THE FORMATION OF SUNSPOTS FROM THE SOLAR TOROIDAL FIELD*

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Received October 18, 1954

ABSTRACT

It is shown that a horizontal magnetic flux tube in an electrically conducting atmosphere is buoyant and will tend to rise. This magnetic buoyancy is large enough to bring an occasional strand of flux from the general solar toroidal field up into the photosphere, if we assume general field densities of a few hundred gauss farther down. Identifying the intersection of such ropes with the photosphere as the source of sunspots, we may deduce several general characteristics of the spots, e.g., east-west orientation, bipolarity, appearance only in low latitudes, migration, reversal of polarity, etc. The linearized static equilibrium equations for a flux tube are developed. With a cooling mechanism, such as that suggested by Biermann (1941), we find from the equilibrium equations that a sunspot group should consist of a diffuse flux tube of 10–100 gauss and 10^5 km extent in the photosphere, forming eventually a number of cool intense cores of several thousand gauss.

I. INTRODUCTION

Hydromagnetic dynamo theory suggests that we should expect dynamo waves just under the surface of the sun in the convective zone migrating from the polar to the equatorial regions. The waves are prevented from diffusing out of the convective zone by the high conductivity of the medium. The waves consist in part of bands of toroidal magnetic field; the sense of the field alternates from one band to the next. The poloidal field is $\pi/2$ out of phase with the toroidal field, essentially occupying the regions between the intense toroidal bands. This is shown schematically in Figure 1. Observations of secondary magnetic phenomena such as sunspots indicate that there are two (or at most three) toroidal bands in each hemisphere at any one time and that about 22 (or 33) years are required for the migration of each band from the pole to the equator. Dynamical considerations indicate that the initial amplification of the relatively weak wave starting at the pole is primarily of the poloidal components; by the time that middle latitudes are reached, the decrease of cyclonic motions and the increase of the nonuniform rotation of the sun shift the amplification to the toroidal field. Thus the poloidal component of the traveling wave predominates from the pole to the middle latitudes; in low latitudes the toroidal field predominates until both the poloidal and toroidal fields finally vanish at the magnetic equator.

The problem before us now is the question of what secondary magnetic effects might be expected around the fringes of the solar dynamo. We have in mind, of course, the obviously magnetic phenomena, such as sunspots and prominences, as well as those occurrences such as spicules, flares, etc., which one suspects must be of magnetic origin because no purely hydrodynamic explanation seems to exist.

In this paper we discuss what we shall call "magnetic buoyancy." Consider a magnetic flux tube running horizontally through a gaseous electrically conducting medium such as one finds in the sun. It is well known that the tensile stress in the tube is $B^2/2\mu$ in mks units, where B is the magnetic-field density. The magnetic field also exerts an outward pressure, and, were the tube not impeded by the surrounding matter, it would expand. As it is, B satisfies the diffusion equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nu_m \nabla^2 \boldsymbol{B} ,$$

* This work was supported by the U.S. Office of Naval Research.

where ν_m is the magnetic viscosity. If the medium is a sufficiently good conductor, ν_m becomes small enough that

$$\frac{\partial B}{\partial t} \cong 0,$$

and the field does not diffuse through the medium. Hydrostatic equilibrium requires that the magnetic pressure p_m be balanced by the gas pressure p_e outside the tube. Thus, if p_i is the gas pressure inside the tube, we must have

$$p_e = p_i + p_m . \tag{1}$$

Now

$$p_m = \frac{B^2}{2\mu} \tag{2}$$

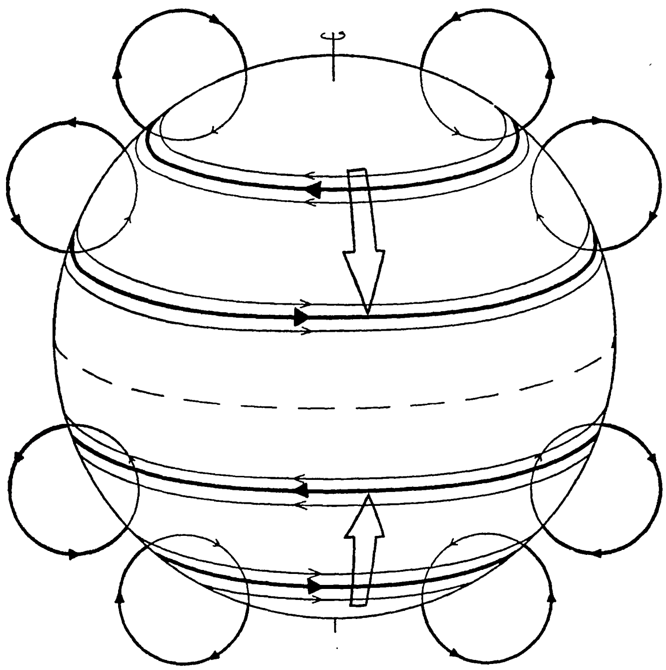

FIG. 1—Schematic drawing of solar toroidal and poloidal magnetic fields in the outer half of the convective zone. The migration toward the equator is indicated by the large arrows.

and is always positive. Thus $p_i < p_e$. Supposing that the temperature of the gas within the flux tube is the same as the temperature outside, we are led to the conclusion that $\rho_i < \rho_e$. Thus the flux tube is, in effect, a bubble and will try to rise: this is the "magnetic buoyancy" referred to earlier.

The buoyant force per unit length of a tube of cross-sectional area A is $g(\rho_e - \rho_i)A$. The tension is $AB^2/2\mu$. Consider a length L of the flux tube clamped at both ends. If the tube is to be able to rise, we must require that the buoyant forces exceed the tension at the ends of the length, which will try to hold the length in place. Thus we must satisfy an approximate relation of the form

$$Lg(\rho_e - \rho_i)A > \frac{2AB^2}{2\mu}. \tag{3}$$

Now

$$p = \frac{kT\rho}{m}, \tag{4}$$

where k is Boltzmann's constant and m is the mass of an individual gas molecule. Using equation (4) for p_e and p_i, equation (1) may be rewritten as

$$\rho_e = \rho_i + \frac{m}{kT}\frac{B^2}{2\mu}; \tag{5}$$

and equation (3) becomes

$$L > \frac{2kT}{mg}. \tag{6}$$

Thus magnetic buoyancy is effective over any length of flux tube exceeding twice the scale height of the medium.

It should be emphasized that magnetic buoyancy is not an instability in the usual sense. The buoyant force per unit volume is the quantity

$$F_b = \frac{mg}{kT}\frac{B^2}{2\mu}, \tag{7}$$

and a long horizontal flux tube can never be in static equilibrium. So long as F_b is large enough not to be overwhelmed by other motions, such as convection and turbulence, the tube will rise.

Conditions within a flux tube that has undergone vertical displacement are investigated at some length in Appendixes II and III; it is found that raising a length of a long flux tube results in a flow of fluid along the tube which enhances the magnetic buoyancy in the raised portion. Thus, once the tube has begun to rise, it will not generally stop.

To obtain a quantitative estimate of the buoyancy force, consider a flux tube of 100 gauss at a depth of 2×10^4 km in the sun. At this level $\rho_e \cong 2.5 \times 10^{-4}$ gm/cm^3, $T_e \cong 2^\circ\!.5 \times 10^5$ K. Equation (5) gives $\rho_e - \rho_i \cong 2 \times 10^{-11}$ gm/cm^3, which is only 10^{-7} of the density ρ_e. A temperature variation of $0^\circ\!.02$ K would produce the same fluctuation in the density. We see, then, that magnetic buoyancy will be negligible for the general solar field. Consider, however, a relatively intense strand of field of, say, 10^3 gauss, produced by an abrupt shearing in the turbulent convective motions at a depth of only 10^3 km. Now $\rho_e \cong 0.8 \times 10^{-8}$ gm/cm^3 and $T_e \cong 1^\circ\!.5 \times 10^4$ K; equation (5) gives $\rho_e - \rho_i \cong 3 \times 10^{-8}$ gm/cm^3. Hence $\rho_e - \rho_i$ is now 0.04 ρ_e and is equivalent to heating the region by 600° K; if the rope is not swept back down into the convective zone by some violent convective flow, it will rise to the surface of the sun.

In the sun, then, we expect to find an occasional strand from the toroidal or poloidal fields bobbing up to the surface of the sun; the main field will be essentially unaffected. We expect these strands to come up where the buoyant force F_b is strongest and can overcome the random velocity and magnetic fields present in the convective zone. Thus strands of the toroidal field are expected to appear only below the middle latitudes. This leads us to a suggestion by Elsasser[1] that sunspots seem most naturally explained as a portion of the toroidal field which has been heaved up to the surface of the sun by some dynamic mechanism. The mechanism here assumed is magnetic buoyancy, supplemented to an unknown extent by the convective forces existing in the convective zone. Strands of the poloidal field may appear much nearer the pole than strands of the toroidal field, because, as was pointed out earlier, the poloidal field is amplified at higher latitudes than the toroidal field. Ropes of flux floating up from the poloidal field may be responsible for the prominence activity observed (Menzel 1953) in the middle latitudes shortly before the onset of a new sunspot cycle.

If we identify sunspots with the strands of the toroidal field breaking through the photosphere, several of the general properties of sunspots follow immediately. The bi-

[1] Unpublished.

polar character of the spots is due to the two passages of the flux tube through the photosphere, one exit and one entrance; the near east-west orientation of an individual group of spots results from the initial east-west direction of the flux tube; the appearance of the spots only below middle latitudes is due to the fact that the toroidal field is not intensely amplified until low latitudes are reached; the migration toward the equator of the region of spot formation is a result of the migration of the underlying toroidal field; the reversal of polarity every half-cycle occurs as a consequence of the alternation of sign of successive bands of toroidal field.

II. FORMATION OF A SUNSPOT

Having shown that several of the general characteristics of sunspots follow from the assumption that the magnetic buoyancy occasionally brings a strand of the solar toroidal field up to the surface, let us now investigate the configuration of such a strand upon reaching the surface.

The flux tube forming a real sunspot is not at all slender but diverges abruptly. To take into account the resulting curvature of the lines of force results in nonlinear equations, which have not yet been solved except for very special cases (Schlüter and St. Temesvary 1953). In this paper we shall confine our attention to slender flux tubes, neglecting the curvature of the lines in force, and so obtain linear equations which are easily solved. In this way we are able to determine how a flux tube will deviate from uniformity, but our quantitative description of the extent to which the tube will taper can be only preliminary. Our purpose is merely to demonstrate that flux tubes do tend toward configurations suggestive of sunspots. A quantitative description of sunspots could follow only from extension of the nonlinear calculations of Schlüter and St. Temesvary.

The relatively long life of a sunspot group suggests that the flux producing each group is near static equilibrium. Now consider a *vertical* flux tube (along the z-axis), with the assumption that, after the tube has broken through the surface, the part located in the upper convective zone has a sufficiently steep inclination to be considered vertical. We shall also assume that, to begin with, the tube does not taper off rapidly. Let the field B be homogeneous across the tube. We shall denote the state of the gas inside the flux tube by p_i, ρ_i, and T_i; outside by p_e, ρ_e, and T_e. For static equilibrium of the gas within the flux tube, we must satisfy equation (1). In addition, we must now require that the net force in the z-direction be zero or at least approximately so. Thus, in mks units,

$$0 = -\frac{dp_i}{dz} - g\rho_i + \frac{1}{\mu}[(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}]_z . \tag{8}$$

Now

$$[(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}]_z = \left[\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right]B_y - \left[\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right]B_x . \tag{9}$$

Our assumption that B is homogeneous across the tube means that

$$\frac{\partial B_z}{\partial y} = \frac{\partial B_z}{\partial x} = 0 ,$$

and equation (9) reduces to

$$[(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}]_z = -\frac{\partial}{\partial z}\frac{B_x^2 + B_y^2}{2} . \tag{10}$$

Hence equation (8) becomes

$$\frac{dp_i}{dz} = -g\rho_i - \frac{\partial}{\partial z}\frac{B_x^2 + B_y^2}{2\mu} . \tag{11}$$

We assume that the gas outside the tube is in equilibrium, so that

$$\frac{dp_e}{dz} = -g\rho_e . \tag{12}$$

Differentiating equation (1) with respect to z and using equations (11) and (12), we obtain

$$\frac{\partial}{\partial z}\left(\frac{B^2}{2\mu}\right) = g(\rho_i - \rho_e) + \frac{\partial}{\partial z}\frac{B_x^2 + B_y^2}{2\mu}$$

or.

$$\frac{\partial}{\partial z}\frac{B^2}{2\mu} = g(\rho_i - \rho_e) . \tag{13}$$

Equation (13) is the equation for longitudinal equilibrium of a vertical flux tube; it states that any change in the longitudinal magnetic stress must be balanced by the local buoyancy.

The assumption of a slowly tapering flux tube implies that $B_x, B_y \ll B_z$; so that, if B is the magnitude of $\boldsymbol{B}$, equations (11) and (13) may be rewritten as follows:

$$\frac{dp_i}{dz} = -g\rho_i + O^2(B_x, B_y) , \qquad \frac{\partial}{\partial z}\frac{B^2}{2\mu} = g(\rho_i - \rho_e) + O^2(B_x, B_y) . \tag{14}$$

Equation (14) applies to an oblique, as well as to a vertical, flux tube, as is shown more generally in Appendix I.

Let us use the static equilibrium equation (14) to investigate the portion of the flux tube rising up from the convective zone through the photosphere. Assume that, as a consequence of the slowness of the rise up to the photosphere, the tube is in thermal equilibrium with its surroundings, $T_i = T_e$. The sunspot which ultimately results from the flux tube is independent of whether $T_i = T_e$ initially; and so we shall not investigate the rate of rise and of radiative transfer to see whether the assumption is entirely justified; observation indicates that it is. Given that $T_i = T_e$, equation (1) may be rewritten to give

$$\rho_e - \rho_i = \frac{m}{kT_e}\frac{B^2}{2\mu} , \tag{15}$$

where m is the mass of a gas molecule in grams and k is Boltzmann's constant. Using equation (15), equation (14) becomes

$$\frac{\partial}{\partial z}\frac{B^2}{2\mu} = -\frac{mg}{kT_e}\frac{B^2}{2\mu} \quad \text{or} \quad \frac{\partial B}{\partial z} = -\frac{mg}{2kT_e}B .$$

Integrating, we have

$$B(z) = B(0)\exp\left[-\int_0^z dz\,\frac{mg}{kT_e}\right] . \tag{16}$$

We see that the magnetic field decreases with height, with a characteristic length of twice the scale height of the atmosphere. In other words, $B \propto p_e^{1/2}$. The width w of the flux tube varies as $p_e^{-1/4}$.

Between the base of the vertical flux tube in the convective zone and the upper end of the tube in the photosphere p_e decreases by one or two powers of 10. Thus, if the flux tube had an initial field density of 100 or 1000 gauss before rising up to the photosphere, $B \propto p_e^{1/2}$ implies that we will find fields of only 10 or 100 gauss at the photosphere. This diffuse field will appear over a region of the order of 10^5 km. The configuration of the field is illustrated in Figure 2, *a;* the line PP' represents the level of the photosphere.

The diameter of the flux tube at the photosphere determines how close together the exit and re-entrance of the tube may be; we expect the halves of the bipolar pair forming a sunspot group to be separated by a distance of the order of 10^5 km. At this stage of development of the flux tube we introduce a cooling postulate (Biermann 1941; Kuiper 1953). We assume that the presence of a magnetic field of the order of 100 gauss or more produces a cooling of the region occupied by the flux tube. Biermann's assumption that the cooling follows as a result of the magnetic field's inhibiting effect on convection seems the most stranghtforward explanation, though our conclusions do not depend critically on details.

We find that the field intensity at the photosphere in a flux tube is remarkably sensitive to the temperature difference between the inside and the outside of the tube. Consider a vertical flux tube in static and thermal equilibrium with its surroundings. Suppose that, at the base of the flux tube, the field intensity is 100 gauss and that $\rho_e = 10^{-5}$ gm/cm^3, $T_e = 4 \times 10^{-4}$ ° K. Then, from equation (1), we find that $(\rho_e - \rho_i)/\rho_e = 10^{-5}$ and $\rho_e - \rho_i = 10^{-10}$ gm/cm^3. The important fact is that $\rho_e - \rho_i$ is a very small quantity.

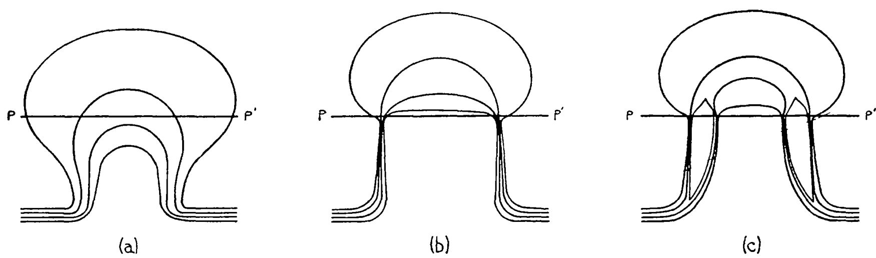

FIG. 2.—The development of a toroidal flux tube into a sunspot. *a*, indicates in a rough way how the tube might look after being borne to the surface by the magnetic bouyancy; *b*, shows the concentration just under the photosphere due to cooling; *c*, indicates splitting of the tube as a consequence of the abrupt tapering above the cool region.

If we decrease the temperature T_i inside the flux tube by 1° K, about two parts in 10^5, then ρ_i, in order to maintain the static equilibrium condition (1), must increase by 2 parts in 10^5. Then $\rho_e - \rho_i$ changes from $+10^{-10}$ gm/cm^3 to -10^{-10} gm/cm^3; from equation (21) we see that $(\partial/\partial z)[B^2/2\mu]$ also reverses its sign without changing its magnitude. Thus, instead of the divergence of the flux tube with height, as indicated in equation (16), where we find that $B_e \propto p_e^{1/2}$, the cooling by 1° K of the interior of the tube results in the tube's converging with height and $B \propto p_e^{-1/2}$. Hence a slight cooling effect in a vertical flux tube can result in a tremendous increase in field intensity at the upper end of the tube, as shown in Figure 2, *b;* this change of the static equilibrium configuration of the tube involves a large change in the volume of the tube, as may be seen by comparing Figure 2, *a* and *b*. The decrease in volume results in considerable flow of gas along the tube and is discussed in Section IV.

To investigate the matter a little further, we write equation (1) in terms of temperature and density. The resulting expression may be rearranged as follows:

$$\rho_i - \rho_e = \frac{\rho_e}{T_i}(T_e - T_i) - \frac{m}{kT_i}\frac{B^2}{2\mu}. \tag{17}$$

For vertical equilibrium we put equation (17) into equation (14) and obtain

$$\frac{d}{dz}\frac{B^2}{2\mu} = \frac{\rho_e g}{T_i}(T_e - T_i) - \frac{mg}{kT_i}\frac{B^2}{2\mu}. \tag{18}$$

As a first approximation, let us assume that the cooling effect $(T_e - T_i)$ is simply proportional to the magnetic stresses. Then we write

$$T_e - T_i = \kappa \left[\frac{B^2}{2\mu}\right], \tag{19}$$

where κ is a constant. Equation (18) becomes

$$\frac{d}{dz}\frac{B^2}{2\mu} = \frac{B^2}{2\mu}\frac{mg}{kT_i}\left[\frac{k\rho_e\kappa}{m} - 1\right]. \tag{20}$$

Upon integration, we obtain

$$\frac{B^2}{2\mu} = \left(\frac{B^2}{2\mu}\right)_0 \exp\left[\int_0^z dz\, \frac{mg}{kT_i}\left(\frac{k\rho_e\kappa}{m} - 1\right)\right]. \tag{21}$$

Then $(k\rho_e\kappa/m) - 1$ is integrated over distances several times the scale height kT_i/mg and appears in the exponent. Thus fields of the order of 2000 gauss are easily obtained from a field of 100 gauss at the base of the flux tube, even though $(k\rho_e\kappa/m) - 1$ may be only slightly greater than zero. We do not need the intense fields throughout the outer layers of the sun that have been postulated by Gurvich and Lebedinsky (1946).

In conclusion, then, we see that our calculations from the local lateral equilibrium equation (1) and the longitudinal equilibrium equation (14) have shown that the cooling and the intense magnetic field of a sunspot are mutually dependent; before cooling becomes effective, we have at the level of the photosphere a diffuse flux tube of 10–100 gauss over 10^5 km diameter. Giving a cooling effect, however, the diffuse tube forms a dense core. In our greatly oversimplified model the density of the core increases up to the level where the cooling is no longer effective; at higher levels $T_i = T_e$, and the flux tube diverges according to equation (16), as is discussed in the next section. We need not assume that the entire cross-section of the flux tube goes into the core, because there is undoubtedly a transition region near the surface of the flux tube where the cooling is not very effective. Thus we may expect a spot to be surrounded by a region of diffuse field.

III. THE EVOLUTION OF A SPOT GROUP

Now consider the field over a sunspot. If Biermann's mechanism is correct, we expect no cooling because there is no convection above the photosphere; observation indicates that the gas in the chromosphere and the corona over a sunspot are at least not cooler than gas at the same level elsewhere in the solar atmosphere. Thus we are led back to $T_i = T_e$ and the resulting divergence with height (eq. [16]) of a flux tube in equilibrium. The diameter of such a tube increases with height as $p_e^{-1/4}$. Thus in a region $T = 6000°$ K, yielding a scale height of 200 km, the flux tube increases its width by a factor of e every 800 km; a spot with a diameter of 2×10^4 km at a height z will have a diameter of 5.5×10^4 km at $z + 800$ km. The walls of the flux tube will make an angle of only 1°.7 with the horizontal. It must be remembered that this result is only approximate, because it was computed from equation (16), which was derived by assuming that the rate of divergence is small. But it serves to show how rapidly the tube is diverging and how the horizontal velocities of the Evershed effect may indeed be gas flowing along the lines of force rather than across them.

The rapid divergence of the tube has interesting dynamic implications. Consider a tube which tapers off abruptly, as shown in Figure 3, *a*. The lines of force have no tendency to stick together and, because of the tension along the lines, will try to separate into several branches at the restriction, as shown in Figure 3, *b*.

The tendency for such a breakup is readily demonstrated by showing that the energy of the field is less after breakup has occurred than before. If we assume that the cooling within the flux tube is not affected seriously or large quantities of matter elevated, it will be sufficient to consider the energy of the two-dimensional periodic field given by

$$B_x = 0\,, \qquad B_y = \left[\frac{2B_1}{ak}\right]\sin ky\left(\frac{z}{a}\right)\exp -\frac{z^2}{a^2}\,,$$
$$B_z = B_0 + B_1 \cos ky \exp -\frac{z^2}{a^2}\,. \tag{22}$$

This represents an initial uniform field B_0 extending in the z-direction. The periodic disturbing field, characterized by B_1, has the effect of bunching the initial field into bundles at intervals of $2\pi/k$ along the y-axis. The field at the center of each bundle is

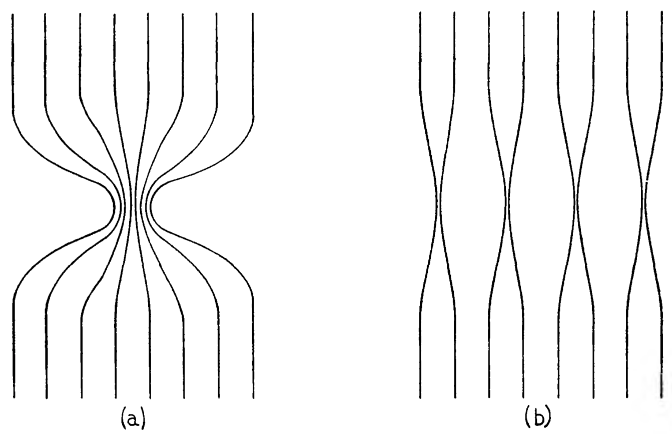

FIG. 3.—Schematic drawing of the splitting of a flux tube at a constricted region

$B_0 + B_1$. These bundles merge into a uniform field a distance of the order of a on each side of the y-axis.

The energy of the bundle lying in $-\pi/k \leqq y \leqq +\pi/k$ is

$$E = \frac{1}{2\mu}\int_{-\infty}^{+\infty} dz \int_{-\pi/k}^{+\pi/k} dy\,(B_y^2 + B_z^2)$$
$$= E_0 + \frac{B^2}{2\mu}\int_{-\infty}^{+\infty} dz \int_{-\pi/k}^{+\pi/k} dy\left[\frac{4}{a^2k^2}\left(\frac{z^2}{a^2}\right)\sin^2 ky + \cos^2 ky\right]\exp\left(-2\frac{z^2}{a^2}\right),$$

where E_0 is the energy of the homogeneous component of the field. The first-order term in B_1 drops out because of the integration over y. We finally obtain the energy due to the bunching as

$$E - E_0 = \frac{\pi^{3/2}}{2^{1/2}}\frac{B_1^2}{2\mu}\frac{a}{k}\left(1 + \frac{2}{a^2k^2}\right).$$

The energy per unit y is

$$\epsilon = \frac{k}{2\pi}(E - E_0) = \frac{\pi^{1/2}}{2^{3/2}}\frac{B_1^2}{2\mu}\,a\left(1 + \frac{2}{a^2k^2}\right). \tag{23}$$

For given values of a and B_1, ϵ is a monotonically decreasing function of k. This demonstrates the tendency for flux tubes to split up where the tube changes size abruptly.

The magnetic stresses at the surface of the sun are of the order of 0.2 times the gas pressure; we conclude that, unless some other effect is of large magnitude and opposite in sense to the magnetic stresses, the flux tube has sufficient potential energy to carry out the branching, pushing the gas out of its way as it does so. Therefore, at the surface of the sun we expect to find, after a time, not one, but several, flux tubes, each producing a small spot. Initially, the field from the spot was of the general configuration shown in Figure 2, *b;* branching results in something like Figure 2, *c*.

Each branch of a flux tube in a spot group extends, on the order of 10^4 km or more, down into the convective zone and so is pushed around by the convective motions there. We should expect the portion of the branch above the surface of the sun to show some of this random motion, with the result that the spots of a group spread out from their initial position. Besides this branching process, one would expect the magnetic buoyancy, which was initially successful in heaving a region of relatively intense field up to the surface, to continue to operate, though more slowly than at first, to pull up more of the toroidal field. The process will not go far, because, as was shown in the first section, the magnetic buoyancy becomes unimportant as one goes to weaker fields and deeper layers. But, in so far as the process operates, it should result in a progressively larger region of the toroidal field rising to the surface, with a subsequent increase in the separation of the two parts of the spot group. We conclude from the two effects discussed in this paragraph that each half of a spot group should slowly diverge within itself and from the other half.

The eventual expiration of a given spot group follows from the fact that a flux tube rising from the convective zone up to the photosphere and descending again to the convective zone, to form an inverted U, does not constitute a regenerative portion of the solar hydromagnetic dynamo; in this prodigal state it dies from diffusion. We must remember that it is not the molecular diffusion, which is negligible, but the eddy diffusivity of the convective zone that is responsible for the decay of the spot; vertical convective velocities of 10 m/sec in the region of the convective zone under the spot give a decay time of only a few months.

I should like to express my gratitude to Dr. Arnulf Schlüter for critical discussion of the linearized sunspot model presented in this paper.

APPENDIX I

KINEMATICS OF A FLUX TUBE

To supplement the rather brief discussion in Section II of the equilibrium of a flux tube, let us consider the stresses within a tube of flux which may not be in equilibrium. Let us idealize the flux tube to be of square cross-section of side w and to contain a magnetic field B uniform over the cross-section. Let us confine the tube to the yz-plane. We assume a gravitational field of acceleration g in the negative z-direction. We let s represent distance measured along the axis of the tube from left to right, and θ the inclination of the tube. We shall take the tube to be sufficiently slender compared to the characteristic lengths of the medium in which it is suspended that

$$\frac{dw}{ds} \ll 1\,, \qquad \frac{d\theta}{ds} \ll \frac{1}{w}\,. \tag{24}$$

Let p_i, ρ_i, T_i, and p_e, ρ_e, T_e represent the state of the material medium inside and outside the flux tube, respectively. Assuming the molecular weight of the medium to be uniform throughout, we write

$$p_i = \frac{k}{m} T_i \rho_i\,, \qquad p_e = \frac{k}{m} T_e \rho_e\,. \tag{25}$$

We shall assume that equation (1) is satisfied, which puts the flux tube in local lateral equilibrium. Finally, we shall assume that the medium outside is in equilibrium, satisfying the barometric relation,

$$\frac{\partial p_e}{\partial z} = -\rho_e g , \qquad \frac{\partial p_e}{\partial x} = \frac{\partial p_e}{\partial y} = 0 . \tag{26}$$

Consider the element $w^2 ds$ of the flux tube shown in elevation and plan view in Figure 4, *a* and *b*, respectively. The angle between the sides and the axis of the flux tube is

$$\alpha = \frac{1}{2} \frac{d w}{d s} . \tag{27}$$

Consider the force $F_s ds$ in the s-direction on the element. The weight of the element is $g\rho_i w^2 ds$. On the left-hand end there is a net pressure $p_i - B^2/2\mu$ on the area w^2; a corresponding

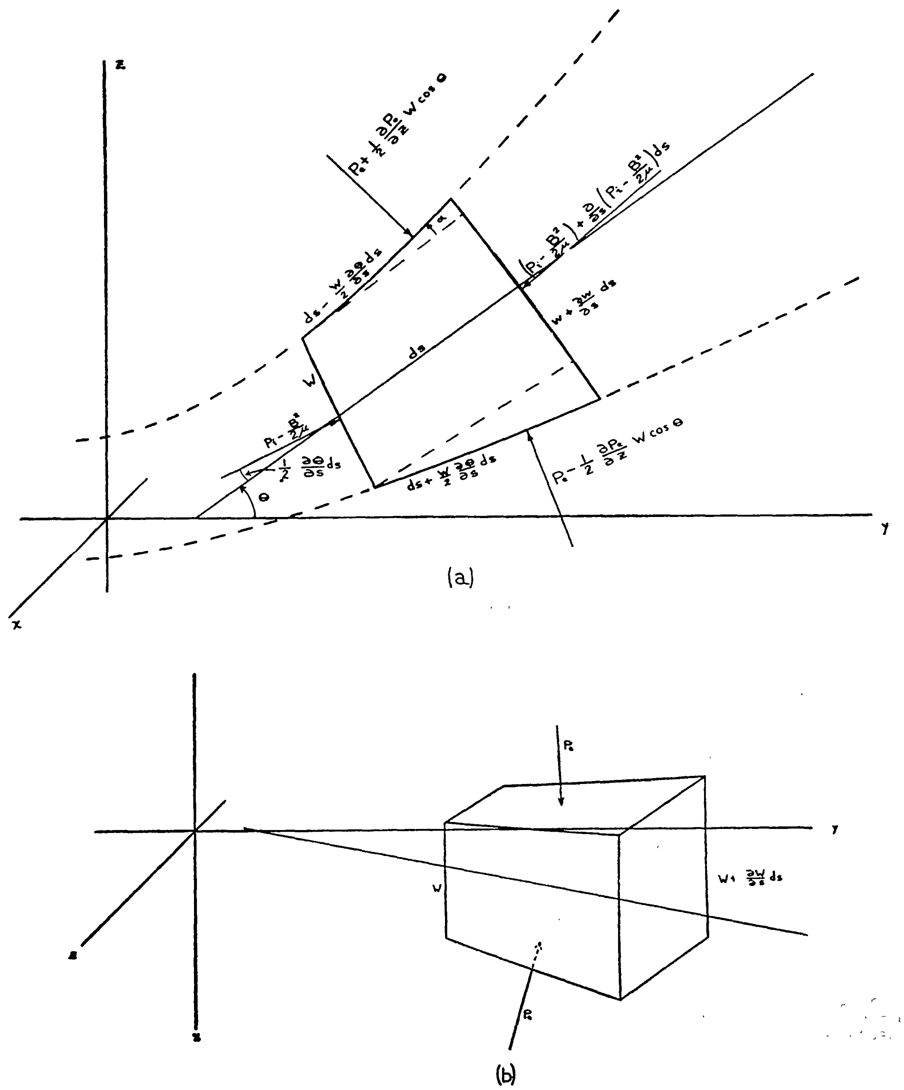

FIG. 4.—*a*, elevation of flux tube. *b*, plan view of flux tube

force is exerted on the opposite end of the element. The sides of the element parallel to the yz-plane experience a pressure p_e inclined at an angle $(\pi/2 - \alpha)$ to the axis of the element. Finally, the remaining pair of sides experience pressures $p_e \pm \frac{1}{2}(\partial p_e/\partial z)w \cos\theta$. Multiplying the foregoing pressures by their respective areas and taking the s component, we obtain

$$\begin{aligned} F_s ds = &-g\rho_i w^2 \sin\theta\, ds + 2p_e w\alpha\, ds \\ &+\left(p_i - \frac{B^2}{2\mu}\right)w^2 - \left[\left(p_i - \frac{B^2}{2\mu}\right) + \frac{\partial}{\partial s}\left(p_i - \frac{B^2}{2\mu}\right)ds\right]\left(w^2 + \frac{\partial w^2}{\partial s}ds\right) \\ &+\left(p_e + \frac{1}{2}\frac{\partial p_e}{\partial z}w\cos\theta\right)w\left(1 - \frac{w}{2}\frac{d\theta}{ds}\right)\alpha\, ds \\ &+\left(p_e - \frac{1}{2}\frac{\partial p_e}{\partial z}w\cos\theta\right)w\left(1 + \frac{w}{2}\frac{d\theta}{ds}\right)\alpha\, ds . \end{aligned} \tag{28}$$

Using equation (1) to eliminate p_i, equations (26) to eliminate $\partial p_e/\partial z$, and equation (27) to eliminate α, we may simplify equation (28) to

$$F_s = \frac{d}{ds}\left(w^2 \frac{B^2}{\mu}\right) + g w^2 \sin\theta\,(\rho_e - \rho_i)\,, \tag{29}$$

neglecting terms of second and higher order in dw/ds and $d\theta/ds$.

Since we are considering a flux tube, it follows that the total flux Φ is independent of s. Then

$$B = \frac{\Phi}{w^2}, \tag{30}$$

and

$$\frac{d}{ds}\left(w^2\frac{B^2}{\mu}\right) = \frac{\Phi^2}{\mu}\frac{d}{ds}\left(\frac{1}{w^2}\right) = \frac{\Phi^2}{2\mu}w^2\frac{d}{ds}\left(\frac{1}{w^4}\right) = w^2\frac{d}{ds}\frac{B^2}{2\mu}.$$

Thus we may write equation (29) as

$$F_s = w^2\left[\frac{d}{ds}\frac{B^2}{2\mu} + g\sin\theta\,(\rho_e - \rho_i)\right]. \tag{31}$$

Consider the forces normal to the axis of the tube. Similar to equation (28), we write

$$\begin{aligned} F_n ds = &-g\rho_i w^2\cos\theta\, ds - 2\left(p_i - \frac{B^2}{2\mu}\right)w^2\frac{1}{2}\frac{d\theta}{ds}ds \\ &-\left(p_e + \frac{1}{2}\frac{\partial p_e}{\partial z}w\cos\theta\right)w\left(1 - \frac{w}{2}\frac{d\theta}{ds}\right)ds + \left(p_e - \frac{1}{2}\frac{\partial p_e}{\partial z}w\cos\theta\right)w\left(1 - \frac{w}{2}\frac{d\theta}{ds}\right)ds , \end{aligned}$$

which reduces to

$$F_n = w^2\left[\frac{B^2}{\mu}\frac{d\theta}{ds} + g\cos\theta\,(\rho_e - \rho_i)\right]. \tag{32}$$

Consider the special case where the tube is in longitudinal equilibrium. Then $F_s = 0$, and equation (31) reduces to

$$0 = \frac{d}{ds}\frac{B^2}{2\mu} + g\sin\theta\,(\rho_e - \rho_i)\,. \tag{33}$$

Now

$$\frac{d}{ds} = \frac{dz}{ds}\frac{d}{dz} = \sin\theta\frac{d}{dz}. \tag{34}$$

Hence, for any inclination of the tube, we obtain

$$0=\frac{d}{dz}\frac{B^2}{2\mu}+g\,(\rho_e-\rho_i)\,, \tag{35}$$

which was obtained in equation (14) for a vertical tube. If we differentiate equation (1) with respect to z and use equation (26), we obtain

$$0=\frac{d}{dz}\frac{B^2}{2\mu}+g\rho_e+\frac{dp_i}{dz}. \tag{36}$$

Comparing equations (35) and (36), we find that equilibrium requires that p_i obey the barometric law,

$$\frac{dp_i}{dz}=-\rho_i g\,. \tag{37}$$

The physically obvious fact that a flux tube is stable against a local constriction or expansion is readily demonstrated from equation (31). Consider a horizontal flux tube of uniform cross-section. Let us pinch the tube over a finite extent of its length. B is increased in the restriction. Approaching the restriction from the left, we have $(d/ds)(B^2/\mu)>0$. Equation (31) gives $F_s>0$, causing the fluid to flow into the restricted region and restoring the tube to its initial uniformity.

APPENDIX II

FLUX TUBE IN THERMAL EQUILIBRIUM

Consider a flux tube in thermal equilibrium with its surroundings. The static equilibrium of such a tube was investigated briefly in Section II. There we found that $B\propto p_e^{1/2}$, so that $p_m\propto p_e$. From equation (1) it follows that $p_i\propto p_e$. Since Bw^2 is the total flux through the tube and is constant, we have $w\propto p_e^{-1/4}$. We note that the mass per unit length $w^2\rho_i$ decreases with height for a tube in either an isothermal or an adiabatic atmosphere.

We shall now inquire into the longitudinal motions within a flux tube and the variation of the magnetic buoyancy as a segment of an initially horizontal flux tube is displaced vertically by the magnetic buoyancy. We should like to know whether the tube can be expected to tend toward the static equilibrium configuration discussed in the foregoing paragraph and whether the magnetic buoyancy continues to function even after large displacement; the fact that $w^2\rho_i$ decreases with height implies that there must be a longitudinal flow away from the elevated portion of the tube. We shall find just such a flow from the following calculations; the flow allows an approach to $B\propto p_e^{1/2}$ and guarantees that the magnetic buoyancy will not fail after some finite displacement.

Consider how conditions will vary when a flux tube, initially horizontal and of uniform cross-section, is displaced vertically by some small but varying amount $\delta z(y)$. We are particularly interested in finding whether an upward bulging of the tube will result in fluid flowing along the tube away from the bulge or toward the bulge. Thus we shall assume that $T_i=T_e=T$ and constrain the fluid within the tube to move only in the z-direction, allowing no flow along the tube; we then investigate the longitudinal force F_s, to see which way along the tube the fluid would flow if the constraints were removed. The tube will have to be held in place, of course, by external forces, because of the magnetic buoyancy and longitudinal stresses.

We shall assume that $d\delta z/ds\ll 1$. Then

$$\sin\theta\cong\frac{d\delta z}{ds}\,,\qquad ds\cong dy\,. \tag{38}$$

Using equations (5), (30), and (36), we may write equation (29)

$$F_s=\frac{\Phi^2}{\mu w^2}\frac{d\delta z}{dy}\left[\frac{mg}{2kT}-\frac{2}{w}\frac{dw}{d\delta z}\right].$$

Consider $(2/w)(dw/d\delta z)$. With the constraint that there be no longitudinal flow within the tube, it follows that

$$\rho_i = \rho_{i0}\left(\frac{w_0}{w}\right)^2, \qquad B = B_0\left(\frac{w_0}{w}\right)^2. \tag{39}$$

Then equation (5) may be written

$$\rho_e\left(\frac{w}{w_0}\right)^4 - \rho_{i0}\left(\frac{w}{w_0}\right)^2 - \frac{mB_0^2}{2\mu kT} = 0. \tag{40}$$

Because we are considering only a small vertical displacement, $\delta z(y)$, we may introduce the approximation

$$\left(\frac{w}{w_0}\right)^2 = 1 + \left[\frac{d}{dz}\left(\frac{w}{w_0}\right)^2\right]_0 \delta z + O^2(\delta z), \qquad T = T_0 + \left(\frac{dT}{dz}\right)_0 \delta z + O^2(\delta z),$$
$$\rho_e = \rho_{e0} + \left(\frac{d\rho_e}{dz}\right)_0 \delta z + O^2(\delta z). \tag{41}$$

Solving equation (25) for ρ_e, differentiating with respect to z, dividing by ρ_e, and using equation (26) to eliminate $\partial p_e/\partial z$, we obtain

$$\frac{1}{\rho_e}\frac{d\rho_e}{dz} = -\frac{mg}{kT_e} - \frac{1}{T_e}\frac{dT_e}{dz}. \tag{42}$$

Putting equations (41) and (42) into equation (40), we finally obtain

$$\frac{d}{d\delta z}\left(\frac{w}{w_0}\right)^2 = \left[\frac{mg}{kT} + \frac{1}{T}\frac{dT}{dz}\left(1 - \frac{p_{m0}}{p_{e0}}\right)\right]\left(1 + \frac{p_{m0}}{p_{e0}}\right)^{-1} + O(\delta z)$$
$$= \left[\frac{mg}{kT}p_{e0} + \frac{1}{T}\frac{dT}{dz}p_{i0}\right](p_{e0} + p_{m0})^{-1} + O(\delta z). \tag{43}$$

But

$$\frac{2}{w}\frac{dw}{d\delta z} = \frac{1}{w_0^2}\frac{dw^2}{d\delta z} + O(\delta z).$$

Thus, to the degree of approximation used in equation (41), equation (43) gives $(2/w)(dw/d\delta z)$; the equation for F_s may be rewritten as

$$F_s = \frac{d\delta z}{dy}\frac{\Phi^2}{\mu w^2}\left\{\frac{mg}{2kT} - \left[\frac{mg}{kT}p_{e0} + \frac{1}{T}\frac{dT}{dz}p_{i0}\right](p_{e0} + p_{m0})^{-1} + O^2(\delta z)\right\}. \tag{44}$$

If the medium through which the flux tube passes is of uniform temperature, so that dT/dz vanishes, then

$$F_s = -\frac{d\delta z}{dy}\frac{\Phi^2}{\mu w^2}\frac{mg}{2kT}\frac{p_{e0} - p_{m0}}{p_{e0} + p_{m0}}. \tag{45}$$

If, on the other hand, it is an adiabatic atmosphere, then

$$\frac{1}{T}\frac{dT}{dz} = \frac{\gamma - 1}{\gamma}\frac{1}{p_e}\frac{dp_e}{dz} = -\frac{(\gamma - 1)}{\gamma}\frac{mg}{kT},$$

and

$$F_s = -\frac{d\delta z}{dy}\frac{\Phi^2}{\mu w^2}\left(\frac{2-\gamma}{\gamma}\right)\left(\frac{mg}{2kT}\right)\left(\frac{p_{e0} - p_{m0}}{p_{e0} + p_{m0}}\right). \tag{46}$$

We see from equations (45) and (46) that, for both an isothermal and an adiabatic atmosphere, $d\delta z/dy > 0$ implies that $F_s < 0$, resulting in a flow of fluid out of the raised portions of the flux tube. This has the effect of increasing p_m relative to p_i and *enhances* the magnetic buoyancy in the upward bulges. Only when dT/dz is sufficiently negative to reverse the sign of F_s, will the longitudinal flow degenerate the magnetic buoyancy; certainly in any large-scale region, T never decreases at a rate significantly greater than the adiabatic rate because of the extreme convective instability that would result from a more rapid decrease.

APPENDIX III

ADIABATIC FLUX TUBE

Consider a flux with an adiabatic interior. Here T_i no longer need be equal to T_e, and many physical relations which are taken for granted in Appendix II are no longer obvious. We have in mind the same questions as when $T_i = T_e$, viz., whether a flux tube displaced vertically by the magnetic buoyancy tends toward the static equlibrium configuration and whether the magnetic buoyancy vanishes after some finite displacement. The latter question no longer has the unambiguous answer it had when $T_i = T_e$.

Consider the static equilibrium of a flux tube with an adiabatic interior; the relation between p_i and ρ_i is, accordingly,

$$p_i = p_{i0}\left(\frac{\rho_i}{\rho_{i0}}\right)^{\gamma}. \tag{47}$$

If the flux tube is in equilibrium, we may combine equation (47) with equation (37). We obtain the familiar relations for an adiabatic atmosphere:

$$\rho_i = \rho_{i0}\left[1 - \frac{\gamma-1}{\gamma}\frac{mg}{kT_{i0}}z\right]^{1/(\gamma-1)}, \qquad p_i = p_{i0}\left[1 - \frac{\gamma-1}{\gamma}\frac{mg}{kT_{i0}}z\right]^{\gamma/(\gamma-1)}. \tag{48}$$

ρ_{i0}, p_{i0}, and T_{i0} are the values of ρ_i, p_i, and T_i at $z = 0$. Using equation (1), the magnetic pressure for static equilibrium is

$$p_m = p_e - p_i = p_e - p_{i0}\left[1 - \frac{\gamma-1}{\gamma}\frac{mg}{kT_{i0}}z\right]^{\gamma/(\gamma-1)}. \tag{49}$$

The width w of the tube may be computed from the fact that it varies as $p_m^{-1/4}$.

Consider the special case that the atmosphere outside the tube is an adiabatic atmosphere and, further, that $T_i = T_e$ at $z = 0$. Now, independently of the latter condition, T_i and T_e vary linearly with height, according to

$$T_i = T_{i0}\left[1 - \frac{\gamma-1}{\gamma}\frac{mg}{kT_{i0}}z\right], \qquad T_e = T_{e0}\left[1 - \frac{\gamma-1}{\gamma}\frac{mg}{kT_{e0}}z\right].$$

Thus, setting $T_i = T_e$ at $z = 0$ implies that $T_i = T_e$ at all heights in the atmosphere; the tube is in thermal equilibrium with its surroundings, even though insulated from them, and the relations worked out in the previous section for thermal equilibrium in an adiabatic atmosphere are valid.

The magnetic buoyancy depends on the sign of $\rho_i - \rho_e$, which must be investigated quantitatively, because, in an adiabatic atmosphere, it depends critically on the initial conditions. Consider, then, a long horizontal uniform flux tube. There will be no longitudinal flow under these conditions, and $B \propto \rho_i$. Thus

$$p_m = p_{m0}\left(\frac{\rho_i}{\rho_{i0}}\right)^2. \tag{50}$$

The zero subscript denotes initial values. Comparing equation (50) with equation (47), it follows that p_m varies more rapidly with the density than does p_i, since $\gamma < 2$. A vertical displacement of the flux tube, with the resulting change in ρ_e, results in ρ_m taking up more than its share of the pressure change. Thus p_i does not have to vary so rapidly, and we conclude that

$$\frac{1}{p_i}\frac{dp_i}{dz}<\frac{1}{p_e}\frac{dp_e}{dz}. \tag{51}$$

The slower rate of variation of p_i may be deduced quantitatively from equation (1). Differentiating with respect to z, we obtain

$$\frac{dp_i}{dz}=\frac{dp_e}{dz}-\frac{B^2}{\mu}\frac{1}{B}\frac{dB}{dz}. \tag{52}$$

But, from equations (39) and (48), it follows that

$$\frac{1}{B}\frac{dB}{dz}=\frac{1}{\rho_i}\frac{d\rho_i}{dz}=\frac{1}{\gamma}\frac{1}{p_i}\frac{dp_i}{dz}. \tag{53}$$

Putting equation (53) into equation (52) and solving for $(1/p_i)(dp_i/dz)$, we obtain

$$\frac{1}{p_i}\frac{dp_i}{dz}=\frac{dp_e}{dz}\left[p_i+\frac{B^2}{\gamma\mu}\right]^{-1}=\frac{1}{p_e}\frac{dp_e}{dz}\left[1+\frac{2-\gamma}{\gamma}\frac{p_m}{p_e}\right]^{-1}. \tag{54}$$

Thus, with $\gamma < 2$, we obtain equation (51).

If the medium outside the flux tube varies adiabatically with height z, then, besides equation (51), we have

$$\frac{1}{\rho_i}\frac{d\rho_i}{dz}<\frac{1}{\rho_e}\frac{d\rho_e}{dz}, \qquad \frac{1}{T_i}\frac{dT_i}{dz}<\frac{1}{T_e}\frac{dT_e}{dz}. \tag{55}$$

To obtain relations between ρ_e and ρ_i, etc., rather than just their derivatives, we put equations (47) and (50) into equation (1), obtaining

$$p_e=p_{i0}\left(\frac{\rho_i}{\rho_{i0}}\right)^{\gamma}+p_{m0}\left(\frac{\rho_i}{\rho_{i0}}\right)^{2}. \tag{56}$$

Equation (56) gives ρ_i in terms of p_e; and p_i, p_m, and w may then be computed, using equations (47), (50), and (39), respectively.

If the external medium is an adiabatic atmosphere, then

$$p_e=p_{e0}\left(\frac{\rho_e}{\rho_{e0}}\right)^{\gamma}, \tag{57}$$

and equation (56) may be written as

$$p_{e0}\left(\frac{\rho_e}{\rho_{e0}}\right)^{\gamma}=p_{i0}\left(\frac{\rho_i}{\rho_{i0}}\right)^{\gamma}+p_{m0}\left(\frac{\rho_i}{\rho_{i0}}\right)^{2}. \tag{58}$$

Consider how ρ_i and ρ_e compare near the top of the atmosphere, where ρ_e approaches zero. Here ρ_i also approaches zero, so that $\rho_i/\rho_{i0} \ll 1$. Equation (58) reduces to

$$p_{e0}\left(\frac{\rho_e}{\rho_{e0}}\right)^{\gamma}=p_{i0}\left(\frac{\rho_i}{\rho_{i0}}\right)^{\gamma}.$$

Thus

$$\frac{\rho_e}{\rho_i}=\frac{\rho_{e0}}{\rho_{i0}}\left(\frac{p_{i0}}{p_{e0}}\right)^{1/\gamma}=\left(\frac{\rho_{e0}}{\rho_{i0}}\right)^{(\gamma-1)/\gamma}\left(\frac{T_{i0}}{T_{e0}}\right)^{1/\gamma}. \tag{59}$$

For the special case that $T_{i0} = T_{e0}$, we have $\rho_{e0} > \rho_{i0}$; equation (59) implies that $\rho_e/\rho_i > 1$ as ρ_e approaches zero. If, on the other hand, $\rho_{e0} = \rho_{i0}$, then $T_{e0} > T_{i0}$, and equation (59) tells us that $\rho_e/\rho_i < 1$, which also follows from equation (55). Thus we have demonstrated that, beginning with a flux tube initially in thermal equilibrium with its surroundings, the magnetic buoyancy is operative at all heights in an adiabatic atmosphere to which the flux tube may be displaced; the presence of flux tubes induces convection in an adiabatic atmosphere. If, on the other hand, the flux tube initially satisfies equation (1) by virtue of $T_{i0} < T_{e0}$ rather than $\rho_{i0} < \rho_{e0}$, then the flux tube is stable against vertical displacement; the presence of flux tubes inhibits convection in an adiabatic atmosphere.

The critical condition giving an initial magnetic buoyancy which vanishes as $\rho_e/\rho_i \to 1$ at the top of the atmosphere may be obtained from equation (59) by putting $\rho_e = \rho_i$. We obtain the relation

$$\rho_{i0} = \rho_{e0} \left(\frac{T_{i0}}{T_{e0}}\right)^{1/(\gamma-1)} . \tag{60}$$

This is the condition that the variation of the matter within the flux tube is adiabatic during the generation of the magnetic field.

To determine p_e/p_i as $\rho_e \to 0$, we note from equations (47) and (50) that p_m decreases more rapidly with decreasing ρ_i than does p_i. Thus p_m/p_i approaches zero as the top of the atmosphere is approached, and equation (1) becomes

$$\frac{p_e}{p_i} = 1 + \frac{p_m}{p_i} \sim 1 . \tag{61}$$

To compute T_e/T_i as $\rho_e \to 0$, we use equations (25), (59), and (61). We find

$$\frac{T_e}{T_i} = \left(\frac{T_{e0}}{T_{i0}}\right)^{1/\gamma} \left(\frac{\rho_{i0}}{\rho_{e0}}\right)^{(\gamma-1)/\gamma} . \tag{62}$$

If, initially, $T_{e0} = T_{i0}$, we have $\rho_{i0} < \rho_{e0}$ and conclude from equation (62) that $T_i > T_e$, the interior of the flux tube becomes hotter than its surroundings, which also follows from equation (55). If, on the other hand, $\rho_{i0} = \rho_{e0}$, then $T_{i0} < T_{e0}$, and we conclude that $T_i < T_e$. Again the critical case leading to $T_i = T_e$ at the top of the atmosphere yields the adiabatic relation (60).

As in the previous section, where $T_i = T_e$, let us investigate F_s as a result of a small vertical adiabatic displacement of an initially horizontal and uniform flux tube in thermal equilibrium with its surroundings. We shall again introduce the constraint that there be no longitudinal flow; the resulting F_s will tell us whether the fluid would flow toward or away from an upward bulge of the tube if the constraint were removed.

From equations (29), (30), and (38) we obtain

$$F_s = \frac{d\,\delta z}{d s} \left[\frac{\Phi^2}{\mu w_0^2} \frac{1}{\rho_{i0}} \frac{d\rho_i}{d\,\delta z} + g w^2 (\rho_e - \rho_i) \right] . \tag{63}$$

We shall compute the quantity in brackets, omitting terms $O(\delta z)$, and obtain F_s omitting terms $O^2(\delta z)$. Thus, using initial values, we obtain, from equation (5),

$$w^2 (\rho_e - \rho_i) = w_0^2 (\rho_{e0} - \rho_{i0}) + O(\delta z) = \frac{m g}{k T_0} \left(\frac{\Phi^2}{2 \mu w_0^2}\right) + O(\delta z) . \tag{64}$$

Since the fluid inside the flux tube varies adiabatically, we obtain

$$\frac{1}{\rho_i} \frac{d\rho_i}{dz} = \frac{1}{\gamma} \frac{1}{p_i} \frac{dp_i}{dz} = \frac{1}{\gamma} \frac{1}{p_e} \frac{dp_e}{dz} \left[1 + \frac{2-\gamma}{\gamma} \frac{p_m}{p_e}\right]^{-1} ,$$

the latter by equation (54). Using equations (25) and (26), we finally obtain

$$\frac{1}{\rho_{i0}} \frac{d\rho_i}{dz} = -\frac{1}{\gamma} \frac{m g}{k T_0} \left[1 + \frac{2-\gamma}{\gamma} \frac{p_m}{p_e}\right]^{-1} + O(\delta z) . \tag{65}$$

Substituting equations (64) and (65) in equation (63), we obtain

$$F_s = -\frac{d\,\delta z}{d\,s}\,\frac{m g}{kT_0}\,\frac{\Phi^2}{2\,\mu w^2}\,\frac{\gamma-2}{\gamma}\,\frac{p_{e0}-p_{m0}}{p_{e0}+(2-\gamma/\gamma)\,p_m}+O^2(\delta z)\,, \qquad (66)$$

which is to be compared with equation (56). Now $d\delta z/ds > 0$ implies that $F_s < 0$, indicating a longitudinal flow along the tube away from an upward bulge, as in the case worked out previously where $T_e = T_i$. The transport of matter from the raised, and therefore expanded, portion of the tube enhances p_m relative to p_i by removing part of the fluid producing p_i and compressing B. This *enhances* the magnetic bouyancy and explains why ρ_e/ρ_i is greater than unity, as shown by equation (59), at the top of the atmosphere, even though p_m, on which the magnetic buoyancy depends, drops off with ρ_i more rapidly than does p_i.

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