variable in radial velocity with a range of several kilometers per second. Velocity-curves and mean periods should be obtained for these stars. Also stars on the main sequence should be investigated for variability. Fath (1935) and Neubauer have shown that α Lyrae, A0 V, is probably variable in light and velocity. The ranges are small (0.03 mag. and 1 km/sec), but the phase correlations are very good. The mean period appears to be roughly 0.07 days, yielding a Q-ratio of 1.1.

In Figure 6 there is a marked tendency for the Q-ratios to be grouped around 1.0 and 2.0. This may be accidental, since an error in bolometric magnitude of 0.8 mag., which is a distinct possibility in some cases, causes a change in the Q-ratio by a factor of 2.

Multiple periods are common among the β Canis Majoris, RR Lyrae, and semiregular M stars. It may be that the cause of the irregularity and of the small ranges among semiregular variables is the superposition of several modes of oscillation. In some cases this has been demonstrated (e.g., Arp and Wallerstein 1956). The classical cepheids, which are almost perfectly regular and have relatively large ranges, probably have no multiple periods due to pulsation (Herbig and Moore 1952).

VIII. CONCLUSIONS

1. Probably all the stars in the region of the H-R diagram above $M_v = +1$ and to the right of the main sequence are variable in light and radial velocity.
2. The variability has the largest range and is most regular in behavior in several well-defined sequences in this region (e.g., β Canis Majoris stars, cepheids, LPV).
3. The variability may well be due to pulsation in some form.
4. The values of the pulsation quantity, $Q = P(\rho/\rho_{\odot})^{1/2}$, agree with that of the classical cepheids within factors of about 2 or 3 except possibly in the case of the M giants.
5. The variability is most pronounced in light on the right side of the H-R diagram and in velocity on the left.

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