THE EARLY PHASES OF STELLAR EVOLUTION

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INTRODUCTION

Comparatively little effort has been directed to the study of the early gravitational phases in stellar evolution. Since during the last few years considerable interest has arisen in the study of what appear to be recently formed groups of stars, added significance may be given to theoretical investigations concerning the early life of a star. The following results deal with the pure gravitational contraction of a stellar configuration and its transition to one deriving its energy from thermonuclear sources.

THE CALCULATIONS

A complete discussion of the technique of calculation will be given elsewhere[1] and only a brief summary is needed in connection with the following discussion. A few remarks are first given concerning the physical effects allowed for in the work and then a summary of the mathematical representation and numerical treatment is presented.

The gas is considered as ideal and the radiative contributions to pressure and internal energy are disregarded. The calculations automatically allow for changes in the gas constant resulting from thermonuclear changes of composition, although in the cases presented here these never became significant.

The opacity we have used is a composite of the Morse[2] values for the Russell mixture, represented by a reasonably good interpolation formula, free-free contributions of hydrogen and helium, both with unit Gaunt factors, and electron scattering. Energy transport by convection is included in the event that the central regions become unstable. No allowance has been made, however, for convection associated with the ionization of hydrogen and helium in the outer layers.

The calculations include energy production by both the *CN* cycle and the p-p chain, each of these being represented by reasonably accurate interpolation formulae. (A summary of such formulae for reaction rates and opacity is in preparation and will be published separately.) The depletion of hydrogen, as well as the resulting increase in helium content, is considered as purely local except in the convective core where the composition is modified homogeneously. Actually, no significant depletion is encountered in the results reported here.

The conversion of gravitational energy is treated by including in the mass energy generation rate the contribution

$$-\left(\frac{\partial E}{\partial t}+P\frac{\partial V}{\partial t}\right),$$

where E is the specific internal energy, V the specific volume, and P the pressure. The derivatives are time rates.

In the calculations difference equations were used, replacing the spatial and time differential operators by appropriate difference operators. In effect, the star is regarded as consisting of a large number of concentric shells, the properties of which enter directly into the difference equations. These were solved by an iterative procedure which is essentially a multidimensional generalization of Newton's method for finding the root of a function. The numerical work was done with the UNIVAC installation of the Atomic Energy Commission at the Livermore site of the Radiation Laboratory of the University of California. The convenient and practically unlimited storage on tape provided by this machine makes it ideally suited for a problem of the great complexity encountered in the automatic calculation of stellar evolution.

THE RESULTS

Figures 1, 2, and 3 display the theoretical tracks in the H-R diagram for a number of starting configurations. These are defined by the mass in terms of the sun; the proportions by mass of hydrogen, X; helium, Y; and heavy mixture, Z; and an initial quasi-equilibrium configuration. This was obtained in each case by a general homology transformation from some other configuration for some case which had already been calculated. The very first of these was similarly derived from a gravitational model calculated by Levée.[3] The method used in starting necessarily leads to the introduction of certain transients which quickly disappear. Actually without an acceptable and detailed theory of star formation it is difficult to formulate an unambiguous prescription for establishing a starting configuration. Fortunately, some check calculations indicate that a considerable latitude in choice leads after a few time steps to very nearly the same results.

Before we discuss the separate tracks individually, a few gen-

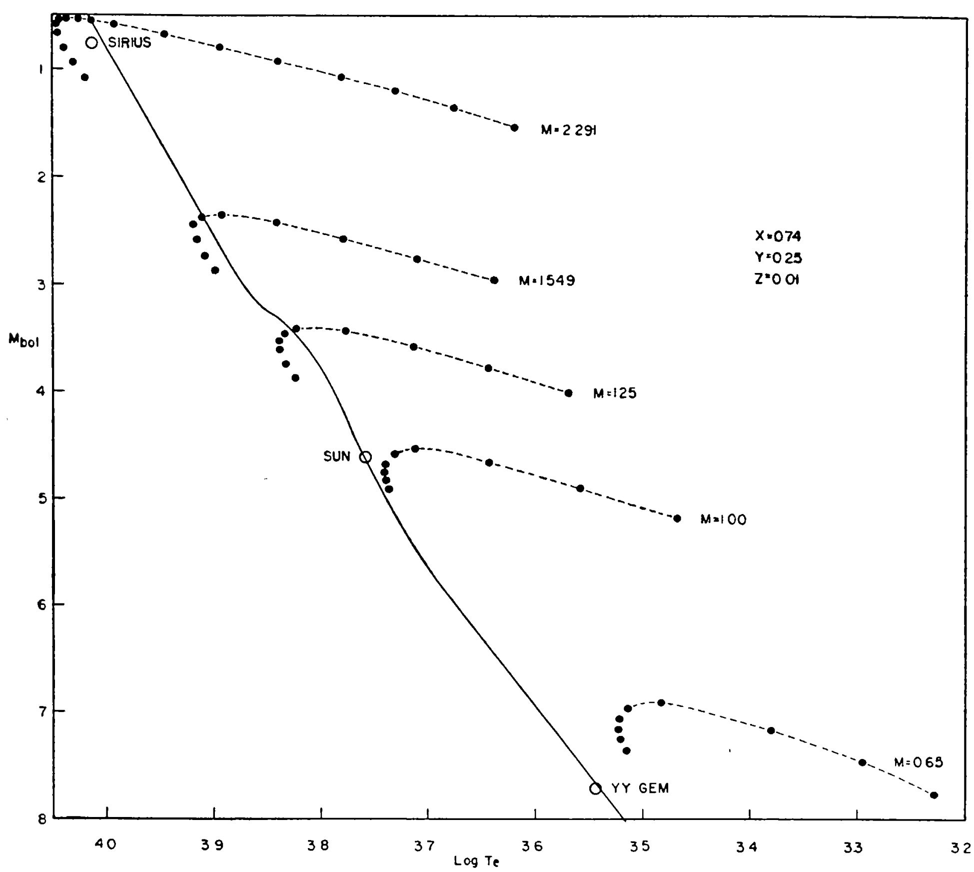

FIG. 1.—Theoretical evolution tracks for different masses.

eral points concerning common characteristics should be noted. The early development involves a slow increase in brightness as the effective temperature rises (radius decreases). In all cases this trend is interrupted by a sudden drop in brightness. The point at which a maximum is reached is essentially that where thermonuclear reactions become incipiently significant. At this point contraction ceases in the central regions, leading to a reduction in the rate of conversion of gravitational potential energy for which only partial compensation is provided by the thermonuclear sources. Gradually contraction ceases even in the outer layers, and finally the time scale for changes increases enormously. Further evolution takes place only as a result of changes in composition. All tracks are stopped where changes on the original time scale become imperceptible. Convection always sets in, if at all, on the downward drop.

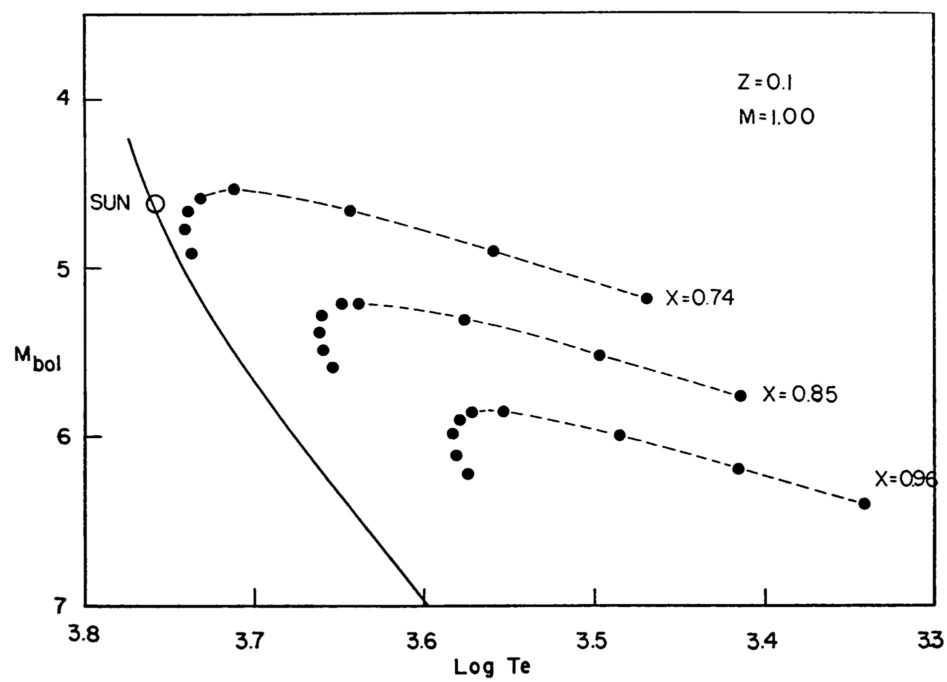

FIG. 2.—Tracks with different proportions of X (hydrogen).

Figure 1 displays tracks for the mixture $X = .74$, $Y = .25$, and $Z = .01$, for various masses. The solid curve represents what is approximately the main sequence as given by Keenan and Morgan.[4] The tracks cross the main sequence for masses above that of the sun while the track for 0.65 solar mass stops short. Possibly allowance for convection in the ionization region would bring this track further to the left. The size of the convective core varies from 20 percent for $M = 2.291$ to 1 percent for $M = 1$ and to nothing for the lowest track.

The significance of the apparent relationship between the ends of the upper tracks and the main sequence (luminosity class V) must be considered in terms of the fact that these stars immediately move up and to the right on a time scale longer than that encountered here but moderately short in terms of the age of the galaxy. In other words, stars somewhat more massive than the sun cannot be expected to remain in the neighborhood of the points falling at the ends of our tracks for times which are significant fractions of the age of the galaxy.

Figure 2 represents the effect of varying the proportions of hydrogen and helium but keeping the heavy constituents constant. Figure 3 shows similar effects when the hydrogen is held constant while the other two vary. All cases in Figures 2 and 3 were calculated for unit mass.

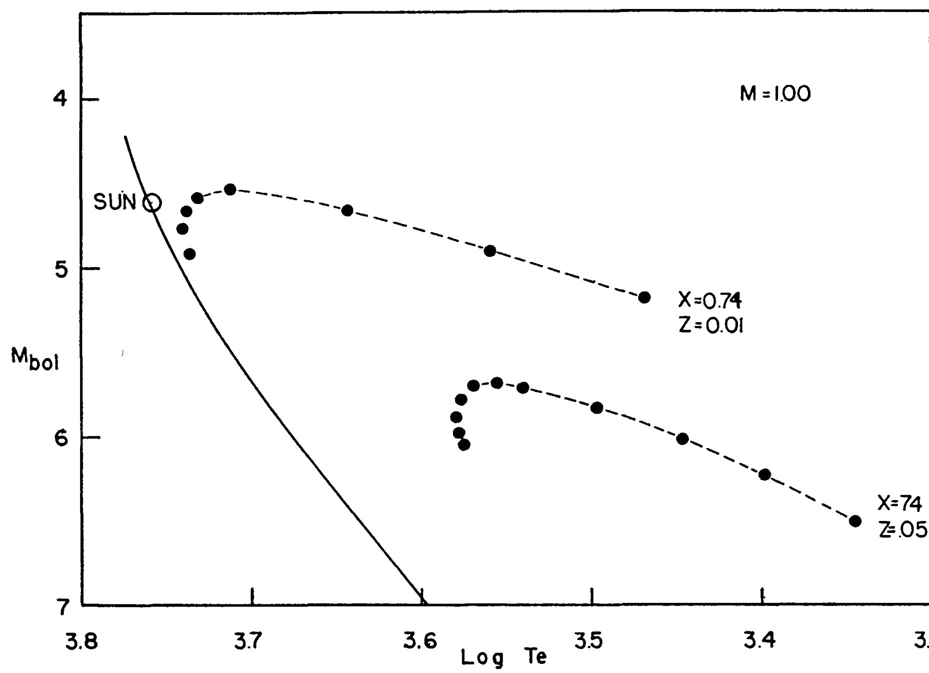

FIG. 3.—Tracks with different proportions of Z (heavy elements).

Since these investigations were started, new results for opacities have become available. Considerable differences exist between these results and the values used here, particularly for the lower temperatures. Indeed the majority of recent investigations are subject to similar discrepancies. Much of the work reported here will, it is hoped, be repeated soon with the newer opacities. The general relationships which have been revealed by this work will, however, probably not be changed basically but only modified numerically.

The time scales associated with each of the tracks in Figure 1 are given in Table I. The times t_1 and t_2 are respectively the time of maximum luminosity and the time at the end of the track.

TABLE I

TIME SCALES AND MASS

Mass	t_1 (maximum) (years)	t_2 (end of track) (years)
0.65	7×10^7	1.5×10^8
1.00	1.6×10^7	3×10^7
1.25	8×10^6	1.4×10^7
1.549	4×10^6	8×10^6
2.291	1.8×10^6	3×10^6

These must be regarded as providing only a rough measure of the scale for two reasons: first, they are both referred to the somewhat arbitrary zero provided by the starting configurations; second, t_2 is poorly defined since changes in terms of a gravitational time scale level off gradually, the various parameters approaching their "final" values only asymptotically.

Figures 2 and 3 indicate that changes in original chemical composition shift the tracks in a direction having the same sense as the main sequence, but having a slope which is somewhat less. This remark applies specifically to the ends of the dips which correspond precisely to homogeneous equilibrium models. Since the evolution of a chemically inhomogeneous configuration is roughly orthogonal to this direction—that is, toward higher luminosity and lower effective temperature—any attempt to represent a star, which, like the sun, has suffered significant depletion of hydrogen in the central regions, by a homogeneous model will be subject to the following serious criticism. The effects of evolution will be taken up by making large fictitious modifications in the composition as suggested by the previous remarks. Extravagant values for the hydrogen content may be encountered. In fact, even if an apparent and plausible representation is achieved with a homogeneous model, it must be regarded as specious and due to some compensating but erroneous effect introduced into the calculations.

We wish to record our deep gratitude to the Atomic Energy Commission and to Drs. Edward Teller, Herbert York, and Sidney Fernbach and Mr. Robert Mainhardt, all of the Radiation Laboratory staff, for the support and co-operation we have enjoyed during the progress of this work and for the use of the UNIVAC in the calculations.

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[1] L. G. Henyey, Lawrence Wilets, and Robert LeLevier; *Ap. J.* (in preparation).

[2] *Ap. J.*, **92**, 36, 1940.

[3] *Ap. J.*, **117**, 200, 1953.

[4] "Classification of Stellar Spectra," in *Astrophysics*, ed. by J. A. Hynek (McGraw-Hill, 1951), p. 12.