1955ApJ...121..750W

A COMET MODEL. III. THE ZODIACAL LIGHT*

FRED L. WHIPPLE
Harvard College Observatory

Received June 10, 1954; revised December 30, 1954

ABSTRACT

The Poynting-Robertson effect is shown to require about 1 ton/sec of small particles to maintain the zodiacal cloud, irrespective of particle shape, density, or dimension within the range of 10^{-4}–1.0 cm. The zodiacal cloud is assumed to be of the nature deduced by van de Hulst and Allen from their studies of the Fraunhofer corona and the zodiacal light. The probable cometary contribution to the zodiacal cloud is here considered on the basis of the icy-comet model. Some 30 tons/sec of meteoritic material contributed continuously in typical comet orbits are mostly lost by the action of the following physical forces or processes as the particles spiral inward toward the sun by the Poynting-Robertson effect: (*a*) interstellar wind, (*b*) Jupiter's random perturbations, (*c*) the Jupiter perturbational barrier, and (*d*) collisional destruction. Of these, *b* and *d* are found to be the most important. The final calculated contribution is about the required amount.

Collisions among the particles appear to be largely responsible for the cutoff in zodiacal particle size above about 0.03 cm, as found by van de Hulst. Corpuscular radiation from the sun will simulate the Poynting-Robertson effect but will simultaneously tend to destroy the particles. No allowance for this effect is included in the calculations because of uncertainties in the numerical quantities involved. Corpuscular radiation, however, if sufficiently powerful, may exceed the Poynting-Robertson effect in importance and may also demand a larger source of material for the zodiacal cloud. If so, the corpuscular radiation will also increase the critical cutoff dimension.

I. INTRODUCTION

From the original investigation by J. H. Poynting (1903) and the relativistic treatment by H. P. Robertson (1937*a*), it has long been known that "a drag, attendant on the pressure due to the solar radiation, constitutes a force which is effective in clearing the neighborhood of the Sun of small particles in astronomically-significant times" (Robertson). If the zodiacal light and Fraunhofer corona arise from the scattering and diffraction of sunlight by small particles somewhat concentrated toward the plane of the ecliptic, as demonstrated by the studies of H. C. van de Hulst (1947) and C. W. Allen (1947), then these particles must be replenished either continuously or sporadically. Sources of such small particles may exist in (*a*) the interstellar medium by solar capture, (*b*) the interplanetary gases by condensation, (*c*) the asteroidal belt by collisional fragmentation, or (*d*) comets by ejection or disintegration. Source *a* has not been observed, while source *b* presents serious theoretical difficulties. Source *c* has been explored by S. L. Piotrowski (1953) and found to be adequate. The purpose of the present paper is to demonstrate the extent to which the cometary source *d* is also adequate.

Any comet model that produces meteoritic material may well be extended to produce zodiacal dust; for example, J. H. Oort, in his valuable contribution (1950) on the dynamics of comets, makes such a suggestion without specifying the detailed structure of a cometary nucleus. V. G. Fessenkov (1914) was perhaps the first to suggest a cometary origin for the zodiacal light. The icy conglomerate model, as presented in the two previous papers of this series (Whipple 1950, 1951), permits the determination of a quantitative relationship between comets and the zodiacal light.

From the comet model we can calculate the rate of loss of meteoritic material. If the cometary nucleus is spherical of radius R_c, contains a fraction β of its mass as nonvolatile meteoritic material and a fraction $1 - \beta$ as ices with a sublimation heat of H ergs per gram and if the efficiency of solar heat transfer to the ices is $1/n$, then at a solar

* An investigation carried out under Contracts Nos. NOrd-10449 and N5ori-07647 with the U.S. Naval Bureau of Ordnance and Office of Naval Research.

distance of r cm the comet will lose per second a mass of meteoritic material, M_k, given by (Whipple 1951),

$$M_k = \frac{R_c^2 \beta E}{4 n r^2 (1-\beta) H}, \tag{1}$$

where E is the total solar energy radiated per second.

We must integrate this loss of mass over all comets. Several comets are observed to pass perihelion per year; over the interval 1801–1850, the average number was 1.8; during 1860–1879, it was 5.4; and in certain recent years as many as 13. The apparent increase must arise largely from the improvement in observing techniques. The range in intrinsic cometary brightnesses is enormous. N. T. Bobrovnikoff (1942) derives the absolute magnitudes of 45 comets ranging from 1.7 to 10.4, where the absolute magnitude of Halley's Comet is 5.6. More than a third of the comets selected by Bobrovnikoff are brighter than Halley's Comet. It is probably pointless to calculate a precise estimate of the "average" brightness and frequency of comets. Since a few comets are intrinsically very much brighter than Halley's Comet, I shall simply assume that the mass-loss by all comets equals the loss by Halley's, were it always at 1 a.u. from the sun. This assumption is tantamount to the assumption that the total contribution by all comets is equivalent to a yearly return of Halley's Comet as it was in 1910.

If, according to estimates discussed previously, we adopt for Halley's Comet $R_c = 10^6$ cm, $\beta = \frac{1}{5}$, $H = 1.88 \times 10^{10}$ erg/gm, $r = 1$ a.u., and $1/n = \frac{1}{2}$ in equation (1), we find, for the total *meteoritic* mass contributed by comets, $M_k = 2.8 \times 10^7$ gm/sec. Hence the total contribution of meteoritic material by the comets is of the order of *30 tons per second*. The material is, of course, left near the orbits of comets, which, on the average, should roughly approximate the present distribution of comet orbits. The contribution of gases alone will be of the order of 100 tons per second.

Various factors, however, require us to correct the calculated cometary contribution in comparing it to the zodiacal rate of mass gain and loss: (*a*) Jupiter perturbs the motions of all the particles and eliminates a large fraction; (*b*) we do not know the distribution in the dimensions of small particles ejected from comets; (*c*) an "interstellar wind" from the interstellar gas through which our system may pass will affect particle motions; (*d*) corpuscular radiation from the sun may assist the Poynting-Robertson effect but may also tend to destroy the particles; (*e*) encounters among the zodiacal particles may be destructive. Effects by planets other than Jupiter, by asteroidal material, and by interplanetary gases are other factors to be considered. In the following sections we shall attempt to evaluate these various factors.

II. THE POYNTING-ROBERTSON EFFECT

Robertson's treatment of the Poynting-Robertson effect for light may be readily generalized to include the corresponding effects by corpuscular radiation from the sun. Suppose the sun loses M_c gm/sec of material (radiation or very small particles) emitted isotropically with velocity V_c. This material then strikes and is momentarily retained by a particle of mass m and cross-sectional area A, moving in an orbit about the sun at a distance r with total velocity V and velocity V_N normal to the radius vector. If $V_c \gg V_n$ (or V), the radial repulsive force on the particle is closely $AM_cV_c/(4\pi r^2)$. If, further, the material is re-emitted from the particle, isotropically about the radius vector, the resisting force normal to the radius vector is equal to the radial force multiplied by V_N/V_c, or $AM_cV_N/(4\pi r^2)$.

Robertson introduces a quantity, α, such that the retarding force normal to the radius vector is given by $\alpha m V_N/r^2$. Equating these two statements of the retarding force, we evaluate the quantity α as follows:

$$\alpha = \frac{AM_c}{4\pi m}. \tag{2a}$$

For the case of radiation alone, let the total energy emitted by the sun per second be given by E and equal to $M_c c^2$ by Einstein's relation, where c is the velocity of light. Then, from equation (2a), we have α given by Robertson's relation,

$$\alpha = \frac{AE}{4\pi m c^2}. \tag{2b}$$

For meteoric particles somewhat greater in dimension than the wave length of maximum energy in the solar spectrum, equation (2b) is correct to the order of 10 per cent for likely materials with relatively low albedos. The result is little changed for perfect reflecting spheres.

Robertson shows generally that α is the proportionality factor between the true anomaly of the orbital particle and the particle's rate of loss of angular momentum (per unit mass) about the sun. In a differential time, dt, the parameter of the orbit, $p = a(1 - e^2)$, will be reduced by an amount dp, given by

$$dt = -\frac{a^{3/2} p^{1/2} dp}{2\alpha p}. \tag{3}$$

Let us now consider the total energy of solar radiation, L, that is scattered or diffracted by a particle from the time that the particle is released in an orbit of parameter, p_0, until finally it spirals close enough to the sun to be vaporized at $p = p_f$. Generally, we may assume that the initial orbit is rather eccentric, while the final orbit is nearly circular (Robertson 1937a; Wyatt and Whipple 1950), although the latter condition will fail for particles from a few comets with very small perihelion distances. The energy, dL, scattered in time, dt, including the time average of $1/r^2$ in an eccentric orbit, is given by

$$dL = \frac{K AE dt}{4\pi a^{3/2} p^{1/2}}, \tag{4}$$

where K is the mean (with respect to both wave length and particle dimension) efficiency factor for total scattering by reflection and diffraction.

Combining equations (4), (2b), and (3), we find the total energy scattered by the particle spiraling through dp to be,

$$dL = -\frac{K m c^2 dp}{2p}. \tag{5}$$

Integrated between the limits of p_0 and p_f, equation (5) gives, for the total energy, L, diverted by the particle during its entire life as an independent body, the equation

$$L = \tfrac{1}{2} K m c^2 \ln\left(\frac{p_0}{p_f}\right). \tag{6}$$

This total scattered radiation is independent of the particle shape, the particle density, and the nature of the solar radiation, except for small indirect effects in K, the scattering coefficient. The energy is numerically little dependent upon the initial and final orbital characteristics within practical limits. The dimensions of the particle, however, are limited if K is to be accepted as independent of particle size over an appreciable range of wave length.

Our application of equation (6) to the zodiacal light (or Fraunhofer corona) depends upon an equilibrium assumption, viz., that the zodiacal light is statistically constant or stable over long intervals of time, of the order of millions of years if particles of the order of 10^{-2} cm radius are to be considered (van de Hulst 1947; Allen 1947). The interval varies directly as the radii of particles that contribute most to the phenomenon.

To follow the consequences of the foregoing equilibrium assumption, let us designate the total energy scattered in all directions by the entire zodiacal cloud per second as L_Z and the relevant mass contributed per second as a constant, M_Z. We may adopt average values of K and log (p_0/p_f) in equation (6). Over a sufficient interval of time, once equilibrium has been established, the average mass-input (and loss) rate will match the average energy-scattering rate according to the relation

$$L_Z = \tfrac{1}{2} K M_Z c^2 \ln\left(\frac{p_0}{p_f}\right). \tag{7}$$

Our assumption of equilibrium thus leads to the conclusion that the total brightness (energy-scattering rate) of the zodiacal light and the Fraunhofer corona is proportional to the rate at which mass, distributed within the proper limits of size, is gained (and lost).

To examine the theoretical space density of the particles with respect to the sun as determined by the Poynting-Robertson effect, let us consider a region near the sun filled largely with particles with almost circular orbits near the end of their spirals. Then $a \simeq p \simeq q \simeq r$, and the time, dt, of fall through a distance, dr, becomes simply

$$dt = -\frac{r\,dr}{2a}. \tag{8}$$

Since the distribution of the inclinations of the orbits will not change appreciably during these later stages of the spiral, the space density, ρ_a, of particles with a given value of a is proportional to dt divided by the volume element, $\pi r^2 dr$, along a given radius, or, from equation (8),

$$\rho_a \sim (a r)^{-1}. \tag{9}$$

Hence, in the neighborhood of the earth, the distribution of particles probably follows a space-density law close to the $1/r$ law, but corrected toward the $1/r^2$ law by the contributions of particles in more elongated orbits. A better evaluation of the space-density law should become available from meteor orbits obtained photographically by the Baker-Super-Schmidt meteor cameras or obtained by electronic techniques. Clearly, the exponent of r must exceed 2 at some moderate solar distance, or the mass integral will become excessive.

III. THE FRAUNHOFER CORONA AND THE ZODIACAL LIGHT

H. C. van de Hulst (1947) and C. W. Allen (1947) agree so well in their quantitative explanations of the Fraunhofer corona and the zodiacal light that only the briefest description of their findings is needed here. Their solutions are not sensitive to the density distribution of the particles with respect to solar distance. Allen prefers a $1/r^{1+}$ law, while van de Hulst adopts roughly a $1/r^2$ law to within 0.1 a.u. of the sun, where sublimation of likely meteoritic compounds over long periods of time would eliminate the small particles. These results are quite in harmony with the Poynting-Robertson results, and I shall adopt 0.1 a.u. as the limiting value, p_f, in applying the theory.

A lower limit of particle size is set at about radius $s = 3 \times 10^{-5}$ cm for metallic-conducting particles, because the pressure of sunlight will exceed the solar gravitational attraction near this value and for smaller dimensions (van de Hulst 1946). For some types of dielectric particles the critical zone occurs near $s = 1.5 \times 10^{-5}$ cm and extends for only a narrow range in dimension, below which Rayleigh scattering sets in and smaller particles may spiral into the sun. Neither Allen nor van de Hulst finds evidence for Rayleigh scattering; Allen's measures indicate that the Fraunhofer corona has the same color or is slightly redder than the sun, while van de Hulst predicts a slight reddening.

Above the limiting particle size, Allen co-ordinates the observations of the Fraunhofer corona and the zodiacal light for various particle sizes, covering approximately the range from 5×10^{-3} to 3×10^{-5} cm, while van de Hulst derives a law in which the number in range, ds, varies as $s^{-2.6}$, with a reduction in this frequency distribution above $s = 0.035$ cm. The resultant density of matter on the ecliptic at $r = 1$ a.u. is given by van de Hulst as 5×10^{-21} gm/cm^3 and by Allen as ($s = 10^{-3}$ cm), 6×10^{-23} gm/cm^3.

As for the integrated intensity of the entire corona and the Fraunhofer corona, respectively, in terms of the integrated intensity of the sun's disk, van de Hulst finds 1.4×10^{-6} and 4.2×10^{-7}, and Allen, 1×10^{-6} and 1.1×10^{-7}. I adopt the value 3×10^{-7} of the sun's intensity for the Fraunhofer corona as seen from the earth.

We wish to derive the value of L_Z, the integrated energy scattered in the Fraunhofer corona and zodiacal light over their entire extent and over all directions. We may crudely approximate L_Z if we increase the foregoing value of the intensity of the Fraunhofer corona because of (*a*) the earth's proximity to the sun and (*b*) the unmeasurable light diffracted within the sun's apparent disk. At the same time, we must reduce the value below that of a spherically symmetrical cloud about the sun because of (*c*) the earth's position in the ecliptic near the plane of greatest particle concentration for diffraction. Since only the correction for *c* can be estimated even roughly and since none of our corrections appears to be much larger than the discrepancy of a factor of 3 between van de Hulst's and Allen's values, I assume simply that the suggested corrections compensate each other and that the total energy, L_Z, diffracted and scattered by the zodiacal particles is 3×10^{-7} of the sun's total energy of radiation per second.

Equation (7) can now be applied to yield the value of M_Z, the mass contribution per second to the zodiacal cloud required to maintain the Fraunhofer corona and the zodiacal light. The average value of the scattering coefficient K may be adopted as 1.1, made up of 1.0 for diffraction and 0.1 for albedo. The effect of diffraction on the Poynting-Robertson effect would be negligibly small. A mean value of $\ln (p_0/p_f) = 3.0$ may be adopted, corresponding to a final orbital parameter, $p_f = 0.1$ a.u., and an initial parameter, $p_0 = 2$ a.u. If, instead, p_0 were taken as 1 or 4 a.u., $\ln (p_0/p_f)$ would be changed only to 2.3 or 3.7, respectively.

A solution of equation (7) with these numerical data yields the result, $M_Z = 0.77 \times 10^6$ gm/sec. Hence an addition of approximately *1 ton per second of meteoritic material will maintain the Fraunhofer corona and the zodiacal light indefinitely* at their present luminosities. By a much less general theory, involving a distribution function of particle sizes and an assumed density for the particles, V. G. Fessenkov (1947) earlier came to a similar numerical conclusion.

IV. EFFECTS OF THE INTERSTELLAR WIND

Although no direct evidence for the fact exists, the solar system is probably moving through an interstellar gas (and dust) cloud. The interstellar wind thus created would tend to blow small distant particles from the gravitational attraction of the sun. Let us assume that the sun is moving through the cloud with velocity V_i, and that there are N_i hydrogen atoms (of mass m_H) per cubic centimeter. We neglect other gases as of a smaller order of magnitude. Suppose the interaction of the H atoms with spherical dust particles of radius s, density ρ_s, and mass m to be one of momentary attachment and subsequent escape at thermal velocities. Then the force on the particles is $\pi m_H s^2 N_i V^2$, and that of solar gravity GmM/r^2, where M is the mass of the sun. The consequent ratio of the force of the wind to that of the solar attraction becomes

$$\frac{\text{Force int. wind}}{\text{Force solar grav.}} = \frac{3\, m_H r^2 N_i V_i^2}{4 \rho_s s G M}. \tag{10}$$

The wind velocity V_i is probably about 20 km/sec, as a reasonable estimate based upon the solar motion. If we adopt the value for the density as $\rho_s = 4$ gm/cm^3, and solve for the solar distance at which the wind force equals solar attraction, we find

$$r\ (\text{a.u.}) = 6.87 \times 10^5 \left(\frac{s}{N_i}\right)^{1/2} . \tag{11}$$

Table 1 lists these critical distances in 1000 a.u. for various particle radii and hydrogen densities at which the solar and wind forces are equal. We should generally expect particles to be lost to the system at distances exceeding those in Table 1. Since *new comets*, according to Oort's study, will usually have aphelion distances exceeding 50,000 a.u., we see that the critical dimension for the retention of meteorite particles from them is near radius 10^{-2} cm, just below the limit for visual meteors. The solution is not extremely sensitive to the interstellar H abundance, a value of 1 atom/cm^3 probably being a reasonable estimate. We conclude, therefore, that *new* and *relatively new* comets, according to the Oort definition, will probably not contribute appreciably to the zodiacal light.

TABLE 1

CRITICAL SOLAR DISTANCES

N_i(H Atoms/ Cm^3) s(Cm)	Critical r (a.u./1000)		
	0.1	1	10
10^{-4}	22	7	2
10^{-3}	69	22	7
10^{-2}	217	69	22
10^{-1}	687	217	69
1	2170	687	217

The interstellar wind will act generally as a resisting medium for particles that are not blown out of the system. The rate of the resulting increase in $1/a$ has been calculated for an orbital plane normal to the direction of the wind, giving the result $\Delta\ (1/a)/\text{period} = 7.9 \times 10^{-11}\ a(\text{a.u.})^{1/2}\ N_H/s$ for the circumstances postulated in this section. Since this rate is smaller than that given in equation (26) by the Poynting-Robertson effect for $a < 4 \times 10^4$ a.u. and $N_H = 1$, the wind-drag effect will be neglected in this discussion.

V. CORPUSCULAR RADIATION FROM THE SUN

The recent auroral studies by C. W. Gartlein (1950), A. B. Meinel (1951), and L. Vegard (1952) demonstrate that hydrogen atoms strike the earth at velocities up to 3000 km/sec during strong aurorae. L. Biermann (1951) finds that such corpuscular radiation from the sun may account for certain phenomena of cometary tails, while V. A. Ambarzumian (1952) holds that such corpuscular ejection is nearly a universal phenomenon for stars. Let us suppose that, on the average, N_c hydrogen atoms per cubic centimeter are crossing the earth's orbit at a constant velocity, V_c, radially from the sun. Under the same physical conditions as those assumed in the previous section, a spherical particle of radius s and density ρ_s will experience an outward force which bears the following ratio to the sun's gravitational attraction:

$$\frac{\text{Force corpuscles}}{\text{Force gravity}} = \frac{3\, m_H N_c V_c^2 (1\ \text{a.u.})^2}{4 \rho_s s G M}, \tag{12}$$

independent of solar distance if V_c remains constant.

For $V_c = 3000$ km/sec, the ratio becomes $1.91 \times 10^{-7} N_c/(\rho_s s)$. Thus the corpuscular radiation will blow away particles of $s = 10^{-4}$ cm and $\rho_s = 4$ gm/cm^3 if N_c exceeds 2.1×10^3 hydrogen atoms/cm^3. We know, however, that N_c is limited to less than this value because the polarization of the zodiacal light sets an upper limit (Whipple and Gossner 1949; Behr and Siedentopf 1953) to the electron density at the earth's distance, about 10^3/cm^3. The hydrogen and other gases expelled from the sun must be ionized so that the number of nuclei will not exceed the number of electrons. Since the calculation is based on extreme values, we may neglect any direct loss of zodiacal particles by corpuscular radiation. Electrostatic effects may possibly change the effective cross-sections of small particles to protons. At velocities of the order of 1000 km/sec or more, however, such effects may be neglected.

For particles of the zodiacal light, the reduction in solar attraction by corpuscular radiation will not yield observable effects, because we cannot yet measure orbital periods.

The fact that corpuscular solar radiation can produce an effect on the orbits of meteoric particles similar in form to the Poynting-Robertson effect was shown in Section II. The ratio of the corpuscular to the P-R effect, J, is given by the ratio of equation (2*a*) to (2*b*), or $M_c c^2/E$, where M_c is the mass of corpuscles emitted radially and isotropically by the sun per second and E is the total radiant energy per second. In terms of the present notation this ratio becomes

$$J = \frac{4\pi m_H c^2 N_c V_c (1 \text{ a.u.})^2}{E}. \tag{13}$$

Thus the corpuscular effect equals the Poynting-Robertson effect when $V_c N_c$ equals 8.9×10^8 (cm/sec)/cm^3. For $V_c = 3000$ km/sec the equality occurs when $N_c = 3$ protons/cm^3 at the earth's distance from the sun. Since we have no adequate measure of the average corpuscular radiation from the sun, we cannot include the corpuscular effect in the present calculations. If later evidence indicates that $V_c N_c$ is comparable to 10^9/cm^2/sec or larger, the consequent effect on the rate of spiraling toward the sun can be introduced into the final results by simply correcting the particle radius, s, for which the calculation has been carried through. For example, if $N_c = 100$/cm^3 and V_c remains 3000 km/sec, then a particle of radius $30s$ will experience the same rate of spiraling as is now calculated for a particle of radius s.

It is of some general interest to explore a bit further some of the consequences of corpuscular radiation from the sun. Under the postulated circumstances, the energy required to maintain the corpuscular radiation is $1.6 \times 10^{-5} N_c$ of the sun's total radiation, not appreciable if N_c is small. Angular momentum changes on the sun, even for tangential expulsion, are small compared to the sun's present angular momentum for 3×10^9 years if N_c is small. For $N_c > 10^3$/cm^3, the effect could become important, however.

The interactions of corpuscles with meteoritic particles are important in a second fashion as cathode sputtering occurs, etching away the surfaces of the particles. The subject of sputtering is well reviewed by H. S. W. Massey and E. H. S. Burhop (1952), who tabulated the measured sputtering rates of various positive ions on various metals. At 500 ev, each H_2^+ ion knocks out 0.4, 0.15, and 0.6–0.7 atoms from the surfaces of *Cu*, *Ni*, and *Ag*, respectively. The sputtering rate increases with atomic (or molecular) weight but at a considerably smaller rate than direct proportionality. With increasing ion energy, up to a few thousand electron volts, the sputtering rate increases nearly linearly. At higher energies, the increase is probably less rapid. Contamination of the surface greatly reduces the sputtering rate; absorbed gases and oxide surfaces appear to be important restraining influences. The rate is apparently much reduced if the impinging ion can combine with an available atom within the surface.

Since sputtering experiments on geological specimens have not come to the writer's attention and since the chemical and physical structure of the surface on a cometary meteoroid is only vaguely surmised, we can only guess at the destructive action by corpuscular radiation. At 3000 km/sec, the energy of our H^+ ions, 50,000 ev, is about 100 times that found in the experiments quoted above. The masses are reduced by half as protons replace H molecules. The crystal structures of the particles must be highly imperfect and well contaminated with residue atoms of C, N, and O combined with hydrogen in a wide variety of compounds and radicals. Thus the incoming H^+ ion might combine easily. In all, the type of meteoritic particle envisaged here seems to present a poor sputtering surface. A sputtering rate of 40 atoms per proton appears to be an excessive upper limit, 10 per proton still a high rate, and 0.01–1 per proton a more probable range of values. A rate of 1 atom per proton corresponds to an energy efficiency of vaporization of about 10^{-4}.

Let us investigate the effect of sputtering by corpuscular radiation on the motions of particles about the sun. The combined spiraling rate of the orbital parameter, p, produced by corpuscular radiation and the Poynting-Robertson effect, will, by equations (3), (2*a*), (2*b*), and (13) take the form

$$\frac{dp}{dt} = -\frac{AWp^{1/2}}{ma^{3/2}}, \tag{14a}$$

where W is defined by the equation

$$W = \frac{E(1+J)}{2\pi c^2}. \tag{14b}$$

Let us assume that each proton of corpuscular radiation that strikes the zodiacal particle removes N_c atoms of mass m_s, with negligibly small velocities. Since the particle has a cross-section A, the rate of mass-loss produced by corpuscular radiation on the particle of mass m will be

$$\frac{dm}{dt} = -\frac{n_c m_s A N_c V_c (1\text{ a.u.})^2}{r^2} = -\frac{AB}{r^2}, \tag{15a}$$

where B is defined by the equation. Averaged over an eccentric orbit, the rate of mass-loss becomes

$$\frac{dm}{dt} = -\frac{AB}{a^{3/2}p^{1/2}}. \tag{15b}$$

Equations (14*a*) and (15*b*) lead to the following relation between the mass-loss and the orbital parameter:

$$\frac{dm}{dp} = \frac{m}{p}\frac{B}{W}, \tag{16}$$

which integrates to the relation

$$m = m_0 \left(\frac{p}{p_0}\right)^{B/W}, \tag{17}$$

where m_0 is the initial mass of the zodiacal particle, corresponding to its constant mass under the Poynting-Robertson effect alone, and p_0 is the initial injection parameter of the orbit, as before.

The rate at which radiant solar energy is totally scattered by the particle, dL/dt, retains its initial form as given by equation (4), although the scattering area A has now become a variable. We change the notation from L to L_c to indicate that the scattering rate will differ in time as A changes with loss of mass. If we now relate dL_c and dp by eliminating dt in equations (4) and (14a), we find, corresponding to equation (6),

$$\frac{dL_c}{dp} = -\frac{KE}{4\pi W}\frac{m}{p}. \tag{18a}$$

We can now introduce the variable mass, m, by means of equation (17), to obtain

$$\frac{dL_c}{dp} = -\frac{KEm_0}{4\pi W p_0^{B/W}}\, p^{(B/W-1)}. \tag{18b}$$

Thus, by integration, we find that the solar radiant energy that is totally scattered by the initial mass m_0 during its spiral from p_0 to p_f, including the momentum effects of both

TABLE 2*

MASS INPUT FOR CORPUSCULAR RADIATION (L/L_c) COMPARED WITH P-R EFFECT

N_c	10	100	1000	10	100	10	100
n_c	0.1	0.1	0.1	1.0	1.0	10	10
$1+J$	1.67	7.7	68	1.67	7.7	1.67	7.7
B/W	0.60	1.30	1.48	6.0	13.0	60	130
L/L_c	3.6	31	306	30	302	302	3020

* Calculated for $V_c = 600$ km/sec. For other velocities correct N_c by $V_c/600$ km/sec.

corpuscular radiation and the Poynting-Robertson effect as well as mass-loss by corpuscular radiation, is

$$L_c = \frac{KEm_0}{4\pi B}\left[1-\left(\frac{p_f}{p_0}\right)^{B/W}\right]. \tag{19}$$

Note that this result is independent of the shape, density, or dimensions of the particle as far as the dimensions lie within the limits specified for the similar Poynting-Robertson result obtained in equation (7). Since the initial mass is linearly related to the total lifetime scattering by the particle in both equations (19) and (7), the rate of mass injection required to maintain the zodiacal light must be increased in the ratio of L/L_c, if corpuscular radiation be active, as assumed here. The ratio of equation (7) to (19) is, then,

$$\frac{L}{L_c} = \frac{2\pi c^2 B}{E \ln(p_0/p_f)}\left[1-\left(\frac{p_f}{p_0}\right)^{B/W}\right]. \tag{20a}$$

The bracketed factor in equation (20a) is nearly unity for practical cases of interest in which the sputtering effect is appreciable and the corpuscular effect exceeds the Poynting-Robertson effect. We may then adopt the earlier value $\ln(p_0/p_f) = 3$, and, from equations (15a) and (20a), obtain the approximate numerical result:

$$\frac{L}{L_c} = 5.0\times 10^{-8} n_c N_c V_c. \tag{20b}$$

Values of L/L_c from equation (20a) are given in Table 2 for various values of N_c and n_c and for $V_c = 600$ km/sec. Corresponding values of $1+J$ and B/W are also included for their general interest. Corpuscular radiation clearly demands a considerably greater supply of material, L/L_c, to maintain the zodiacal cloud than does the Poynting-Robertson effect alone, particularly if n_c exceeds 1.0. Note in Table 2 that $1 + J$ is the factor of increase in mass rate of $n_c = 0$.

We shall neglect corpuscular radiation in the following sections because of the great uncertainties in its amount and in calculating its effects. Its possible important role, however, must be continuously borne in mind.

VI. THE PERTURBATIONS OF PARTICLES IN LONG-PERIOD COMETARY ORBITS

In this section we shall calculate the fraction of particles that attain a value of $1/a = 0.2/(\text{a.u.})$ after release in orbits with the same distribution as is observed for cometary orbits of smaller $1/a$.

Particles ejected in long-period comets will be subjected to perturbations by the planets and by the Poynting-Robertson effect. The latter perturbations increase $1/a$ secularly, while the former exhibit a random character. Particles with $1/a > 0.2/(\text{a.u.})$ will be perturbed in a different manner, particularly by Jupiter and Saturn, than those in much longer orbits, so that our approach to their subsequent history must be altered after we bring them to $1/a = 0.2/(\text{a.u.})$.

Attempts to cope with the large perturbations arising from relatively close approaches of bodies to Jupiter have led to a number of extensive studies by H. A. Newton (1893), M. O. Callandreau (1902), H. N. Russell (1920), A. J. J. von Woerkom (1948), and E. Öpik (1951). These large perturbations are generally of little significance in our problem for $1/a < 0.2/(\text{a.u.})$ and will be neglected here.

The concept of "random" perturbations as suggested by Russell and utilized so effectively by Oort appears to present a sound approach to the problem for aphelia well beyond Jupiter's orbit. Let us assume with Russell and Oort that the arithmetic mean value of random perturbations in $1/a$ produced by Jupiter is zero for a particle with a very long period, a random inclination, and perihelion within Jupiter's orbit. As an approximation to a Gaussian distribution in $\Delta(1/a)$ per revolution, Oort adopts the mean absolute perturbation $|\Delta(1/a)| = 0.00058\,(\text{a.u.})^{-1}$ per perihelion passage for "direct" orbits and 0.00038 for "retrograde" orbits; the first value is very close to Russell's determination from fifteen passages of Halley's Comet. We may assume that a comet undergoes such a perturbation (either positive or negative) in $1/a$ during each period. For small particles, there is superimposed upon this random change in $1/a$, a systematically positive change produced by the Poynting-Robertson effect. To determine the fraction of the particles ejected from very long-period comets that may spiral into short-period orbits, we can apply the theory of the one-dimensional random walk, explicitly the problem of the *Gambler's Ruin*.

Since we utilize a binomial-type expression for the Gaussian distribution in $\Delta 1/a$ per period, an application of the well-known De Moivre-Laplace limit theorem indicates that the individual steps should be taken as equal to the standard deviation, $\sigma(1/a)$, of the perturbation in $1/a$ per period, not the mean absolute value. Let ϵ represent the ratio of the Poynting-Robertson change in $1/a$ per period to the standard deviation, or

$$\epsilon = \frac{\delta 1/a}{\sigma(1/a)}. \tag{21}$$

So long as ϵ remains well below unity, we may accept a good approximate solution to our problem by considering the steps in $1/a$ per period as constant. Let Z be the ratio in probabilities of a positive change per period in $1/a$ to a negative change, as follows:

$$Z = \frac{1-\epsilon}{1+\epsilon}. \tag{22}$$

Then a particle released in an orbit of $1/a_0$ will have a certain probability, P_x, of attaining an orbit of $1/a_x$ without being lost from the system ($a_0 > a_x$). Let the number of steps from $1/a = 0$ to the two boundaries, $1/a_0$ and $1/a_x$, respectively, be

$$A_0 = \frac{(1/a_0)}{\sigma(1/a)} \quad \text{and} \quad A_x = \frac{(1/a_x)}{\sigma(1/a)}. \tag{23}$$

The theory of the Gambler's Ruin (W. Feller 1950) gives the desired probability as follows:

$$P_x = \frac{Z^{A_0} - 1}{Z^{A_x} - 1} \quad \text{if} \quad Z \neq 1, \quad \text{and} \quad P_x = \frac{A_0}{A_x} \quad \text{if} \quad Z = 1. \tag{24}$$

We are interested in the average number of revolutions of the particles, or the average duration of the process, *only* in the *successful* cases, i.e., when the particles succeed in attaining $a = a_x$. In the usual presentation of the Gambler's Ruin problem, both successful and unsuccessful cases are included in the duration. After a certain amount of manipulation, we find that the average duration, D_x, of a successful run is given by

$$D_x = \frac{A_x}{\epsilon} \frac{1 + Z^{A_x}}{1 - Z^{A_x}} - \frac{A_0}{\epsilon} \frac{1 + Z^{A_0}}{1 - Z^{A_0}}, \quad \text{if} \quad Z \neq 1,$$

and

$$D_x = \tfrac{1}{3}(A_x^2 - A_0^2), \quad \text{if} \quad Z = 1. \tag{25}$$

The duration in our problem is, of course, the average number of revolutions about the sun made by a particle released in an orbit of $a = a_0$ until it attains an orbit of $a = a_x$. Those particles lost in hyperbolic orbits are neglected in this average.

The rate of increase in $1/a$ ($\delta\, 1/a$ per period) by the Poynting-Robertson effect can be expressed conveniently in the form

$$\delta \frac{1}{a} = \frac{2\pi \alpha (2 + 3e^2)}{(GM)^{1/2} q^{3/2} (1 + e)^{3/2}}, \tag{26}$$

where M is the mass of the sun, G the constant of gravity, and α the quantity of equation (2a).

Since q, the perihelion distance, remains practically constant until the eccentricity is greatly reduced from nearly unity and since the ratio $(2 + 3e^2)/(1 + e)^{3/2}$ varies only from 1.77 to 1.50 as e changes from 1.0 to 0.5, we may adopt the ratio as 1.7, and q = Constant in equation (26) for orbits of long period. As a consequence, for spherical particles of density 4 gm/cm^3, we may evaluate α from equation (2a) to derive the following numerical result:

$$\delta \frac{1}{a} = \frac{1.5 \times 10^{-8}}{s\, q^{3/2}}, \tag{27}$$

where s is the radius of the particle in centimeters and a and q are expressed in astronomical units.

Hence Z can be determined from equations (22), (21), and (27). Let us accept $\sigma(1/a) = 0.000476/(\text{a.u.})$ for retrograde orbits and 0.000727 for direct orbits, and calculate the probabilities of particle survival and duration from longer period orbits to $1/a_x = 0.2$ (period = 11.2 yr.). For retrograde orbits $\epsilon S = 3.15 \times 10^{-5}$ cm, and $A_x = 420$ steps, while for direct orbits $\epsilon S = 2.06 \times 10^{-5}$ cm, and $A_x = 275$, where S is defined by

$$S^{-1} = \frac{4\ \text{gm/cm}^3}{s\, \rho_s\, q^{3/2}}, \tag{28}$$

where q is expressed in astronomical units.

Thus for $q = 1$ a.u. and $\rho_s = 4$ gm/cm^3, we have S equal to s, the radius of the spherical particle. For a particle of some other perihelion distance or density, the proper relation between s and S is given by equation (28). In case the particle is not spherical, the appropriate value of s to be used here can be obtained by equating $3/(4s\rho_s)$ to the ratio, A/m, of the average cross-sectional area of the particle to its mass.

Typical results of the calculations for probabilities and durations of various-sized particles in direct and retrograde orbits from $1/a_0$ to $1/a_x = 0.2/(\text{a.u.})$ are presented in Table 3. Perhaps the most interesting fact shown by Table 3 is the greater probability of a successful run for retrograde than for direct orbits among the smaller particles. Since the Poynting-Robertson effect is a greater fraction of the average random perturbation for retrograde than direct orbits, the spiraling is relatively more effective. On the other

TABLE 3

PROBABILITIES (P_x) AND AVERAGE NUMBERS OF REVOLUTIONS (D_x) FROM $1/a_0$ TO $1/a_x = 0.2/(\text{A.U.})$

	$1/a_0$	≤ 0.0005	0.001	0.01	0.1
		$S^* = 10^{-4}$ Cm			
Direct	P_x	0.342	0.567	0.995	1.000
	D_x	1310	1310	1260	670
Retro	P_x	0.479	0.729	1.000	1.000
	D_x	1320	1320	1260	670
		$S^* = 10^{-3}$ Cm			
Direct	P_x	0.039	0.075	0.408	0.996
	D_x	10,700	10,700	10,600	6600
Retro	P_x	0.061	0.118	0.720	1.000
	D_x	12,300	12,300	12,100	6700
		$S^* = 10^{-2}$ Cm			
Direct	P_x	0.006	0.012	0.081	0.630
	D_x	24,700	24,700	24,600	17,600
Retro	P_x	0.007	0.013	0.295	0.770
	D_x	53,000	53,000	52,700	38,000
		$S = \infty$			
Direct	P_x	0.002	0.005	0.050	0.500
	D_x	25,200	25,200	25,100	18,900
Retro	P_x	0.002	0.005	0.050	0.500
	D_x	58,600	58,600	58,500	44,000

* The quantity S is roughly one-tenth the particle radius, s.

hand, more revolutions are required for retrograde particles to obtain $1/a_x = 0.2$ because the number of perturbation steps is greater.

We have assumed that the orbit of a particle is identical with the orbit of the parent comet at the time of ejection. In fact, the particle will leave the comet with a small, but finite velocity of ejection, affecting $1/a$. By neglecting this factor, we increase somewhat the calculated losses of particles in extremely long orbits because some particles will be shifted immediately into orbits with larger values of $1/a$. The result of this neglect is not serious, however, as can be seen from Table 3, because the fraction of such particles that attain large values of $1/a$ is small unless the Poynting-Robertson effect is large, in which case neglect again is not important.

VII. THE JUPITER BARRIER

For larger particles, the nature of the perturbations changes markedly as their aphelia approach more and more closely to that of Jupiter. Since the frequency of comet orbits (on a log $1/a$ scale) shows a dip near $1/a = 0.15/(\text{a.u.})$, our calculations in Section VI have carried the particles until they are essentially members of Jupiter's "family," analogous to the short-period comets. Here we meet the *Jupiter barrier* problem. Bodies in such orbits are "bounced about" by the relatively large perturbations of Jupiter. The Poynting-Robertson spiral must operate between the relatively close approaches to Jupiter until aphelion is reduced below Jupiter's perihelion or until the particle meets disaster by (*a*) collision with Jupiter, (*b*) expulsion from the system, (*c*) collision with the sun or other planets, or (*d*) collisions with a sufficient number of particles or corpuscles.

Öpik (1951) has discussed the problem of the Jupiter barrier for particles in nearly circular orbits. His analysis, however, must be altered considerably for particles with small perihelion distances. With its aphelion not far beyond Jupiter's orbit, the particle will suffer changes in aphelion distance until eventually, in almost all cases, the aphelion lies very close to Jupiter's orbit. Close approaches then may not occur for a number of revolutions because of the motion of the node, usually regressive for direct orbits and forward for retrograde orbits. A particle for which $1/a$ increases about $0.01/(\text{a.u.})$ during the interval that the node turns progressively one-half revolution will withdraw through the range from Jupiter's aphelion to perihelion, and thus be relatively safe from the Jupiter menace. The perturbation in the line of apsides may in some cases also play a role in these effects.

The average value of $\Delta\Omega$ per revolution for eleven typical short-period comets is $0\overset{\circ}{.}3$. Thus a half-revolution of the node requires some 600 revolutions. The Poynting-Robertson effect on $1/a$ is 1/70, 1/700, and 1/7000 of $0.01/(\text{a.u.})$ for $S = 10^{-4}$, 10^{-3}, and 10^{-2} cm, respectively. The probability of penetration is practically unity for $S = 10^{-4}$ cm, and very high for $S = 10^{-3}$ cm, because several good opportunities for crossing occur for each particle. The probabilities for crossing the Jupiter barrier are also appreciable for larger values of S, because the aphelion may be left near Jupiter's perihelion after a close approach.

For large particles in direct orbits, we may look to the comet statistics. Of the 63 individual comets recognized since 1850 with $1/a > 0.116/(\text{a.u.})$, in the list by F. Baldet and Miss G. de Obaldia (1952) and in the selected list by J. G. Porter (1952), 8 (or 13 per cent) have aphelion less than 4.951 a.u., Jupiter's perihelion, and 3 (or 5 per cent) approximately equal to it. Of the 17 recent comets not included in the catalogue by I. Yamamoto (1936), 6 (or 35 per cent) are included in these two categories. This remarkable distribution of cometary aphelia is shown in Figure 1 for the 63 comets of $a < 8.62$ a.u. The circles in the diagram represent the frequency distribution for intervals centered halfway between those represented by the connected points. Almost half of the aphelia (29) lie within Jupiter's perihelion and aphelion. The frequency-curve passes fairly smoothly through Jupiter's perihelion but suffers a dip near its aphelion.

We must conclude, in fact, that the periodic comets have little trouble in penetrating the Jupiter barrier, despite their relatively short lives. To what extent secular changes in period, as evidenced by Comet Encke, and others (Whipple 1950, 1951) may be responsible for great reductions in q' is not known at present. More accurate determinations of secular or long-period changes in the periods of short-period comets should clarify this question. In any case, it appears reasonable to adopt the following as empirical probabilities that long-lived particles in direct orbits may cross the barrier: for $S = 10^{-4}$ cm, 1.00; $S = 10^{-3}$, 0.95; $S = 10^{-2}$, 0.67; $S = 0.1$, 0.30. For retrograde particles, the motions of the node are roughly $-\frac{1}{2}$ those for direct orbits; we adopt as intentionally large values of the retrograde probabilities: $S = 10^{-4}$ and 10^{-3} cm, 1.00; $S = 10^{-2}$, 0.80;

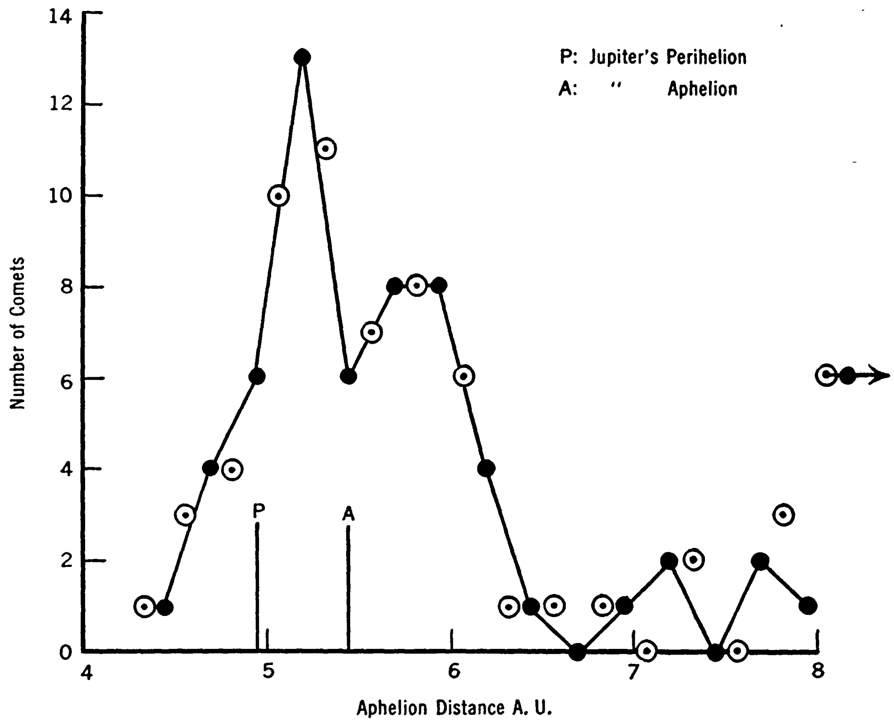

FIG. 1.—Aphelia of 63 comets $a < 8.62$ A

and $S = 0.1$, 0.67. These adopted probabilities are larger than those derived by Öpik, as they should be; Öpik was dealing with nearly circular orbits rather than elongated ones.

The probability of capture by Jupiter appears not to be serious for particles less than about 1 cm in radius. Öpik's theory, when applied to the orbits here considered, makes the probability of capture about 0.8×10^{-6} per revolution for low-inclination direct orbits, 10^{-7} for average direct orbits, and 3×10^{-8} for average retrograde orbits. The maximum average duration in Table 3 is 6×10^{4} revolutions, making the probability of a capture by Jupiter rather small. Hence no correction for such captures has been included. For large particles the end result will be some sort of catastrophe, since the Poynting-Robertson spiral will be too slow to be effective in 3×10^{9} years. This limit on particle size is roughly $s = 10$ cm for particles in short-period orbits.

VIII. THE COMETARY CONTRIBUTION TO THE ZODIACAL CLOUD

The adopted distribution of comet orbits into which potential zodiacal particles are initiated is shown in the second column of Table 4. It is based on Oort's compilation for $1/a$ up to 0.04/(a.u.). His distribution is proportional (not equal) to the number of comets passing perihelion per year and amounts to 44 in all. I have added the shorter-period comets by a count of all comets observed in the interval 1840 to 1900 inclusive. This largely prephotographic interval was chosen as one in which the orbits are reasonably well determined, but in which the extremely faint short-period comets do not overweigh the distribution. These faint comets must contribute little to the zodiacal cloud. In all, 258 comet appearances are listed in Yamamoto's catalogue (1936), an average of 4.2 per year. A correction factor of 0.271 brings the comet distributions of 1840–1900 into agreement with Oort's compilation for $1/a < 0.04/(\text{a.u.})$. The total now becomes 70; a correction factor of 4.2/70 or 0.06 reduces the distribution in Table 4 to the average yearly number between 1840 and 1900. Comet Encke is the sole contributor in the interval $1/a > 0.41/(\text{a.u.})$. In the Table 4 listing of the proportional number of comets of various $1/a$, I have reduced the numbers of short-period comets by a factor of 0.1, to allow for a smaller contribution of zodiacal particles per revolution by these relatively faint comets. The factor of 0.1 is based roughly on determinations of the absolute magnitudes of comets. Thus the effective number of *brighter* comets is reduced from 70 to 50.92 in the table summation.

Table 4 lists the calculated contributions to the zodiacal cloud by the average comets in terms of the probabilities that particles ejected by the comets will attain aphelion distances less than Jupiter's perihelion distance, 4.94 a.u. These probabilities are listed under the heading of "Prob.," for particles with S values of 10^{-4}, 10^{-3}, 10^{-2}, and 10^{-1} cm and for comets with $1/a$ values as given in the first column of the table. Listings for direct and retrograde orbits are separated. It will be demonstrated below that these probabilities roughly approximate the fraction of the total sunlight that would be totally scattered by given masses of particles released by the comets in terms of the light they would have scattered, had no losses been incurred. The effects of collisional destruction of the particles will be discussed in the next section. Neglecting these collisional losses in Table 4, we may take the listed probabilities as proportional to the masses contributed continuously to the zodiacal cloud by the observed distribution of comets in terms of $1/a$ (second column). Hence the columns headed "Contr." list the effective amounts of the mass contributions to the zodiacal cloud by the comets, on the assumption that no further losses among the particles occur after an aphelion distance of 4.95 a.u. is attained.

The probabilities in Table 4 are the products of three probabilities—A, B, and C—explained in more detail in the following paragraphs.

Probability A.—A factor of zero for those particles swept away by the interstellar wind, as listed in Table 1 for $N_i = 1\ H$ atom/cm^3, or a factor of unity for other particles.

Probability B.—A factor, P_z, calculated by means of the random-walk theory of Section VI to allow for the losses of particles from their Poynting-Robertson spirals as a result of the random perturbations by Jupiter. Table 3 lists a few typical values of this factor. That this factor is a fair measure of the fraction of the mass of the particles in so far as their effective light-scattering is concerned can be demonstrated as follows: Equation (5) shows that the total sunlight scattered by a particle during its Poynting-Robertson spiral through an orbital parameter change of dp is proportional to dp/p, which equals $-qd(1/a)(2 + 2e)/(2 + 3e^2)$, where q is the perihelion distance. Since q changes slowly and e remains large in the early spiral period, the total light-scattering is closely proportional to $d(1/a)$. Now from the solution of the Gambler's Ruin problem (Sec. VI), as applied to the random Jupiter perturbations, we find that the average number of revolutions for particles before they either gain the desired value of $1/a$ or are lost at $1/a = 0$ is given by $(A_z P_z - A_0)/\epsilon$. Without perturbations, the number of revolutions to $1/a$ would have been $(A_z - A_0)/\epsilon$. The average ratio in number of revolutions is, therefore, $(P_z - A_0/A_z)(1 - A_0/A_z)^{-1}$. This ratio is P_z if $P_z = 1$ or if $A_0/A_z \ll P_z$. Thus P_z is a good measure of the average fraction of the particles' masses used in light-scattering when P_z is near unity, and an overestimate when P_z is small, over the range of change in $1/a$ when $1/a$ is small. Since P_z is, by definition, the fraction of the mass

TABLE 4

COMETARY CONTRIBUTIONS TO APHELION <4.95 A.U.

DIRECT ORBITS

0.1 PARTICLE RADIUS* (1/a)	No. COMETS	$S=10^{-4}$ CM		$S=10^{-3}$ CM		$S=10^{-2}$ CM		$S=10^{-1}$ CM	
		Prob.	Contr.	Prob.	Contr.	Prob.	Contr.	Prob.	Contr.
0–0.00005	6	0	0	0	0	0.004	0.024	0.001	0.006
0.00005–.0001	2	0	0	0.036	0.072	.004	0.008	.001	.002
.0001–.0002	2	0.171	0.342	.037	0.074	.004	0.008	.001	.002
.0002–.0005	2	0.342	0.684	.037	0.074	.004	0.008	.001	.002
.0005–.001	1.5	0.492	0.738	.059	0.088	.007	0.010	.002	.003
.001–.002	1.6	0.650	1.040	.089	0.142	.010	0.016	.003	.005
.002–.004	1.8	0.800	1.440	.156	0.272	.016	0.029	.004	.007
.004–.006	0.7	0.910	0.637	.221	0.155	.027	0.019	.008	.006
.006–.01	2.1	0.980	2.058	.304	0.638	.040	0.084	.011	.023
.01–.02	1.6	1.000	1.600	.509	0.814	.078	0.125	.022	.035
.02–.03	0.8	1.000	0.800	.675	0.540	.129	0.103	.038	.030
.03–.04	0.3	1.000	0.300	.797	0.239	.184	0.055	.054	.016
.04–.07	1.4	1.000	1.400	.903	1.264	.257	0.360	.080	.112
.07–.1	1.0	1.000	1.000	.942	0.942	.378	0.378	.128	.128
.1–.2+	1.6	1.000	1.600	.950	1.520	.589	0.942	.240	.384
.2+–.41	1.63	1.000	1.630	.096	1.565	.080	1.304	.045	.734
0.41–0.82	0.49	1.000	0.490	0.100	0.490	0.100	0.490	0.100	0.490
Σ	28.52	....	15.759	....	8.889	....	3.963	....	1.985
Fraction direct surviving	......	....	0.553	....	0.313	....	0.139	....	0.070

* S is approximately 0.1, the true particle radius.

RETROGRADE ORBITS

0.1 PARTICLE RADIUS* (1/a)	No. COMETS	$S=10^{-4}$ CM		$S=10^{-3}$ CM		$S=10^{-2}$ CM		$S=10^{-1}$ CM	
		Prob.	Contr.	Prob.	Contr.	Prob.	Contr.	Prob.	Contr.
0–0.00005	6	0	0	0	0	0.005	0.030	0.002	0.012
0.00005–.0001	2	0	0	0.060	0.120	.005	0.010	.002	.004
.0001–.0002	2	0.240	0.480	0.061	0.122	.005	0.010	.002	.004
.0002–.0005	2	0.492	0.986	0.065	0.130	.006	0.012	.002	.004
.0005–.001	1.5	0.730	1.095	0.118	0.177	.011	0.016	.003	.005
.001–.002	1.6	0.830	1.328	0.176	0.282	.016	0.026	.005	.008
.002–.004	1.8	0.980	1.764	0.322	0.580	.032	0.058	.010	.018
.004–.006	0.7	1.000	0.700	0.468	0.328	.054	0.038	.017	.012
.006–.01	2.0	1.000	2.000	0.597	1.194	.077	0.154	.024	.048
.01–.02	1.3	1.000	1.300	0.856	1.113	.155	0.202	.050	.065
.02–.03	0.5	1.000	0.500	0.955	0.478	.240	0.120	.084	.042
.03–.04	0.2	1.000	0.200	1.995	0.199	.332	0.066	.121	.024
.04–.07	0.6	1.000	0.600	1.000	0.600	.434	0.260	.177	.106
.07–.1	0.2	1.000	0.200	1.000	0.200	0.581	0.116	0.284	.057
0.1–0.82	0.0	....	0.000	....	0.000	....	0.000	....	0.000
Σ	22.4	....	11.153	....	5.523	....	1.118	....	0.409
Fraction retrograde surviving	......	....	0.159	....	0.079	....	0.016	....	0.006
Direct/retrograde	1.27	....	1.41	....	1.61	....	3.54	....	4.85
Fraction total surviving	......	....	0.529	....	0.283	....	0.100	....	0.047

* S is approximately 0.1, the true particle radius.

that attains a chosen value of $1/a$ after which losses are not assumed, P_z is nearly proportional to the mass contribution of a comet to the zodiacal cloud and to the zodiacal light.

Probability C.—The probability factor that a particle with aphelion near Jupiter's orbit will cross Jupiter's perturbational barrier to an aphelion distance within Jupiter's orbit (Sec. VII).

The contributions to the zodiacal light and to the zodiacal cloud within Jupiter's orbit are listed in Table 4 as the product of the probability for each particle size and the number of cometary passages per unit time (16.7 years). The sums in the contribution columns represent the relative contribution by the various size groups, direct or retrograde. Direct orbits prevail with an increasing ratio for larger particles. The survival fractions must still be corrected for collision losses before and after the theoretical arrival within Jupiter's orbit.

The quantity S in Table 4 is the particle radius for a specific density of 4 gm/cm^3, and $q = 1$ a.u. (see eq. [28]). The mean value of $q^{-3/2}$ for 67 individual comets from 1840 to 1870 inclusive is 2.16(a.u.)$^{-3/2}$ or 5.67 if Comet 1865 I is included ($q = 0.0258$). Perhaps 3.0(a.u.)$^{-3/2}$ is a fair compromise between these quite different means. A specific density of 4 gm/cm^3 is probably too great for cometary particles (Whipple 1952*c*). The true average value may well lie below 1 gm/cm^3. Since the exact value is not known, we adopt $\rho_s = 1.2$ gm/cm^3 to make $s = 10\,S$ by equation (28). Hence the true radii applying to Table 4 are approximately ten times the tabulated values of S.

A large fraction of the most fragile and most porous meteoroids probably break up during their many revolutions from long-period to zodiacal-cloud orbits. A correction to Table 4 is needed for this effect. Much of the broken matter, however, may well be in the range $s > 3 \times 10^{-5}$ cm, so that it can still contribute to the zodiacal light. Perhaps the greater probability of a successful orbital reduction for the surviving sizable pieces will compensate for the matter lost in finer dust. We make no correction for the surviving pieces because of the difficulty of estimating the gain, but we do use maximal values in calculating the destruction losses by the mutual collisions of zodiacal particles, in the following section.

IX. COLLISIONAL DESTRUCTION

H. Jeffreys (1916) has demonstrated the importance of collisons in revolving clouds of particles such as the zodiacal cloud and the Rings of Saturn. We adopt van de Hulst's distribution of particle sizes and densities for a uniform zodiacal cloud in order to investigate collisional effects. He finds that the number of particles per cubic centimeter, $n(s)$, is given by the equation

$$n(s)\,ds = C_s^{-x}\,ds\,, \tag{29}$$

where $C = 3.5 \times 10^{-20}$ and $x = 2.6$ for particles with radii in the range $3 \times 10^{-5} < s < 0.035$ cm.

A particle of radius s_0, in traveling 1 cm through the cloud, will collide with the following number of particles at rest in the range of radius s_1 to s_2:

$$N(s_1, s_2) = \pi C \int_{s_1}^{s_2} (s_0 + s)^2 s^{-x} ds\,. \tag{30}$$

If the particle s_0 moves at velocity V and encounters cloud particles with a mean relative velocity V_R, then the velocity correction factor to $N(s_1, s_2)$ is V_R/V. The encounter with a particle of radius s and mass m will, for $s \ll s_0$, release kinetic energy equal to $0.5\,mV_r^2$. At velocities of several km/sec, both s and s_0 will suffer destructive action in vaporization, fusion, and fragmentation. The experiments of J. S. Rinehart and W. C. White (1951) suggest that the loss of mass by the more massive body is comparable to the kinetic energy divided by the heat of fusion, ζ (Whipple 1952*a*). Thus the relative mass, $\Delta m_0/m_0$, lost by the particle in a collision with a similar, but much smaller, particle of mass m is given by

$$\frac{\Delta m_0}{m_0}=\frac{V_r^2 s^3}{2\zeta s_0^3}. \tag{31}$$

We assume that the particle s_0 is destroyed when $\Delta m_0/m_0 = 1$. Thus destruction occurs by collision with a particle of radius greater than s_1, given by

$$s_1 = s_0 (2\zeta V_r^{-2})^{1/3}. \tag{32}$$

Hence our particle, in traveling a distance D through the zodiacal cloud, is subject to the probability $P(s_1, s_2)$, of destruction, as given by

$$P(s_1, s_2) = DN(s_1, s_2) V_r V^{-1}, \tag{33}$$

where s_1, as given in equation (32), is applied in equation (30). Attrition by smaller particles and the ready shattering of fragile meteoroids are to some extent included in the relatively large collisional cross-section, $\pi(s_0 + s)^2$, that we have adopted.

For our potential zodiacal particles in elongated orbits near perihelion, let us accept $D = 5$ a.u., $V = 50$ km/sec^{-1}, $s_2 = 10^{-1.5}$ cm, and $\zeta = 10^{10}$ ergs/gm. We may average the nearly circular velocities of the true zodiacal particles, as combined vectorially with V to give roughly

$$V_r = V(1 \pm 2^{-3/2}), \tag{34}$$

where the + sign applies to retrograde and the − sign to direct motions. These means are weighted slightly in favor of low i, as compared to random orientations at perihelion.

In finally correcting the cometary contributions of Table 4 for collisional destruction, we adopt as the true particle radius, s_0, ten times S listed in Table 4. Values of s_0 are given in the first line of Table 5. The lower collisional radius, s_1, in the third line is derived from equations (33) and (34) for direct and retrograde orbits. No particle radii below $10^{-4.5}$ cm are permitted. Table 5 then lists (fourth line) the probability that a particle will be destroyed in a single revolution according to equation (33). In the fifth line follows the mean number of revolutions required for particles starting in long-period orbits to attain $a^{-1} = 0.2$ and (sixth line) 0.6/(a.u.). The probability of survival (seventh line) is given by exp(−Prob. loss/rev. × number of revolutions), and listed to $a^{-1} = 0.6$ and also from 0.2 to 0.6/(a.u.) in the eighth line. The number of cometary particles surviving with no collisional destruction follows in the ninth line, as copied from Table 4. The corresponding number (tenth line) to $a^{-1} = 0.6$/(a.u.) is then derived from the ninth line and the earlier probabilities in Table 5, in terms of 50.92 comets as total. The fractions of the 50.92 surviving are then listed in the eleventh and twelfth lines, followed by the ratio of survivors, "Direct/retrograde," in the last line.

X. CONCLUSIONS

The rapid calculated drop in particle frequency should occur near $s_0 = 0.035$ cm, according to the deduction by van de Hulst. In Table 5 there is a striking cutoff near this value, beginning with some decline slightly below $s = 0.01$ cm. For more exact comparison between van de Hulst's and the present distribution of particle sizes, his function $n(s) \backsim s^{-2.6}$ should be corrected by s^{-1} to give the injection distribution; the Poynting-Robertson rate of loss varies as s^{-1}. The distribution must further be changed to a distribution function in mass, resulting in a law of the form $s_2^{0.4} - s_1^{0.4}$. This law gives an increase of 2.512 in the mass between successive intervals spaced by factors of 10 in s.

The agreement between the present calculations and those of van de Hulst is quite satisfactory in view of the various uncertainties, particularly in the original distribution of particle sizes from comets. The agreement could be improved by including the destruction rates. Piotrowski (1952), however, uses an even larger crushing rate for asteroidal material, by adopting ζ in the range of 10^7–10^9 ergs/gm instead of 10^{10} ergs/gm. I believe that the present values of the collisional destruction are high rather than low because of the high velocities involved.

The considerable fraction of retrograde particles predicted is not necessarily inconsistent with the observed ecliptic concentration of the zodiacal light, in view of the fact that, as the orbits become smaller and more circular, the losses in direct orbits continually decrease, while those in high-inclination orbits remain nearly constant. Nevertheless, the present calculations indicate that the zodiacal cloud should contain a small but appreciable fraction of fairly small particles in high-inclination orbits, although the concentration to the ecliptic will be higher than suggested by the "Direct/retrograde" ratios of Table 5. Among larger particles ($s_0 > 10^{-2.5}$ cm), only low-inclination direct motions will remain.

TABLE 5

COMETARY CONTRIBUTIONS TO THE ZODIACAL CLOUD

Particle Radius (Cm)	10^{-4}		10^{-3}		10^{-2}		10^{-1}	
S (Cm)	10^{-5}		10^{-4}		10^{-3}		10^{-2}	
	Dir.	Retro.	Dir.	Retro.	Dir.	Retro.	Dir.	Retro.
$\log_{10} s_1$ (cm)	−4.50	−4.50	−3.91	−4.12	−2.91	−3.12	−1.91	−2.12
Prob. loss $(\text{rev.})^{-1} \times 10^5$	0.45	0.95	1.26	4.43	2.56	9.83	4.06	19.7
No. rev. to $a^{-1} = 0.2 \times 10^{-3}$	<1.000	<1.0	1.31	1.32	10.7	12.3	25	53
No. rev. to $a^{-1} = 0.6 \times 10^{-3}$	<3.0	<3.0	3.98	3.99	37.4	39.0	292	320
Prob. surv. $a^{-1} = 0.6$	0.99	0.97	0.95	0.84	0.38	0.022	7×10^{-6}	10^{-28}
Prob. surv. $a^{-1} = 0.2$ to 0.6	0.99	0.98	0.97	0.89	0.50	0.072	2×10^{-5}	10^{-23}
Table 4 surviving	....	....	15.8	11.2	8.9	5.52	3.96	1.12
No. surviving	18.2	11.9	15.1	9.4	4.1	0.12	0.010	10^{-28}
Fraction tot. surv.	0.358	0.234	0.296	0.184	0.081	0.002	10^{-4}	10^{-30}
Fraction dir.+retro.	0.592		0.480		0.083		10^{-4}	
Direct/retrograde	1.52		1.60		35		10^{24}	

To estimate the fraction of the total cometary contribution of meteoritic material that actually becomes a part of the zodiacal cloud, we must assume an original distribution of particle sizes. The distribution, $n(s)$, proportional to s^{-4}, derived for meteors by F. G. Watson (1941), leads to the simplest result, a constant mass in each logarithmic interval of s. If we limit the radii to ten logarithmic steps from atomic dimensions, 10^{-8} cm, to 10 cm, Table 5 indicates that the fraction, 0.116, of the cometary meteoritic material effectively contributes to the zodiacal light. Thus 3.5 tons of the estimated 30 tons are added effectively to the zodiacal cloud per second. The agreement with our prediction of 1 ton sec^{-1} required by the observations is excellent indeed.

If we adopt van de Hulst's distribution function in mass, corrected as shown, we find that the fraction depends appreciably upon the selected upper limit of particle radius. For upper limits $s = 10$, 1, and 0.1 cm, we find that Table 5 leads to a cometary contribution to the zodiacal cloud of 0.7, 1.7, and 4.2 tons/sec, respectively. Thus the cometary hypothesis provides about the proper quantity of matter for maintenance of the zodiacal cloud. The effective mass may be somewhat larger than the present calculations indicate, because our "destruction" of particles involves considerable breakage only, not entirely a loss of mass to vapor.

The present theory explains another discrepancy; van de Hulst pointed out the fact that his distribution of particle sizes leads to some 10^4 times the rate of terrestrial meteoritic accretion predicted by Watson. The prediction by van de Hulst has powerful confirmation in the observations of deep-sea sediments by H. Patterson and H. Rotsche (1950). Other less conclusive confirmation is also available from rocket soundings (Whipple 1952*b*) and the collection of micrometeorites (Hoffleit 1952).

In fact, Watson's low prediction (corrected for an error) of about 5 tons per day for the entire earth is based on an extrapolation below the particle sizes observed as meteors. The perturbational and collisional losses sustained by larger particles before they can complete their Poynting-Robertson spirals accounts for a discontinuity in the distribution function of particle sizes between radii of $10^{-1.5}$ and $10^{-2.5}$ cm, almost exactly as predicted by van de Hulst. Öpik's explanation of this phenomenon as a consequence of the Jupiter barrier alone appears inadequate, at least for cometary contributions including retrograde orbits.

Piotrowski (1953) calculates that the asteroidal belt should produce between 20 and 600 tons/sec of finely divided material by collisional crushing. Even though his estimates of the total mass and crushing rate may be much too high, the asteroids appear also to offer an adequate source of zodiacal material.

Whether the comets or the asteroids predominate in zodiacal contribution must be decided on the basis of other criteria than those described here, presumably from meteoric and micrometeoritical information as well as from the shape of the zodiacal cloud. The recent researches of A. Behr and H. Siedentopf (1953) on the zodiacal light should be invaluable in this latter respect. Certain available data, however, do bear on this problem.

There can be little doubt that asteroids constitute the major source of meteorites. As we consider smaller bodies, we find that comets contribute about 90 per cent of the bright photographic meteors (Whipple 1954). Is it possible that the asteroids again become the main contributor of extremely fine dust? Piotrowski's frequency distribution law of s^{-3} for asteroidal particles cannot compete well with Watson's s^{-4} law for meteoroids in the region of small s, if the actual frequencies match at some moderate value of s. In fact, the preponderance of cometary particles among the photographic meteors as compared to the meteorites may well be a measure of the difference between these two frequency laws. If so, the asteroidal contribution to the zodiacal cloud must be vanishingly small.

The present theory has not yet been tested to determine whether it predicts the proper distribution of the zodiacal light on the sky. The author intends to use the results of the present paper as a first approximation to improve the calculations of the rate of collisional destruction. Then all the calculations can be improved, particularly by more subdivision of the orbits with respect to inclination, so that a valid prediction can be made concerning the average spatial distribution of the particles and their total scattering. Perhaps, also, better values of the pertinent parameters of corpuscular radiation and sputtering will permit a better estimate of the effects produced by corpuscular radiation.

It is interesting to note that an increase of the spiraling rate introduced by corpuscular radiation would require more mass input to maintain the zodiacal cloud and would also increase the critical dimension of zodiacal particles at the cutoff. Thus the observed value of the cutoff dimension sets a limit to the amount of the corpuscular radiation. Part of the required increase in the required mass input would be provided by the increased rate of spiraling and consequent reduced loss of larger particles by perturbations and collisions. Both the cutoff dimension and the mass input would be increased theoretically by sputtering. Laboratory data relevant to the sputtering problem are badly needed.

The author is grateful to Dr. H. C. van de Hulst for his critical review of this paper in manuscript and for his suggestions that have served both to increase the clarity of presentation and to eliminate certain errors.

REFERENCES

Allen, C. W. 1947, *M.N.*, **106**, 137.
Ambarzumian, V. A. 1952, "Introductory Discourse at the Symposium on the Evolution of Stars," paper read at the Eighth General Assembly of the I.A.U., Rome.
Baldet, F., and Obaldia, G. de. 1952, "Catalogue général des orbites de comètes de l'an − 466 à 1952," *Obs. de Paris.*
Behr, A., and Siedentopf, H. 1953, *Zs. f. Ap.*, **32**, 19.
Biermann, L. 1951, *Zs. f. Ap.*, **29**, 274.
Bobrovnikoff, N. T. 1942, *Contr. Perkins Obs.*, No. 16.
Callandreau, M. O. 1902, *Ann. Obs. Paris*, **22**, Sec. D, 1.
Feller, W. 1950, *An Introduction to Probability Theory and Its Applications* (New York: John Wiley & Sons, Inc.), Vol. **1**, chap. 14.
Fessenkov, V. G. 1914, *A.N.*, **198**, 465.
———. 1927, *Astr. J. U.S.S.R.*, **24**, 39.
Gartlein, C. W. 1950, *Trans. Amer. Geophys. Union*, **31**, 18.
Hoffleit, D. 1952, *Harvard Reprint*, Ser. II, No. 44.
van de Hulst, H. C. 1946, *Res. Astr. Obs. Utrecht*, **11**, Part 1, 72.
———. 1947, *Ap. J.*, **105**, 471.
Jeffreys, H. 1916, *M.N.*, **77**, 84.
Massey, H. S. W., and Burhop, E. H. S. 1952, *Electronic and Ionic Impact Phenomena* (Oxford: Clarendon Press), chap. ix.
Meinel, A. B. 1951, *Ap. J.*, **113**, 50.
Newton, H. A. 1893, *Mem. Nat. Acad. Sci.*, **6**, 7.
Oort, J. H. 1950, *Bull. Astr. Netherlands*, **11**, 91.
Öpik, E. 1951, *Proc. Roy. Irish Acad.*, **54**, Sec. A, 165–199.
Patterson, H., and Rotsche, H. 1950, *Nature*, **166**, 308.
Piotrowski, S. L. 1953, *Acta astr.*, Ser. A, **5**, 115.
Porter, J. G. 1952, *Comet and Meteor Streams* (New York: John Wiley & Sons, Inc.).
Poynting, J. H. 1903, *Phil. Trans. R. Soc. London, A*, **202**, 525.
Rinehart, J. S., and White, W. C. 1951, U.S. Nav. Ord. Test Sta., *Tech. Mem.*, No. 399, p. 1.
Robertson, H. P. 1937*a*, *M.N.*, **97**, 423.
———. 1937*b*, *ibid.*, eq. (3.7), p. 431.
Russell, H. N. 1920, *A.J.*, **33**, 49.
Vegard, L. 1952, *Norske Vid.-Akad. Oslo, Geofys. Pub.*, Vol. **18**, No. 5.
Watson, F. G. 1941, *Between the Planets* (Philadelphia: Blakiston Co.).
Whipple, F. L. 1950, *Ap. J.*, **111**, 375.
———. 1951, *ibid.*, **113**, 464.
———. 1952*a*, *Physics and Medicine of the Upper Atmosphere*, ed. C. S. White and O. O. Benson, Jr. (Albuquerque: University of New Mexico Press), chap. x, p. 161.
———. 1952*b*, *Bull Amer. Meteorol. Soc.*, **33**, 22.
———. 1952*c*, *A.J.*, **57**, 28 (abstr.).
———. 1954, *ibid.*, **59**, 201.
Whipple, F. L., and Gossner, J. L. 1949, *Ap. J.*, **109**, 380–390.
Woerkom, A. J. J. von. 1948, "On the Origin of Comets" (Thesis, Leiden).
Wyatt, S. P., and Whipple, F. L. 1950, *Ap. J.*, **111**, 134.
Yamamoto, I. 1936, *Pub. Kwasan Obs.*, Vol. **1**, No. 4.