CINEMATOGRAPHY OF PARTIAL SOLAR ECLIPSES

III. Analysis of the observations made at Mombasa, 1948 November 1

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Summary

The observations at Mombasa were the first to be made by this method. The paper describes the method of measuring the photographs, and of determining the zero of P.A.'s; the method of correcting the P.A.'s for the effects of lunar irregularities; the derivation of suitable normal equations; and a number of systematic corrections which should in principle be considered in all eclipse work. The interactions between the Washington limb-traces (which were used to derive the limb-effects) and the eclipse results are discussed, and a direct check on the difference between the mean P.A. system of the two traces used, and that of the eclipse, gave $0°\cdot030 \pm 0°\cdot018$. The "geodetic" probable errors of the result correspond to ± 34 metres and ± 45 metres in the two directions parallel to the fundamental plane, or to $\pm 0''\cdot019$ in $\alpha \cos \delta$ and $\pm 0''\cdot025$ in δ, for (roughly) the difference between the Moon's place and the Sun's. The probable error of the Moon's mean longitude is much larger ($\pm 0''\cdot064$), owing to the uncertainty in that of the Sun. There are in the present case several known sources of possible systematic error, but they appear quite capable of elimination in future work.

The observational material.—The material brought back from the Mombasa eclipse (**1, 2**) consisted of about 59 metres of 35-mm film, showing the rotation and diurnal motion of the unobscured solar crescent continuously from roughly 96 seconds before mid-eclipse to 76 seconds after (**2**, p. 646, Plate 10), together with a chronograph tape showing the instants of the individual exposures directly against radio signals from the Royal Observatory's Time Station at Abinger (via Rugby, MIK, 9725 kc/s). There was also the record of the latitude, longitude, and height above sea level of the station.

The film was developed commercially; as received back, it consisted of a short stretch of pre-eclipse "sky" exposures, moderately blackened over the frame area, followed by 3100 "eclipse" frames, on each of which there was a fully exposed crescent on a blank background. Occasionally there was also the black patch near one corner which showed that the "identification-mark" shutter (**2**, p. 641) had been open; this appeared on one frame only on one occasion, on two consecutive frames on twelve occasions, and on longer stretches on two occasions, when this shutter had evidently stuck for periods of roughly a second each. The mark never impinged on the crescent image. The crescent was about 9 mm long; under a magnifying glass, a fair amount of roughness of the lunar limb was clearly discernible, while the Sun's limb showed no comparable roughness, and since lunar limb-features run only to $\pm 2''$, both the optical definition of the apparatus and the atmospheric seeing must have been good.

The chronograph tape (one pen for the camera shutter and one for the radio signal) showed the start of the run unmistakably, and the radio time marks were clear and were only rarely accompanied by any visible atmospherics. The shutter contacts (about 18 per second) were all well separated except for the brief periods when the camera pen was held over by the "mark" contact which closed whenever the identification-mark shutter was open. This contact had suppressed one "break" on five occasions, two breaks on eight, and longer stretches on two others, evidently in rough agreement with the marks on the film.

Measurement of the chronograph tape.—The tape was measured by Mr C. C. Harris, in the Time Department at Greenwich, according to the standard Observatory procedure. This relates each shutter contact to the two "seconds" contacts which immediately precede and follow it, provided that there is no clear indication that either of these was displaced by an atmospheric. Eight of the 176 time marks used were manifestly displaced (one was advanced by a large amount, and seven were retarded by a total of 125 milliseconds); these were all, in effect, treated as though they had been missing altogether, the interpolation being carried over two seconds (or, in one case, three) instead of one. The possibility that minor atmospherics would more often bias all the signals late than early can never be entirely ruled out when radio time signals are being received; but the net effect on the mean observed "time of mid-eclipse" which the seven visibly late contacts would have produced, if they had been used, would have been less than one millisecond, and it is thought unlikely that there can be any systematic deviation exceeding a very few milliseconds for the *average* of the 168 contacts used.

The two stretches of the camera-pen trace, where the identification signal had been prolonged, were filled by simple arithmetical interpolation, and were found to involve 20 and 13 exposures respectively; the camera ran so uniformly that it would be quite unsatisfactory to suppose that either of these counts was wrong by one, and they also checked with the phase relationships on the 13 other occasions, before and afterwards, when the signal had operated correctly. Fifty-four camera contacts in all had therefore to be supplied by interpolation (mostly of ones and twos), and the loss of weight, out of 3 100 contacts, is negligible.

In 1948, all Greenwich time signals were still of the "rhythmic" pattern, with 61 impulses per minute, and the interpolated shutter-times were thus in "diminished" seconds as they stood. It would have been possible to maintain these units until the final "time" and "rate of change" had been obtained, and then to convert these two values only; but it was preferred to convert earlier. The 3 100 "times" were therefore meaned by tens, and each of these 310 values was converted into true time by adding to the fractional part the integral number of "diminished" seconds elapsed since $04^h 26^m 00^s$, multiplying by 60/61, and then adding $04^h 26^m 00^s$. To these provisional times, the following corrections were then applied:—

(*a*) Correction for signal error. The signal was monitored at Abinger in the normal way, and was treated just as all standard signals are in evaluating the "final corrections" published regularly by the Time Department. The total correction in this case was +9 ms (signal late).

(*b*) Correction for transmission time. This was computed to be +25 ms.

(*c*) A computed correction of −7 ms was applied for difference of time-constant as between a circuit consisting of radio output plus chronograph and a circuit consisting of chronograph plus contacts only. (The radio circuit's high impedance makes the time signals record relatively early.)

Corrections for polar variation, for annual variation in the Earth's rotation, and for the difference in light-time as between the station and the Earth's centre, are discussed below (pp. 94–95). The difference between G.M.T. and the mean of the 14 "best" observatories, as published by the Bureau de l'Heure, was zero for November 1.

The 310 normal "observed" times were therefore all increased by 27 ms, and these corrected values were adopted as the independent parameter of the analysis; however, in the final analysis they were all further increased by 56 ms (corresponding to a mis-count of one frame) for the following reason.

When the phase relationship between frames on which a "mark" appeared and contacts which were suppressed on the tape was subjected to a final check, it was found that there was consistently a discrepancy of one frame, compared with what was expected (and what had originally been found). The phase relationships in the camera are very complicated; the mark is not imprinted at the time of exposure, but on a frame that has already moved since the exposure, and with a time lag which is roughly a half-integral number of camera cycles; the re-assessment showed that the difference between the serial number of the first frame on which a mark appears and the serial number of the first contact suppressed on the tape should usually be two but could be one; and similarly (in a slightly different proportion) for the last mark and the last contact, each time that the mark mechanism is operated. However, comparison of the actual film and tape showed that these differences were usually three and sometimes two, but never one. This means that the first contact of all, on the tape, corresponds to a crescent image not found on the film; re-examination of the film verified that this might well be the case. The last of the pre-eclipse "sky" exposures was very heavily over-exposed, indicating that the shutter had stopped in (or very close to) the "open" position; when this happened, the observer would turn the hand-knob until it closed, and if in this particular case he inadvertently turned it backwards instead of forwards the first crescent image would be superimposed on the last "sky" frame and could be lost. It is impossible to discover any trace of a crescent on the frame in question, but the over-exposure is such that this is not surprising. (This point was incorrectly reported in **2**, p. 642; the evidence of the "marks" is however quite conclusive, and the original decision to make provision for such a check has proved to be justified.)

Measurement of the crescent position-angles.—For this purpose a special film-holder had to be constructed; it was designed to position the film so accurately that the direction of its edge could be regarded as constant throughout the entire process of measurement, and need not therefore be measured on every frame. The holder consisted essentially of two spools for the film, about 32 cm apart, with a brass "bridge" leading from one to the other. Along one edge of this bridge there was a fixed steel guide-rail which determined the position of one edge of the film; the central 5 cm of this rail were cut away, so that the angular control depended on two lengths of film edge about 4 cm long each, with their centres about 9 cm apart. The film was held lightly against these by the pressure of a movable rail, bearing on its other edge and controlled by a light leaf-spring. Over the central region of the bridge, a hole was cut out of the brass plate, and a glass window was let in flush with the brass; the film was held down onto the window by a glass pressure-plate, just less than 35 mm wide, which was pressed down onto it, and prevented from sliding with it, by arms acting at its four corners. A couple of small rollers a few centimetres away held the film down almost to the plane of the window even if the pressure-plate was removed. The flanges of the spools stood just clear of the film, and it was in effect located only by the fixed guide-rail, the pressure-plate, and the extent to which it was paid out from one spool, and taken up by the other; this was done by hand, the spool flanges being 12 cm in diameter.

The entire assembly was rigidly mounted on the turntable of a measuring machine originally designed for measuring sunspots on large-scale solar photographs. The turntable was about 45 cm in diameter, with a good angular scale engraved on silver all round its circumference; the scale could be read, by a pair of fixed verniers, to the nearest 1′ directly. The verniers demonstrated that the centring error, and the random division-errors, were negligible; systematic errors other than the centring error have been assumed to be negligible also. A low-power microscope, with a single fixed fiducial thread (spider-web) in its focal plane, could be traversed above the central region of the circle, riding on stationary precision "ways" which were supported by the base of the instrument; the microscope had to be very gradually traversed during the measurements, as the image worked over from near one edge of the film to near the other.

It had been intended, from the first, to measure the P.A. of the actual line of cusps, and not (as in most previous work) to attempt to locate the centre of the Sun (or Moon) and measure the position-angles of the separate cusps from that point. The procedure was therefore to draw the film along, and slide the microscope as necessary, until a crescent was central in the field of view; rotate the turntable until the line of cusps was almost parallel to the thread; and then by further adjustments in film travel and in orientation to reach a situation in which the extreme tips of the two cusps could both just barely be seen projecting (equally) beyond the thread. The circle was then read. In order to read the film edge, the microscope had to be slid along the bridge until the edge appeared; the edge and the thread were then laid up close together and the circle was adjusted until they were parallel. The edge readings so obtained were very consistent; when the complete film was finally measured (an operation which took a number of days) a couple of edge readings were taken, after two consecutive frames, about once in every 25 frames, and the 248 values so obtained were distributed as follows:—

Circle reading:	162° 18′	162° 19′	162° 20′
No.	55	159	34

The mean is 162°·315; the partial means for successive sixths of the complete run were 162°·313, 162°·314, 162°·316, 162°·315, 162°·317 and 162°·316, and the slight drift thus indicated is small enough to ignore, since the total change in crescent P.A. was about 76°. The above scatter included the effects of (*a*) direct errors of observation, (*b*) departure of the film edge from straightness, (*c*) failure of the edge to be held exactly against the guide rail, and (*d*) any lack of permanence in the position of the film-holder itself on the circle, or of the fixed supports which carried the microscope. It is clear that the film-holder worked reliably, and the mean edge reading constitutes a well-determined datum from which the crescent position-angles can be reckoned.

The zero of position-angles had to be determined with reference to the propulsion holes, not the film edge, and it was therefore necessary to verify that the line joining a pair of holes was perpendicular to the edge; the magnification and field were not such that the full width of the film was visible at one viewing, and the verification was made by special measurements in which the microscope was traversed from one hole to the other, but the effect of the traversing was cancelled by repeating the measurements film down. A fairly long series of tests, on different lengths of the film, demonstrated that the line of holes was indeed perpendicular to the edge to better than 1′, and with a satisfactorily small scatter.

FIG. 1.
• *Individual frames.*
⊙ *Overlapping means of three.*
+ *Straight means of ten* (*cf. Fig.* 9).

A preliminary survey of the film was then made (by R. d'E. A.) by measuring the crescent position-angle (reckoning from the film edge, i.e. a still undetermined zero) for every tenth frame only, together with measurements of every frame over short selected stretches, from the start of the run down to about frame 2200, when it had to be interrupted. As was expected, the results showed some quite violent fluctuations ($\pm 1^\circ$) from a smooth run, due to the roughness of the lunar limb and the smallness ($3\frac{1}{4}^\circ$ at smallest) of the cusp angle; but in general there was very little doubt about any particular frame. In most cases both cusps were uniquely defined, and the probable error, for repeated re-measurement of one frame, was then about $\pm 1'$; the same value was subsequently obtained by other observers. This meant, of course, that the effects of fairly fine lunar limb-detail might be observable, and some tests were therefore made of the consistency from frame to frame. Fig. 1 shows the actual readings obtained over a suitable range where they were not disturbed by (visible) cusp irregularities; an arbitrary smooth ephemeris has been removed for convenience. It is evident that the variations from frame to frame are not purely random, and quite fine-grain detail is observable in principle. No actual use was found for this, however, since it is not reproduced in the E curve (see below); it may well be that this degree of detail cannot be correlated with limb-traces unless the librations are almost identical in the two cases. The oscillations observed may however also be of non-lunar origin, arising, for example, from slight ripples in the seeing, not in phase at the two cusps.

Not all stretches of the film showed clean-cut cusps; occasionally, one cusp (or even both at once) showed an anomaly, usually lasting for 5 to 15 frames but sometimes up to 50, and changing quite consistently from frame to frame; this took the form either of a reasonably well detached Baily's bead or of a thread-like extension without noticeable neck. In such cases, more than one position-angle could be read, depending on whether the tip of the main cusp or the tip of the appendage was set on; but at least one of the values was still pretty well defined in itself and compromise settings were not made. If one measured every frame, one had sooner or later to make a quite definite change from setting on what had been the cusp, and to transfer to the appendage which had developed beyond it and had grown too strong to ignore; or (at the other cusp) to change from what had started as the cusp, but had gradually become a very faint appendage, and to set on a "main cusp", further in. Such changes naturally involved discontinuous jumps in the run of readings, and a little consideration will show that these jumps must always be "forwards", in the numerical values, no matter which cusp is involved. There is a certain arbitrariness in saying just when a jump ought to be made, but this is paralleled in the "theoretical" limb-effects derived (below) from the known lunar profile.

In addition to the sudden forward jumps in the run of readings, which thus occurred, there were many well-marked (and longer) stretches where the rate of advance was little more than half its overall average for that part of the run; such stretches tended to be rather smooth. These "slow runs" are to be expected: the "leader" cusp will move slowly whenever it is working up any fairly steep slope, and the "follower" will move slowly whenever it is moving down one; in either case the rate of advance, in position-angle, of the line joining the two will be much reduced, and irregularities of the limb at that cusp where the slope is steep will also have less effect than usual. When a smooth ephemeris is removed, the residuals thus show a "sawtooth" pattern, which may be considered typical of this method. (Figs. 8 and 9; the points plotted are now means of ten.) Neither these sawtooth features nor any others, in the run of the observed position-angles, bear any superficial resemblance whatever to the lunar profile which causes them.

A number of tests were made to estimate the magnitude of possible systematic errors. Since the two cusps diverged by about 30°, any residual astigmatism in the observer's eye might perhaps tend to weaken one cusp more than the other; and small differences in P.A. might indeed be expected between different observers, and between the two eyes of one observer, whether a clear reason could be given or not. The fiducial wire was necessarily kept fixed, and the orientation of the crescent thus appeared the same in the eyepiece throughout the entire run, so that any such effect might be systematic. Differences were indeed found, both between the two eyes of one observer and between different observers, amounting to 1′, or even 2′, and such differences are rather too large to ignore if the internal probable error of the final results is the criterion; in the present instance, however, they have been ignored, since there are other systematic errors which may be larger; the main purpose of this first attempt was to establish the general serviceability of the method, especially as regards the closeness with which the measurements would agree with what the lunar profile demanded, i.e. the smallness of the internal probable errors. The most satisfactory way of studying, and eliminating, such systematic errors, in future cases, without increasing the large amount of measuring that is already necessary, would probably be by incorporating a reversing prism in the eyepiece, and rotating it (say) 18° between each frame and the next, so that only every tenth image appears in any one orientation; there might also be some advantage in measuring all the odd frames film up, and then all the even ones film down, which would involve no real increase in total labour.

Systematic error may also be expected when one cusp is blunter than the other; this, however, should not occur, at least with a good optical system, except temporarily, and then with equal probability at either cusp. It thus becomes essentially a random error, though of relatively long period.

A considerable amount of time was devoted to studying these matters. It was finally decided that for the purpose of this first attempt it would be sufficient to measure the entire film straight through, in one arrangement only. It seemed improbable that the total systematic error so introduced could reach 0°·05, and the uncertainty which appears (in the present instance) to have affected the zero of position-angles was thought at the time to be larger.

On this basis, the systematic measurement of the entire film was then entrusted to Mr E. A. Whitaker, of the Department of Astrometry. A good deal of effort and strain can be saved if one is content with a probable error of possibly 2′ instead of 1′, and the large number of frames was considered to justify some relaxation; even so, however, the work took many hours. A direct comparison of his figures with those obtained by R. d'E. A. in his preliminary survey, for 208 frames measured by both and not marked as showing some appendage, gave a mean difference of 1′·4 with a formal probable error (per single frame) of ±2′·9; the differences actually showed long-period systematic oscillations, but this probable error has been computed as though they were random. Any variations could almost certainly be made nearly random, in future work, by measuring the odd frames in the "direct" order and the even ones in the "reverse"; it seems likely, however, that a reversing eyepiece would in any case eliminate most sources of personal error.

The 3 100 crescent P.A. values so obtained were meaned by tens, working formally to thousandths of a degree. (In 27 cases in the last thousand frames the image appeared double or poor and was not measured; a value was supplied by rough interpolation, taking account of the general run of the ten and the position within it of the missing one.) The true position-angles increased during the run, while the readings of the measuring machine decreased; the 310 values had thus to be subtracted from the mean reading for the film edge (162°·315), and the results converted to true position-angles by adding the position-angle of the direction in the sky which corresponded to the film edge, together with certain small corrections, which will be described below.

Determination of the zero of position-angles.—It was intended to determine the zero of position-angles by comparing the direction of the line (nearly E–W) joining a pair of propulsion holes with the direction of the diurnal motion as inferred from the common tangent to all the 3 100 south limbs of the Sun, the south limb being unobscured throughout the run. For this purpose the complete film-holder was transferred to a one-screw plate-measuring machine ordinarily used for astrometric plates, and was set with the film edge (i.e. approximately the N–S direction) nearly parallel to the screw. By suitably advancing the film one could bring a crescent into the field of view and place it with its south limb close to the measuring wire; and by traversing in the cross-ways of the machine (i.e. parallel to the wire) one could then bring either one of the appropriate pair of holes into view instead, keeping the film stationary in the holder. It was thus possible to take three readings for each frame; on the limb ($\odot$), on the appropriate "east" hole (E), and on the corresponding "west" hole (W); and so to form for each frame the "north–south" coordinate of the south limb with respect to the holes, $\frac{1}{2}(E+W)-\odot=G$, say. The machine read to microns, and the edge of a hole, being straight and parallel to the measuring wire, could be set on with an uncertainty of a very few microns. The east–west coordinate could also be measured roughly; its total travel was about 11·2 mm. For actual use, however, the length travelled was computed from the focal length (935 mm), the declination, and the total time; the value so obtained was 11·38 mm.

The 3 100 values of G were not all worked out individually; the values of $\odot$, E, and W were summed by pages (40's) and a mean value of $\frac{1}{2}(E+W)-\odot$ for each page was obtained directly from these sums. However, a general check on the individual values was readily available, since the measuring procedure was to set the micrometer back to a standard figure after each measurement of W, pull the film along until the limb of the next crescent was very close to the wire, and then measure $\odot$, E, and W in that order; it followed that the readings for $\odot$ were always close to the standard one, and any large anomalies in G could be picked up by inspection, in both the E and W columns. A complete page of individual G-values was computed occasionally, and half the values ordinarily showed residuals, from the mean for the page, numerically less than about 9 to 12 microns; the distribution was not normal (usually showing a rather sharp cut-off), but the scatter can be qualitatively described by a "probable error" of $\pm 12\,\mu$ or less. All residuals greater than $\pm 30\,\mu$ were marked, with an additional mark when they exceeded $\pm 70\,\mu$; in the first 1653 frames there were only 33 of the former class and none at all of the latter. Indeed, there were none even of the former from frame 745 to frame 1183, and only one of each sign in the further stretch from frame 1208 to frame 1653, and there can be little doubt that the consistency shown by these 900 frames represents the performance which the camera is designed to give. Even if fiducial marks were not introduced in future work, this would be a sufficiently small scatter if it were purely random; it would lead to an uncertainty of a small fraction of a minute, in the zero of position-angles.

Unfortunately, this consistency did not continue in the second half of the run. Frame 1654 showed a residual of $-138\,\mu$, and occasional negative residuals lying between $-100\,\mu$ and $-140\,\mu$ occurred on most pages from that point on. In general, the number of residuals (of either sign) which exceeded $30\,\mu$ but did

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not reach $70\,\mu$ remained small, with no large positive residuals; the conclusion appears inescapable that large negative residuals represent an anomaly, liable to occur on individual frames but not disturbing the general distribution in the other cases, and that the proper procedure is simply to exclude the anomalous frames and accept the resulting loss of weight.

There seems no doubt that in the anomalous cases the film had failed to pull quite as completely into place, in the camera, as it did in the normal ones. At the end of each "pull" it was clamped by the pressure-plate, with one hole forced down on to a fixed peg; but it was now found that there was a play of over $\frac{1}{10}$ mm on this peg if the pressure-plate was released, and it seems clear that the film was always near one or other of the two possible limiting positions, and in the first half of the run always near one of them. The number of anomalous frames became seriously large towards the end of the run, but it still appeared that the proper treatment was simply to exclude them; for example, the page of frames 3021–3060 showed six negative residuals exceeding $70\,\mu$, reckoning from the original mean of the page, and it also showed four positive ones $> +70\,\mu$; but the fact that two compact groups (now nearly equal) were really involved was made clear by the circumstance that only one of the entire 40 residuals now fell in the range between $-30\,\mu$ and $+30\,\mu$. Thus even in this extreme case it was possible to distinguish with certainty between "normal" and "anomalous" frames; when they had been sorted out, the "normal" group comprised 21 of the original 40, with only one residual (from their mean) numerically greater than $30\,\mu$ $(-35\,\mu)$, and with half of them numerically less than $11{\cdot}5\,\mu$ (i.e. with an acceptable scatter), while the anomalous 19 frames had a mean value of $119\,\mu$ smaller, and formed a group fully as compact as the normal one, the extreme range of the residuals (from the mean of this group) being from $-27{\cdot}3\,\mu$ to $+30{\cdot}2\,\mu$, with half of them numerically less than $13{\cdot}3\,\mu$. The gap between the largest value in the low group and the smallest in the high one was $53\,\mu$, and there is nothing arbitrary in the criterion for separating them. The diurnal run should thus be determinable if the "normal" frames only are used, the "anomalous" ones being all rejected. (In principle, one might also use only the "anomalous" ones; the result would be identical if their mean difference was constant, but it would have very much less weight. The mean difference was not actually evaluated; if it had been, and if it had proved constant, it could have been applied to all the anomalous values, but the labour would have been considerable and the gain in weight rather small.)

The mean values of G are plotted in Fig. 2 against frame number; both the uncorrected means are given and also the "corrected" ones, obtained by simply dropping the anomalous frames. It is at once apparent that the corrected points not only scatter much less than the uncorrected ones, but also follow on considerably better from the first half of the run. The fact that the Sun's declination was not zero means that there should be a slight concavity upwards, and the corrected points do also indicate this, though too strongly; the uncorrected ones forbid it altogether.

When the "corrected" points are re-plotted with a smooth ephemeris of $-11\,\mu$ per 400 frames removed (Fig. 3), it is seen that the general appearance of the run really suggests two different straight lines. It is certain that there cannot have been any sudden sideways tilt of the camera bench, rotating the film in its own plane by $0^\circ{\cdot}4$ or thereabouts in a short interval of time near

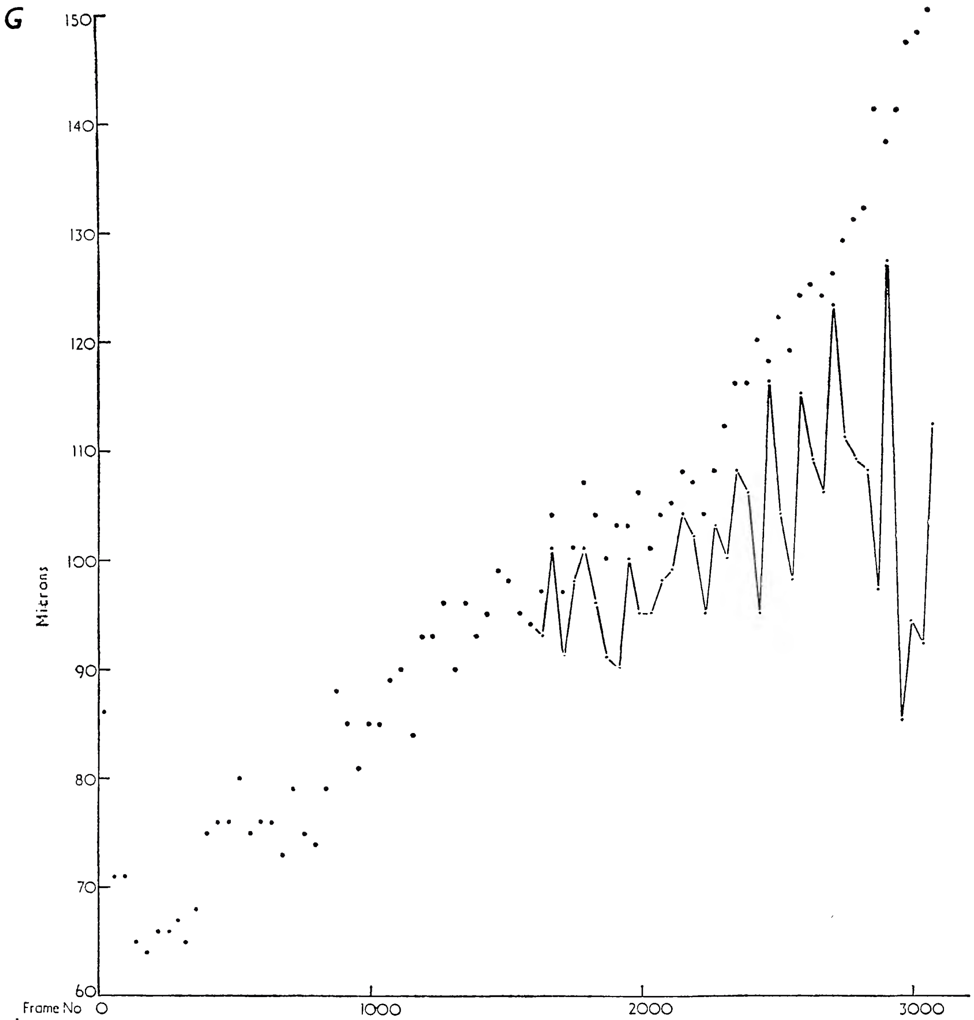

FIG. 2.—*Zero of position-angles.*
. — . *Uncorrected points.*
. . . *Corrected points.*

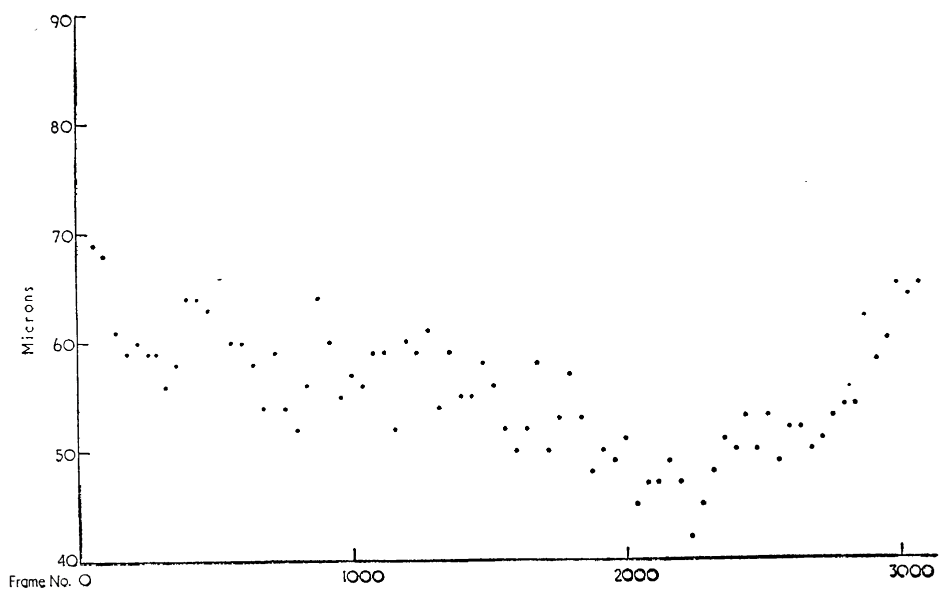

FIG. 3.—*Zero of position-angles.*
Corrected points with linear ephemeris removed.

frame 2200. It is not merely that since the light was first reflected by an oblique mirror, carried on the same bench, such a twist would be expected to produce very large discontinuities in the absolute values of both coordinates in Fig. 3, as well as in the slope; even if these discontinuities happened both to be zero, a real rotation of the camera would necessarily introduce a discontinuity of $0^\circ{\cdot}4$ into the measured position-angles themselves and, as will be seen below, there is evidently no such discontinuity in these values at all.

In the early stages of the work, since there seemed no good reason to favour one part of the diurnal run at the expense of the other, a straight line was drawn (by eye) through the entire array of "corrected" points in Fig. 2; its slope was $+75\,\mu$ in 3100 frames. The computed length of the diurnal run being 11·38 mm, the slope of the "common tangent" so drawn was $0^\circ{\cdot}378$, but with serious uncertainty. However, at a late stage of the work several independent considerations strongly suggested (p. 85) that it might definitely be the final stretch of the run which depicted the true diurnal direction, some disturbing factor having been operative until frame 2200 or thereabouts. A discontinuity in slope, with none in the actual ordinates, could result from a slow drift in the longitudinal tilt of the whole bench (or possibly of the prism only), which persisted at a nearly constant rate until about frame 2200 and then definitely stopped; a drift totalling about $10''$ (instead of $0^\circ{\cdot}4$) would now be adequate, and such a small drift cannot be certainly excluded in the Mombasa case.

It was decided, therefore, to adopt the run of the last 900 frames exclusively, for determining the zero of position-angles, and to evaluate this slope rigorously, since the small scatter here permits this. For this purpose, the abscissa scale was changed from frame numbers (though the camera did in fact run with great uniformity) to the stricter

$$X = \frac{15 f \cos\delta}{3438}(t - t_0), \tag{1}$$

where f is the focal length (935 mm) and $t - t_0$ the time from (computed) mid-eclipse in minutes. The ordinates G were also diminished by the computed effect, $\Delta G = X^2/(2f \cot\delta)$, of the curvature of the diurnal circle, so that the slope refers strictly to the moment of mid-eclipse. A least-squares solution was then made for the 23 normal points representing the means of 40-frame stretches centred on 2200, 2240, ... 3080, with approximate weights when the totals departed appreciably from 40 owing to the omission of anomalous ("A") frames. The result was

$$G - G_0 = (0^\circ{\cdot}683 \pm 0^\circ{\cdot}022)X \tag{2}$$

and the value $0^\circ{\cdot}683$ was adopted for the final work. In principle, it is still subject to an error caused by differential refraction, but the diurnal track was almost vertical in the present case, and the computed error was only $0^\circ{\cdot}002$ and was ignored. The difference between this system and all the earlier ones is about $0^\circ{\cdot}3$ in position-angle, which is equivalent to about 0·6 sec change in t_0, or $0''{\cdot}3$ in the Moon's place; the systems thus differ very substantially.

It is evident that the above probable error does not really represent the reliability of the figure, in the present case, but the weakness should not recur in future work. Not only have fiducial marks since been arranged in the cameras, so that the difference between "anomalous" and "normal" frames effectively disappears; the horizontal position in which the bench can generally be used also allows it to be properly clamped down onto flat metal bedplates, and with a good concrete pier a much higher stability should then be possible than could be guaranteed in the Mombasa case. It seems probable that the zero of position-angles should in future be obtainable with a formal probable error well below $\pm 0^\circ{\cdot}01$, and with good reason to hope that this would represent the true uncertainty.

The fact that the film sometimes failed to pull completely into place, in the camera, now suggested that it might also take up an anomalous direction at the end of the pull; the position-angle of an image, measured from the direction of the film edge, would then be anomalous as well as the actual position itself. If this happened at random, it might be hard to detect, though it would certainly cause a scatter; but the question whether it happened preferentially with the frames already noted as anomalous is easily investigated. For this purpose, all the frames on which the value of G had been anomalous, in the work on the zero, were now marked "A" in the record of the position-angle readings themselves, and the circle reading for each "A" frame (already recorded) was compared with the mean of the preceding and following "regular" ones. Where two or more "A" frames were consecutive, their mean was compared with the mean of a correspondingly increased number of adjacent "regular" ones, and where two or more "A" frames alternated with regular ones the mean of the odd ones was compared with the mean of the even ones; in all cases the differences were given a weight equal to the number of "A" frames involved. On the last three pages (frames 2950–3100) there were so many "A" frames that these procedures were unsuitable; the last 150 P.A. readings were therefore all plotted, smooth curves were drawn through the "A" points and the regular ones separately, and the mean differences between the two curves were read off. (These curves are necessarily rather irregular, since they contain the full effect of the lunar limb detail.) The systematic differences, in the sense circle reading "A" *minus* "regular", were thus found to be as follows:—

frame	1654 to 2150	2151 to 2950	2951 to 3000	3001 to 3050	3051 to 3100
difference	$+5'{\cdot}0$	$+6'{\cdot}5$	$+7'{\cdot}2$	$+5'{\cdot}0$	$+5'{\cdot}8$

The variations in the values from frame 2951 on are barely significant, and a mean value of $+6'{\cdot}0$ is sufficiently near for actual use; even when there are five "A" frames in one set of ten (which happened only towards the very end), the difference between $5'$ and $6'$ is a matter of only half a minute in that one of the 310 normal points. It was therefore decided simply to decrease the value for any given mean of ten circle readings by $0^\circ{\cdot}010g$, if there were g "A" frames in that ten, throughout the entire range affected by anomalies; in what follows, the values of P' are the 310 means of ten circle readings, as so corrected. (It was found that most of the 27 cases in which the image had been too poor to measure were "A" cases; as already explained, values for these had been supplied when the means were first formed, and since these supplied values were as a rule little affected by the systematic difference now discovered, they were not corrected for it.)

It was verified by inspection of the camera mechanism that the sign of the correction is what one would expect if the error arose through forcing the film down onto the positioning peg; it seems reasonably certain that the film would lie in the "normal" direction whenever the following edge of the hole was pressed against the peg, and in an abnormal direction when the leading edge of the hole was, so that it really would only be in those cases in which the position of the image was anomalous that one would expect its position-angle to be so. These disturbances will of course also disappear in any camera fitted with fiducial marks.

The angle $0^\circ{\cdot}683$ obtained above (eqn. (2)) represents strictly the extent to which the "mid-eclipse" tangent to the slightly curved diurnal run failed to be parallel to the cross-ways of the measuring machine, as the film was actually mounted in the machine. It was mounted with the line joining an "E" hole to a "W" one very nearly parallel to the cross-ways, but not exactly; on the average, the value of the difference $E-W$ was about $+5{\cdot}3\,\mu$. (The scatter from this showed some small systematic features, but these have been ignored.) Since the image approached the W edge as the run proceeded, and since the holes are 28 mm apart, the angle $0^\circ{\cdot}683$ is larger by $57^\circ{\cdot}3\times 5{\cdot}3/28000=0^\circ{\cdot}011$ than it would have been if the film had been mounted quite correctly; the tangent therefore failed to be perpendicular to the film edge by $0^\circ{\cdot}672$. (A little consideration shows that lack of perpendicularity between the two "ways" of the measuring machine does not enter as well.) Since increasing readings corresponded, on any one frame, to increasingly north coordinates, the holes ran increasingly north of the Sun as the eclipse proceeded.

The diurnal run was not, however, due west, owing to the change in the Sun's declination. The "diurnal" slant Y is $\dfrac{\sec\delta\,.\,\Delta\delta}{15\,.\,\Delta t}$ (radians) if $\Delta\delta$ is the change in δ (in seconds of arc, say) in the time Δt (in seconds), and δ is the declination of the Sun's centre. (The point set on was always a constant distance from the Sun's centre.) We thus have

$$Y=-\frac{1{\cdot}0324\times 1156''{\cdot}4\times 57^\circ{\cdot}3}{15''\times 86400}=-0^\circ{\cdot}0528.$$

The diurnal track ran south of true west, while the line of holes ran north of the track, so that the line of holes ran $0^\circ{\cdot}672-0^\circ{\cdot}053=0^\circ{\cdot}619$ north of true west, or in P.A. $270^\circ{\cdot}619$. (See Fig. 4.) Accordingly, the P.A. of the

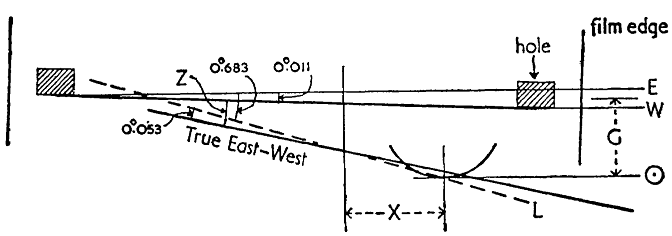

FIG. 4.

film edge, as judged by the tangent to the diurnal motion at mid-eclipse, was $0^\circ{\cdot}619$, and the figure from which the (means of) circle-readings, P', have to be subtracted, in order to convert them to position-angles of the line of cusps, is $162^\circ{\cdot}315+0^\circ{\cdot}619=162^\circ{\cdot}934$.

The absolute "observed" position-angles.—So far we have assumed that there are no systematic falsifications of the P.A. due to lens distortion or to projection effects. The lens was tested for distortion by strapping it to the Astrographic mounting and using it to photograph the Pleiades. This was done through a Wratten 58 (green) filter, to parallel the eclipse conditions, and the images were "extremely good" all over the field ($1\frac{1}{2}°$). The computed standard coordinates were compared (in the Department of Astrometry) with the measured places for a central region, for an intermediate zone, and for the outer region of the field separately, and no detectable difference of scale was found; it was therefore concluded that distortion was too small to have any influence on the results.

The lens thus gives a simple gnomonic projection, with all hour-circles projecting strictly as straight lines; within the small field used, these diverge at $15' \sin\delta$ per minute of time. It is readily verified that over this field, in view of the fact that projection errors do not affect the position-angle when the two cusps are either in line with the centre of the field or both equidistant from it, it is sufficiently accurate to apply the above rate of rotation to the entire field; projection errors can be neglected in all cases. Accordingly, the apparent position-angles differ from $162°{\cdot}934 - P'$ by

$$\Delta P = 15' \sin\delta\,(t-t_0) = -0°{\cdot}0622(t-t_0). \tag{3}$$

It is not necessary (as one might imagine) to correct for the fact that the centre of the line of cusps runs ahead of, or behind, the Sun's centre by an amount (nearly) proportional to $\sin M \sec(M-M_0)$, where M is the P.A. of the line of centres; the measurements can be considered as though they referred to a line through the Sun's centre truly parallel to the line of cusps, and since it is convenient to compute M at the Sun's centre it is strictly correct also to use the rotation of field appropriate to that point.

Since the Sun's declination was negative, the convergence makes the position-angles at the start of the eclipse appear larger than they should, and at the end smaller, so that ΔP must be subtracted from the measured values, these having been obtained with reference to a fixed direction. It is important to include this correction for "rotation of the field", however uncertain the absolute zero of position-angles might be, for this term goes straight into the observed rate of change of $(M-M_0)$, (eqn. (4), etc.) and the rate of change is well observed even if there is no good zero at all. The coordinate of the Moon's place perpendicular to its direction of apparent relative motion is inferred from this rate of change, and both the present term and the one discussed in the following section must be strictly applied in all cases.

These apparent position-angles have now to be converted to true "observed" position-angles by removing the effect of differential refraction. This should strictly be computed for the actual P.A. of the line of cusps, as affected (sometimes to the extent of a degree) by lunar limb detail, but it is sufficient to take the P.A. for a smooth Moon, and amply sufficient to use for computing this the formula

$$\tan(M-M_0) = k(t-t_0) \tag{4}$$

with preliminary values of k and of the mid-eclipse constants M_0 and t_0 (**1**, p. 28, eqn. 9). Since M is the P.A. of the line of centres, and Q (the parallactic angle) is the P.A. of the vertical, the inclination of the line of cusps to the horizontal is $M-Q$, and the correction R', which naturally vanishes when $M-Q=0$ or $\pm 90°$, can be computed in the usual way. The actual values are summarized in Table I.

TABLE I

$t-t_0$	$-1^{m}{\cdot}5$	$-1^{m}{\cdot}0$	$-0^{m}{\cdot}5$	$0^{m}{\cdot}0$	$+0^{m}{\cdot}5$	$+1^{m}{\cdot}0$	$+1^{m}{\cdot}5$
R'	$+0°{\cdot}010$	$-0°{\cdot}009$	$-0°{\cdot}031$	$-0°{\cdot}046$	$-0°{\cdot}049$	$-0°{\cdot}040$	$-0°{\cdot}028$

The final expression for the observed position-angles of the line of cusps is then

$$P'' = 162^\circ\cdot 934 + 0^\circ\cdot 0622(t-t_0) + R' - P'$$

or, for the line of centres (as falsified, still, by limb-effects),

$$P = 252^\circ\cdot 934 + 0^\circ\cdot 0622(t-t_0) + R' - P', \tag{5}$$

where P' is the mean of ten circle readings, corrected for "anomalous" frames (where necessary) as already described. If M is the computed position-angle of the line of centres, the quantities $P-M$ may thus be called the "observed limb-effects" in P.A.

When suitable lunar profiles are available, the "theoretical limb-effects", E, can be computed, and the residuals $P-M-E$ will form the basis of any least-squares solution for the Moon's place; it is useful to split them up for graphical study, and to plot separately a curve of $P-M$ values and a curve of E values. (The final curves will be seen below, Fig. 9, p. 100.) If the available limb profiles are exactly suitable, and if there is no error in the observations, these two curves should be identical in shape; and whether or not they are identical they can be adjusted to have the same mean height and mean slope (both will be nearly zero) by adjusting the Moon's place, since this changes the mean height and mean slope of the function M. (Alternatively, of course, one might adjust the coordinates of the station, or the Sun's place.)

The necessary equation of condition for this adjustment was obtained for only two variables, which can be taken as the corrections to the two coordinates of the Moon's place, and only their effect on M was allowed for. They have also some small effects on E, but it is inconvenient to include these explicitly and they are best dealt with separately. The further analysis thus falls into three main parts: (i) the computation of the M-values, i.e. of the P.A. of the line of centres, for the 310 absolute times involved, using some assumed place (the Almanac one, or a later approximation) for the Moon; (ii) the computation of the 310 limb-effects, E, which involves computing crescent lengths and cusp angles, and measuring the appropriate (Washington) limb-traces; and (iii) the formation of the equation of condition, and of the normal equations.

The computed position-angles.—The approximate formula (4) gives values of $M-M_0$ which are good, but not quite sufficient for the observational accuracy reached, and a rigorous formula is necessary. In the standard Besselian coordinate system the observer is at the point (ξ, η, ζ) where

$$\left.\begin{aligned} \xi &= \rho\cos\phi' \,.\, \sin(\mu-\lambda), \\ \eta &= -\rho\cos\phi' \,.\, \cos(\mu-\lambda) \,.\, \sin d + \rho\sin\phi' \,.\, \cos d, \\ \zeta &= \rho\cos\phi' \,.\, \cos(\mu-\lambda) \,.\, \cos d + \rho\sin\phi' \,.\, \sin d. \end{aligned}\right\} \tag{6}$$

ρ and ϕ' are here the geocentric distance (in units of the Earth's equatorial radius) and geocentric latitude of the observer, deduced in the usual way from the true latitude ϕ, the assumed ellipticity $(\frac{1}{297})$, and the height of the station; λ is the longitude of the observer; μ and d are the Greenwich hour angle and declination of the point on the celestial sphere towards which the axis of the shadow is instantaneously directed; and the "fundamental plane", $\zeta=0$, is perpendicular to that axis. μ, $\sin d$, and $\cos d$ are tabulated in the Almanac, at ten-minute intervals; $\ddot{\mu}$ and $\ddot{d}$ are completely negligible.

If $\dot{\mu}$ and $\dot{d}$ denote the rates of change of μ and d in *radians* per minute, we have, neglecting $\dot{d}^2$, $\ddot{\mu}$ and $\ddot{d}$,

$$\left.\begin{aligned} \dot{\xi} &= \dot{\mu}(\zeta\cos d - \eta\sin d), & \ddot{\xi} &= -\dot{\mu}^2\xi, & \dddot{\xi} &= \text{etc.} \\ \dot{\eta} &= \dot{\mu}\xi\sin d - \dot{d}\zeta, & \ddot{\eta} &= \dot{\mu}(\dot{\xi}\sin d + 2\dot{d}\xi\cos d), & \dddot{\eta} &= \text{etc.} \\ \dot{\zeta} &= -\dot{\mu}\xi\cos d + \dot{d}\eta, & \ddot{\zeta} &= \dot{\mu}(-\dot{\xi}\cos d + 2\dot{d}\xi\sin d), & \dddot{\zeta} &= \text{etc.} \end{aligned}\right\} \qquad (7)$$

where $\dot{\xi}$, $\ddot{\xi}$, $\dddot{\xi}$, etc. are first, second, and third derivatives with respect to the time (in minutes). We are thus able to express ξ, η, ζ as polynomial functions of the time from any desired zero, preferably an epoch, t_1 say, fairly near that of mid-eclipse. As a check we may use

$$\xi^2 + \eta^2 + \zeta^2 = \rho^2 \quad \text{and} \quad \xi\dot{\xi} + \eta\dot{\eta} + \zeta\dot{\zeta} = 0,$$

both of which are of course valid at all times.

The coordinates of the axis of the shadow on the fundamental plane, (x, y), are tabulated in the ephemerides for every ten minutes, and are thus also readily expressed as polynomial functions of the time, measured from the same epoch t_1 as the expressions for ξ and η; we thus have polynomials for $x-\xi$ and $y-\eta$, reckoned from this origin.

In the standard notation

$$\left.\begin{aligned} x-\xi &= m\sin M, & \dot{x}-\dot{\xi} &= n\sin N, \\ y-\eta &= m\cos M, & \dot{y}-\dot{\eta} &= n\cos N, \end{aligned}\right\} \qquad (8)$$

where (m, M) is the vector (perpendicular distance in Earth-radii, and direction from north), from the observer to the axis of the shadow, and (n, N) is the vector rate of change (Earth-radii per minute, and angle from north) of (m, M). M is also the position-angle of the line of centres of Sun and Moon, as seen by the observer, and $m(\pi_{☾} - \pi_{\odot})$ is their apparent separation. We may extend this notation, and write

$$\left.\begin{aligned} \ddot{x}-\ddot{\xi} &= r\sin R, & \dddot{x}-\dddot{\xi} &= s\sin S, \\ \ddot{y}-\ddot{\eta} &= r\cos R, & \dddot{y}-\dddot{\eta} &= s\cos S, \end{aligned}\right\} \qquad (9)$$

and taking the angles M, N, etc. to be in radians, the polynomials for $x-\xi$ and $y-\eta$ then appear in the convenient form

$$\begin{aligned} y-\eta+\mathrm{i}(x-\xi) &= m\,\mathrm{e}^{\mathrm{i}M} \\ &= m_1\,\mathrm{e}^{\mathrm{i}M_1} + n_1\,\mathrm{e}^{\mathrm{i}N_1}(t-t_1) + \tfrac{1}{2}r_1\,\mathrm{e}^{\mathrm{i}R_1}(t-t_1)^2 \\ &\qquad + \tfrac{1}{6}s_1\,\mathrm{e}^{\mathrm{i}S_1}(t-t_1)^3 + \ldots, \end{aligned} \qquad (10)$$

where the suffix 1 indicates that the corresponding differential coefficients (etc.) have been evaluated at time t_1.

The time t_1 is arbitrary, and a similar formula can of course be written down for t_0, the estimated time of mid-eclipse. Since mid-eclipse is defined by $N_0 - M_0 = \pm\pi/2$ (indicating that m has reached its minimum value), we can eliminate N_0 and write

$$m\,\mathrm{e}^{\mathrm{i}M} = m_0\,\mathrm{e}^{\mathrm{i}M_0} \pm \mathrm{i}n_0\,\mathrm{e}^{\mathrm{i}M_0}(t-t_0) + \tfrac{1}{2}r_0\,\mathrm{e}^{\mathrm{i}R_0}(t-t_0)^2 + \tfrac{1}{6}s_0\,\mathrm{e}^{\mathrm{i}S_0}(t-t_0)^3 \ldots, \qquad (11)$$

where the suffix 0 indicates that the corresponding differential coefficients have now been evaluated at time t_0; and, multiplying through by $\mathrm{e}^{-\mathrm{i}M_0}$,

$$\begin{aligned} m\,\mathrm{e}^{\mathrm{i}(M-M_0)} &= m_0 \pm \mathrm{i}n_0(t-t_0) + \tfrac{1}{2}r_0\,\mathrm{e}^{\mathrm{i}(R_0-M_0)}(t-t_0)^2 \\ &\qquad + \tfrac{1}{6}s_0\,\mathrm{e}^{\mathrm{i}(S_0-M_0)}(t-t_0)^3 \ldots \end{aligned} \qquad (12)$$

Separating this into real and imaginary parts, and dividing the latter by the former, we have

$$\tan(M-M_0) = \pm \frac{n_0}{m_0}(t-t_0) + \tfrac{1}{2}\frac{r_0}{m_0}\sin(R_0-M_0)(t-t_0)^2$$
$$\mp \tfrac{1}{2}\frac{r_0}{m_0}\frac{n_0}{m_0}\cos(R_0-M_0)(t-t_0)^3 + \ldots. \qquad (13)$$

The convergence of this series is not immediately evident, since $|t-t_0|$ can be as large as $1^{\mathrm{m}}{\cdot}6$; however, subsequent coefficients are all of the order s_0/m_0 or of the order $\left(\frac{r_0}{m_0}\right)^2$ or of still smaller ones, and in the present case we are dealing with approximately the following values:—

$$\left.\begin{array}{ll} m_0 \simeq {\cdot}0158; & r_0 \simeq 1{\cdot}8\times 10^{-5}; \\ n_0 \simeq {\cdot}0086; & s_0 < 4\times 10^{-8}; \end{array}\right\} \qquad (14)$$

so that $(r_0/m_0)^2 \simeq 10^{-6}$ and $s_0/m_0 \simeq 2\times 10^{-6}$. Since $\frac{1}{6}(t-t_0)^3$ is less than 1, and $\frac{1}{4}(t-t_0)^4$ less than 2, we may neglect all further terms, whatever the phase-angles R_0 and S_0, and still be sure that $\tan(M-M_0)$ is correct to five places of decimals. Since, in addition, $\sec^2(M-M_0)$ becomes appreciable whenever $|t-t_0|$ is large, $M-M_0$ is guaranteed to half a thousandth of a degree at all times; and as the thousandths are only required for rounding-off and checking, formula (13) is as it stands the (sufficiently) rigorous substitute for (4).

The upper signs in (11), (12), and (13), which correspond to $N_0-M_0 = +\pi/2$, must be used when the station is south of the centre-line (as it was in the present case), and the lower signs must similarly be used when it is north; except for some rare and unimportant cases where $|\mu-\lambda| > \pi/2$.

To determine the actual values of M we require also a knowledge of M_0 and of t_0; these are of course obtained directly from the polynomials in $x-\xi$ and $y-\eta$, essentially in the usual manner.

The computed effect of limb irregularities.—It was fortunate for this work that the investigations of Dr C. B. Watts, of the U.S. Naval Observatory, had reached a fairly advanced stage; he had a large number of limb-traces available, and his kindness in selecting and lending suitable ones is very gratefully acknowledged. The limb-traces produced by his machine are of a quality (and quantity) immensely superior to anything available in work on earlier eclipses; important and valuable as the information is which can be obtained from Hayn's classical "Selenographische Koordinaten", the present work gains greatly from being able to substitute the large-scale and very detailed traces obtained by Watts, and especially also from being able to use his "revised" base-lines (see below, p. 84).

The ultimate aim of Watts' work is to produce a collection of lunar data from which the limb-contour for any desired pair (l, b) of libration coordinates can be evaluated; this stage has not yet been completed, but his original traces afford more complete and detailed information, for the particular librations at which they were actually obtained, than could easily be compressed into a general "library" of data of manageable size at all; naturally, he will have no traces exactly corresponding to any arbitrary (l, b) pair that may be required, but he has a very large number of traces altogether, and several came close to having the right librations for the present case. The geocentric librations for the time of mid-eclipse (1948 November 1·186) were $l_0 = +4^\circ{\cdot}12$, $b_0 = +0^\circ{\cdot}38$, and the differential corrections for Mombasa, computed by Atkinson's formulae (3), were $\Delta l = +0^\circ{\cdot}89$, $\Delta b = -0^\circ{\cdot}26$, so that the topocentric values were $l = +5^\circ{\cdot}01$, $b = +0^\circ{\cdot}12$. Dr Watts was able to supply traces as follows:—

No.	43	94	275/6	J65007/8	J65081/2
Date	1948 June 17·1	1949 Feb. 17·4	1950 Aug. 2·4	1950 Aug. 30·0	1950 Sept. 25·9
Limb	West	East	East	East	West
l	$+5^\circ{\cdot}23$	$+5^\circ{\cdot}07$	$+5^\circ{\cdot}04$	$+4^\circ{\cdot}68$	$+4^\circ{\cdot}80$
b	$-0^\circ{\cdot}52$	$+0^\circ{\cdot}27$	$+0^\circ{\cdot}48$	$-1^\circ{\cdot}04$	$-0^\circ{\cdot}51$

and those for 1948 June 17·1 and 1950 August 2·4 were finally selected for use. (No. 94 was from a plate described as "poor", and was rejected for this reason.) Some thought was given to the possibility of tracing a mean between No. 43 and No. J65081; but the Johannesburg plates (J-numbers), in contrast to the Washington ones, have no independent zero of position-angles on them, and in addition they were often rather underexposed for Watts' machine. (They had not originally been taken for this purpose.) The idea was therefore abandoned. The maximum distance of the limb, on trace No. 43, from the limb at the eclipse occurs in lunar P.A. 235° (reached by the "leader" cusp at the start of the run), and is about $0^\circ{\cdot}56$ of lunar surface distance; the maximum distance for trace 275 is in lunar P.A. 156° (reached by the "follower" cusp at the end of the run), and is about $0^\circ{\cdot}34$. The large-scale fit with the eclipse results is undeniably poorer towards the end of the run, but it is by no means certain that the major part of this misfit is even due to this cause at all; on the whole, it seems to be true that misfits of this magnitude cause little real difficulty, though no doubt they would affect the detail shown in Fig. 1.

The method of using the traces is as follows. For any given time, reckoned from mid-eclipse, the half-length, ψ, of the crescent (expressed as an angle at the Moon's centre, not the Sun's) is calculated, and also the value of $M-M_0$. Since M_0, and also C, the position-angle of the Moon's axis (as viewed at Mombasa), are both known, the lunar position-angles, Λ and Φ, of the leader and follower cusps for a smooth Moon, can be computed for this time from

$$\Lambda,\ \Phi = M_0 + 180^\circ - C + (M - M_0) \pm \psi, \tag{15}$$

where the upper sign refers to the leader, since position-angles were increasing. In order to establish the effect of the limb features at the leader cusp (say) on the P.A. for this time, one turns to the appropriate trace (west limb), and lays down, through the point Λ on the baseline, a line which slopes upwards at the correct angle to represent the Sun's limb. One then reads off the difference in lunar P.A., $E'(\Lambda)$ say, between Λ and the P.A. where this line cuts the trace. One does the same for the follower cusp, at the Φ corresponding to the same $t-t_0$ value; and the quantity $E' = \frac{1}{2}[E'(\Lambda) + E'(\Phi)]$ is (except for a small correction) the net effect which the lunar limb irregularities have caused. It should be directly comparable with the value of $P-M$ for this same $t-t_0$ value.

The procedure may be clearer if a couple of practical examples are taken. Figs. 5 and 6 show two stretches of Watts' traces; the continuous lines represent the trace and its baseline, with true lunar position-angles as marked off by Watts, and the short straight lines are lines ruled on a protractor of transparent plastic, of which a number were specially prepared for this work. The horizontal protractor lines are merely guides for orientation. The vertical lines are ruled at separations of $\frac{1}{10}$ degree on the mean scale of Watts' abscissae; his actual scale was very slightly variable, on account of differential refraction and other minor factors, but was sufficiently uniform to make it easy to adjust by eye, within any one degree, for the slight error involved in using a constant protractor spacing. The slanting line represents the Sun's limb; the slant depends on the sharpness of the cusp (reckoned for a smooth Moon), and this varies during the eclipse and is also changed systematically if the minimum apparent separation of

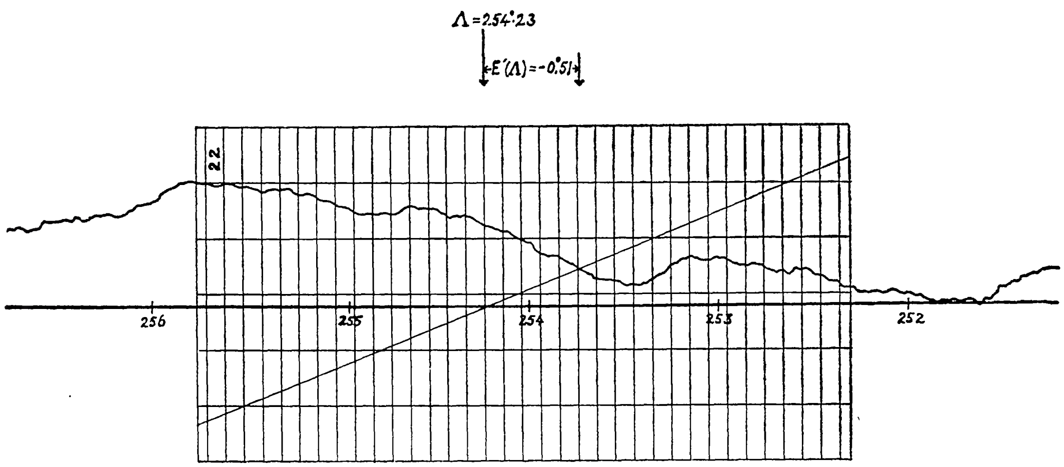

FIG. 5.

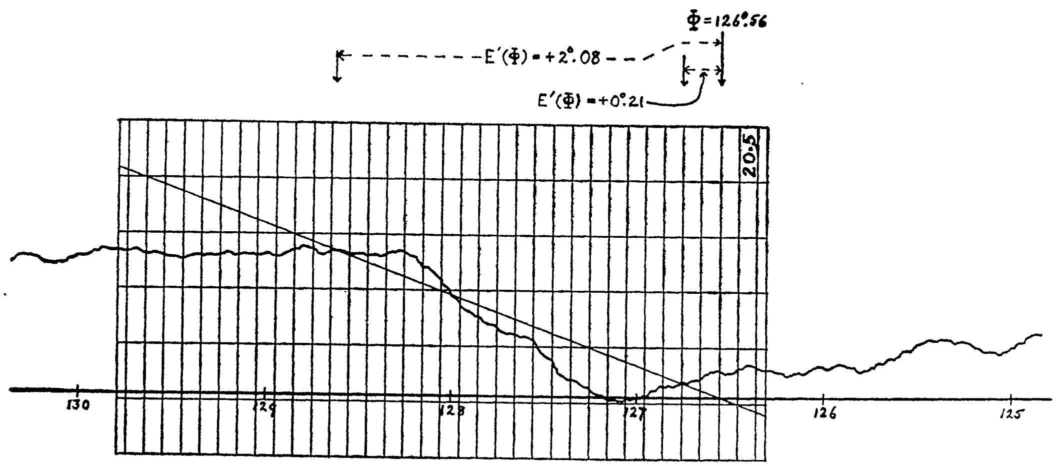

FIG. 6.

centres is changed; in addition it depends on the ratio of vertical and horizontal scales for the trace in question, which depends on the Moon's distance at the time the plate was taken. The computed slants (both traces included) ran from $20^\circ\cdot5$ to $27^\circ\cdot5$, and experience showed that the nearest half degree was sufficiently accurate; 15 protractors in all were therefore prepared, and a critical table was computed for each of the two traces, to indicate the ranges within which each protractor should be used.

Fig. 5 shows the leader cusp at a point where it is working its way up a fairly steep slope. The value of Λ, which would in practice be computed for the $t-t_0$ corresponding to one of the 310 normal times of the chronograph record, is here taken as $254^\circ\cdot23$ and the appropriate protractor is laid down so that (a) a horizontal line is conveniently close to, and strictly parallel to, the baseline;

and (*b*) the intersection of the slant line and a vertical line coincides with the point Λ on the baseline. The difference between Λ and the P.A. where the slant line cuts the trace (P.A. of the true cusp) is then read off directly, in tenths and hundredths of a degree of protractor-ruling, as $-0^\circ{\cdot}51$; this is $E'(\Lambda)$ for the time in question. It is clear that the P.A. of the true cusp will be increasing, in this region, more slowly than that of the "smooth" one, and that the rate will go up to that for the smooth Moon again as soon as the profile becomes level, whether it is then high or low.

In Fig. 6 the follower cusp is shown at a point where a Baily's bead has separated off from the cusp but has not yet disappeared. The computed P.A. for the "smooth" cusp is taken as $\Phi = 126^\circ{\cdot}56$, and the distance to the actual one is either $+2^\circ{\cdot}08$, if the "main" cusp is read, or $+0^\circ{\cdot}21$ if the bead is. (The inner edge of the bead, at $E'(\Phi) = +1^\circ{\cdot}45$ in this case, was very seldom a good object on the film.) It is evident that in determining E'-values, just as in measuring the film, as the computed Φ increases one will at some point or other be forced to jump discontinuously forward, from what was once the main cusp but has now become a negligible bead, and to set on a new "main cusp" which had previously been only a nick in the crescent. The decision when to jump was guided (on both the leader and follower traces) by what had been done in measuring the film originally; but a further special study was afterwards undertaken (see below) which disposes of the risk that one might in this way force the E curve into agreement with the $P-M$ one to an extent which the data themselves did not warrant. (Of the 310 normal P-values, only about 26 were subject to any uncertainty on this account, so that the maximum error which could be introduced is not in any case large; but this revision practically eliminates it). It is clear that steep down-slopes on this trace have the same effect as steep up-slopes on the other, i.e. they will introduce a downwards slant in the E-curve.

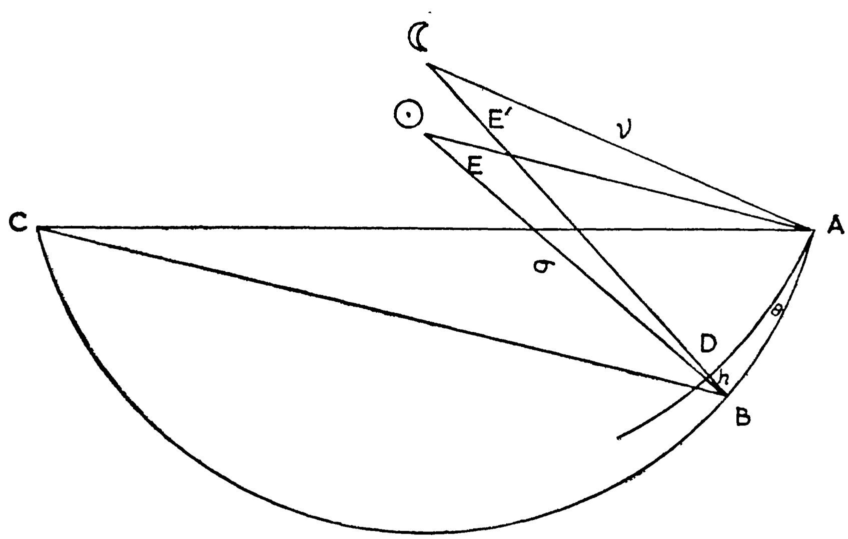

FIG. 7.

The true effect of limb features is slightly different from E'. This may be seen as follows. In Fig. 7, ☉ and ☾ are the centres of the Sun and Moon, σ and ν are their radii, θ is the cusp-angle (computed for a smooth Moon), and h is the height of a lunar mountain above the smooth level. A is where the (leader) cusp would be if the Moon were smooth, B is the actual cusp, and C is the other cusp, which need not lie on the smooth Moon's limb but necessarily does lie on the Sun's. If the Moon's limb were smooth at the leader cusp, AC would be the line of cusps, while the actual line is BC; the true effect produced in the P.A. of the line of cusps, by the mountain of height h, is thus $\mathrm{A}\widehat{\mathrm{C}}\mathrm{B}$ or $\frac{1}{2}\mathrm{A}\widehat{\odot}\mathrm{B}=\frac{1}{2}E(\Lambda)$ say. The angle read off from the trace, $E'(\Lambda)$, is $\mathrm{A}\widehat{☾}\mathrm{B}$, and if BD is drawn perpendicular to the smooth limb, we have

$$\mathrm{AD}/\nu=(h/\nu)\cot\theta=E'(\Lambda)$$

and

$$\mathrm{AB}/\sigma=(h/\sigma)\operatorname{cosec}\theta=E(\Lambda),$$

so that

$$E/E'=(\nu/\sigma)\,.\sec\theta; \tag{16}$$

the effect introduced by the mountain at the leader cusp is thus

$$\tfrac{1}{2}E(\Lambda)=\tfrac{1}{2}E'(\Lambda)\,.\,(\nu/\sigma)\,.\sec\theta.$$

Exactly the same argument applies, independently, to the follower cusp, and the total effect on the line of cusps is thus

$$E=\tfrac{1}{2}[E'(\Lambda)+E'(\Phi)](\nu/\sigma)\,.\sec\theta. \tag{17}$$

For this eclipse, ν/σ was 1·015, constant to the accuracy needed here; θ varied from $4^\circ\!\cdot4$ down to $3^\circ\!\cdot2$ and up to $4^\circ\!\cdot2$ again, and sec θ may thus also be considered constant, at 1·002. We have then

$$E=0\cdot5085[E'(\Lambda)+E'(\Phi)]. \tag{18}$$

Although the difference between 0·5085 and the simpler $\frac{1}{2}$ is small, it can enter systematically into the residuals P–M–E, since neither the mean height nor the mean slope of the E curve is necessarily zero; the rigorous factor has therefore been used in the reductions. It is readily verified that we may ignore the fact that the slant lines on the protractors should ideally be slightly curved (since $\nu\neq\sigma$); even when $E=1^\circ$, the error introduced into E by this is less than $0^\circ\!\cdot001$, and it is quasi-random.

The question which of the features at the leader cusp will combine with which at the follower depends on the crescent length, 2ψ, and in fact the form of the function E is fairly sensitive to ψ; ψ itself is decidedly sensitive to the difference, $\nu-\sigma$, of the lunar and solar semi-diameters. In the first trial solutions which were made, for restricted stretches only, by R. d'E. A., the "eclipse" value of the lunar semi-diameter was used, and the resulting E curve displayed discouragingly little detailed resemblance to the P–M one. Inspection showed that a shorter crescent would probably combine the "leader" and "follower" features more suitably, and a re-trial with a semi-diameter 1″ larger confirmed this. The "eclipse" value, given by $\sin\nu=0\cdot272274\sin\pi_☾$, was derived (**4**) from eclipse observations, in a quite deliberate attempt to represent the effect of the "average valley", at second and third contacts, by means of a circular but undersize Moon. The preoccupation with second and third contacts (**5**) even led to its re-introduction after it had been abandoned, and that preoccupation still persists, of course; but a false semi-diameter is not the most suitable way to cater for it if detailed contours are available, since these must be applied to the true mean. (A false value smaller than the true one is in any case unsuitable for first and fourth contacts, and even for second and third contacts in annular eclipses, since these are all concerned with the "average mountain" and not with the "average valley".)

As soon as a larger semi-diameter than the "eclipse" one was seen to be demanded by the present work, it was decided to use the "occultation" one, given by $\sin \nu = 0{\cdot}2724953 \sin \pi_☾$. (The one used in the lunar ephemeris, given by $\sin \nu = 0{\cdot}272481 \sin \pi_☾$, or $\nu = 0{\cdot}272446\pi_☾ + 0''.079$, would be equally satisfactory.) Fig. 8 indicates, for one particular feature, how much the agreement between E and $P-M$ is improved by the change; in general (except where one limb was nearly featureless), a similar improvement resulted throughout.

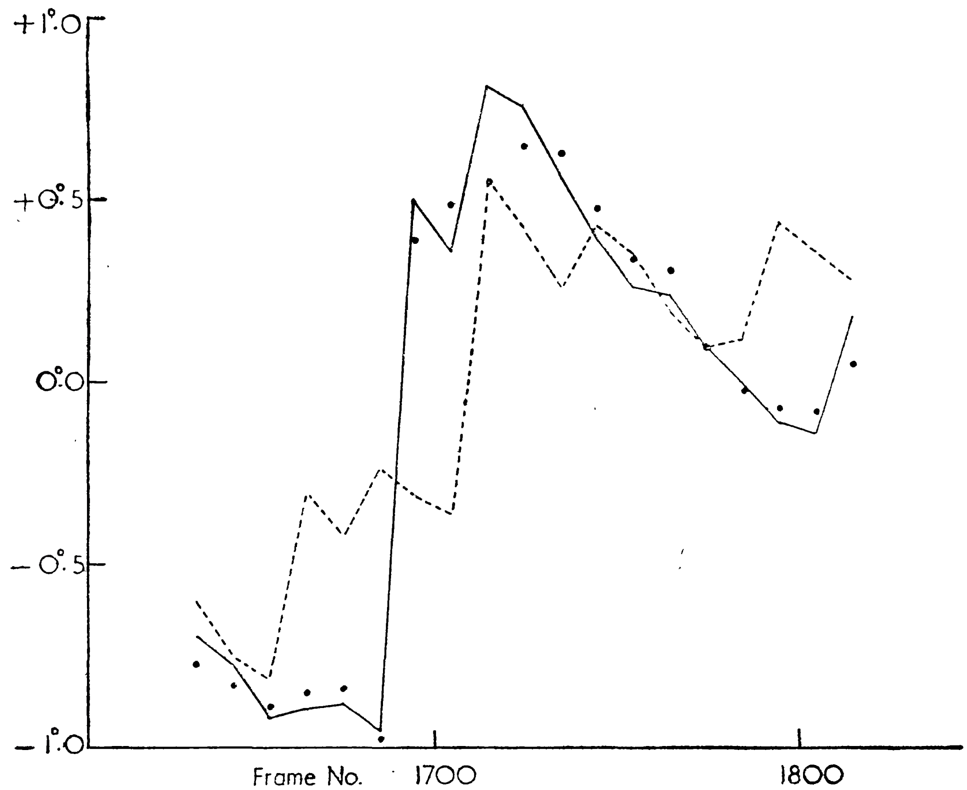

FIG. 8.—*Comparison between "Eclipse" and "Occultation" semi-diameters.*
——— *E curve (occultation semi-diameter).*
– – – – – *E curve (eclipse semi-diameter).*
• • • *P—M values (Mombasa).*

The values of ψ and θ can be computed as in (**1**), but it is more satisfactory to work exclusively with the data given in the "eclipse" pages of the Almanac, using the formulae given in the N.A. "Explanation" in 1940 and previous years. The necessary amendment, to take account of the larger value of ν, is to increase l_1, and decrease l_2, by $(0{\cdot}2724953-0{\cdot}2722740) \sec f/(1-b)$, where $b = \sin \pi_\odot / \sin \pi_☾$, whether l_2 is positive or negative. (It is negative in all eclipses which are total throughout.) $\tan f$ is not affected, to the accuracy required. The eclipse pages then contain all the necessary data, and they obviate the need for an appreciable amount of interpolation, as well as being free from irradiation terms, and already incorporating the special eclipse corrections (**6**).

Accordingly we have

$$\cos \psi = \frac{m^2 - L_1 L_2}{m(L_1 - L_2)}, \tag{19}$$

where $L_1 = l_1 - \zeta \tan f_1$, $L_2 = l_2 - \zeta \tan f_2$.

6

A large number (310, in the present case) of ψ-values are needed for insertion in (15); however, it is not necessary to compute a large number of them from this formula; it is appreciably simpler to compute only enough to determine the constants in the formula

$$\psi = a_0 + a_1 \cos(M - M_0) + a_2 \cos 2(M - M_0) + b_1 \sin(M - M_0) + b_2 \sin 2(M - M_0) \quad (20)$$

which fits the rigorous values with ample accuracy, and then to compute from this an ephemeris of ψ-values for the whole degrees of $M - M_0$ (or for every alternate one) and to interpolate for the fractions. For this purpose it is convenient to obtain $\cos\psi$ as an explicit function of $M - M_0$; this may be done as follows.

Neglecting s_0, we have from the real part of (12)

$$m = (m_0 + dm) \sec(M - M_0), \quad (21)$$

where $dm = \frac{1}{2} r_0 \cos(R_0 - M_0)(t - t_0)^2$. Writing, in this small term,

$$t - t_0 = + \frac{m_0}{n_0} \tan(M - M_0),$$

we have

$$m \simeq m_0 \sec(M - M_0)\left[1 + \tfrac{1}{2}\frac{r_0 m_0}{n_0^2} \cos(R_0 - M_0)\tan^2(M - M_0)\right], \quad (22)$$

which can be used in both the numerator and the denominator of (19). The quantity $L_1 - L_2$ is sensibly constant. L_1 and L_2 both vary slowly and almost linearly with time, and $d(L_1 L_2)/dt$ can be readily evaluated; we have then

$$L_1 L_2 = (L_1 L_2)_0 + \frac{m_0}{n_0} \tan(M - M_0) \frac{d}{dt}(L_1 L_2), \quad (23)$$

so that $\cos\psi$ can be computed for any desired $M - M_0$ values. It is sufficient to compute it for the values 0, $\pm v$, $\pm 2v$, where $\pm 2v$ lie near the ends of the run; the five values of ψ then determine the five coefficients in (20) very simply.

The angles of slant of the protractors are computed as follows. If one imagines a lunar feature which slopes upwards, with respect to the smooth limb, at an angle θ, this slope will of course be unaltered by changing the Moon's distance. But since Watts' abscissae are position-angles, i.e. are not affected by distance, while his ordinates are seconds on the sky, the mountain will appear both higher and steeper, *on his traces*, when the Moon is near. His horizontal scale, being 1° of P.A. to 4·06 cm, is 1 cm $= \nu/232{\cdot}7$ when the semi-diameter is ν (in seconds of arc). His vertical scale being 1 cm $= 0''{\cdot}605$ (for the focal length of the Washington telescope), a mountain whose true slope is θ will have a slope, γ, on his trace given by

$$\tan\theta = (140{\cdot}8/\nu)\tan\gamma, \quad (24)$$

where ν is the topocentric semi-diameter when the Washington plate was taken. If now we interpret θ as the cusp angle at the eclipse, we can interpret γ as the appropriate protractor slant for reading the trace, since the tangents of all slopes are affected in the same proportion.

Since

$$\sin\theta = \frac{2m}{L_1 + L_2} \sin\psi, \quad (25)$$

we may compute an empirical formula for $\sin\theta$ in terms of $M-M_0$, by methods analogous to those of the last section. In practice, it appeared that a symmetrical formula was sufficient, i.e.

$$\begin{aligned}\sin\theta &= a'_0 + a'_1\cos(M-M_0) + a'_2\cos 2(M-M_0)\\ &= a'_0 - a'_2 + a'_1\cos(M-M_0) + 2a'_2\cos^2(M-M_0),\end{aligned} \tag{26}$$

so that $\cos(M-M_0)$ can be expressed as a function of $\tan\gamma$, thus enabling critical tables to be explicitly computed, for selected values of the protractor angle γ.

Historical.—In the present case, all of this analysis, and the accompanying measurement of the limb traces, was carried through several times. The zero of position-angles which was originally used demanded a large change from the 'Almanac' place of the Moon; Dr Watts made important revisions of his 'baselines'; the change to the final zero of position-angles cancelled most of the original change in the Moon's place; and each of these changes was so large that the limb-effects had to be completely re-assessed. In any future work, however, one measurement should be fully sufficient, and the total amount of labour is then not excessive, considering the accuracy which appears to be attainable as a result.

The first analyses were made on the basis of limb-traces supplied, by Dr Watts, before his own work had reached the stage of collating different traces. It was then evident that although (once the "occultation" semi-diameter of the Moon had been adopted) there was a great similarity in the features of the *P–M* curve and the *E* curve, there was a systematic term in the ordinate differences, the central regions being a considerable fraction of 1° apart if the ends coincided; there was also a general tendency for the features of the *E* curve to lie to the right of those of the *P–M* curve. Owing to the "sawtooth" form of the *P–M* and *E* curves, and to the fact that at this stage the steep up-slopes had practically no weight, since on both curves there was some arbitrariness in deciding just when they should occur (p. 65 and p. 79 above), it was difficult to estimate the phase shift, even when the general run of the separation in ordinates had been approximately assessed from the various horizontal stretches; but it was clear that the phase displacement lay roughly between 0°·1 and 0°·5, i.e. that there was a discrepancy of this magnitude between the zero of position-angles used in this work and that used by Dr Watts, in addition to some unknown systematic factor which falsified the ordinates.

When Dr Watts saw these results, at the time of the Rome I.A.U. meetings, he immediately realized that the systematic difference in ordinates might be connected with the difference between the original baselines of his traces and the "revised" ones which he was then engaged in obtaining. His original "baseline" (zero from which mountains reckon upwards and valleys down) is a nearly straight line, computed and drawn by him on the recording paper, and representing in effect the best arc of a circle that can be fitted to the particular limb photograph in question, after correcting for the distortion produced by differential refraction; it is obtained by a straightforward least-squares process based on that trace alone, without reference to any others. Very near full Moon, a large fraction of the whole perimeter may be illuminated (though even then there is almost always some "defective illumination", and when the libration in latitude is large there is always a good deal); at most times of the month, however, the usable arc is much shorter, often only 160° or so, and as there are very extensive regions which lie entirely above, or below, the mean height, the arc of a circle which best fits any particular photograph may differ considerably from the complete circle which would be fitted to the complete perimeter, supposing that this could be, and were, all illuminated at the same time. The regions beyond the poles never can be illuminated when they are on the limb, and thus although one can try to construct complete circles, by fitting together overlapping arcs taken at different phases but with the same l and b values, these circles cannot be closed, however many photographs one may take. The best complete circle for any (l, b) pair may also differ from any central section of that sphere which best fits the Moon as a whole, and it is clearly desirable to refer all profiles to this sphere if possible. This involves some intricate considerations, and a great deal of laborious intercomparison between profiles photographed when the libration coordinates (l, b) were different. Dr Watts is still engaged on this difficult task; he very kindly advanced the particular traces used in the present work ahead of his regular schedule, and communicated "revised" baselines for them specially, the general framework of his intercomparisons being sufficiently far advanced to make this possible, though a further revision (to take account of the difficult problem of "dead ground") is still in progress. The "revised" baselines depart from the original ones over fairly long stretches, sometimes falling as much as $0''\cdot35$ above or below them; it is important to realize that the employment of detailed limb-traces, even of the high quality which the Washington ones possess, will therefore not necessarily eliminate discrepancies of this magnitude between the Moon's place at different librations, at least if the "place" is based on observations of only a short length of limb; it is only the "revised" baselines which will make it possible to eliminate them, and even then not completely unless the centre of the sphere to which they are all referred coincides with the centre of gravity.

A re-computation using the new baselines gave a somewhat different place for the Moon (the mean values of the changes not being zero), and also a considerably better overall fit; for what it is worth, this improvement undoubtedly confirms the general correctness of Watts' revision, but it is clear from Fig. 9 that some systematic discrepancies remain. The longer the arcs of the Moon's limb that are observed in eclipses, the more chance there is, of course, for any residual curvature error in the baselines to show up; if only a short stretch of limb is used, such errors cannot be detected, and the apparent probable error may be correspondingly less than the real uncertainty. However, the small curvature difference still remaining in Fig. 9 may also have its origin in the present work.

The considerable improvement which had now been obtained only hardened the discrepancy in abscissae already noted, and there was extensive correspondence with Dr Watts on the subject. He had not originally expected that his position-angles would be so reliable, and so valuable, as they have proved to be, and his original method of determining them had been somewhat less rigorous, and less complete, in its analysis than his data actually turned out to warrant. In the course of the correspondence, he discovered and communicated several small systematic corrections of P.A. which, if they had all had their maximum values, and in the right sense, in the two traces in question, could almost have accounted for the whole discrepancy; however, the net effect (which varies from trace to trace) is less, and on both traces it proved to be definitely insufficient, though it did produce some improvement. Ultimately, both Dr Watts and the senior author were satisfied that no further corrections could be expected; the remaining error must then be looked for in the zero of position-angles used in the present work. If this zero is increased, all the values of P are increased also, and it becomes necessary to increase the Moon's longitude in order to lower the $P-M$ curve to the old level again; the values of M for all the times t are thus increased, and the values of Λ and Φ are increased with them; any given feature on the lunar limb is therefore reached at an earlier time, so that the E curve is displaced to the left.

The possibility that this might be the correct thing to do was confirmed by the fact that the analyses had so far all resulted in a lunar longitude smaller by several tenths of a second of arc than the value obtained from occultation results; it became increasingly clear, as the occultation reductions themselves proceeded, and were supported by meridian work, that this discrepancy was probably real, and unacceptable. Both the discrepancy with Watts' position-angles and the discrepancy in the Moon's place thus suggested that the zero used so far was too small, i.e. the values of P associated with any given absolute time ought to be larger. Serious consideration was given to the possibility of discarding the zero based on the supposed diurnal run altogether, and basing the analysis on that of Watts' traces; but at that time his estimate of the probable error of his position-angles was about $\pm 0^\circ\cdot 1$, which would mean $\pm 0''\cdot 1$ in the corresponding coordinate of the Moon's place, and this is much larger than the present method appears capable, in principle, of yielding.

It was at this stage that a detailed reassessment showed that the slope of the second part of Fig. 3 indicated a zero which would very nearly cancel both the above stubborn discrepancies at the same time. Since the scatter of individual points was also less in the final stretch than in the earlier part, it was decided to adopt this latter run as the true direction of diurnal motion, ignoring the first part altogether. The solution (2) obtained in this way is formally rigorous, with a reasonably small probable error, and is independent of Watts' zero (though in harmony with it); it contains an element of arbitrariness in its basic assumption, but any other treatment of the points in Fig. 3 (including discarding them altogether) would involve some arbitrariness, and all would result in much larger formal uncertainties.

Before the final analysis was undertaken, it was decided to eliminate as far as possible the arbitrary factor (pp. 65, 79) in the moments of transfer from "cusp" to "appendage", or the reverse, both for the upward jumps in P and for those in E. Accordingly, at all points where either a bead or a conspicuous thread-like appendage appears at the leader-cusp (or disappears, at the follower) the film was now examined frame by frame, to see which was the extreme frame on which it could be detected. The $(t-t_0)$ for this particular frame was taken, and the Λ (or Φ) value for this was computed just as it had been for the 310 means of ten; this Λ (or Φ) was then compared with the extreme Λ (or Φ) at which the appropriate protractor had to be laid down on the limb-trace, so as to bring the bead or thread to the verge of appearance (or disappearance) there also. The differences between the values indicated by the photographs and those indicated by the Washington trace were meaned for the Λ trace, and similarly for the Φ trace, and these two means were adopted as corrections to the position-angle systems of those traces.

It should perhaps be stressed that whatever these corrections may be, they do not directly affect the final result. Their mean indicates a systematic rotation of the Moon's outline (as compared with Watts), but *not* a rotation of the crescent and therefore not a change in the Moon's place; their difference indicates either a small difference between the P.A. systems of the two traces, as determined by Watts, or else a correction to the computed ψ values, which would mean a correction to $\nu-\sigma$; but the different possibilities cannot well be separated. For any trace obtained (as No. 275 was) near the opposite node to the eclipse one, the P.A. correction now derived is also not effectively separable from a correction to the inclination of the Moon's axis. The present method is therefore not suitable for determining these corrections; but this was never its purpose.

The adoption of these two corrections means that the details of the shape of the E curve are now practically "frozen"; the phase relationship between the leader trace and certain critical frame-numbers has been discovered, and similarly for the follower, so that their (mean) relationship to each other can no longer be varied. Even though the least-squares solution may lead to an appreciable value of the correction Δm_0, and so to a change in the minimum apparent separation of centres and in the computed crescent length 2ψ, the only effect will be corresponding changes (of opposite signs) in the corrections that must then be inferred for the P.A. systems of Watts' two traces; similarly, a change in the mid-eclipse time will necessitate changes of the same sign in both corrections. The E curve will be essentially unchanged and undisplaced, and in particular it need not be recomputed, after the least-squares solution, in order to evaluate the residuals (at least if the corrections to M_0 and m_0 are as small as they are here); the E curve of Fig. 9 is that on which the least-squares solution was based, and also that from which the final residuals were obtained, for computing probable errors. These considerations thus result in an appreciable saving of labour, at the same time that they also remove the uncertainty in the phase relation between the steep up-slopes of the E curve and those of the $P-M$ curve; the difference between the ordinates of the two curves has now just as much weight at these places as it has elsewhere. The method is, of course, only practicable if the definition in the photographs is sufficient to show detail at the cusps.

The actual values for the corrections to Watts' systems, after the M_0 and ψ values used in (15) had been corrected by the results of the least-squares solution, became $-0^\circ{\cdot}03 \pm 0^\circ{\cdot}03$ for trace 43 and $+0^\circ{\cdot}09 \pm 0^\circ{\cdot}02$ for trace 275; the mean of these would clearly be acceptable even if the probable errors of both of Watts' systems and of the Mombasa one were all zero. Both the corrections and their probable errors were slightly smaller, after the least-squares solution, than they had been at the "first approximation". They would, of course, both have to be about $-0^\circ{\cdot}3$ if the original zero of position-angles were correct.

The above argument assumes that there is no systematic error due to irradiation, as between a "thread" or other faint cusp feature, on the one hand, and a blunt cusp caused by steep limb-slopes "towards" the crescent, on the other. Blunt tips correspond at both cusps (p. 65) to down-slopes in the P–M curve, and if it were the case that the photographic image of a blunt tip was always somewhat extended, by irradiation, as compared with the image of a thread (which, in the limit, is underexposed), the values of P would be abnormally great when the leader cusp was blunt, i.e. on those down-slopes for which the leader cusp was responsible, and abnormally small for the follower cases. There clearly are some down-slopes in Fig. 9 where $P-M>E$, and others where $P-M<E$, systematically; but examination showed no clear correlation between the sign of $P-M-E$ and the question whether it is the $E(\Lambda)$ values, or the $E(\Phi)$ ones, which show a strong downwards drift in these regions. It thus seems satisfactory to adjust the phase relations of $E(\Lambda)$ and $E(\Phi)$ on the basis of the sudden up-slopes only; observationally, they are the only directly identifiable features.

The equation of condition.—Neglecting higher-order terms, eqn. (10) is

$$m\,e^{iM}=m_1\,e^{iM_1}+n_1\,e^{iN_1}(t-t_1), \tag{27}$$

and we require the effect on M, at any given t, of making small changes in the parameters m_1, M_1, etc., which are the values taken by m, M, etc. at the fixed arbitrary time t_1. Accordingly, if Δ denotes the effect, at any given frame-number (or time) of all such changes as we may make in the positions of the Moon, observer, etc., we have

$$\left(\frac{\Delta m}{m}+i\Delta M\right)m\,e^{iM}=\left(\frac{\Delta m_1}{m_1}+i\Delta M_1\right)m_1\,e^{iM_1}+\left(\frac{\Delta n_1}{n_1}+i\Delta N_1\right)n_1\,e^{iN_1}(t-t_1). \tag{28}$$

Having differentiated, we may change all the subscripts to zeros, i.e. we may write the equation for mid-eclipse. It should be noted, however, that Δm_0, ΔM_0, etc. are not then (in general) the changes in the mid-eclipse values; they are the changes in the values at the fixed time t_0. In particular, the change in the mid-eclipse value of M is almost identically zero, since mid-eclipse is defined by $M=N\pm\pi/2$ and both n and N are very insensitive to any change in the basic data, or in the time. Indeed, this insensitivity is such that we may at once drop Δn and ΔN in (28), and write

$$\Delta m+im\Delta M=\left(\frac{\Delta m_0}{m_0}+i\Delta M_0\right)m_0\,e^{-i(M-M_0)}. \tag{29}$$

In order to eliminate m from this, it is convenient to define the angle T by the equation (essentially eqn. (4))

$$\tan(T-M_0)=\frac{n_0}{m_0}(t-t_0), \tag{30}$$

which differs from (13) only by terms in r_0. T is thus a good approximation for M; the difference amounts to only a very few minutes at the ends of the 76° run. Dropping the terms in r_0 and s_0 from (12), and replacing M by T, the real part becomes

$$m\simeq m_0\sec(T-M_0). \tag{31}$$

Inserting this in (29) and replacing M by T there also,

$$\frac{\Delta m}{m_0}+i\sec(T-M_0)\Delta M=\left(\frac{\Delta m_0}{m_0}+i\Delta M_0\right)e^{-i(T-M_0)}. \tag{32}$$

The imaginary part of this is

$$\Delta M=\cos^2(T-M_0)\Delta M_0-\cos(T-M_0)\sin(T-M_0)\frac{\Delta m_0}{m_0}, \tag{33}$$

and this yields the necessary equation of condition, since we may equate ΔM to the residuals, $P-M-E$, obtained by comparing the observations with the Washington traces. The equation is therefore

$$\cos^2(T-M_0)\Delta M_0-\cos(T-M_0)\sin(T-M_0)\Delta m_0/m_0=P-M-E, \tag{34}$$

where M is computed from (13), i.e. including terms in n_0 and r_0, and T is defined by (30).

The normal equations which result from this are

$$[c^4]\Delta M_0 - [c^3 s]\Delta m_0/m_0 = [c^2(P-M-E)]$$
$$+[c^2 s^2]\Delta m_0/m_0 = -[cs(P-M-E)], \tag{35}$$

where we have written c for $\cos(T-M_0)$ and s for $\sin(T-M_0)$; the two "unknowns" ΔM_0 and $\Delta m_0/m_0$, with their weights, may then readily be evaluated.

In order to use the result, we return to (8). Evidently

$$\Delta(x-\xi) = m\cos M\Delta M + \sin M\Delta m$$
$$= (y-\eta)\Delta M + (x-\xi)\Delta m/m,$$

and in particular

$$\Delta(x_0-\xi_0) = (y_0-\eta_0)\Delta M_0 + (x_0-\xi_0)\Delta m_0/m_0. \tag{36}$$

Similarly $$\Delta(y_0-\eta_0) = -(x_0-\xi_0)\Delta M_0 + (y_0-\eta_0)\Delta m_0/m_0,$$

and we can thus correct either x_0 or ξ_0, and either y_0 or η_0, from the results of (35).

The real part of (32) also gives an equation of condition, and this would be applicable to observations which gave m (essentially the apparent separation of centres of Sun and Moon) as a function of time, instead of to observations of M. However, the two "unknowns" are the same as before, namely ΔM_0 and $\Delta m_0/m_0$; thus, since all information which eclipses can give about the relative places of Sun, Moon, and observer must be expressible in terms of corrections to the run of m and M, it appears that eclipse observations must in effect always yield only the combined corrections $\Delta(x_0-\xi_0)$ and $\Delta(y_0-\eta_0)$, without separating Δx_0 from $\Delta\xi_0$ or Δy_0 from $\Delta\eta_0$, at least unless higher-order terms of sufficient weight can be introduced, so as to get further unknowns in the equation of condition. This seems unlikely, so far as methods confined to a total time of a few minutes near mid-eclipse are concerned. Equation (32) is not dependent at all on the present method of observation; it is equally applicable to any method whatever, inside the shadow track as well as outside, subject only to the limitations within which the approximation $M \simeq T$ holds, i.e. to the condition that the time shall be close to mid-eclipse. (The approximations $\Delta N_0 = \Delta n_0 = 0$ will always hold if the corrections to the places of Moon, Sun, or observer are small; they represent second-order terms.) The question whether to set Δx_0 or $\Delta\xi_0$ to zero (or to some other figure) in order to derive a value for $\Delta\xi_0$ or Δx_0, as the case may be, will thus usually have to be made on other grounds.

Numerical values.—For the final analysis one small further change was made. The Besselian elements used hitherto had been those of the A.E., very slightly smoothed so as to fit them to appropriate polynomials. However, Mrs Gossner, of the A.E. staff, reported that there was an approximation still in use when the 1948 Ephemeris was computed which had a systematic effect in the sixth decimal place. It is known, of course, that the sixth place may be several units in error on account of the limitations of Brown's Tables themselves; but it seemed preferable to remove, even so, an error which represented a systematic departure from those Tables. Mrs Gossner very kindly re-computed the Besselian elements for 03^h, 04^h, and 05^h U.T., correcting this error; similar computations were then carried out for 06^h, and for $04^h\,30^m$, by C.A.M. (The elements for $04^h\,30^m$ were computed from α and δ values interpolated to this time from the hourly values.) The values of α and δ used were those furnished by the N.A.O. for eclipse purposes, which go formally to $0''\cdot01$ in both coordinates, and for all five times the special eclipse corrections to the places of Sun and Moon, as adopted in the Almanacs, were also included, but carried to $0''{\cdot}01$ throughout instead of rounding off to $0''{\cdot}1$ at an early stage. (They were still taken as constants in α and δ though in principle it is $L_{\odot}$, $L_{☾}$, and $B_{☾}$ which should be taken as constant.)

The x and y values so obtained were as follows:—

U.T. =	$03^{h}\,00^{m}$	$04^{h}\,00^{m}$	$04^{h}\,30^{m}$	$05^{h}\,00^{m}$	$06^{h}\,00^{m}$
$x=$	$-1{\cdot}6765355$	$-1{\cdot}1628069$	$-0{\cdot}90587562$	$-0{\cdot}6489093$	$-0{\cdot}1349067$
$y=$	$+0{\cdot}3744145$	$+0{\cdot}1410343$	$+0{\cdot}02440527$	$-0{\cdot}0921730$	$-0{\cdot}3251808$

(37)

They were fitted to polynomials in at least seven figures, to facilitate checking and to minimize rounding-off errors; the polynomials were obtained using $04^{h}\,30^{m}$ as origin, with a fourth-power term included, and after transformation to the origin $t_1=04^{h}\,28^{m}$, close to the time ($04^{h}\,27^{m}{\cdot}9$) of mid-eclipse, they were

$$\left.\begin{aligned} x &= -{\cdot}9230056+{\cdot}00856493(t-t_1)+1{\cdot}98\times10^{-8}(t-t_1)^2-5\times10^{-11}(t-t_1)^3,\\ y &= +{\cdot}0321790-{\cdot}00388692(t-t_1)+2{\cdot}83\times10^{-8}(t-t_1)^2+2\times10^{-11}(t-t_1)^3,\end{aligned}\right\} \quad (38)$$

where t is in minutes.

The following values were also obtained directly, for $t=t_1$:—

$$\mu=251^{\circ}{\cdot}0884;\ \dot{\mu}=4{\cdot}36378\times10^{-3}\ \text{(radians/min)}; \quad (39)$$

$$\sin d=-{\cdot}2485986,\ \cos d=+{\cdot}9686066,\ \dot{d}=-3{\cdot}73\times10^{-6}. \quad (40)$$

For l_1, l_2, f_1, and f_2, the A.E. values were used without change except for that required by the adoption of the "occultation" semi-diameter of the Moon in place of the "eclipse" one. The values were (**6**)

$$\left.\begin{aligned} l_1 &= +{\cdot}5441576+2{\cdot}355\times10^{-6}(t-t_1);\ \tan f_1=+{\cdot}0047120;\\ l_2 &= -{\cdot}0021925+2{\cdot}355\times10^{-6}(t-t_1);\ \tan f_2=+{\cdot}0046885.\end{aligned}\right\} \quad (41)$$

The adopted geographical coordinates of the station were

$$\lambda=-39^{\circ}{\cdot}6800;\quad \phi=-4^{\circ}{\cdot}0702;\quad h=14{\cdot}7\ \text{metres};$$

and using the ellipticity $(\frac{1}{297})$ adopted in the Almanac these lead to

$$\rho\cos\phi'=+{\cdot}9974972,\ \rho\sin\phi'=-{\cdot}0705028 \quad (42)$$

and to

$$\xi=-{\cdot}93268 12+{\cdot}00154349(t-t_1)+8{\cdot}8804\times10^{-6}(t-t_1)^2-4{\cdot}90\times10^{-9}(t-t_1)^3, \quad (43)$$

$$\eta=+{\cdot}0196408+{\cdot}00101314(t-t_1)-0{\cdot}8226\times10^{-6}(t-t_1)^2-3{\cdot}22\times10^{-9}(t-t_1)^3, \quad (44)$$

$$\zeta=+{\cdot}3601268+{\cdot}003942 17(t-t_1)-3{\cdot}2658\times10^{-6}(t-t_1)^2-12{\cdot}51\times10^{-9}(t-t_1)^3. \quad (45)$$

Thus

$$x-\xi=+{\cdot}0096756+{\cdot}00702144(t-t_1)-8{\cdot}8606\times10^{-6}(t-t_1)^2 +4{\cdot}85\times10^{-9}(t-t_1)^3, \quad (46)$$

$$y-\eta=+{\cdot}0125382-{\cdot}00490006(t-t_1)+0{\cdot}8509\times10^{-6}(t-t_1)^2 +3{\cdot}24\times10^{-9}(t-t_1)^3, \quad (47)$$

from which we may deduce, in the usual way, the "Almanac" time of mid-eclipse, $t_a = 04^h\,27^m{\cdot}91120$ and the "Almanac" mid-eclipse values

$$(x_a - \xi_a) = +{\cdot}0090520, \quad (y_a - \eta_a) = +{\cdot}0129734.$$

The difference between these values and the corresponding ones based on the original A.E. elements is of course slight.

The corrections (to the original A.E. elements) demanded by all the earlier analyses were rather large. When the zero of position-angles was finally changed, a new "first" approximation was reached by adding estimated (arbitrary) corrections of the opposite sign, which were also rather large; at the same time, the elements due to Mrs Gossner were adopted. The net effect is that the final least-squares solution was based on a system given rigorously by

$$\left.\begin{aligned} x-\xi &= (46)+\Delta_1(x-\xi), \\ y-\eta &= (47)+\Delta_1(y-\eta), \end{aligned}\right\} \tag{48}$$

where

$$\left.\begin{aligned} \Delta_1(x-\xi) &= -74\times 10^{-7}, \\ \Delta_1(y-\eta) &= -250\times 10^{-7}, \end{aligned}\right\} \tag{49}$$

and (46) and (47) are the "Gossner" values, i.e. are derived from Newcomb's and Brown's Tables, as modified by the corrections $+1''{\cdot}00$ to the mean longitude of the Sun, $-1''{\cdot}50$ to the mean longitude of the Moon, and $-0''{\cdot}50$ to the latitude of the Moon. There is no advantage in (48), as compared with (46) and (47); either pair could serve equally well as the first approximation, and in all future cases it seems likely that the Almanac values will be perfectly satisfactory as a basis for the only least-squares solution that will be needed. In this particular case it happened that the solution was based on the intermediate system (48), but this involves no loss of rigour.

The above values lead, in the usual way, to the mid-eclipse time, for this "first approximation", $t_0 = 04^h\,27^m{\cdot}91023 = t_1 - 0^m{\cdot}08977$, and transforming (48) to this origin we have

$$\begin{aligned} x-\xi &= +{\cdot}0090378 + {\cdot}00702303\,(t-t_0) - 8{\cdot}862\times 10^{-6}(t-t_0)^2 + 5\times 10^{-9}(t-t_0)^3, \\ y-\eta &= +{\cdot}0129531 - {\cdot}00490021(t-t_0) + 0{\cdot}850\times 10^{-6}(t-t_0)^2 + 3\times 10^{-9}(t-t_0)^3. \end{aligned} \tag{50}$$

The cubic terms have now no further importance, since they do not affect $x-\xi$ and $y-\eta$ for $|t-t_0| < 1^m{\cdot}6$ (the extreme values), and therefore cannot affect $\tan(M-M_0)$; they were finally dropped at this stage. Differentiating as necessary we have

$$\left.\begin{aligned} &m_0 = {\cdot}0157945; \quad n_0 = {\cdot}00856359; \quad r_0 = 1{\cdot}78\times 10^{-5}; \\ &M_0 = 34^\circ{\cdot}905; \quad N_0 = 124^\circ{\cdot}905; \quad R_0 = 275^\circ{\cdot}5; \end{aligned}\right\} \tag{51}$$

and so from (13)

$$\tan(M-M_0) = +0{\cdot}542188(t-t_0) - 4{\cdot}91\times 10^{-4}(t-t_0)^2 + 1{\cdot}50\times 10^{-4}(t-t_0)^3, \tag{52}$$

from which the values of M to be used in (34), (15), etc. were computed; also, eqn. (30) evidently becomes

$$\tan(T-M_0) = +0{\cdot}542188(t-t_0). \tag{53}$$

Five values of ψ were computed, from (19), etc., for the values $M-M_0 = 0^\circ$, $\pm 18^\circ$, $\pm 36^\circ$, and were fitted by the formula

$$\begin{aligned} \psi = 83^\circ{\cdot}099 - 7^\circ{\cdot}642\cos(M-M_0) - 1^\circ{\cdot}287\cos 2(M-M_0) \\ - 0^\circ{\cdot}093\sin(M-M_0) - 0^\circ{\cdot}010\sin 2(M-M_0), \end{aligned} \tag{54}$$

from which a table of the 80 values of ψ for the whole-degree values of $M-M_0$ from $-42°$ to $+37°$ was readily computed. The values of Λ and Φ for the 310 values of $M-M_0$ were then obtained from (15), interpolating from this table to get the actual ψ-values, and using $C=+17°{\cdot}194$ as the topocentric P.A. of the Moon's axis at mid-eclipse at Mombasa (3).

The topocentric semi-diameters of the Moon, for the two Washington traces used, were ν (leader)$=954''{\cdot}4$ and ν (follower)$=922''{\cdot}9$; accordingly from (24)

$$\left.\begin{array}{ll} 0{\cdot}148\tan\gamma=\tan\theta & \text{for the leader trace,} \\ 0{\cdot}153\tan\gamma=\tan\theta & \text{for the follower.} \end{array}\right\} \tag{55}$$

Equation (26) took the form

$$\sin\theta=+{\cdot}19113-{\cdot}21010\cos(M-M_0)+{\cdot}07540\cos^2(M-M_0), \tag{56}$$

and critical tables for γ were prepared accordingly. The two traces were then each measured at the 310 Λ (or Φ) values, with the protractors appropriate to each region, as shown by these tables, and the 310 values of E were obtained from (18).

The normal equations.—The 310 values of $P-M-E$ were combined into 31 normal values (each of which was taken as having unit weight), and these were converted to radians. Thirty-one values of c and s were computed from (53), for the appropriate means of ten values of $t-t_0$ (i.e. of 100 chronograph times). Equations (35) then became

$$\left.\begin{array}{r} 22{\cdot}683\Delta M_0+1{\cdot}043\Delta m_0/m_0=-{\cdot}031\,48, \\ 1{\cdot}043\Delta M_0+3{\cdot}550\Delta m_0/m_0=-{\cdot}002\,70, \end{array}\right\} \tag{57}$$

from which

$$\Delta M_0=-{\cdot}001\,371 \text{ (wt } 22{\cdot}38), \tag{58}$$

$$\Delta m_0/m_0=-{\cdot}000\,357 \text{ (wt } 3{\cdot}50). \tag{59}$$

A smooth ephemeris of "calculated" corrections, ΔM_c was prepared from (33), using these values, and was applied to the 31 normal values of $P-M-E$ to obtain the corrected ones. The sum of the squares of the latter was found to be $6{\cdot}047\times10^{-5}$ radians^2 ($=0{\cdot}1985$ deg.^{2}); the standard deviation is thus $\pm{\cdot}001\,444$ radians ($\pm0°{\cdot}083$), and the (formal) probable errors of ΔM_0 and $\Delta m_0/m_0$ are $\pm{\cdot}000\,206$ (radians) and $\pm{\cdot}000\,520$ respectively. The former of these values, i.e. $\pm0'{\cdot}71$, indicates the accuracy with which the P.A. of the crescent at any stated time near mid-eclipse is determined by this method. Also, since

$$\Delta\left[\frac{d}{dt}\tan(M-M_0)\right]\simeq-\frac{n_0}{m_0}\frac{\Delta m_0}{m_0}$$

(n_0 being in effect not subject to correction), the probable error of the quantity $\dfrac{n_0}{m_0}\dfrac{\Delta m_0}{m_0}$, $\pm{\cdot}000282$, represents the uncertainty with which the rate of change of $\tan(M-M_0)$ is determined; near mid-eclipse this corresponds to an uncertainty in the rate of change of P.A. of $\pm0'{\cdot}97$/min.

Inserting (58) and (59), together with the above probable errors, in (36), and taking $x_0-\xi_0$ and $y_0-\eta_0$ from (50), we find

$$\left.\begin{array}{l} \Delta(x_0-\xi_0)=(-210\pm54)\times10^{-7}, \\ \Delta(y_0-\eta_0)=(+78\pm70)\times10^{-7}. \end{array}\right\} \tag{60}$$

If we multiply these probable errors by the Earth's radius, we obtain ± 34 metres and ± 45 metres for the "geodetic" probable errors, parallel to the fundamental plane. The values on the Earth's slanting surface are of course larger, by the secants of the slants, but these will seldom exceed 2 or 2·5.

Although they are not actually required, we may readily obtain a new mid-eclipse time, and new mid-eclipse values of M, etc., if we correct the polynomials (50) by these amounts. Denoting the new values by the subscript 2, we find by the usual methods

$$\left.\begin{aligned} t_2 - t_0 &= +0^{\mathrm{m}}{\cdot}002\,54 \pm 0^{\mathrm{m}}{\cdot}000\,38, \\ t_2 &= 04^{\mathrm{h}}\,27^{\mathrm{m}}{\cdot}912\,77, \\ M_2 &= 34^{\circ}{\cdot}905, \\ N_2 &= 124^{\circ}{\cdot}905, \end{aligned}\right\} \qquad (61)$$

thus directly confirming the practical invariability of the mid-eclipse value of M. The probable error of the time of mid-eclipse (± 23 milliseconds) makes it clear that systematic timing errors less than 5 ms or so would be quite unimportant; the time system actually used thus appears of fully adequate accuracy.

Adding the corrections (49) to (60), so as to reduce the results to the Almanac, we have

$$\left.\begin{aligned} \Delta(x_0 - \xi_0) &= (-284 \pm 54) \times 10^{-7}, \\ \Delta(y_0 - \eta_0) &= (-172 \pm 70) \times 10^{-7}, \end{aligned}\right\} \qquad (62)$$

and the same values of t_2, M_2, N_2 will of course result if these are applied to the polynomials (46) and (47).

Differential corrections.—The basic data which may have to be changed, in order to produce the changes Δ, are as follows:

(a) The R.A. and Dec. of the Sun and Moon. The effects of changes in these on the tabular x and y are well known, and (approximate) formulae for them are quoted in the Almanacs. A change in the Sun's place does however introduce, in addition, a practically identical change in the coordinates, μ and d, of the apex, and this causes changes in ξ and η. We shall need to correct the Sun's place (on independent grounds), and we therefore include terms in $\Delta\mu_0$ and Δd_0 as well as in Δx_0 and Δy_0.

(b) The latitude, longitude, and height of the station will be "unknowns" in geodetic cases, but must be regarded as given in the present work. Corrections $\Delta\phi$, $\Delta\lambda$, and Δh are however included in the analysis, to allow for the possibility that the coordinates of Mombasa may have to be adjusted at some future date on independent grounds; incidentally, they indicate the effect (very small, but readily calculable) of polar variation at Mombasa.

(c) The Earth's ellipticity and the lunar parallax are related; the parallax with respect to the Earth's "mean" radius may be regarded (7) as not subject to correction, but for this reason the "equatorial" parallax is necessarily changed if the ratio of equatorial radius to mean radius is changed, i.e. if the ellipticity is. Brown used the relation given by Newcomb,

$$\pi_{\mathrm{e}} = \pi_{\mathrm{m}}(1 + \tfrac{1}{3}k), \qquad (63)$$

where k is the ellipticity, to connect the equatorial parallax, π_{e}, with the mean one, π_{m}. The situation is a little unsatisfactory, since Brown's Tables are based on a parallax corresponding to the ellipticity $\frac{1}{294}$, while the S and C of the

Almanacs are computed for $\frac{1}{297}$; if $\frac{1}{297}$ is adopted (as it nominally is), Brown's parallax should be diminished by $0''\cdot04$, and the other consequential changes listed by him should also be made. It would be simpler, in the present case, to use S and C values based on $\frac{1}{294}$ and to keep the Almanac x and y values (**8**); the discrepancy between the two ellipticities was however not noticed until the analysis had been completed, and it has not been thought worth while to repeat it all with self-consistent data since the effect on the probable errors would clearly be negligible. The question is rather more complex than this discussion indicates (**9**), and until it has been fully clarified there seems little point in allowing for corrections to the parallax.

(*d*) The time system. A correction (Δt) to the entire time system is provided for, principally so that polar variation and the "annual" term in the Earth's rotation can be dealt with differentially. The correction is without effect on $t-t_0$ and thus does not affect the equation of condition; it is sufficient to include it at the present stage.

Accordingly, if we defer for the moment the explicit treatment of Δx_0 and Δy_0, we may re-write (62) as

$$\Delta x_0-\frac{\partial\xi_0}{\partial\phi}\Delta\phi-\frac{\partial\xi_0}{\partial(\mu_0-\lambda)}\Delta(\mu_0-\lambda)-\frac{\partial\xi_0}{\partial h}\Delta h+(\dot{x}_0-\dot{\xi}_0)\Delta t=(-284\pm54)\times10^{-7},$$

$$\Delta y_0-\frac{\partial\eta_0}{\partial\phi}\Delta\phi-\frac{\partial\eta_0}{\partial(\mu_0-\lambda)}\Delta(\mu_0-\lambda)-\frac{\partial\eta_0}{\partial d_0}\Delta d_0-\frac{\partial\eta_0}{\partial h}\Delta h+(\dot{y}_0-\dot{\eta}_0)\Delta t$$
$$=(-172\pm70)\times10^{-7}. \tag{64}$$

From (6) and the standard formulae for $\rho\cos\phi'$ and $\rho\sin\phi'$ we have

$$\frac{\partial\xi_0}{\partial\phi}=-SC^2\sin\phi\sin(\mu_0-\lambda)=-0\cdot065\,9, \tag{65}$$

$$\frac{\partial\xi_0}{\partial(\mu_0-\lambda)}=\xi_0\cos d_0-\eta_0\sin d_0=+0\cdot353\,3, \tag{66}$$

$$\frac{\partial\xi_0}{\partial h}\simeq\xi_0/6378=-1\,462\times10^{-7}\text{ if }\Delta h\text{ is in km}, \tag{67}$$

$$\frac{\partial\eta_0}{\partial\phi}=+SC^2[\cos\phi\cos d_0+\sin\phi\sin d_0\cos(\mu_0-\lambda)]=+0\cdot965\,9 \tag{68}$$

$$\frac{\partial\eta_0}{\partial(\mu_0-\lambda)}=\xi_0\sin d_0=+0\cdot231\,9, \tag{69}$$

$$\frac{\partial\eta_0}{\partial d_0}=-\zeta_0=-0\cdot359\,8, \tag{70}$$

$$\frac{\partial\eta_0}{\partial h}\simeq\eta_0/6378=+30\times10^{-7}. \tag{71}$$

Thus if $\Delta\phi$ and $\Delta(\mu_0-\lambda)$ are understood to be in seconds of arc, and Δt in seconds of time,

$$10^7\Delta x=-284\pm54-3\cdot20\Delta\phi+17\cdot13\Delta(\mu-\lambda)-1462\Delta h-1170\Delta t,$$
$$10^7\Delta y=-172\pm70+46\cdot83\Delta\phi+11\cdot24\Delta(\mu-\lambda)-17\cdot44\Delta d+30\Delta h+816\Delta t, \tag{72}$$

where we have now dropped the subscripts; to the accuracy we are using, these corrections refer to any fixed time near mid-eclipse, and are constant.

If we differentiate the formulae which define x and y, (**10**), treating $\pi_{☾}$ as constant and setting $\cos(\alpha_{☾}-a)\simeq\cos(\delta_{☾}-d)\simeq 1$, we obtain

$$\left.\begin{aligned}\sin\pi_{☾}\Delta x&=\cos\delta_{☾}(\Delta\alpha_{☾}-\Delta a)-\sin\delta_{☾}\sin(\alpha_{☾}-a)\Delta\delta_{☾},\\ \sin\pi_{☾}\Delta y&=\Delta\delta_{☾}-\Delta d+\cos\delta_{☾}\sin d\sin(\alpha_{☾}-a)(\Delta\alpha_{☾}-\Delta a).\end{aligned}\right\}\tag{73}$$

In these, we may write with sufficient accuracy,

$$\Delta\alpha_{☾}-\Delta a=\frac{1}{1-b}(\Delta\alpha_{☾}-\Delta\alpha_{\odot})=1\cdot0025(\Delta\alpha_{☾}-\Delta\alpha_{\odot})\tag{74}$$

and
$$\Delta\delta_{☾}-\Delta d=\frac{1}{1-b}(\Delta\delta_{☾}-\Delta\delta_{\odot})=1\cdot0025(\Delta\delta_{☾}-\Delta\delta_{\odot}),\tag{75}$$

and if $\Delta\alpha$ and $\Delta\delta$ are in seconds of arc we then have

$$\cos\delta_{☾}(\Delta\alpha_{☾}-\Delta\alpha_{\odot})-\cdot0041\Delta\delta_{☾}=3568''\Delta x,\tag{76}$$

$$\Delta\delta_{☾}-\Delta\delta_{\odot}+\cdot0041\cos\delta_{☾}(\Delta\alpha_{☾}-\Delta\alpha_{\odot})=3568''\Delta y,\tag{77}$$

or, solving explicitly for $\Delta\alpha_{☾}$ and $\Delta\delta_{☾}$,

$$\left.\begin{aligned}\cos\delta_{☾}(\Delta\alpha_{☾}-\Delta\alpha_{\odot})-\cdot0041\Delta\delta_{\odot}&=3568''(\Delta x+\cdot0041\Delta y),\\ \Delta\delta_{☾}-\Delta\delta_{\odot}&=3568''(\Delta y-\cdot0041\Delta x).\end{aligned}\right\}\tag{78}$$

Combing (78) with (72), and writing $\Delta\mu\simeq-\Delta\alpha_{\odot}$, $\Delta d\simeq+\Delta\delta_{\odot}$, we have as our experimental results

$$\left.\begin{aligned}\cos\delta_{☾}(\Delta\alpha_{☾}-0\cdot994\Delta\alpha_{\odot})-\cdot004\Delta\delta_{\odot}&=-0''\cdot102\pm0''\cdot019\\ &\quad-\cdot0010\Delta\phi-\cdot0061\Delta\lambda-\cdot522\Delta h-\cdot417\Delta t,\\ \Delta\delta_{☾}-0\cdot994\Delta\delta_{\odot}+\cdot004\Delta\alpha_{\odot}&=-0\cdot''061\pm0''\cdot025\\ &\quad+\cdot0167\Delta\phi-\cdot0040\Delta\lambda+\cdot013\Delta h+\cdot293\Delta t,\end{aligned}\right\}\tag{79}$$

where, as already stated, $\Delta\phi$ and $\Delta\lambda$ are in seconds of arc, Δh is in km, and Δt is in seconds of time.

Polar variation.—The international coordinates of the pole, interpolated to 1948·835, are $-0''\cdot032$, $+0''\cdot196$. These introduce corrections to the coordinates of the station of $\Delta_p\phi=-0''\cdot15$, $\Delta_p\lambda=+0''\cdot01$, together with a time correction $\Delta_p t=-16\cdot4$ ms, since the time used was that determined at Greenwich. Accordingly, we must add to the right-hand side of (79)

$$\left.\begin{aligned}\cos\delta_{☾}\Delta_p\alpha_{☾}&=+0''\cdot007,\\ \Delta_p\delta_{☾}&=-0''\cdot007.\end{aligned}\right\}\tag{80}$$

Annual variation.—This can be treated in a similar way, i.e. by a time change combined with a longitude change, but in one sense it is a matter of opinion whether one actually should apply this correction or not, since the annual variation is only one part of the complete E.T.–U.T. difference, while it is the whole difference which is connected with $\Delta\alpha_{☾}$. Since, however, occultation results show a number of oscillations which are not yet fully explained, so that nothing less than an annual mean can be relied on for estimating E.T.–U.T. from them, it appears best to remove the annual variation, as determined by clocks, in the present case. Indeed, the application to lunar observations (whatever the method) of an independently determined correction for short-period E.T.–U.T. fluctuations may still remain for some years preferable to any attempt to infer those fluctuations from the lunar observations directly; lunar observations by themselves are not very well suited for separating E.T.–U.T. fluctuations from errors in the Moon's elements.

If the correction to be added to the time system used, in order to reduce it to "uniform" time for the year in question, is $\Delta_v t$ (seconds), we must insert this, and also the longitude correction $\Delta_v\lambda = +15''\Delta_v t$, in (79). $\Delta_v t$ for 1948 is not particularly well determined; we adopt -58 ms for November 1 (from Finch (**11**), using the 1948–9 figures only and ignoring any $\Delta_\alpha\alpha$ term that may affect the FK3 places), and so

$$\left.\begin{aligned}\cos\delta_{☾}\Delta_v\alpha_{☾} &= +0''\!\cdot\!030,\\ \Delta_v\delta_{☾} &= -0''\!\cdot\!014,\end{aligned}\right\} \tag{81}$$

must be added to the right-hand side of (79).

The equinox.—Eclipse observations refer the Moon to the true Sun, whose place is rigorously computed with respect to the true equinox. The present observations, however, have used a time system based on the FK3 equinox. The formula for "Sidereal Time of 0^h U.T." is not itself affected by an equinox change, but the experimental determinations of U.T. are: if the R.A.'s of the FK3 stars are too small by a quantity E (seconds of time), the U.T. employed in the observations will require a correction $\Delta_E t = +E$. The Washington observations probably afford the best recent determinations of E: the value adopted from the 9-inch T.C. observations 1935–45 (**12**) is $E = +0^s\!\cdot\!022$, with a probable error (based on the discrepancies and weights) which is readily found to be $\pm 0^s\!\cdot\!0043$; the value adopted from the 6-inch T.C. observations 1941–49 (**13**) was $+0^s\!\cdot\!023$ with a p.e. which may similarly be obtained as $\pm 0^s\!\cdot\!0051$. We adopt, therefore, $\Delta_E t = +0^s\!\cdot\!022 \pm 0^s\!\cdot\!003$ (**14**), so that the quantities

$$\left.\begin{aligned}\cos\delta_{☾}\Delta_E\alpha_{☾} &= -0''\!\cdot\!009 \pm 0''\!\cdot\!001,\\ \Delta_E\delta_{☾} &= +0''\!\cdot\!006 \pm 0''\!\cdot\!001,\end{aligned}\right\} \tag{82}$$

must be added to the right-hand sides of (79).

Light-time and aberration.—Brown's Tables take no explicit notice of aberration at all, and the Moon's place in the Almanacs is computed without applying it; this caused some difficulty at the start of the present work. The question was raised with Dr G. M. Clemence, who replied that he was already engaged on a review of several related matters and would include this one. The point has since been fully clarified (**15**); almost all of the effects of aberration are taken up (in the tables) by a slight falsification of the lunar elements, but there remain certain small correction-terms which should be applied. The result should then strictly be the Moon's geocentric place; however, all observations (including those on which Brown based his tables) are also falsified by the fact that the light-time to the observer is less than that to the centre of the Earth. The difference amounts to $21\cdot3\zeta$ milliseconds, or $7\cdot6$ ms in the present case. The need for this correction appears to have been first pointed out by Banachiewicz (**16**); it is insignificant unless ζ is large, but we insert it here along with the other small terms. We have $\Delta_L t = +7\cdot6$ ms, $\Delta_L\lambda = +0''\!\cdot\!114$ and so

$$\left.\begin{aligned}\cos\delta_{☾}\Delta_L\alpha_{☾} &= -0''\!\cdot\!004,\\ \Delta_L\delta_{☾} &= +0''\!\cdot\!002.\end{aligned}\right\} \tag{83}$$

The additional corrections (15), to deal fully with the ordinary aberration, and the correction pointed out by Woolard (17), are omitted here, because the final result is expressed as a departure of the observed place of the Moon from that of Brown's Tables as they stand; for 1948, they do of course still contain even the Great Empirical Term, and the tabular places based on them are not in any case really adequate for geodetic work.

The Sun's place.—It does not appear to be generally realized that a correction to the Sun's mean longitude (which is applied in the Almanacs) is not the only correction which may be important. Corrections to the obliquity, eccentricity, and longitude of perihelion all produce changes in the Sun's R.A. and Dec., which at some times of the year can add up to very appreciable amounts, and these corrections seem relatively well known. For simplicity, and because the Cape results are more self-consistent than any others, we take the Cape corrections alone; the figures can readily be modified if an alternative choice were preferred. For the four periods 1907–11, 1912–16, 1925–32, and 1932–36, the corrections Δh and $\Delta k (h = e\cos\pi,\ k = e\sin\pi)$ have weighted means $\Delta h = -0''\cdot141 \pm 0''\cdot010$ and $\Delta k = +0''\cdot136 \pm 0''\cdot010$. A correction to the obliquity need not be applied; it affects the Moon's tabular place as much as the Sun's, and although the left-hand sides of (79) are not exactly $(\Delta_{\text{☾}} - \Delta_{\odot})$, they are sufficiently near this to make any admissible value of the correction cancel out. The corrections to the Sun's place, in addition to the correction $\Delta L_{\odot}$ which is already incorporated in the Almanacs (and which there seems no good reason to modify) are

$$\left.\begin{aligned} \Delta\alpha_{\odot} &= +2\sin\alpha\sec\delta\Delta h - 2\cos\alpha\sec\delta\cos\epsilon\Delta k, \\ \Delta\delta_{\odot} &= +2\cos\alpha\sin\delta\Delta h - 2\cos^2\alpha\cos\delta\sin\epsilon\Delta k, \end{aligned}\right\} \tag{84}$$

or for 1948 November 1·186,

$$\left.\begin{aligned} \Delta\alpha_{\odot} &= +0''\cdot380 \pm 0''\cdot021, \\ \Delta\delta_{\odot} &= -0''\cdot125 \pm 0''\cdot007, \end{aligned}\right\} \tag{85}$$

and these should also be inserted in (79). By comparison, we may note that the corrections already included, on account of $\Delta L_{\odot}$, are

$$\left.\begin{aligned} \Delta_L\alpha_{\odot} &= +1''\cdot00 \pm 0''\cdot060, \\ \Delta_L\delta_{\odot} &= -0''\cdot33 \pm 0''\cdot020. \end{aligned}\right\} \tag{86}$$

The probable errors in (86) are based on a p.e. for $\Delta L_{\odot}$ of $\pm 0''\cdot060$; this value depends on questions of weighting which are to some extent a matter of opinion, but it seems that the values in (86) are at any rate considerably larger than those in (85).

The Moon's place.—Collecting eqns. (79), (80), (81), (82), (83), and (85) we obtain

$$\left.\begin{aligned} \cos\delta_{\text{☾}}\Delta\alpha_{\text{☾}} &= +0''\cdot288 \pm 0''\cdot019 - 0\cdot0010\Delta\phi - 0\cdot0061\Delta\lambda - 0\cdot522\Delta h, \\ \Delta\delta_{\text{☾}} &= -0''\cdot200 \pm 0''\cdot025 + 0\cdot0167\Delta\phi - 0\cdot0040\Delta\lambda + 0\cdot013\Delta h, \end{aligned}\right\} \tag{87}$$

where we have assumed that no further corrections to the Sun's place, or to the time system, will be required, but have still kept the terms which would be needed in geodetic work; we have also, for the moment, omitted the probable errors of the Sun's place, since these are irrelevant in geodetic applications. (The probable errors of all the other corrections will ordinarily be too small to have any effect, by comparison with $0''\cdot02$.) In order to express this result in the form of corrections to Brown's Tables, we must now add the special eclipse corrections that were applied (by Mrs Gossner and C.A.M., in this case) when computing the Besselian elements. These were $-1''\cdot720$ in $\alpha_☾$ (i.e. $-1''\cdot666$ in $\alpha_☾ \cos\delta_☾$) and $+0''\cdot190$ in $\delta_☾$. Accordingly

$$\left.\begin{aligned} \cos\delta_☾ \Delta_B \alpha_☾ &= -1''\cdot378 \pm 0''\cdot019 - 0\cdot0010\Delta\phi - 0\cdot0061\Delta\lambda - 0\cdot522\Delta h, \\ \Delta_B \delta_☾ &= -0''\cdot010 \pm 0''\cdot025 + 0\cdot0167\Delta\phi - 0\cdot0040\Delta\lambda + 0\cdot013\Delta h, \end{aligned}\right\} \quad (88)$$

are the corrections to Brown's Tables demanded by the present results.

We now transform these into orbital components. At $04^{\mathrm{h}}\,28^{\mathrm{m}}$, the hourly rates of change of $\alpha_☾$ and $\delta_☾$ were $+135^{\mathrm{s}}\cdot84$ and $-888''\cdot0$, while $\delta_☾$ was $-14^\circ\,21'\cdot6$; the P.A. of the Moon's (geocentric) orbit was thus $90^\circ+24^\circ 13'$, and its orbital speed was $2\,164''\cdot5$/hr. Rotating axes in (88) through $24^\circ 13'$ we have the orbital corrections

$$\Delta_B L = -1''\cdot253 \pm 0''\cdot020 - 0\cdot008\Delta\phi - 0\cdot004\Delta\lambda - 0\cdot481\Delta h, \quad (89)$$

$$\Delta_B B = -0''\cdot574 \pm 0''\cdot024 + 0\cdot015\Delta\phi - 0\cdot006\Delta\lambda - 0\cdot202\Delta h, \quad (90)$$

and multiplying (89) by $1\,976''\cdot5/2\,164''\cdot5$ we obtain the mean longitude correction as

$$\Delta_B L_m = -1''\cdot144 \pm 0''\cdot018 - 0\cdot007\Delta\phi - 0\cdot004\Delta\lambda - 0\cdot439\Delta h. \quad (91)$$

The coefficients of the correction terms can of course be justifiably carried to several more places, if $\Delta\phi$ and $\Delta\lambda$ seem likely to be large.

In geodetic work, a pair of equations of the form (90) and (91) would be obtained for each station, and the same values of $\Delta_B B$ and $\Delta_B L_m$ would then be adopted for all, probably by assuming that the values of $\Delta\phi$, $\Delta\lambda$, and Δh for some one station were zero, or, more generally, by weighting all the absolute terms according to the supposed certainties of their geographical coordinates. Unfortunately, it seems that some arbitrary decision of this sort would always have to be made; for even if it is assumed that Δh is zero for all stations, there are still $2n+2$ unknowns for n stations, namely ΔL_m and ΔB in addition to the two coordinates of each station, and there are only $2n$ equations. It thus appears that even in order to get a geodetic result it is necessary to derive (or assume) *absolute* corrections to the Moon's place (and the Sun's); it is not, for example, true that changing the longitudes of two stations by equal amounts would alter the absolute terms of all the eqns. (90), or (91), by the same amounts, though such a change would leave the geodetic separation of the stations unaffected.

Probable errors.—The probable errors of (90) and (91) are those which apply in geodetic work. In many respects it is also these which are most directly comparable with those obtained by other methods. It is true that in the present case the p.e. of ΔM_0 should be increased, on account of the uncertainty in the zero of position-angles; the p.e. there (2) was $\pm 0^\circ\cdot022 = \pm\cdot000\,384$ radians, and neglecting the fact that there is a varying weighting, $\sec^2(M-M_0)$, involved, roughly this figure should be combined with the $\pm\cdot000\,206$ quoted for ΔM_0. In future cases, with the scatter of Fig. 3 much reduced by the use of fiducial marks, and with three or four times as long a diurnal run to rely on if the bench is properly stabilized, this contribution should be much reduced; it may, however, always remain a factor to be considered. The p.e. of $\Delta m_0/m_0$ ($\pm\cdot000\,520$) is practically unaffected by uncertainty in the P.A. zero, and those of $\Delta(x-\xi)$ and $\Delta(y-\eta)$ are thus not greatly affected by it, even in the present case.

A more serious objection is that the residuals $P-M-E$ still show some appreciable systematic features, so that the formulae for the probable errors (and indeed the least-squares process itself) are of doubtful validity. The 310 values of $P-M$ plotted in Fig. 9 are those which result from the least-squares solution, i.e. the original $P-M$ have been corrected by the smooth ephemeris ((33), (58), (59)) already mentioned; it is evident that in addition to several short stretches there is one longer one, at the end, where they depart systematically from the E values. The least-squares process is valid only on the assumption that successive residuals are independent, and this is not so here.

The question how to obtain a "probable error" in such cases is peculiarly liable to arise in work involving the Moon's limb. If, for example, the coordinates of the centre of an observed arc of lunar limb are deduced by a least-squares process from the lengths of a number of radii measured from a provisional centre, the probable errors of the result are sometimes quoted without any special discussion of possible systematic runs in the residuals. Obviously if the number of radii is made large enough for many to fall within the same "mountain" or "valley", systematic trends will appear; the number of radii used will then have an appreciable effect on the formal probable errors obtained, even though it has very little on the coordinates of the centre themselves, and if too many are used the formal probable errors may fall considerably below the true ones. In general, it is for this reason unprofitable to use more than 15 or 20 radii for a lunar semi-circumference; and since the mountains and valleys can run to $\pm 2''$, the probable errors of the Moon's place at any one instant cannot be less than $0''\cdot 1$ or $0''\cdot 2$ whatever the method of observation, so long as limb corrections are not applied.

When they are applied, of course, the true probable errors can be much reduced; on the one hand, the residuals themselves are much reduced, and on the other, their independence of each other should persist even if many more are used. The question how many one can use is not altogether self-evident, but the following consideration can provide some guidance (**18**). If w residuals were all completely independent, the number of changes of sign if they were run through in order would be $w/2$ on the average; it is thus reasonable to consider that the number of residuals which can fairly be regarded as independent is not greater than twice the number of their changes of sign. However, this condition, though it appears to be necessary, is not sufficient; the 310 residuals of Fig. 9 change sign about 87 times, which suggests a fairly high degree of independence, but the true extent is much less, owing to the way the changes are distributed, and there can be no doubt that the uncertainty of the final result is greater than is suggested by the probable errors obtained, even though these have only assumed that the 31 normal points are independent.

The residual systematic effects which are still present in Fig. 9 may be partly due to occasional slow waves of atmospheric distortion; they are perhaps due in part to instability of the bench, of some other kind than that already taken into account (and to this extent it seems likely that they can be eliminated in future work); they may to a small extent be due to the fact that the librations of the Washington traces were not exactly those of the eclipse (though such comparisons as could be made suggested that these discrepancies were not very serious); and they may also be due to the fact that Dr Watts has still some further systematic baseline revisions to carry out. These could not account for any of the short systematic divergences in Fig. 9, but they might in principle almost entirely remove such a trend as is shown by the final 42 points.

The absolute probable errors of the Moon's place are greater than the "geodetic" ones of (89)–(91), owing to the rather large uncertainty which still attaches to the Sun's corrections. For this purpose we must include the probable errors of both (85) and (86); these combine to $\pm 0''{\cdot}067$ in ecliptic longitude and, of course, $\pm 0''{\cdot}000$ in ecliptic latitude, and rotating axes to the direction of the Moon's orbit we get $\pm 0''{\cdot}067$ in orbital longitude and $\pm 0''{\cdot}006$ in latitude. Accordingly we may write (89), (90), and (91) as

$$\Delta L = -1''{\cdot}25 \pm 0''{\cdot}070, \tag{92}$$

$$\Delta L_m = -1''{\cdot}14 \pm 0''{\cdot}064, \tag{93}$$

$$\Delta B = -0''{\cdot}57 \pm 0''{\cdot}025, \tag{94}$$

together with the same terms in $\Delta\phi$, $\Delta\lambda$, and Δh as before, if these are needed.

It is these results which should be most nearly comparable with those obtained from occultations. Even these, however, cannot be directly compared with either the lunation means or the quarterly ones. If they are compared with the annual ones, they should in principle give the effect, at the time of the eclipse, of those errors in the tables whose period is comparable with a year, or is much shorter. As they stand, however, the occultation results have not been corrected for polar variation or the FK3 equinox.

The largest single uncertainty, so far as the above probable errors are concerned, is that in the Sun's mean longitude; until this is decreased, the accuracy with which the Moon's place can be determined from eclipses cannot now be much reduced by any further experimental advances. In the present case there are undoubtedly still some systematic errors also; in particular, there is the uncertainty in the zero of position-angles (though the good agreement with the mean of Watts' systems suggests that there is little actual error here), and the fact that the crescents were only measured in one orientation of the film. Further, Dr Watts' baselines are not yet final, and the geographical coordinates of Mombasa may also need some revision. If these uncertainties can be eliminated in future work (as seems indeed quite possible), the probable errors in (90) and (91) should indicate the geodetic reliability to be obtained by this method.

Royal Observatory,
Greenwich:
1954 *November* 30.

Royal Greenwich Observatory,
Herstmonceux:
1954 *November* 30.

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