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THE CLASSIFICATION OF CLOSE BINARY SYSTEMS

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ABSTRACT. — *An analysis of the observational data pertaining to two-spectra close binary systems which happen to be eclipsing variables reveals that all such systems possessing at least one (the primary) component on the Main Sequence can be divided naturally into three principal groups of the following characteristics :*

I. Main-Sequence (detached) Systems : *Both components do not deviate appreciably from the Main Sequence, and obey statistically a mass-luminosity relation* $L \sim m^{\alpha}$, *where* $\alpha = 2.8$ *for massive stars* ($m > 2 \odot$), *and* 4.7 *for* $m \ll 2 \odot$. *In addition, they appear to obey quite closely a statistical relation between mass and (mean) radius* R *of the form* $m \sim R^{\beta}$, *where* $\beta = 1.5$ *for* $m > 2 \odot$, *and* 1.0 *for small masses. The break in direction of the mass-luminosity and mass-radius relations turns out to take place approximately in the region where also the Main Sequence alters its slope in the H. R. — diagram. The primary (more massive) component is the* larger *of the two and of earlier spectral type (which may place the star anywhere along the Main Sequence) ; if the system happens to be an eclipsing variable, the primary (deeper) minima are, therefore, due to eclipses of* transit *type. The orbital periods are as a rule constant.*

The values of the total potential over free surfaces of both components are sensibly equal *— in spite of the fact that the absolute values of such potentials vary by a factor of more than ten from system to system.*

The volumes of both components are smaller than those of the largest closed equipotential capable of containing the masses of the two stars (hereafter called the Roche limit) for a given value of their mass-ratio.

II. Semi-detached Systems (Algol type) : *The secondary component lies* above *the Main Sequence ; and while the primary (Main-Sequence) stars of such pairs continue to obey the same statistical mass-luminosity and mass-radius relations as stars of Group I, the secondary (subgiant) components possess masses which are as a rule too small for their observed luminosities. The primary (more massive) component is as a rule the* smaller *of the two, and of* earlier *spectral type. The primary (deeper) minima of such eclipsing systems are, therefore, due to* occultation *eclipses.*

The values of the potential over surfaces of the primary components appear to be normal for stars of their spectra, but the potentials of the secondaries are abnormally low. *Moreover, in the majority of known cases, the secondary components appear to fill* exactly *the largest closed equipotentials capable of containing the whole mass of the respective star.*

A complementary type of semi-detached systems, with primary components at their Roche limits and secondaries well below it, seems conspicuous by its absence.

III. Contact Systems : — *perhaps the most numerous type of close binaries in the stellar population. Both components lie (though not very closely) on the Main Sequence — their spectral types exhibiting a marked concentration from late* A's *to early* K's (*W UMa-type stars*). *These show, however, — individually or statistically* — no vestige of any relation between mass and luminosity.

The primary (more massive) component is usually the larger *of the two, but as a rule of* later *spectral type than that of its mate, and quite frequently also of* lesser *luminosity. The primary (deeper) minima in eclipsing systems of this type are, therefore, likely to be due to* occultations.

The potentials over free surfaces of the two components appear to be sensibly equal, with each component just about filling completely the respective branch of their Roche limits. Both components are, therefore, probably in contact at the inner Lagrangian point L_1, *and may actually exchange mass through the conical end of the critical equipotential at* L_1.

In addition, contact systems are encountered also among certain early-type and very massive binaries, which are as rare, per unit volume of space, as dwarf contact binaries are frequent and which may be appropriately called ,, stars of the β Lyrae type ", although β Lyr itself, as a supergiant, deviates significantly from the Main Sequence.

In contrast to the behaviour of detached systems of Group I, the orbital periods of semi-detached and contact systems do often — though not always — undergo abrupt changes.

The bearing of the above characteristics of different groups on evolutionary trends of close binary systems is then briefly discussed. It is pointed out that the equality of potentials over free surfaces of the components of systems Group I and III may be indicative of the way of their formation : of current theories, only the fission hypothesis would seem to be able to account logically for this particular fact.

With regard to systems of Group II, the observed statistical clustering of the secondaries of their Roche limits indicates that these stars are secularly expanding *and represent, possibly, an evolutionary stage subsequent to the Main Sequence. The relative scarcity of intermediate cases (i.e., of subgiant components occupying volumes smaller than their Roche limit) indicates, moreover, that this secular expansion may proceed at a relatively rapid rate. Once the maximum distension permissible on dynamical grounds has been attained, a continuing tendency to expand would bring about a secular loss of mass through the conical and of the critical equipotential. It is suggested that the observed abnormally small masses of the secondary components (leading to abnormally large mass-ratios* m_1/m_2) *in systems of Group II are probably due to this fact.*

Резюме. — *КЛАССИФИКАЦИЯ ТЕСНЫХ ДВОЙНЫХ СИСТЕМ.* — Зденек КОПАЛЬ, Разбор наблюдений над тесными двойными звездами, имеющими двойной спектр, которые в то-же время являются затменными переменными, показывает, что все такие системы, имеющие по крайней мере одну (первую) компоненту на главной последовательности, естественно разделяются на три основных группы со следующими свойствами :

1) *Раздельные системы главной последовательности.*

Ни одна, ни другая из двух компонент не отклоняется значительно от главной последовательности, и обе статистически подчиняются отношению масса-светимость : $L \sim m^\alpha$, где $\alpha = 2.8$ для тяжелых звезд ($m > 2\odot$) и 5.7 если $m << 2\odot$. Сверх того, они повидимому подчиняются довольно точно статистическому соотношению между массой и (средним) радиусом R, имеющим форму $m \sim R^\beta$, причем $\beta = 1.5$ для $m > 2\odot$, и 1.0 для звезд меньших масс. Оказывается, что отклонение в направлении диаграмм масса-светимость и масса-радиус совершается приблизительно в том же месте, где главная последовательность изгибается в диаграммах яркость-спектр. Первая (более тяжелая) компонента является более крупной и более раннего спектрального типа (эта звезда

может быть изображена любой точкой главной последовательности); в случае, если система является затменной переменной — то главные (более глубокие) минимумы являются результатами прохождений.

Как правило, периоды обращения остаются постоянными.

Потенциалы на свободных поверхностях обоих компонент более или менее равны, несмотря на то, что абсолютная их величина изменяется фактором выше десяти от одной системы до другой.

По объему обе компоненты меньше, чем объем самого крупного замкнутого эквипотенциала, могущего вместить массу обеих звезд при данном отношении масс (предел Роша).

2) *Полураздельные системы (тип Алголя).*

Вторая компонента изображается *над* главной последовательностью. В то время, как первые (главной последовательности) звезды таких пар продолжают подчиняться тем же статистическим отношениям масса-светимость и масса-радиус, как и звезды первой группы, вторые компоненты (субгиганты) имеют массы, которые обыкновенно слишком малы для наблюдаемых у них светимостей. Первая (самая тяжелая) компонента обыкновенно является *меньшей* и более раннего спектрального типа. Главные (более глубокие) минимумы таких затменных систем таким образом являются результатом полных затмений. Потенциалы на поверхностях первых компонент являются нормальными по сравнению с звездами тех-же спектров, но потенциалы вторых компонент ненормально низки. Кроме того, в большинстве известных случаев вторые компоненты, повидимому, точно наполняют наибольший замкнутый эквипотенциал, могущий содержать всю массу данной звезды.

Обратный тип полураздельных систем с главными компонентами, достигающими предел Роша и со вторыми, не достигающими этого предела, кажется, не существует.

3) *Контактные системы.*

Контактная система, вероятно, самый многочисленный тип тесных двойных звезд. Обе компоненты находятся недалеко от главной последовательности, — их спектры заметно сосредотачиваются между поздними A и ранними K (звезды типа W.U.Ma). Среди них не замечается ни индивидуально, ни статистически *никакого отношения между массой и светимостью.*

Первая (более тяжелая) компонента является бо́льшей, но обыкновенно более позднего спектрального типа, чем ее спутник, и довольно часто обладает меньшей светимостью. Главные (более глубокие) минимумы в затменных системах этого типа поэтому могут являться результатами покрытия.

Потенциалы на свободных поверхностях двух компонент, повидимому, являются равными, и каждая компонента приблизительно наполняет данную часть предела Роша. Таким образом обе компоненты вероятно соприкасаются во внутренней точке Лагранжа L_1, и могут обменивать материю через конический конец критического эквипотенциала L_1.

Сверх того, контактные системы встречаются также между некоторыми двойными раннего типа и большой массы. Эти системы встречаются гораздо реже, чем карликовые контактные двойные и для них подходящим названием будет звезды типа « β Лиры », хотя сама β Лиры, как сверхгигант, значительно отклоняется от главной последовательности.

В отличие от раздельных систем первой группы, периоды вращения полуотдельных и контактных систем часто, хотя не всегда, подвергаются резким переменам.

Автор рассматривает влияние выше-упомянутых характеристик разных групп на эволюцию тесных двойных систем. Он указывает, что равенство потенциалов на свободных поверхностях компонент — первой и третьей групп — может служить указанием на образ их происхождения : из существующих теорий только гипотеза разделения могла бы, повидимому, логически объяснить этот факт.

Относительно систем второй группы замеченная статистическая группировка вторых звезд в их пределах Роша указывает, что эти звезды *беспрерывно расширяются* и представляют, может быть, эволюционную стадию, следующую за главной последовательностью.

К тому же, относительная редкость промежуточных типов (например, субгигантских компонент, занимающих объемы меньше объема предела Роша) показывает, что это расширение может происходить сравнительно быстро. После того как максимальный объем возможный с динамической точки зрения достигнут, дальнейшая тенденция к расщирению вызвала бы безпрерывную потерю массы через конический конец критического эквипотенциала. По мнению автора, это явление может служить объяснением замеченных, ненормально маленьких масс вторых компонент в системах второй группы (вызывающих необыкновенно большие отношения масс m_1/m_2).

I. Introduction.

Close double stars which manifest themselves as spectroscopic or eclipsing binaries represent a very common phenomenon among the stellar population of our galactic system. For example, Plaskett and Pearce in their *Catalogue of the Radial Velocities of O and B Type Stars* [1] state that, of the 1 010 objects listed,.... " there are 319 stars which are definitely assigned as having variable velocity " ; and while in some few of these (such as short-periodic stars of the β CMa type) the velocity variation may not be due to orbital motion, it seems certain that at least 30 per cent of stars on the upper part of the Main Sequence are close doubles. For stars of more advanced spectral type this percentage seems to drop (1), but climb to new heights among late F and G stars. Close binary systems are encountered in almost all populated portions of the Hertzsprung-Russell diagram ; and of all possible combinations of their components only pairs consisting of two white dwarfs are so far unknown. The percentage of such systems which, by accident of the orientation of their orbits in space, happen also to exhibit eclipses is rather difficult to estimate with any degree of accuracy. There are at least seven such systems within thirty parsecs from the Sun (2) ; and since this volume is known to contain some 3 000 stars, eclipsing variables would seem to form about 0.2 per cent of the total population. Despite the evident scarcity of data, this order of magnitude is probably close to the truth, and throws some light on the relative frequency of eclipsing variables in the Universe around us. If a similar ratio extends over the whole galactic system, the total number of close or eclipsing binaries in it is enormous and quite beyond the hopes of discovery.

Up to the present time, a few thousand individual stars have been recognized as close binaries by virtue of their variation of light and (or) radial velocity. For

(1) Harper in his catalogue of radial velocities of stars of spectral class A (*Publ. D. A. O.*, **7**, 1937, 1), states that among 917 such stars, there are 23 recognized spectroscopic binaries, and 55 additional objects suspected of duplicity (about a half of which are likely to prove binary systems). Hence, it would appear that only 2 — 5 % of A-type stars are close binaries — in marked contrast to the much higher percentage encountered among earlier spectral classes. This disparity cannot, moreover, be explained away by observational selection (unless the masses of the companions of A stars as a class bear a much smaller ratio to those of their primaries than is the case for O and B stars).

(2) These objects are : β Aur, i Boo, RS CVn, R CMa, VW Cep, α CrB, and YY Gem.

over five hundred of them available, spectrographic observations proved sufficient for the derivation of orbits of one (or both) component(s) [2], while KUKARKIN and PARENAGO's *Catalogue of Variable Stars* [3] with its supplements lists over 2 000 eclipsing variables with known orbital elements and light curves are available for many hundreds of them. This material — no doubt a very tiny fraction of the total galactic supply — has served as a basis for numerous statistical studies of close binary systems and, in particular, for setting up an appropriate system for their *classification.* The immediate objective of classification may be segregation of those stars with certain common characteristics. In order for such a classification to possess deeper meaning, the characteristics on which it is based must not only be unambiguous, and be concerned with no mere superficial qualities, but rather with properties sufficiently fundamental to render each group a definite physical association. In point of fact, while the immediate aim of a classification may be a purely practical one, its ultimate aim should be to throw light on the origin and generic relation of the individual groups of close binary systems, and to identify their proper place within the general framework of stellar evolution.

Let us examine, from this point of view, the systems of classification suggested so far for close binary systems ; and in doing so, we shall be led almost from the outset to confine our attention to eclipsing variables. This is not only because the binaries which reveal their nature by light variation outnumber those for which radial velocity alone is known to vary at least four-to-one ; but also because an inspection of the light curve (or its appropriate analysis) reveals as a rule many more details concerning each particular system than its radial-velocity curve. It goes, however, without saying that both these kinds of binaries constitute the same physical group, and differ in their observable manifestations only by accident of orientation of their orbits.

It has been customary, for many years, to divide the eclipsing variables into three principal groups — with Algol, β Lyrae, and W Ursae Maioris as their respective prototypes. It is, however, easy to show that any such classification — still current in most catalogues today — lacks proper physical basis and can be justified, if at all, only on historical grounds. Its principal characteristic, — the presence or absence of the photometric ellipticity effect between minima — is the natural resultant of a combination of the fractional dimensions, masses, and relative luminosities of the two components, and can be predicted from them in terms of a reliable theory. Statistical investigations reveal, however, no apparent break in the frequency distribution of fractional radii, or mass and luminosity ratios in eclipsing binary systems. As a result, with increasing separation of the components, stars of the β Lyrae type merge gradually with the Algol systems : any dividing line between the two being essentially a matter of observational precision with which the amount of photometric ellipticity can be ascertained.

This fact renders, however, any such classification not only inexact, but ambiguous as well ; for the dependence of the mean photometric ellipticity of a system on the relative luminosities of the constituent components, which are, generally, of different spectral types, renders this ellipticity dependent on the effective wave length of observation. In order to illustrate this point consider, for instance, the well-known eclipsing systems of Algol and *u* Herculis. The former, the prototype of its group, exhibits a light-curve which, in visible light, is virtually free from any ellipticity effect ; while that of *u* Herculis is conspicuously convex between minima — thus rendering the system one of the β Lyrae type according to the conventional classification. Geometrically, however, both these systems are very similar, and so are their mass-ratios : in both, the primary component of the earlier spectral type, is several times as massive as its mate, and its form departs relatively little from a sphere. In each system the secondary component is slightly larger than the primary (the ratios k of their radii being equal to 0.95 ± 0.02 and 0.99 ± 0.04 for Algol and *u* Herculis, respectively), and the inequality of their masses renders the less massive star conspicuously distorted — so much, in fact, as to effectively fill the largest closed equipotential capable of containing its whole mass. The principal difference between the two systems reduces, indeed, to a difference in the ratio of the surface brightnesses of their primary and secondary components : whereas, in photoelectric light (λ 4 500 Å), the ratio J_1/J_2 of the surface brightnesses of the components of Algols is equal to 18.0 ± 1.5, in *u* Herculis the same ratio happens to be 2.50 ± 0.02. Since the ratios of the radii k in both systems are very nearly equal, the disparity in the values of J_1/J_2 reveals, therefore, a corresponding disparity in the luminosities of the two components. The secondary components in both systems are known to be highly distorded ; but owing to its low fractional luminosity, the secondary of Algol can influence the combined light of the system very much less than does the secondary of *u* Herculis in ordinary frequencies ; hence a conspicuous difference in the appearance of their light curves.

A disparity between the surface brightnesses of the two stars of dissimilar spectral types is, however, known to diminish with the increasing wave length of observation. A recourse of Planck's law discloses that the value of $J_1/J_2 = 18$ for Algol at λ 4 500 Å is expected to decrease to 2.5 at about $\lambda = 2.7\,\mu$ — i.e., at an effective wave length accessible to modern infrared detectors. In point of fact, there is every reason to expect that an infrared light curve of Algol, obtained with the aid of a lead-sulphide or lead-telluride cell through the atmospheric " window " between 2.0-2.5 μ would markedly resemble that of *u* Herculis, and thus earn for Algol a claim to be included in the β Lyrae group. Moreover, a generalization of this point makes it clear that whereas eclipsing systems which consist of components of similar spectral types will exhibit (more or less) similar light curves when observed in light of any frequency, those consisting of dissimilar components may exhibit very different light curves at different effective wave lengths if their components are also different in form. These facts show that a simple division of eclipsing systems into Algol and β Lyrae-type stars is not only *inaccurate* for lack of a suitable dividing line between the two types, but also *ambiguous* ; for it may refer at best only to observations carried out in a certain frequency range.

It has been pointed out by previous investigators (4) that a simple classification of eclipsing binaries in two or three groups is insufficient to reveal any physical or evolutionary connection, and more complex systems have been elaborated from time to time. Thus KRAT [5] proposed a classification of eclipsing variables into eight groups (A, A_1, B, C, ... G) according to their spectral type, the form of the light curve, and the orbital period ; while more recently STRUVE [6] has worked out a different classification, consisting of five groups (I, II, ... V), for systems whose primary components were of spectrum earlier than F0. In so far as KRAT's classification is based on the form of the light curve, it suffers from the same limitations as the simple two-group classification criticised above ; and a closer scrutiny [7] reveals that there are indeed continuous transitions between KRAT's groups — thus rendering his classification again essentially an arbitrary one. On the other hand, STRUVE's criteria are based largely on spectroscopic phenomena which are, so far, known only for a limited number of systems, and pertain to the motions of gas streams between the two components rather than to the properties of the constituent stars themselves.

Quite recently, PLAUT pointed out on the basis of a statistical study of eclipsing variables brighter than 8.5 apparent photographic magnitude at maximum [7] that most such binaries can be divided into the following three groups :

(1) Systems with both components on the Main Sequence (excluding the W UMa-type stars) ;

(2) Systems with one component outside the Main Sequence ; and

(3) Systems of the W Ursae Maioris type.

PLAUT noticed, moreover, that almost all well-observed systems of group (2) show spectral peculiarities ; that variable periods occur more frequently in group (2) than in group (1) ; and that " all W UMa-type systems probably will show variable periods, variable light curves, and peculiar spectra, if observed accurately ".

The aim of the present investigation will be to open up the problem of classification of close binary systems *de novo*, in order to ascertain whether or not a suitable system of classification can be devised which may enable us to discern at least some *evolutionary trends* among eclipsing variables, which might possibly throw some light on the *origin* of such systems. In doing so, we shall consider all fundamental aspects of the photometric as well as spectroscopic evidence, and shall pay particular attention to such elements as are likely to remain invariant over astronomically long intervals of time. The material at our disposal, and our method of analysis, will be set forth in subsequent sections following this introduction. Our conclusion will, on the whole, justify a classification system as simple as PLAUT's, and a closer scrutiny will reveal that the systems within each group have several and so far unnoticed properties in common — some of which may indeed be indicative of evolutionary trends. The main results of this investigation have already been summarised in the abstract ; so that, in what follows, our task will be to substantiate them in detail.

II. OBSERVATIONAL DATA.

The observational data which will form the basis of the present investigation are summarised in the following Tables I and II. The first one contains a list of all eclipsing binaries, with known photometric elements, for which the ratio of the masses of the two components has been determined by spectroscopic observations. The list is believed to include all such systems known at the time of writing, though for some of them (marked so in the References column of Table I) the observational data are uncertain or incomplete.

The headings of the individual columns of Table I are self-explanatory. The letters A, B in column (1) refer to the more massive and less massive component, respectively ; though the former need not necessarily be the more luminous of the two. The masses *m* and radii *R* as given in columns (2) and (3) have been expressed in solar units. The spectra of column (4) adopted for the secondary components are in harmony with the observed ratios of surface brightnesses of the two components. Whenever direct reliable spectroscopic determination was not available (or was difficult to obtain in the disparity in luminosities of the two stars was too large) a determination of the secondary's spectral type from its relative surface brightness by a recourse to PLANCK's formula has been preferred. All spectra determined in this way have been placed in brackets.

The logarithms $\log_{10}$ T of the effective temperatures (in degrees K) of both components of each system, as given in the next column (5), have been adopted for spectral types of column (4) in accordance with the following procedure : for spectra later than A0 the corresponding temperatures can nowadays be regarded as well-established, and a temperature scale (essentially KUIPER's) as quoted by KEENAN and MORGAN in HYNEK's *Astrophysics* [8] has been adopted ; the only correction required being an increase of temperature at the extreme end of the spectral sequence. For spectral types earlier than A0, however, several lines of observational evidence based on eclipsing variables indicate that KUIPER's 1938 temperatures tend to be too high. The present writer has already discussed this evidence in an earlier investigation [9], but certain supplementary data may be added here:

There exist three early-type eclipsing systems for which a determination of the effective temperatures of their principal components can be wholly based on empirical data ([1]) : namely β Aur, β Per, and μ^1 Sco ; and the pertinent quantities are summarised in the following tabulation :

	β Aur A	β Per A	μ^1 Sco A
	—	—	—
Spectrum	A0	B8	B3
Parallax	0″037 ± 0″004	0″042 ± 0″002	0″0053 ± 0″0004
Abs. vis. magn....	+ 0.6 ± 0.2	+ 0.3 ± 0.1	— 1.7 ± 0.2
Bol. Corr.	— 0.7	— 1.0	— 1.7
Radius	2.5 ± 0.1	2.7 ± 0.2	4.8 ± 0.2
Temperature	10 700° ± 500°	12 200° ± 500°	16 600° ± 700°

The parallax of β Aur was taken from Jenkins' *General Catalogue of Trigonometric Stellar Parallaxes* (No. 1373), while that of β Persei is due to van de Kamp, Smith and Thomas [10]. The absolute radius of Algol A is taken from Kopal [11]. The star μ^1 Sco A should be adjoined to the preceding two on the strength of Blaauw's recent group parallax [12] deduced from the membership of this system in the Scorpio-Cenaturus stream ([2]). In view of the relatively high precision of the effective temperatures of the three early-type stars as determined from the foregoing data, it seems reasonable to adopt a temperature scale which coincides, at the respective spectra, with those determinations : namely,

Spectrum	Log T
—	—
O8	(4.35)
B0	(4.30)
B2	4.25
B3	4.22
B5	4.17
B8	4.09
A0	4.03

These temperatures are somewhat higher than those proposed earlier by the writer, but still significantly lower than Kuiper's.

The only other correction to Kuiper's scale, the need of which is indicated by new observational data, concerns the M stars. The effective temperature scale at the extreme lower end of the Main Sequence has been based by Kuiper largely on

([1]) The spectra of the secondary components in these systems have so far not been accurately classified.

([2]) According to Blaauw (*Groningen Publ.*, 1946, No. 52), μ^1 Sco is to be regarded as " certain member of the cluster " (*op. cit.*, p. 119).

the data furnished by the dwarf eclipsing system YY Gem. Since 1938, however, these data were superseded by the results of a more recent careful photoelectric study of the system by KRON [1] at the Lick Observatory. The parallax of YY Gem (which is a physical member of the Castor system) is known to be $0''.073 \pm 0''.002$ (ALLEGHENY, 1938 [2]), leading to an absolute visual magnitude of either component of $M_{vis} = 9.14 \pm 0.06$ [1]. On the other hand, their absolute radii are 0.62 ± 0.02 $\odot$. If, therefore, ΔC stands for the bolometric correction appropriate for stars of spectral class dM1, an application of STEFAN's law leads to the following relation

$$\log T = \underset{\pm .02}{3.42} - 0.1\,\Delta C,$$

between T and ΔC, which, in turn, furnishes the results summarized in the following tabulation :

Log T	ΔC	M_{bol}
3.53	— 1.1	8.0
3.56	— 1.4	7.7
3.59	— 1.7	7.4

Should we adopt, with KEENAN and MORGAN, log T for dM1 stars to be 3.53, the corresponding bolometric correction of — 1.1 comes out positively too small. On the other hand should we adopt, with KUIPER, $\Delta C = -1.7$ for dM1 stars, the corresponding effective temperature results much too high (in comparison with the data based on BETEIGEUZE and other more indirect evidence). For the purpose of the present study we have, therefore, adopted the pair of intermediate values of $\log T = 3.56$ and $\Delta C = -1.4$, corresponding to an absolute bolometric magnitude M_b of each component of YY Gem of $+ 7.7 \pm 0.2$.

With the aid of a temperature scale thus established the absolute bolometric magnitudes M of the individual components of systems included in Table I have been evaluated (by a recourse to STEFAN'S LAW) as

$$M = 4.7 - 5 \log R - 10 \log (T/5\,730^{\circ}), \tag{1}$$

where $+ 4.7$ stands for the adopted bolometric absolute magnitude of the Sun, and 5 730° (corresponding to $\log T_{\odot} = 3.76 \pm 0.01$) for its effective temperature. Such values of M are listed in the penultimate column (6) of Table I, while the ultimate column (7) contains references to the basic observational data. Of the pairs of such references given in each case, the first pertains always to the source of photometric elements ; and the second to that of the spectrographic data.

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(1) *Ap. J.*, **115**, 1951, 301.
(2) *A. J.*, **53**, 1948, 137.

TABLE I

Star	m	R	Sp	Log T	M
RT And A	1.5	0.76	G0	3.78	5.1
RT And B	1.0	1.36	K1	3.65	5.1
AB And A	1.7	0.9	G5	3.74	5.1
AB And B	1.1	0.9	G4	3.75	5.0
S Ant A	0.76	1.4	A8	3.90	2.6
S Ant B	0.42	1.1	A3	3.96	2.5
V 599 Aql A	12	7.8	B4	4.20	— 4.2
V 599 Aql B	6.4	4.4	B8	4.09	— 1.8
σ Aql A	6.8	4.2	B8	4.09	— 1.7
σ Aql B	5.4	3.3	B9	4.06	— 0.9
SX Aur A	10.5	4.8	B3.5	4.21	— 3.2
SX Aur B	5.6	4.3	B6	4.15	— 2.4
TT Aur A	6.7	3.3	B3	4.22	— 2.5
TT Aur B	5.3	3.2	B7	4.12	— 1.4
WW Aur A	1.82	1.9	A7	3.91	1.8
WW Aur B	1.75	1.9	A8	3.90	1.9
AR Aur A	2.6	1.8	B9	4.06	0.4
AR Aur B	2.3	1.8	A0	4.03	0.7

TABLE I (*Continuation*)

Star	References
RT And	Payne-Gaposchkin, *Ap. J.*, **103**, 1946, 291.
AB And	: Gaposchkin, *Berlin Bab. Veröff.*, **9**, 1932, Heft 5.
	Struve and others, *Ap. J.*, **111**, 1950, 658.
S Ant	Hogg and Bowe, *M. N.*, **110**, 1950, 373.
	Joy, *Ap. J.*, **64**, 1926, 287.
V 599 Aql	: : Gaposchkin, *Harv. Bull.*, 1943, No. 917.
	* Pearce, *Publ. D. A. O.*, **4**, 1927, 75.
σ Aql	Wylie, *Ap. J.*, **56**, 1922, 232.
	Luyten, Struve, Morgan, *Yerkes Publ.*, VII, 1939, pt. 4.
SX Aur	Oosterhoff, *B. A. N.*, **7**, 1933, 107.
	** Popper, *Ap. J.*, **97**, 1943, 394.
TT Aur	Joy and Sitterly, *Ap. J.*, **73**, 1931, 77.
WW Aur	Huffer and Kopal, *Ap. J.*, **114**, 1951, 297.
	* Slocum, *Lick Bull.*, **19**, 1942, 147.
AR Aur	Huffer and Eggen, *Ap. J.*, **106**, 1947, 106.
	* Harper, *Publ. D. A. O.*, **6**, 1937, 311.

TABLE I (*Continued*)

STAR	m	R	Sp	LOG T	M
—	—	—	—	—	—
EO Aur A	27	13	B3	4.22	— 5.5
EO Aur B	27	16	B8	4.08	— 4.5
β Aur A	2.4	2.49	A0	4.03	0.0
β Aur B	2.3	2.28	A0	4.03	0.2
i Boo A	1.07	0.66	dG2	3.76	5.6
i Boo B	0.54	0.64	dG1	3.77	5.6
SZ Cam A	21	9.8	O9.5	4.31	— 5.8
SZ Cam B	6.1	4.3	(B2)	4.25	— 3.4
TX Cnc A	1.5	0.74	F8	3.79	5.0
TX Cnc B	0.8	0.78	F7	3.80	4.8
RS CVn A	1.9	1.8	F4	3.83	2.7
RS CVn B	1.7	5.1	gK4	3.60	2.8
TV Cas A	1.7	2.6	A0	4.03	— 0.1
TV Cas B	1.0	2.4	(gF9?)	3.77	2.7
AO Cas A	30	14	O8.5	4.33	— 6.7
AO Cas B	28	10	O9	4.32	— 5.9
CC Cas A	21	9.3	O8	4.35	— 6.0
CC Cas B	10	4.5	B0	4.30	— 4.0

TABLE I (*Continuation*)

STAR	REFERENCES
—	—
EO Aur	: GAPOSCHKIN, *P. A. S. P.*, **55**, 1943, 193.
	* PEARCE, *Journ. R. A. S. C.*, **40**, 1946, 139.
β Aur	PIOTROWSKI, *Ap. J.*, **108**, 1948, 510.
	* SMITH, *Ap. J.*, **108**, 1948, 504.
i Boo	EGGEN, *Ap. J.*, **108**, 1948, 15.
	** POPPER, *Ap. J.*, **97**, 1943, 394.
SZ Cam	WESSELINK, *Leiden Ann.*, **17**, 1941, pt. 3 (rediscussed by KOPAL).
TX Cnc	HAFFNER, *A. N.*, **276**, 1948, 233.
	POPPER, *Ap. J.*, **108**, 1948, 490.
RS CVn	KELLER and LIMBER, *Ap. J.*, **113**, 1951, 637.
	JOY, *Ap. J.*, **72**, 1930, 41.
TV Cas	HUFFER and KOPAL, *Ap. J.*, **114**, 1951, 297.
	* PLASKETT, *Publ. D. A. O.*, **2**, 1922 ***, 141.
AO Cas	WOOD, *Ap. J.*, **108**, 1948, 28 ; HILTNER, *Ap. J.*, **110**, 1949, 443.
	* PEARCE, *Publ. D. A. O.*, **3**, 1926, 283.
CC Cas	: GAPOSCHKIN, *Publ. A. A. S.*, **10**, 1939, 12.
	* PEARCE, *Publ. D. A. O.*, **4**, 1927 ***, 67.

TABLE I (*Continued*)

STAR	m	R	Sp	LOG T	M
U Cep A	2.9	2.4	B8	4.09	— 0.5
U Cep B	1.4	3.9	gG8	3.67	2.6
VW Cep A	1.1	0.97	dK1	3.69	5.5
VW Cep B	0.35	0.58	G5	3.74	6.1
WX Cep A	1.0	2.0	A2	3.99	0.9
WX Cep B	1.0	2.0	A5	3.94	1.4
AH Cep B	16.5	6.1	B0	4.30	— 4.6
AH Cep A	14.2	6.1	B0.5	4.28	— 4.4
CW Cep A	10.0	4.5	B3	4.22	— 3.2
CW Cep B	9.8	4.0	B3.5	4.22	— 2.9
RZ Com A	1.6	0.9	K0	3.71	5.4
RZ Com B	0.8	0.9	G9	3.72	5.3
U CrB A	6.4	1.9	B5	4.17	— 0.8
U CrB B	2.4	3.6	(gG0†)	3.76	1.9
Y Cyg A	17.4	5.9	O9.5	4.31	— 4.6
Y Cyg B	17.2	5.9	O9.5	4.31	— 4.6

TABLE I (*Continuation*)

STAR	REFERENCES
U Cep	DUGAN, *Princ. Contr.*, 1920, No. 5.
	HARDIE, *Ap. J.*, **112**, 1950, 542.
VW Cep	HUFFER, *Ap. J.*, **103**, 1946, 1.
	* POPPER, *Ap. J.*, **108**, 1948, 490.
WX Cep	: : SCHNELLER, *Veröff. Berl. Bab.*, **8**, 1931, Heft 6.
	SAHADE and CESCO, *Ap. J.*, **102**, 1945, 128.
AH Cep	HUFFER and EGGEN, *Ap. J.*, **106**, 1947, 313.
	* PEARCE, *Journ. R. A. S. C.*, **29**, 1935, 411.
CW Cep	: GAPOSCHKIN, *Per. Zvezdy*, **7**, 1949, 34.
	* PETRIE, *Publ. D. A. O.*, **7**, 1947, 305.
RZ Com	: : GAPOSCHKIN, *Harv. Mono.*, 1946, No. 5, 70.
	STRUVE and GRATTON, *Ap. J.*, **108**, 1948, 497.
U CrB	: BAKER, *Laws Bull.*, 1921, No. 29.
	* SAHADE and STRUVE, *Ap. J.*, **102**, 1945, 480 (cf. also PEARCE, *Publ. A. A. S.*, **8**, 1935, 219).
Y Cyg	DUGAN, *Princ. Contr.*, 1931, No. 12.
	* REDMAN, *Publ. D. A. O.*, **4**, 1931, 341.

TABLE I (*Continued*)

STAR	m	R	Sp	LOG T	M
GO Cyg A	1.12	1.8	B9	4.06	0.4
GO Cyg B	0.94	1.6	A0	4.03	0.9
MR Cyg A	3.0	3.4	A0	4.03	— 0.6
MR Cyg A	2.6	2.6	F7	3.80	2.2
V 453 Cyg A	17.8	7.6	B2	4.25	— 4.6
V 453 Cyg B	13.8	6.3	B3	4.22	— 3.9
V 470 Cyg A	12.5	7	B2	4.25	— 4.3
V 470 Cyg B	11.1	6	B4	4.20	— 3.8
V 477 Cyg A	2.3	1.5	A3	3.96	1.8
V 477 Cyg B	1.6	1.2	F5	3.82	3.7
V 478 Cyg A	14.4	7.6	B0.5	4.29	— 5.0
V 478 Cyg B	14.2	7.6	B0.5	4.29	— 5.0
TW Dra A	2.2	3.5	A6	3.92	0.4
TW Dra B	0.62	5.1	K2	3.63	2.4
YY Eri A	0.97	0.89	G5	3.74	5.2
YY Eri B	0.57	0.81	G4	3.75	5.3
YY Gem A	0.64	0.62	dM1	3.56	7.8
YY Gem B	0.64	0.62	dM1	3.56	7.8

TABLE I (*Continuation*)

STAR	REFERENCES
GO Gyg	OVENDEN, *M. N.*, **114**, 1954, 569.
	PEARCE, *Journ. R. A. S. C.*, **27**, 1933, 62.
MR Cyg	KAMINSKI, *Per. Zvjozdy*, **9**, 1953, 285.
	PEARCE, *Journ. R. A. S. C.*, **29**, 1935, 411.
V 453 Cyg	SMIRNOV, *Per. Zvjozdy*, **6**, 1946, 13.
	* PEARCE, *Publ. A. A. S.*, **10**, 1941, 233.
V 470 Cyg	: GAPOSCHKIN, *A. J.*, **53**, 1948, 112.
	* PEARCE, *P. A. S. P.*, **58**, 1946, 247.
V 477 Cyg	WALLENQUIST, *Uppsala Medd.*, 1949, No. 96.
	PEARCE, *A. J.*. **57**, 1952, 22.
V 478 Cyg	: GAPOSCHKIN, *Harv. Bull.*, 1949, No. 919.
	McDONALD, *Publ. D. A. O.*, **8**, 1949, 135.
TW Dra	BAGLOW, *Publ. David Dunlap Obs.*, **2**, 1952, No. 1.
	SMITH, *Ap. J.*, **110**, 63, 1949.
YY Eri	HURUHATA, DAMBARA and KITAMURA, *Ann. Tokyo Obs.*, (2), **3**, 1953, 227.
	STRUVE, *Ap. J.*, **106**, 1947, 92.
YY Gem	KRON, *Ap. J.*, **115**, 1952, 301.
	STRUVE, HERBIG and HORAK, *Ap. J.*, **112**, 1950, 216.

TABLE I (*Continued*)

Star	*m*	R	Sp	Log T	M
—	—	—	—	—	—
Z Her A	1.5	1.8	F2	3.85	2.4
Z Her B	1.3	2.8	dG4	3.74	2.7
RX Her A	2.1	2.2	A0	4.03	0.4
RX Her B	1.9	1.8	A1	4.01	0.8
TX Her A	2.1	1.8	A5	3.94	1.6
TX Her B	1.8	1.5	A7	3.91	2.2
DI Her A	3.7	2.1	B4	4.20	— 1.3
DI Her B	3.3	2.4	B5	4.17	— 1.3
u Her A	7.9	4.5	B3	4.22	— 3.2
u Her B	2.8	4.3	B7	4.12	— 2.1
VZ Hya A	1.2	1.5	F5	3.82	3.2
VZ Hya B	1.1	1.2	F6	3.81	3.8
SW Lac A	1.08	0.82	G3	3.75	5.2
SW Lac B	0.93	0.89	G1	3.77	4.9
AR Lac A	1.32	1.54	G5	3.73	4.1
AR Lac B	1.31	2.86	K0	3.67	3.3
CM Lac A	2.0	1.8	A2	3.99	1.1
CM Lac B	1.5	1.3	F2	3.85	3.2

TABLE I (*Continuation*)

Star	References
—	—
Z Her	Baglow, *Publ. David Dunlap Obs.*, **2**, 1952, No. 1.
	* Adams and Joy, *Ap. J.*, **49**, 1919, 192.
RX Her	Wood, *Ap. J.*, **110**, 1949, 465.
	* Sanford, *Ap. J.*, **68**, 1928, 51.
TX Her	Plaut, *Groningen Publ.*, 1953, No. 55.
	* Plaskett, *Publ. D. A. O.*, **1**, 1920 ***, 207.
DI Her	Jacchia, *Harv. Bull.*, 1940, No. 912.
	* McKellar, *Publ. D. A. O.*, **8**, 1950, 235.
u Her	Kopal and Shapley (unpublished).
	* Smith, *Ap. J.*, **102**, 1945, 500.
VZ Hya	Pierce, *Princ. Contr.*, 1946, No. 21.
	Struve, *Ap. J.*, **102**, 1945, 74.
SW Lac	: Schilt, *B. A. N.*, **2**, 1924, 175.
	Struve, *Ap. J.*, **109**, 1949, 436.
AR Lac	Wood, *Princ. Contr.*, 1946, No. 21.
	Harper, *Journ. R. A. S. C.*, **27**, 1933, 146 ; Sanford, *Ap. J.*, **113**, 1951, 299.
CM Lac	: Wachmann, *A. N.*, **259**, 1936, 323.
	* Sanford, *Ap. J.*, **79**, 1934, 95.

TABLE I (*Continued*)

STAR	m	R	Sp	LOG T	M
UV Leo A	1.36	1.21	G0	3.78	4.1
UV Leo B	1.25	1.22	G2	3.77	4.2
IM Mon A	9.0	3.8	B5	4.17	— 2.3
IM Mon B	6.0	2.7	B8	4.09	— 0.8
U Oph A	5.30	3.4	B5	4.17	— 2.1
U Oph B	4.65	3.1	B6	4.15	— 1.7
WZ Oph A	1.4	0.88	G0	3.78	4.8
WZ Oph A	1.35	0.86	G0	3.78	4.8
V 502 Oph A	1.2	1.51	G2	3.76	3.8
V 502 Oph B	0.49	1.04	F8	3.79	4.3
ER Ori A	0.58	0.72	G1	3.77	5.3
ER Ori B	0.35	0.72	G1	3.76	5.4
δ Ori A	26	17	B1	4.27	— 6.5
δ Ori B	10	10	B2	4.25	— 5.2
U Peg A	1.25	1.18	F3	3.84	3.6
U Peg B	1.00	1.18	F2	3.85	3.4
AG Per A	5.01	3.1	B3	4.22	— 2.4
AG Per B	4.47	2.8	B4	4.20	— 1.9

TABLE I (*Continuation*)

STAR	REFERENCES
UV Leo	WELLMANN, *Zs. f. Ap.*, **34**, 1954, 99.
	GAPOSCHKIN, *Ap. J.*, **104**, 1946, 370.
IM Mon	GUM, *M. N.*, **111**, 1951, 634.
	* PEARCE, *Publ. D. A. O.*, **6**, 1932 ***, 70.
U Oph	HUFFER and KOPAL, *Ap. J.*, **114**, 1951, 297.
	* PLASKETT, *Publ. D. A. O.*, **1**, 1919, 138.
WZ Oph	GAPOSCHKIN, *Harv. Bull.*, 1938, No. 907.
	SANFORD, *Ap. J.*, **86**, 1937, 157.
V 502 Oph	: NEKRASSOVA, *Astr. Circ. U. S. S. R., Acad. Sci.*, 1943, No. 21.
	STRUVE and GRATTON, *Ap. J.*, **108**, 1948, 497.
ER Ori	: TAYLOR, *P. A. S. P.*, **56**, 1944, 112.
	STRUVE, *P. A. S. P.*, **56**, 1944, 34.
δ Ori	: STEBBINS, *Ap. J.*, **42**, 1915, 133.
	LUYTEN, STRUVE and MORGAN, *Yerkes Publ.*, VII, 1939, pt. 4.
U Peg	: LA FARA, *Ap. J.*, **115**, 1952, 14.
	STRUVE and others, *Ap. J.*, **111**, 1950, 658.
	EGGEN (unpublished).
AG Per	* PLASKETT, *Publ. D. A. O.*, **3**, 1925, 184.

TABLE I (*Continued*)

STAR	m	R	Sp	LOG T	M
ξ Pho A	2.8	2.5	B7	4.12	— 0.9
ξ Pho B	2.0	1.5	B9	4.06	0.8
V Pup A	16.6	6.0	B1	4.27	— 4.3
V Pup A	9.8	5.3	B3	4.22	— 3.5
U Sge A	6.7	4.1	B9	4.06	— 1.4
U Sge B	2.0	5.4	gG6	3.70	1.6
V 356 Sgr A	12.1	4.9	B3	4.23	— 3.4
V 356 Sgr B	4.7	12.7	A2	3.99	— 3.2
μ^1 Sco A	14.0	4.8	B3	4.22	— 3.3
μ^1 Sco B	9.2	5.3	B7	4.12	— 2.5
RZ Tau A	2.2	1.4	F0	3.88	2.8
RZ Tau B	1.2	1.3	F1	3.87	3.0
W UMa A	1.29	1.1	dF8	3.79	4.2
W UMa B	0.65	0.61	dF6	3.80	5.4
TX UMa A	2.9	2.16	B8	4.09	— 0.3
TX UMa B	0.9	3.79	gG4	3.71	2.3
AH Vir A	1.36	1.3	dK0	3.71	4.6
AH Vir B	0.57	0.75	dG6	3.74	5.5
Z Vul A	5.25	4.7	B3	4.22	— 3.3
Z Vul B	2.34	4.2	A2	3.99	— 0.7

TABLE I (*Continuation*)

STAR	REFERENCES
ξ Pho	HOGG, *M. N.*, **111**, 1951, 315.
	COLACEVICH, *P. A. S. P.*, **47**, 1935, 84.
V Pup	VAN GENT, *B. A. N.*, **8**, 1939, 319 (rediscussed by KOPAL).
	** POPPER, *Ap. J.*, **97**, 1943, 394.
U Sge	IRWIN (unpublished).
	JOY, *Ap. J.*, **71**, 1930 ***, 336.
V 356 Sgr	POPPER, *Ap. J.*, **121**, 1955, 56.
μ^1 Sco	STIBBS, *M. N.*, **108**, 1948, 398.
	STRUVE, *Strömgren Festschrift*, Copenhagen, 1940, 258.
RZ Tau	HURUHATA and KITAMURA, *Publ. Astr. Soc. Japan*, **5**, 1953, 102.
	STRUVE and others, *Ap. J.*, **111**, 1950, 658.
W UMa	PLAUT, *Groningen Publ.*, 1953, No. 55.
	* STRUVE and HORAK, *Ap. J.*, **112**, 1950 178.
TX UMa	PIOTROWSKI, *Ap. J.*, **106**, 1947, 472.
	HILTNER, *Ap. J.*, **101**, 1945, 108 (cf. also PEARCE. *Publ. A. A. S.*, **8**, 1936, 251).
AH Vir	: : CHANG, *Ap. J.*, **107**, 1948, 96.
Z Vul	BAKER, *Laws Obs. Bull.*, 1916, No. 26.
	* PLASKETT, *Publ. D. A. O.*, **1**, 1920, 251.

TABLE I (*Continued*)

STAR	m	R	Sp	LOG T	M
—	—	—	—	—	—
RS Vul A	4.6	4.2	B5	4.17	— 2.5
RS Vul B	1.4	5.4	(F4)	3.83	0.3

† Spectral type of the secondary component uncertain (cf. SAHADE and STRUVE, *Ap. J.*, **102**, 1945, 480).

TABLE I (*Continuation*)

STAR	REFERENCES
—	—
RS Vul	BAGLOW, *Publ. David Dunlap Obs.*, **2**, 1952, No. 1.
	* PLASKETT, *Publ. D. A. O.*, **1**, 1919 ***, 141.

: The light curve on which the photometric solution is based is inadequate and should be re-observed (: : indicates highly uncertain objects).

* The ratio of luminosities of both components has been determined spectroscopically by PETRIE (*Publ. D. A. O.*, **7**, 1939, 205 ; and **8**, 1950, 319).

** The ratio of luminosities of both components determined spectroscopically by POPPER (*Ap. J.*, **97**, 1943, 394).

*** Spectroscopic elements re-discussed by LUYTEN (*Ap. J.*, **84**, 1936, 53).

TABLE II (1)

STAR	PERIOD	m_2/m_1	A	C_0	W_0
—	—	—	—	—	—
RT And	0.629	0.65 ± 0.03	4.2	3.979 ± .004	1.18
AB And	0.332	0.62	2.80	3.974	1.94
S Ant	0.648	0.55 ± 0.06	3.32	3.959 ± .009	0.70
V 599 Aql	1.849	0.53 ± 0.02	16.4	3.953 ± .005	2.22
σ Aql	1.950	0.79 ± 0.01	15.2	3.993 ± .001	1.60
SX Aur	1.210	0.53 ± 0.02	12.0	3.953 ± .005	1.13
TT Aur	1.333	0.80	11.7	3.994	2.05
WW Aur	2.525	0.96 ± 0.02	11.9	4.000 ± .000	0.600
AR Aur	4.135	0.88 ± 0.03	18.6	3.998 ± .001	0.527
EO Aur	4.066	1.00 ± 0.02	40	4.000 ± .000	2.66
β Aur	3.960	0.97 ± 0.01	17.5	4.000 ± .000	0.537
i Boo	0.268	0.50 ± 0.01	2.05	3.944 ± .003	1.55
SZ Cam	2.698	0.29 ± 0.02	24.1	3.84 ± .02	2.17
TX Cnc	0.383	0.52 ± 0.05	2.8	3.950 ± .013	1.64
RS CVn	4.798	0.93 ± 0.02	18.3	3.999 ± .001	2.9
TV Cas	1.813	0.56 ± 0.03	8.61	3.963 ± .007	0.621
AO Cas	3.523	0.93 ± 0.02	38	3.999 ± .001	3.06
CC Cas	3.369	0.49 ± 0.01	30	3.941 ± .003	2.04
U Cep	2.493	0.49	12.6	3.941	0.672
VW Cep	0.278	0.32 ± 0.04	2.01	3.86 ± .03	1.39
WX Cep	3.378	1.0 ± 0.1	8.4	4.000 ± .000	0.48
AH Cep	1.775	0.86	18.7	3.997	3.28
CW Cep	2.729	0.98 ± 0.03	22.0	4.000 ± .000	1.75

TABLE II (2)

STAR	PERIOD	m_2/m_1	A	C_0	W_0
RZ Com	0.339	0.48 ± 0.05	2.72	3.94 ± .02	1.71
U CrB	3.452	0.38 ± 0.01	12.9	3.898 ± .004	1.33
Y Cyg	2.996	0.99 ± 0.02	28.5	4.000 ± .000	2.43
GO Cyg	0.718	0.85	4.3	3.997	0.96
MR Cyg	1.677	0.85	10.5	3.997	1.06
V 453 Cyg	3.890	0.77 ± 0.02	32.9	3.992 ± .002	1.92
V 470 Cyg	1.874	0.88	18.5	3.997	2.55
V 477 Cyg	2.347	0.67	11.8	3.982	0.658
V 478 Cyg	2.881	0.99 ± 0.02	27.9	4.000 ± .000	2.04
TW Dra	2.807	(0.28) (1)	16.7	3.830	0.323
YY Eri	0.321	0.55 ± 0.06	2.26	3.959 ± .014	1.35
YY Gem	0.814	1.00 ± 0.03	3.96	4.000 ± .000	0.646
Z Her	3.993	0.87	15.1	3.997	0.371
RX Her	1.779	0.89 ± 0.06	9.7	3.998 ± .001	0.824
TX Her	2.060	0.85 ± 0.03	10.6	3.997 ± .001	0.735
DI Her	10.550	0.90 ± 0.02	38.7	3.998 ± .000	0.362
u Her	2.051	0.34 ± 0.02	15.0	3.881 ± .012	1.384
VZ Hya	2.904	0.89 ± 0.09	13.5	3.998 ± .001	0.34
RT Lac	5.074	0.53 ± 0.01	17.9	3.953 ± .003	0.32
SW Lac	0.321	0.85 ± 0.09	2.48	3.997 ± .002	1.61
AR Lac	1.983	0.99 ± 0.04	9.13	4.000 ± .000	0.576
CM Lac	1.605	0.75	8.8	3.991	0.795
UV Leo	0.600	0.93 ± 0.13	4.1	3.999 ± .001	1.22
TU Mon	5.049	0.43 ± 0.08	16.5	3.92 ± .03	0.404
IM Mon	1.190	0.67 ± 0.02	11.6	3.982 ± .003	2.57
U Oph	1.677	0.88 ± 0.03	12.8	3.998 ± .000	1.56
WZ Oph	4.183	0.96	10.9	4.000 ± .000	0.505
V 502 Oph	0.453	0.40 ± 0.03	2.97	3.907 ± .012	1.22
ER Ori	0.423	0.61 ± 0.11	2.16	3.97 ± .02	0.709
δ Ori	5.733	0.38 ± 0.01	45	3.898 ± .005	1.56
U Peg	0.375	0.80	2.93	3.994	1.54
AG Per	2.029	0.88 ± 0.03	14.3	3.998 ± .001	1.33
ξ Pho	1.670	0.70	10	3.986	0.96
V Pup	1.454	0.58 ± 0.02	16.2	3.966 ± .004	3.23
U Sge	3.381	0.30 ± 0.02	19.4	3.847 ± .014	0.683
V 356 Sgr	8.896	0.38 ± 0.03	47	3.898 ± .013	0.697
μ^1 Sco	1.446	0.66 ± 0.02	15.4	3.980 ± .003	3.00
RZ Tau	0.416	0.54	2.2	2.956	3.07
W UMa	0.334	0.50 ± 0.02	2.52	3.944 ± .006	1.52
TX UMa	3.063	0.30 ± 0.02	13.7	3.847 ± .014	0.534
AH Vir	0.408	0.42	2.9	3.915	1.33
Z Vul	2.455	0.45 ± 0.02	15.1	3.927 ± .007	0.987
RS Vul	4.478	0.31 ± 0.03	20.9	3.85 ± .02	0.553

(1) See footnote on p. 412.

Of the individual columns of Table II, the headings of the first three are again self-explanatory. The fourth lists the radius (or, for eccentric systems, semi-major axis) A of the relative orbit of each individual system, expressed in terms of the solar radius (i.e., 6.95×10^{10} cm) taken as unit. The last column gives the values, in solar units (i.e., 1.905×10^{15} g cm^2 sec^{-2}), of the potential

$$W_0 = \frac{m_1 + m_2}{2A} C_0 \tag{2}$$

of a particle of unit mass placed on an equipotential surface consisting of two contact loops (i.e., over the surfaces of both components expanded so as to develop a common point on the axis joining their centres). The values of the characteristic constants C_0, which depend solely on the mass-ratio of each individual system, have been taken from a recent work by the writer [13] and their values are listed in the penultimate column of Table II for further use.

III. Characteristic parameters.

A discussion of the material contained in the foregoing tables reveals several noteworthy facts of considerable significance, which have so far escaped detection. In order to expose them, let us begin by asking ourselves the following question : *how many parameters are necessary and sufficient for a complete specification of the form of both components in a close binary system ?* This form should, in principle, be specified by the nature of the forces acting on the surface (of centrifugal and tidal origin) ; and provided that the free periods of nonradial oscillations of both components are short in comparison with that of their orbit, the distortion of both components should be governed by the equilibrium theory of tides. The level-surfaces of constant density then coincide with those of constant potential ; and the boundary of zero density becomes a particular case of surfaces over which the potential arising from all forces acting upon it remains constant.

A complete theory of the form of such surfaces for stars of arbitrary structure has not so far been developed. If, however, the density concentration in the components of binary systems is so high that their actual gravitational potentials can be approximated by those of central mass-points, the total potential of forces acting at any other point can readily be expressed in a closed form in the following manner. Suppose that the positions of the two components of a binary system of masses $m_{1,2}$ separated by a distance D, are considered in relation to a rectangular system of coordinates with origin at the centre of the more massive component m_1 ($> m_2$), whose x-axis coincides with the line joining the centres of the two stars, while the z-axis is perpendicular to the plane of the orbit. If so, the total potential W of forces acting at an arbitrary point $P(x, y, z)$ clearly becomes

(3) $$W = G\frac{m_1}{r} + G\frac{m_2}{r'} + \frac{\omega^2}{2}\left\{\left(x - \frac{Dm_2}{m_1 + m_2}\right)^2 + y^2\right\},$$

where

(4) $$r = \{x^2 + y^2 + z^2\}^{1/2}$$

and

(5) $$r' = \{(D - x)^2 + y^2 + z^2\}^{1/2}$$

represent the distance of P from the centres of gravity of the two components, and ω denotes the angular velocity of rotation of the system about its centre of gravity ; G being the gravitational constant. The first term on the right-hand side of (3) stands for the potential arising from the mass of the primary component ; the second for the disturbing potential of its mate ; and the third represents the centrifugal potential.

In close binary systems it is reasonable to identify ω with the KEPLERIAN angular velocity

(6) $$\omega^2 = G\frac{m_1 + m_2}{D^3};$$

and in the case of two stars in actual contact this identification becomes inevitable. Suppose, therefore, that we introduce (6) in (3) and adopt the combined mass of the system $m_1 + m_2$ and the separation of its component as our units of mass and length, respectively. If so, equation (3) will assume the form

(7) $$(1 + q)C = 2r^{-1} + 2q(r'^{-1} - x) + (1 + q)(x^2 + y^2) + q^2(1 + q)^{-1},$$

where

(8) $$C = \frac{2DW}{G(m_1 + m_2)} \quad \text{and} \quad q = \frac{m_2}{m_1}$$

are nondimensional parameters. The surfaces generated by setting C = constant on the left-hand side of (7) will hereafter be referred to as the *Roche equipotentials*, and the C's themselves as *Roche constants*. Provided only that the density concentration in the interiors of the components of our binary system are sufficiently high (as there is indeed every physical reason to expect), the Roche equipotentials should approximate the actual shape of such bodies with a high degree of accuracy *irrespective of their proximity*.

The shape of surfaces defined by equation (7) will evidently depend on the adopted value of C. If C is large, the corresponding Roche equipotentials will consist of *two* separate ovals enclosing each one of the two mass-points [1] and differing but slightly from spheres — the less they do to, the greater C becomes.

[1] The right-hand side of equation (7) can be large only if r (or r') becomes small ; and if the left hand side of (7) is to be constant, so must r or r' be (very nearly).

With diminishing values of C the oval defined by (7) will become increasingly elongated in the direction of the centre of gravity of the system until, for a certain critical value $C \equiv C_0$, characteristic of each mass-ratio q, both ovals will unite at a single point on the x-axis (identical with the inner Lagrangian point L_1 of the restricted problem of three bodies). This limiting equipotential will hereafter be called the *Roche Limit* ; and C_0. the limiting Roche constant. Beyond this limit, for $C < C_0$, we have no longer any right to regard the respective configuration as a model of a binary system (since both parts have coalesced into a single body) ; but for each value of $C \geqslant C_0$ and q equation (7) will define two detached equipotentials which may be invoked to approximate to the forms of the centrally-condensed components of a close binary system to a very high degree. It is, furthermore, clear that given the mass-ratio, the fractional size and form of each component will be uniquely specified characterized by a *single* value of C. Therefore, the question raised at the outset of this section turns out to admit of the following answer : the minimum number of parameters sufficient for a complete geometrical description of both components of close binary systems is *three*, and consists of the values of C_1, C_2 and q. Each one of the values of $C_{1,2}$ introduced in (7) defines, to be sure, a pair of the equipotentials for a given value of q — of which only the one enveloping the centre of gravity of the respective (primary or secondary) component is relevant. A properly determined trio of $C_{1,2}$ ($\geqslant C_0$) and q can be made to specify the complete geometry of a close binary system very much more simply and accurately than could be accomplished, more artificially, by an introduction of any number of particular semi-axes of the individual components [1] ; moreover, the quantities C_1, C_2 and q possess the additional advantage of a direct and simple physical meaning.

A determination of the ratio q from the spectroscopic data is straightforward. On the other hand, the values of $C_{1,2}$ are not accessible to observation and cannot be deduced directly from an analysis of the light curves of eclipsing binary systems. A knowledge of the coordinates of any point $P(x, y, z)$ on the surface of a Roche equipotential would, however, be sufficient to specify C uniquely. In actual practice, it is expedient to confine our attention to a pair of points at the intersection of the $\pm\, y$-axis with the respective equipotential ; for the absolute value of the y-coordinate of such points is essentially [2] identical with the fractional " radii " $r_{1,2}$ of the primary and secondary components obtainable from an analysis of the light curves of the respective eclipsing systems by well-known methods [14].

(1) If the latter were in contact (or nearly so), a conventional model consisting of three-axial ellipsoids would, in particular, constitute almost as poor an approximation to their actual shape as one consisting of two spheres.

(2) The geometrical error of this identification if (in most cases) considerably less than the uncertainty within which the radii $r_{1,2}$ can be deduced from the existing photometric data.

Since, then, along the y-axis, $y = r_{1,2}$ while $x = 0$ or 1 (again within an insensible error [1]) and $z = 0$, it follow from (7) that

$$\text{(9)} \quad \begin{cases} C_i = \dfrac{2(1+\mu_i)}{r_i} + \dfrac{2\mu_i}{\sqrt{1+r_i^2}} + r_i^2 + \mu_i^2, \\ \mu_i = \dfrac{m_j}{m_i + m_j}, \quad \begin{cases} i = 1,2 \\ j = 2,1 \end{cases} \end{cases}$$

where $r_{1,2}$ stands for the conventional radii of the two components [2].

The foregoing equations can, therefore, be used to determine the values of C_i for all eclipsing systems whose geometrical elements and mass-ratio are known. For binary systems exhibiting circular orbits the quantities C_i over each free surface are constant at any phase. Should, however, the orbit happen to be eccentric, the C_i's will remain constant (within the scope of the equilibrium theory of tides) over free surfaces at any particular moment, but their values will vary with the phase (since the radius-vector D, in terms of which the fractional radii r_i have conventionally been expressed, so varies). If, in particular, we set $D = A$, the values of C evaluated from (9) in terms of so normalized fractional radii will be relevant to such phases at which the radius-vector of the relative orbit becomes equal to its semi-major axis. We may also add that the *errors* δC_i within the C_i's can be evaluated from (9) are to a first approximation, equally distributed between the errors δr_i and $\delta \mu_i$ in the basic data and follow from the equation

$$\text{(10)} \quad \left(\frac{\delta C}{C}\right)^2 = \left(\frac{\delta\mu}{\mu}\right)^2 + \left(\frac{\delta r}{r}\right)^2,$$

while the errors of the limiting Roche constants C_0 (as given in column 5 of Table II) depend on those of the mass-ratio alone.

IV. Main Sequence Systems.

The first result which emerges from such a discussion can be briefly stated as follows : *in systems consisting of Main-Sequence stars the potentials over free surfaces of both components are sensibly equal.* This fact is borne out by an inspection of the following Table III, containing all systems whose components exhibit spectra and absolute magnitudes (as given in cols. (2) and (6) of Table I) distributing them along the Main Sequence, as demonstrated on the accompanying Figure 1, where the primary (more massive) components of each pair are represented by full circles ;

(1) For the actual amount of this error cf. the writer's study of the Roche model in Jodrell Bank Annals, 1, 1954, 37 (Table I, cols. 4 and 6).

(2) In the case of distorded stars approximated by prolate spheroids in the course of light curve analysis, the present radii $r_{1,2}$ should be identified with $a_{1,2}\sqrt{1-z}$, where $a_{1,2}$ are the (customarily given) semi-major axes of the (similar) spheroids and $z = \varepsilon^2 \sin^2 i$ (ε^2 being the equatorial ellipticity and i, the angle of orbital inclination).

the secondaries, as open circles ; and the dashed line represents the average trend of the Main Sequence according to KEENAN and MORGAN [8]. The headings of the individual columns of Table III are again self-explanatory. Columns (2) and (3) give the fractional radii $r_{1,2}$ of both components, taken from the source as listed in

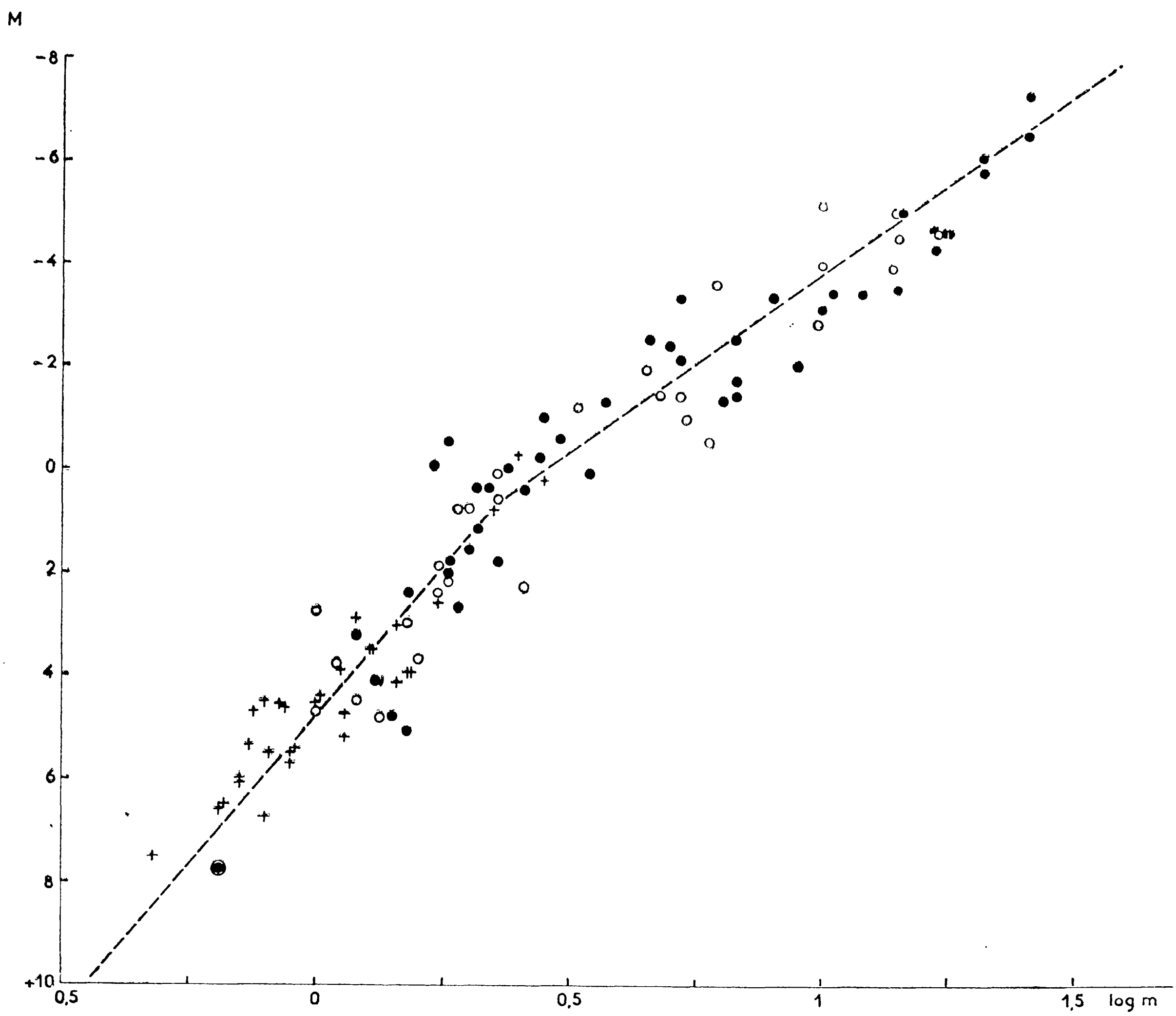

FIG. 1. — Mass-Luminosity Relation for Detached Components of Close Binary Systems.

the ultimate column of Table I, column (4) the corresponding ratio of the radii r_1/r_2 ; column (5) the ratio J_1/J_2 of surface brightnesses of the two components, and columns (6) and (7) give the values of $C_{1,2}$ together with their uncertainty as computed with the aid of equations (9) and (10) ; while column (8) lists their ratios C_1/C_2.

An inspection of this latter column reveals that, throughout the whole table, the ratio C_1/C_2 — or, what is the same, the ratio of the surface potentials W_1/W_2 — remains remarkably constant and does not deviate from unity by more than 10 % — whereas the absolute values of the potential W from system to system in

TABLE III

DETACHED (MAIN-SEQUENCE) SYSTEMS

STAR	r_1	r_2	r_2/r_1	J_1/J_2	C_1	C_2	C_1/C_2	$(m_2/m_1)_c$	O-C
σ Aql	0.278 ± 0.011	0.218 ± 0.010	**0.78 ± 0.06**	**1.3**	**5.1** ± 0.2	5.5 ± 0.3	0.93 ± 0.07	0.69	+ 0.10
TT Aur	0.283 ± 0.015	0.28 ± 0.03	**1.0 ± 0.1**	**1.8**	**5.1** ± 0.3	4.7 ± 0.5	1.08 ± 0.11	0.95	— 0.15
WW Aur	0.161 ± 0.002	0.160 ± 0.002	**1.00 ± 0.01**	**1.19**	**7.6** ± 0.2	7.4 ± 0.2	1.02 ± 0.03	0.99	— 0.03
AR Aur	0.098 ± 0.002	0.098 ± 0.002	**1.00 ± 0.03**	**1.16**	**12.0** ± 0.5	10.9 ± 0.4	1.10 ± 0.06	1.00	— 0.12
β Aur	0.142 ± 0.005	0.130 ± 0.004	**0.92 ± 0.05**	**1.14**	**8.4** ± 0.3	8.9 ± 0.3	0.95 ± 0.05	0.90	+ 0.07
SZ Cam	0.426 ± 0.001	0.180 ± 0.001	**0.44 ± 0.01**	**1.26**	**4.5** ± 0.3	4.6 ± 0.3	0.96 ± 0.10	0.26	+ 0.03
TV Cas	0.307 ± 0.015	0.283 ± 0.014	**0.92 ± 0.05**	**11.1**	**5.0** ± 0.3	4.3 ± 0.2	1.2 ± 0.1	0.74	— 0.18
CC Cas	0.31	0.15	**0.5**	**1.3**	**5.2**	6.1	0.85	0.43	+ 0.16
WX Cep	0.24	0.24	**1.0**	**1.2**	**5.5**	5.5	1.00	1.00	0.00
AH Cep	0.324 ± 0.005	0.324 ± 0.005	**1.00 ± 0.03**	**1.12**	**4.5** ± 0.1	4.3 ± 0.1	1.06 ± 0.03	1.00	— 0.14
CW Cep	0.20	0.18	**0.9**	**1.1**	**6.3**	6.8	0.94	0.86	+ 0.12
Y Cyg	0.206 ± 0.015	0.206 ± 0.015	**1.0 ± 0.1**	**1.00**	**6.2** ± 0.5	6.1 ± 0.5	1.01 ± 0.11	1.00	— 0.01
MR Cyg	0.30	0.22	**0.7**	**8.4**	**4.6**	5.2	0.88	0.61	+ 0.24
V 453 Cyg	0.23	0.19	**0.8**	**1.2**	**6.0**	6.1	0.99	0.76	+ 0.01
V 477 Cyg	0.124 ± 0.002	0.099 ± 0.002	**0.80 ± 0.03**	**4.05**	**10.6** ± 0.2	9.7 ± 0.2	1.10 ± 0.03	0.77	— 0.10
V 478 Cyg	0.27	0.27	**1.0**	**1.0**	**5.0**	5.0	1.00	1.00	0.00
YY Gem	0.157 ± 0.001	0.157 ± 0.001	**1.00 ± 0.01**	**1.00**	**7.6** ± 0.2	7.6 ± 0.2	1.00 ± 0.04	1.00	0.00
RX Her	0.229 ± 0.005	0.186 ± 0.009	**0.81 ± 0.05**	**1.1**	**5.8** ± 0.4	6.4 ± 0.5	0.91 ± 0.11	0.74	+ 0.15
TX Her	0.173 ± 0.005	0.138 ± 0.006	**0.80 ± 0.05**	**1.8**	**7.4** ± 0.4	8.0 ± 0.5	0.92 ± 0.08	0.75	+ 0.10
DI Her	0.055	0.061	1.1	**1.2**	20.3	16.9	1.20	1.12	— 0.22
VZ Hya	0.110	0.092	0.84	**1.2**	10.7	11.7	0.91	0.81	+ 0.08
CM Lac	0.20	0.15	0.75	**5.2**	6.8	7.2	0.94	0.72	+ 0.02
UV Leo	0.295 ± 0.006	0.26 ± 0.02	0.87 ± 0.02	**1.00**	4.8 ± 0.6	5.1 ± 0.7	1.0 ± 0.2	0.83	+ 0.10
IM Mon	0.33	0.235	0.71	**1.7**	4.7	5.0	0.94	0.57	+ 0.09
U Oph	0.263 ± 0.003	0.247 ± 0.003	0.94 ± 0.02	**1.12**	5.2 ± 0.2	5.2 ± 0.2	1.02 ± 0.06	0.90	— 0.02
WZ Oph	0.081 ± 0.004	0.079 ± 0.004	0.98 ± 0.07	**1.00**	13.8 ± 0.7	13.7 ± 0.7	1.01 ± 0.07	0.97	— 0.01
δ Ori	0.38	0.23	0.6	**1.1**	4.5	4.4	1.03	0.42	— 0.04
AG Per	0.220 ± 0.006	0.197 ± 0.005	0.90 ± 0.04	**1.2**	6.0 ± 0.2	6.1 ± 0.2	0.99 ± 0.06	0.85	+ 0.03
ζ Pho	0.262	0.163	0.62	**1.8**	5.5	6.6	0.84	0.51	+ 0.19

Table III varies by a factor of the order of 10. This fact, which does not seem to have been noticed before, is certainly not the result of a chance (the size of our sample is such that the probability of it being fortuitous is vanishingly small) and its evolutionary significance suggests itself : for how otherwise could the near-equality of the surface potentials of the components of binaries under consideration have come about without both surfaces having once been two loops of the *same* equipotential — i.e., parts of the same body ? Of all hypotheses of the origin of close binary systems which have so far been considered, only the fission theory would seem to account logically for this fact ; and for this reason alone it may merit further consideration. It is true that, in the original form as advanced by DARWIN and JEANS several decades ago, the fission theory has become the target of grave criticisms — discussed in recent years, particulary by LYTTLETON [15] — and, as far as the original form of this theory is concerned, most of LYTTLETON's criticisms are indeed well founded. The original DARWIN-JEANS theory involved, however, several simplifying assumptions (the stars to consist of homogeneous liquid, possessing no sources of internal energy generation, etc.) which were made for the sake of mathematical tractability and which would be quite unacceptable to-day as a basis for any physically realistic discussion of the subject. So extreme a case of stellar hydrodynamics as a fission, under stress, of an originally single star in two components of comparable masses has never so far been properly formulated — let alone solved. As long as this is true, it seems impossible to rule out the fission from the theories contesting for the explanation of the origin of close binary systems. The purely empirical evidence adduced earlier in this section suggests indeed that the fission theory may deserve further consideration (difficult as it may prove from the mathematical point of view) ; but this is not the place to do so in any detail.

Instead, we propose to confine ourselves, in this section, to pointing out that the statistical near-equality of surface potentials of the components of Main Sequence binary systems lends itself to one eminently practical use : namely, to a *statistical determination of mass ratios of single-spectrum binaries.* For the (straight) arithmetic mean of all values of the ratio C_1/C_2 in column (6) of Table III turns out to be 0.992 ± 0.010, with the standard deviation of an individual value of C_1/C_2 from the mean being ± 0.05. In view of the smallness of the latter, the idea suggests itself to consider what could be obtained by setting

$$C_1 = C_2. \tag{11}$$

An appeal to equations (9) reveals that (11) represents, in fact, a *relation between the fractional radii* $r_{1,2}$ *and the mass-ratio* m_2/m_1 *of the system* ; for inserting (9) in (11) we can solve the outcome for m_2/m_1 to find that, for systems obeying (11),

$$(12) \quad \frac{m_2}{m_1} = \frac{r_2}{r_1} \left\{ \frac{2 - r_2 + r_1(r_1^2 - r_2^2) - 2r_1(1 + r_2^2)^{-1/2}}{2 - r_2 + r_2(r_2^2 - r_1^2) - 2r_2(1 + r_1^2)^{-1/2}} \right\} = \frac{r_2}{r_1} \left\{ \frac{2 - 3r_1 + r_1^3 + \dots}{2 - 3r_2 + r_2^3 + \dots} \right\},$$

the error of which should be sensibly equal to that of a linear function

$$(13) \quad \delta\left(\frac{m_2}{m_1}\right) = 2 \left\{ \frac{\delta r_2}{r_2(2 - 3r_2)} - \frac{\delta r_1}{r_1(2 - 3r_1)} \right\},$$

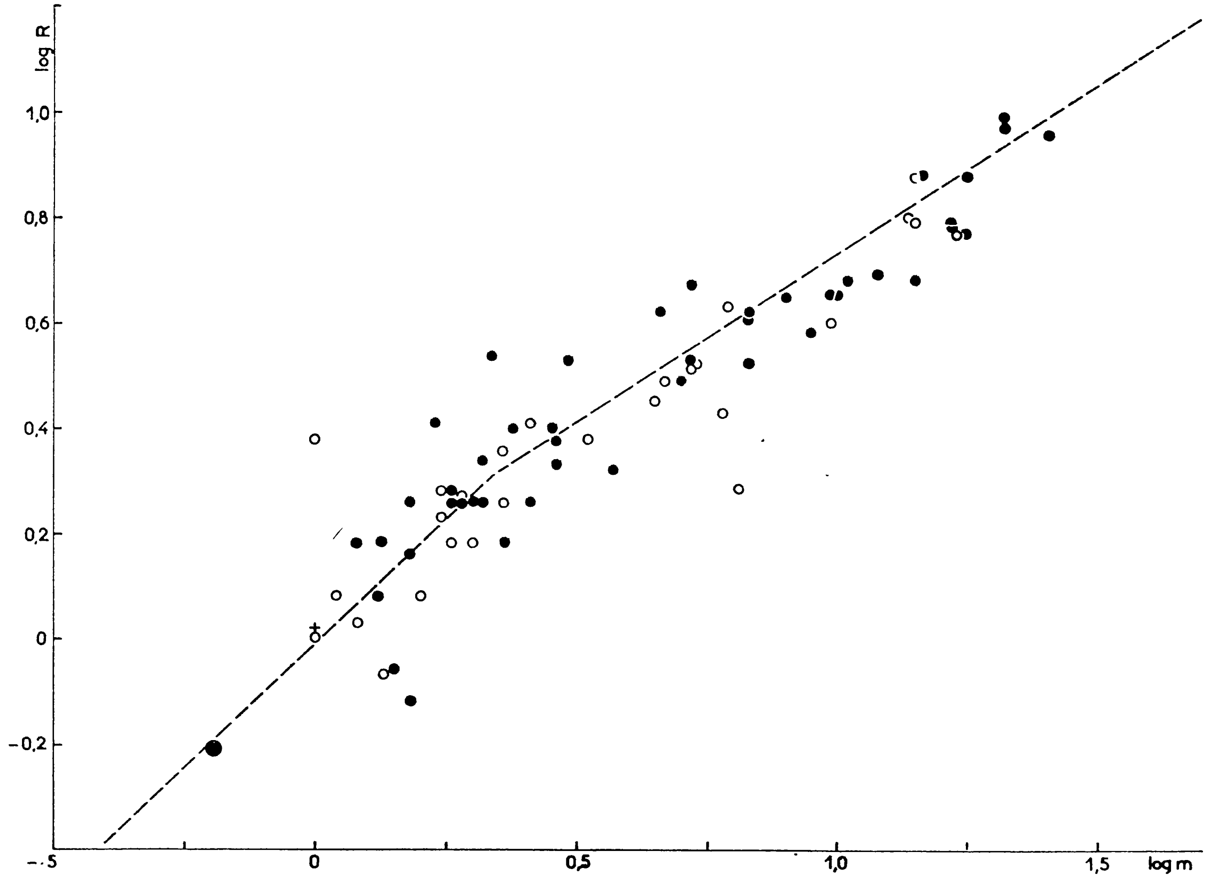

Fig. 2. — Mass-Radius Relation for Detached Components of Close Binary Systems.

where the errors δr_1, and δr_2 of the respective fractional radii (deducible from an analysis of the light curve) are, of course, not mutually independent [14]. If, moreover, we are willing to ignore in (12) the term $3r_{1,2}$ in comparison with 2, the error of the mass-ratio based on (12) becomes identical with that of the ratio k of the two fractional radii.

Column (9) of Table III lists the mass-ratio m_2/m_1 of Main-Sequence binary systems computed from their fractional radii with the aid of equation (12), and the ultimate column (10) gives the respective O-C residuals obtained by subtracting the computed m_2/m_1's from their observed values as compiled in column (3) of Table II. As the reader may early verify, the r.m.s. value of all the O-C residuals becomes $\pm$ 0.07 — i.e., comparable with the r.m.s. value of the uncertainty of spectroscopic determinations of the respective mass-ratios. This fact does not exclude the possibility that, in the case of systems considered in this section, equation (11) may be individually true — though it is rather difficult to envisage a mechanism which could maintain its validity long after an originally single star may be split up in two. It is more likely that, as time elapses, equation (11) may tend to become only approximately true — and to fail completely in advanced evolutionary stages, as we shall have an opportunity to discuss in the next section.

Before doing so we wish, however, to exhibit two statistical properties of the Main Sequence components of close binary systems : namely, the relations between their masses and radii or luminosities. The accompanying Figure 2, on which the absolute bolometric magnitudes M (taken from column (6), Table I) have been plotted against the logarithms of the masses of all stars whose Roche constants C_j are significantly greater than their limiting values C_0, (primary components : full circles ; secondaries : open circles), reveals the existence of a strong statistical correlation between M and log m which is essentially *linear* but characterized by *different* slope depending on whether (approximately) $m \gtrless 2\odot$. For more massive stars ($m > 2\odot$), the broken line drawn through the observed points satisfies the equation

$$\log m = 0.45 - 0.143\,\mathrm{M}\,; \tag{14.0}$$

while the stars of small masses ($m \ll 2\odot$) obey statistically the relation

$$\log m = 0.42 - 0.086\,\mathrm{M}. \tag{14.1}$$

The transition between the two appears to be fairly abrupt and, moreover, to take place approximately in the region in which also the Main Sequence itself happens to change its slope (cf. again Fig. 1). The slope of the line (14.0) is in good agreement with the results of most previous investigations ; but line (14.1) appears to be considerably steeper than was realized hitherto, though its present slope appears to be fully borne out also by the outcome of a recent discussion of the relevant visual binary data by STRAND and HALL (1). The individual components of visual binary systems with known parallaxes and mass-ratios, compiled by STRAND and HALL on p. 323 of their paper just quoted, have been plotted as crosses on Fig. 2, and are seen to be perfectly consistent with the eclipsing binary data. The failure of previous investigations to distinguish two distinct slopes (2) was probably due to the admixture of W UMa-type stars and subgiant components of Algol-type systems which, as we shall demonstrate more fully in subsequent sections (see Fig. 5), are systematically overluminous and exhibit little or no

(1) *Ap. J.*, **120**, 1954, 322.

(2) The visual binary data compiled by STRAND and HALL indicate that, for stars of small masses, (late M-type objects), the slope of the mass-luminosity relation may again become less steep. As, however the bolometric corrections for stars of such spectra are still uncertain within wide limits, the reality of this second change in direction of the mass-luminosity relation is yet open to doubt.

relation between mass and luminosity on account of their dynamical instability.

The second statistical relation we wish to exhibit is shown on Figure 3, on which the absolute radii R (taken from column (3) of Table I) of stable ($C > C_0$) components of close binary systems have been plotted against log m. A glance at

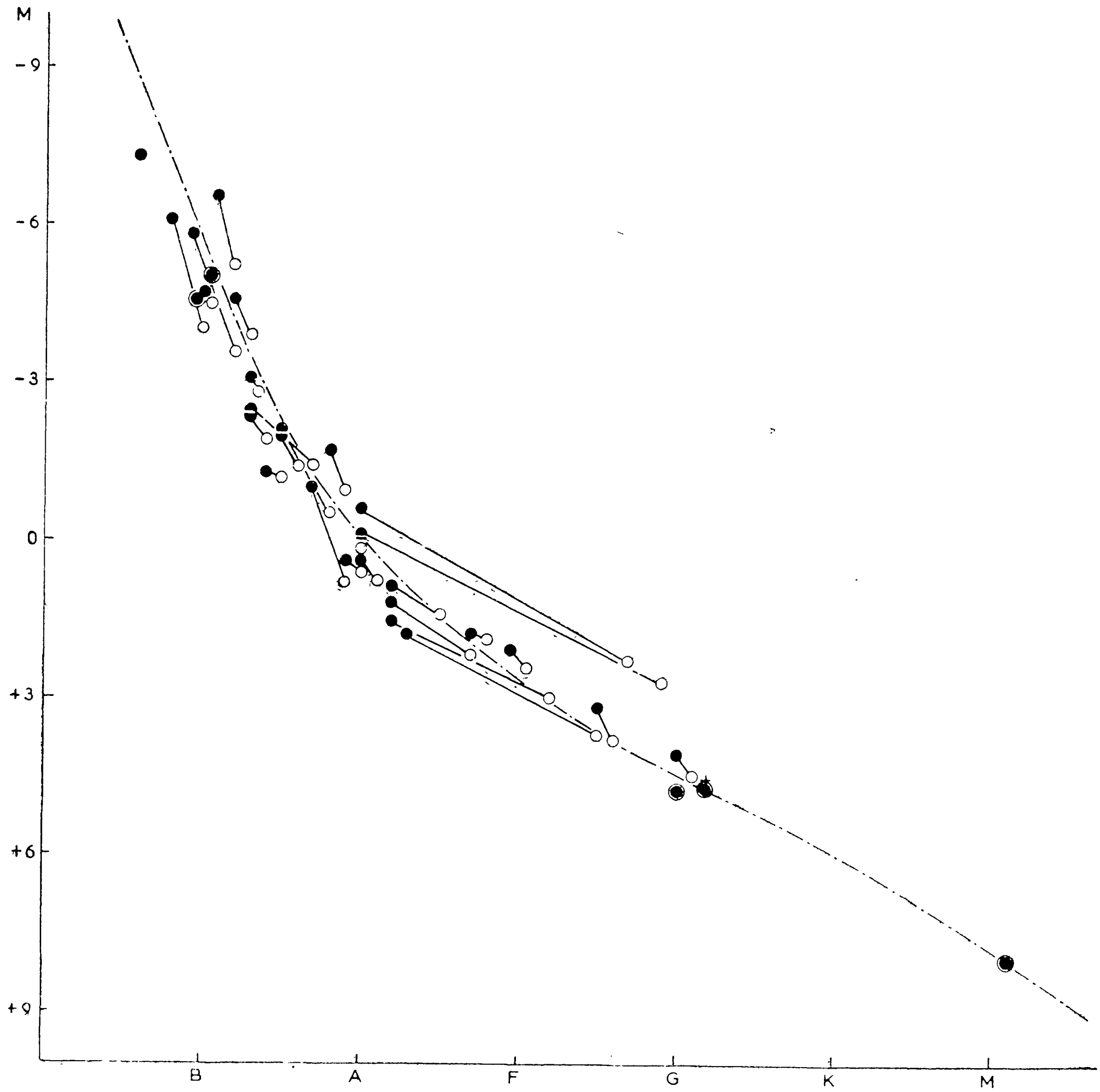

FIG. 3. — Hertzsprung-Russell Diagram of Detached Binary Systems.

this Figure reveals again the existence of a *linear* statistical relation between log m and log R whose slope changes rather abruptly with increasing mass et about the same place as the mass-luminosity relation : whereas, for massive stars ($m > 2\odot$), the broken line drawn through the observed points satisfies the equation

$$\log m = 1.57 \log \mathrm{R} - 0.15, \tag{15.0}$$

stars of smaller masses ($m \ll 2\odot$) appear to obey the relation

$$\log m = 1.02 \log \mathrm{R}, \tag{15.1}$$

which is borne out not only by the evidence based on eclipsing variables, but also by that relating to the Sun (plotted on our Fig. 3 as $\odot$) and other dwarf stars of known characteristics.

It may be noted that, eliminating log m between equations (14.0) and (15.0) or (14.1) and (15.1), and combining this eliminant with the Stefan's law (1) employed to compute our absolute magnitudes, we find that equations (14) and (15) jointly are tantamount to the following statistical relations

$$(16) \qquad \left\{ \begin{aligned} \log R &= 1.67 \log M - 6.44, \quad (m > 2\odot) \\ &= 1.51 \log T - 5.97, \quad (m \ll 2\odot) \end{aligned} \right.$$

between the radii and effective temperatures of Main Sequence stars, in which a difference between the behaviour of the two branches has largely disappeared.

It may further be added that the statistical relations (15) between mass and radius imply the existence of another statistical relation between the mass-ratios and other observable properties of single-spectrum binary systems. The reader may note that the coefficients of log R on the right-hand sides of equations (15) are so-nearly equal to 1.5 and 1, respectively, that their differences from these round values may be due to observational errors. On the other hand, if $K_{1,2}$ denote the amplitudes of the radial-velocity curves of the respective components of a close binary system ; P the period of its orbit, and i its inclination to the celestial sphere, it is well known that (for circular orbits)

$$(17) \qquad m_{1.2} = [3.016 - 10]\,(K_1 + K_2)^2 K_{2.1}\, P \csc i \odot$$

and

$$(18) \qquad R_{1.2} = [8.296 - 10]\,(K_1 + K_2)\, r_{1.2}\, P \csc i \odot$$

if the K's are expressed in kms/sec, and P, in mean solar days. Inserting these relations in (15.0), rounding off the latter's coefficients, and remembering that $K_1/K_2 = m_2/m_1$ we find that

$$(19.0) \qquad 1 + \frac{m_1}{m_2} = [8.56]\, P \left(\frac{r_2 \sin i}{K_1}\right)^3$$

for massive binary systems (when both m_1 and m_2 are in excess of 2 $\odot$), and

$$(19.1) \qquad 1 + \frac{m_1}{m_2} = [5.28]\, r_2 \left(\frac{\sin i}{K_1}\right)^2$$

for less massive stars. In both these equations K_1 denotes the radial-velocity amplitude of the brighter component, observable in single-spectrum systems. It should, however, also be noted that a dispersion of the statistical relations (15) affects the values of the mass-ratios m_1/m_2 computed with the aid of equations (19) more sensitively than the inaccuracy inherent in the use of (11) — so that, for close binaries situated on the Main Sequence, equation (12) should generally offer a more reliable way than (19) for statistical determination of the mass-ratio of single-spectrum systems.

V. The problem of the subgiants.

Having surveyed some properties of close binary systems whose both components belong to the Main Sequence, let us turn next to such systems of Table I

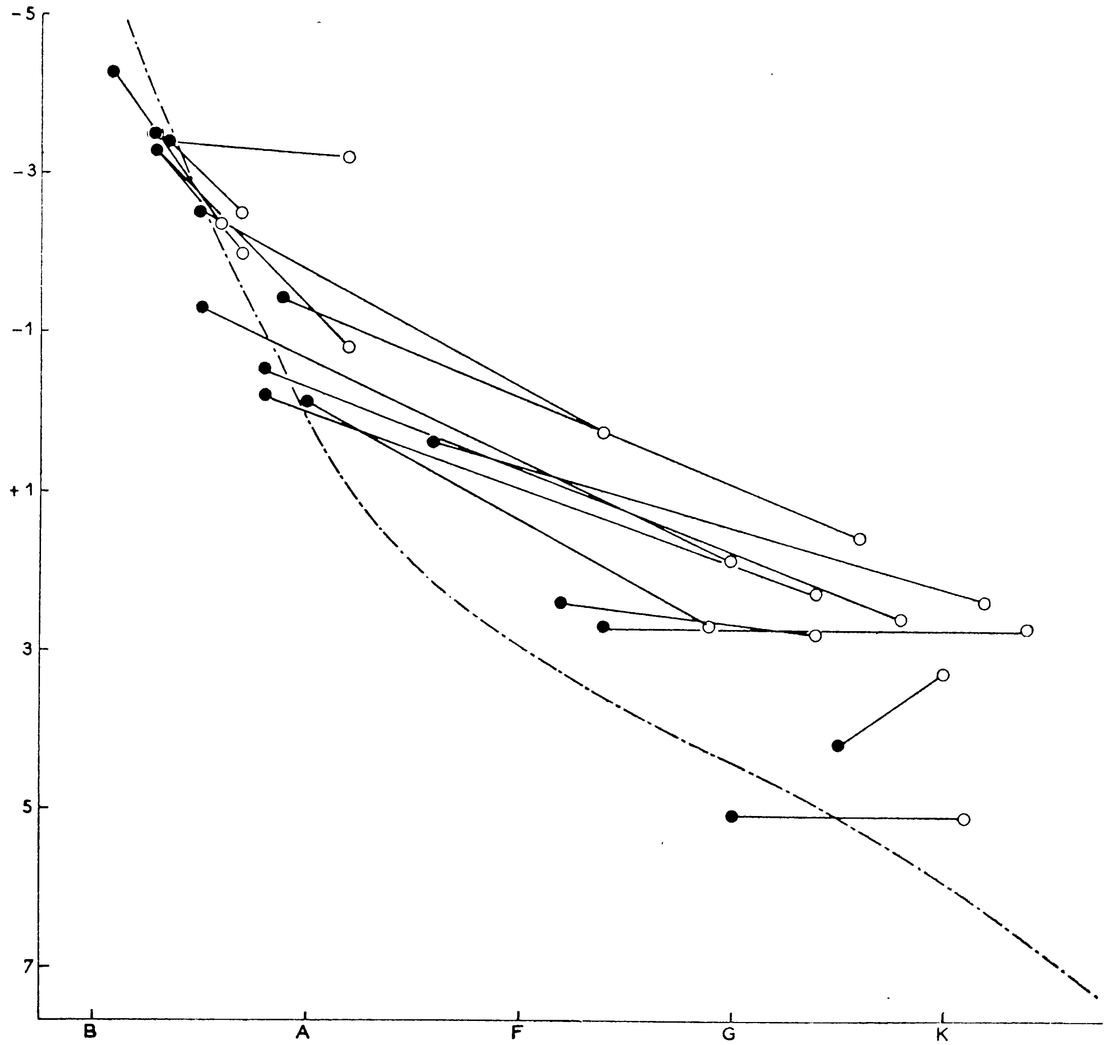

Fig. 4. — Hertzsprung-Russell Diagram of Semi-Detached Binary Systems.

whose secondary (less massive) components lie *above* the Main Sequence (being too luminous for their mass). Such stars exhibit also certain common spectroscopic characteristics and are usually classified as *subgiants.* The Hertzsprung-Russell diagram of close binary systems of known absolute properties and containing subgiant secondaries are shown on the accompanying Figure 4, on which the primary components are plotted as full circles, and the secondaries, as open circles ; the Morgan-Keenan Main Sequence being plotted — as on Figure 1 — in the form of a dashed line.

TABLE IV

SEMI-DETACHED BINARY SYSTEMS

STAR	r_1	r_2	r_1/r_2	J_1/J_2	C_1	C_2	C_0/C_2
RT And	0.18 ± 0.01	0.325 ± 0.004	0.55 ± 0.06	6.2	7.7 ± 0.5	4.1 ± 0.2	0.98 ± 0.05
SX Aur	0.40	0.36	1.1	1.6	4.2	3.7	1.06
U Cep	0.19 ± 0.01	0.31 ± 0.01	0.61 ± 0.09	18	7.9 ± 0.4	3.9 ± 0.1	1.02 ± 0.03
U CrB	0.15	0.28 ± 0.01	0.54	14	10.3	4.0	0.98
TW Dra	0.212 ± 0.001	0.306 ± 0.001	0.70 ± 0.02	19	7.9 ± 0.1	3.63 (1)	1.05 (1)
u Her	0.301 ± 0.003	0.287 ± 0.003	1.05 ± 0.02	2.50	5.6 ± 0.2	3.9 ± 0.2	1.01 ± 0.05
V Pup	0.368 ± 0.003	0.327 ± 0.004	1.13 ± 0.02	1.36	4.4 ± 0.1	4.0 ± 0.2	1.00 ± 0.04
U Sge	0.210 ± 0.002	0.278 ± 0.003	0.76 ± 0.01	14	7.9 ± 0.5	3.8 ± 0.3	1.01 ± 0.06
V 356 Sgr	0.106 ± 0.008	0.278 ± 0.007	0.38 ± 0.04	4.2	14 ± 1	4.0 ± 0.2	1.03 ± 0.05
μ^1 Sco	0.313 ± 0.003	0.347 ± 0.004	0.90 ± 0.01	1.92	4.9 ± 0.1	3.9 ± 0.1	1.02 ± 0.03
TX UMa	0.158 ± 0.001	0.277 ± 0.001	0.57 ± 0.01	29	10.3 ± 0.6	3.8 ± 0.3	1.01 ± 0.06
Z Vul	0.26	0.301 ± 0.002	0.86	10	6.2	4.1	0.96
RS Vul	0.20 ± 0.01	0.26 ± 0.01	0.77 ± 0.06	18	8.2 ± 0.8	4.0 ± 0.5	0.98 ± 0.10
RS CVn	0.097 ± 0.005	0.280 ± 0.005	0.35 ± 0.02	25	12.1 ± 0.7	4.7 ± 0.1	0.84 ± 0.02
Z Her	0.12 ± 0.01	0.185 ± 0.005	0.65 ± 0.08	6.3	10.1 ± 0.8	6.4 ± 0.2	0.63 ± 0.02
AR Lac	0.169 ± 0.005	0.313 ± 0.007	0.54 ± 0.02	3.4	7.2 ± 0.4	4.5 ± 0.2	0.89 ± 0.04

Further properties of subgiant binary systems are listed in the following Table IV, the headings of which are again self-explanatory. The first seven columns contain the same data as the corresponding columns of Table III ; and column (8) gives the ratios C_0/C_2 of the Roche constants at the secondary's surface (C_2) to its limiting value C_0 listed already in column (5) of Table II.

A glance at the data given in the sixth column of Table IV reveals an arresting fact : namely, with a very few significant exceptions, *the ratios of* C_0/C_2 *for subgiant secondaries are sensibly equal to unity* — i.e. the secondary components appear to fill *exactly* (or, rather, within the limit of observational errors) the largest closed volumes which they can possibly occupy in systems of given mass-ratios. Stars whose Roche constants characterizing their surface are equal to C_0 will hereafter be termed *contact components.* Subgiant secondaries in close binary systems represent the first group of such stars which we have met so far in our survey ; and the present section will be devoted to a closer discussion of their properties and significance.

In order to demonstrate the degree to which subgiant secondaries cling to the Roche limits of their respective system, let us form the arithmetic mean of the

(1) Cf. footnote on p. 412.

values of C_0/C_2 for all systems listed in Table IV, with the exception of the last three stars (RS CVn, Z Her, and AR Lac) whose secondaries are probably significantly smaller than the respective critical equipotentials : we find that

$$\frac{C_0}{C_2} = 1.008 \pm 0.006 \text{ (p. e.)}, \tag{20}$$

the standard deviation of a single value of C_0/C_2 from the mean being $\pm$ 0.02, which is certainly within the errors of observational determination of the individual values of C_0 and C_2.

TABLE V

FRACTIONAL DIMENSIONS OF SECONDARY (CONTACT) COMPONENTS IN SEMI-DETACHED BINARY SYSTEMS

STAR	Sp	m_2/m_1	$(r_2)_{comp}$	$(r_2)_{obs}$
RT And B	K1	0.65 ± 0.03	0.333 ± 0.007	0.325 ± 0.004
U Cep B	gG8	$0.49 \pm$	$0.31 \pm$	0.31 ± 0.01
U CrB B	gG0	0.38 ± 0.01	0.290 ± 0.003	0.28 ± 0.01
u Her B	B7	0.35 ± 0.02	0.285 ± 0.004	0.287 ± 0.003
V Pup B	B3	0.58 ± 0.02	0.324 ± 0.004	0.327 ± 0.004
U Sge B	gG6	0.30 ± 0.02	0.272 ± 0.005	0.278 ± 0.003
V 356 Sgr B	A2	0.38 ± 0.03	0.292 ± 0.007	0.28 ± 0.01
μ^1 Sco B	B6	0.66 ± 0.02	0.337 ± 0.004	0.347 ± 0.004
TX UMa B	gG4	0.30 ± 0.02	0.272 ± 0.004	0.277 ± 0.001
Z Vul B	(A2)	0.45 ± 0.02	0.303 ± 0.003	0.301 ± 0.002
RS Vul B	(F4)	0.31 ± 0.03	0.274 ± 0.007	0.26 ± 0.01

In order to impress the reader further with the exactitude with which the secondary components actually fill the largest closed equipotentials, attention is invited to the data collected in the foregoing Table V : in which the observed diametral radii (col. 5) of the secondary (subgiant) components of systems listed in column 1 are compared with the maximum theoretical radii of closed configurations (col. 4) corresponding to the observed mass-ratios (col. 3). An inspection of the foregoing data reveals that the dimensions of the secondaries tend to coincide with those of the corresponding Roche limits the more closely, the greater the precision of the underlying observational data. This fact suggests convincingly that this coincidence is probably *exact* in these systems, and possibly in many others as well.

As to significance of the observed clustering of subgiant values of C_0/C_2 in the neighbourhood of unity, its reason can scarcely be in doubt. It is certainly not the result of a random distribution of the fractional dimensions of stars in static equilibrium ; for the probability of so peculiar a frequency distribution in so large a sample is negligibly small. Suppose, therefore, for the sake of argument that such stars are not equilibrium configurations, but are (secularly) expanding or contracting. If they were contracting, the frequency distribution of the C_0/C_2's — if originally uniform — should tend to exhibit an increase towards large values, which should become pronounced in the course of time. There is, in particular, no reason why there should be any clustering of their sizes around the Roche limit. If, however, these stars are secularly expanding, there is a compelling reason why their size cannot exceed that of their Roche limit : for no larger *closed* equipotential exists which would contain the whole mass of the respective configuration. Therefore, the growth of a slowly expanding star is bound to be *arrested* at its Roche limit ; and if this tendency is characteristics of such stars as a group, they should indeed be expected to exhibit clustering at the Roche limit which should be marked the more, the longer this tendency has been operative (or the more rapidly the expansion progresses).

The observed facts pointed out above can, therefore, scarcely be accounted for otherwise than by an assumption that *the subgiant components in close binary systems are secularly expanding.* This conclusion, which was announced by the present writer some time ago [16], was independently arrived at also by CRAWFORD [17] on the basis of a statistical study of a number of single-spectrum systems whose mass-ratios were estimated previously by PARENAGO [18] by assuming their primary components to be average Main-Sequence stars of masses appropriate for their spectra. In his paper referred to above, CRAWFORD listed 18 single-spectrum binaries whose secondary components are likely to possess subgiant characteristics and whose fractional dimensions cluster around their respective Roche limits. Although the statistical nature of PARENAGO's estimates of mass ratios makes it impossible to ascertain that this coincidence is true in each individual case (1), the statistical existence for clustering around the Roche limit is altogether strong. It should be stressed, in this connection, that the lists of objects at the basis of CRAWFORD's discussion and ours show no overlap ; yet both point to essentially the same conclusion — namely, a secular expansion of subgiants

(1) In particular, an excess of the size of a few secondaries on CRAWFORD's list over their Roche limits is no doubt the consequence of an inaccurate estimate of the mass ratio. On dynamical grounds it is extremely unlikely that any centrally-condensed star in quasi-static equilibrium could exceed its Roche limit ; and no single instance of such an excess has so far been established in any system whose mass-ratio is reliably known.

An apparent exception to this rule would seem to be the case of TW Dra B, for which BAGLOW (*David Dunlap Publ.*, 2, 1952, No. 1) established an unusually accurate value of its fractional radius $r_2 = 0.3064 \pm 0.0002$, corresponding (on contact hypothesis) to a mass-ratio of $m_2/m_1 = 0.43$ within an error of less than 0.01 — whereas, according to PEARCE (*Publ. A. A. S.*, **9**, 1937, 131), "three observations of the 10th magnitude companion during totality gave an approximate value of the mass-ratio of 0.28 ". PEARCE did not estimate the uncertainty of his result ; but the total eclipse of TW Dra lasts only 1.8 hours, and a determination of the slope of the secondary's velocity-curve from three points secured within 2,7 % of a cycle is likely to be so uncertain as to bring the above theoretical value 0.43 of the mass-ratio within the limits of observational errors.

— which can probably be regarded to be as well established by existing evidence as we can reasonably expect.

Do systems containing such contact components form a compact group, well-separated from the Main Sequence systems discussed in the preceding section, or is there a smooth transition between the two ? In an attempt to answer this question we may observe that, among the stars of Group I (Main Sequence systems) the frequency-distribution of the values of C_j/C_0 is fairly uniform up their lower limit. Consider, for instance, the systems of TV Cas, AH Cep, or δ Ori, all listed in Table III, as their components on the basis of available observational data appear to be stable. The values of C_2 characterizing their secondary components are, however, so close to their limiting values C_0 that a relatively small expansion would bring such stars in contact with their Roche limits. Are these stars indeed on their way to become subgiants, or will a parallel expansion of their primaries transform them, in time, into contact systems of the β Lyr-type (cf. Section VII) ? Both AH Cep and δ Ori are massive, early-type eclipsing systems, similar in many respects to the members of β Lyr-group. On the other hand, the secondary component of TV Cas, whose relative dimensions are quite accurately known, should (for PLASKETT's spectroscopic mass-ratio of 0.56) be significantly smaller than its Roche limit. Its position in the H. R.-diagram (see Fig. 1) well above the Main Sequence suggests, however, that this star may already be in contact with its Roche limit, as it would be for a mass-ratio of 0.34. Hence, either the spectroscopic mass-ratio as given by PLASKETT is too large (1), or the subgiant characteristics of TV Cas B may be on the way of developing before the Roche limit has been reached — as it appears to be the case for the secondary components of RS CVn, Z Her, or AR Lac.

The second question arising in this connection concerns the *relative proportion of semi-detached systems*, whose secondary components have attained their Roche limits, to Main-Sequence systems whose both components are significantly smaller than this limit. Our Table III lists 30 Main-Sequence systems whose components are (apart from two or three somewhat dubious cases) stable, in contrast with 13 semi-detached systems of Table IV. A direct comparison of these two numbers is, however, bound to exaggerate the relative proportion of semi-detached systems because of observational selection. As the secondary component expands and its fractional size increases, its relative surface brightness declines in comparison with that of its mate : hence, the primary minima become both deeper and wider ; in addition, with increasing value of the sum $r_1 + r_2$ eclipses will take place for diminishing values of orbital inclination. All these factors favour discovery of the system as an eclipsing variable, and thus exaggerate the number of known systems

(1) It may be pertinent to mention, in this connection, that SAHADE and STRUVE (*Ap. J.*, **102**, 1945, 480) were unable to detect the lines of the secondary component in the combined spectrum of the system.

of this class out of proportion to their actual abundance. The appropriate allowance for observational selection operative in this field is rather difficult to express in quantitative terms ; but it leads us to surmise that the actual proportion of Main-Sequence systems to semi-detached ones is likely to be not less than 10 : 1, and possibly even greater. Eclipsing systems with secondaries at their Roche limits are, therefore, likely to be relatively rare ; it is the high probability of their discovery which has provided us with so many examples of this intriguing group of close binaries among relatively bright stars.

A secular expansion of subgiant components of close binary systems implies certain further consequence of high astrophysical interest. As was already pointed out, the existing evidence shows a large number of such stars clinging to their Roche limits ; but once this maximum distension permissible on dynamical grounds has been attained, a continuing tendency to expand is bound to bring about a *secular loss of mass*, streaming out of the conical end of the critical equipotential at the inner Lagrangian double-point L_1 (at which this equipotential begins to open up). The subsequent behaviour of matter thus escaping at L_1 represents a separarate problem which it outside the scope of our present discussion (1) ; but whatever it may be, the secondary component will keep losing mass and this loss is bound, in time, to increase the disparity in masses between the two components of the same pair. Now close eclipsing systems possessing subgiant components have been known for some time to exhibit abnormally large mass-ratios — a fact discussed particularly by STRUVE [19, 20] — due to abnormally small masses of the companions. All such companions in systems of known mass ratios having now been identified to possess fractional dimensions coinciding with their Roche limits, there remains but little room for doubt that the relative smallness of their present masses is but the consequence of a secular loss caused by continued expansion.

The loss of mass in such systems leads one naturally to speculate concerning a possible connection of this fact with the notorious period changes which are only too well known in many eclipsing systems. This is indeed an intriguing possibility which has already been considered by WOOD [21]. In re-opening this question we should, however, be careful not to jump (like WOOD) to conclusions which, attractive though they may appear on first sight, do not stand the light of closer scrutiny. In point of fact, it is easy to show that all evidence available at present is very much less conclusive than one would wish.

In order to demonstrate it, let us recall that, of 19 stars discussed by WOOD in support of his thesis, spectroscopic mass-ratios are available only for 8 of them and for the rest (i.e., a majority of 11) the mass-ratios were estimated from the ratios of luminosities of the two components combined with an appeal to mass-luminosity

(1) This study is being undertaken separately by the present writer, and preliminary results have been presented at the I. A. U. Symposium on Non-stable Stars in Dublin, 1955.

relation. Now it is well known that subgiant components of close binary systems are as a rule overluminous, and deviate conspicuously from the statistical mass-luminosity relation. In consequence, the ratios of luminosities of the components in such systems offer a very shaky clue to their mass-ratios. For this reason, it is not known whether their secondary components of the majority of systems discussed by Wood have actually attained their Roche limits ; and their semi-detached nature remains so far but conjectural.

Among the 11 semi-detached systems whose secondaries have been established to have reached their Roche limits in the present study (cf. Table V), only 4 are known to exhibit period changes (i.e., U Cep, U CrB, U Sge, TX UMa) to a varying extent, and 7 do not. The absence of appreciable period changes for the majority of such systems is, moreover, an established fact, not stemming from any lack of observational supervision. Of 18 stars which Crawford [17] suspects of possessing contact secondaries, 7 exhibit fluctuating periods, and 11 are known not to do so. Moreover, of the three known stable but subgiant-possessing systems (RS CVn, Z Her, AR Lac), two may exhibit period fluctuations (RS CVn and AR Lac (1)), while the period of Z Her appears to be constant. On the other hand, out of 29 stable Main Sequence systems listed in our Table III, 25 exhibit constant orbital periods (in fact, 28 of them if we disregard Y Cyg whose orbit is eccentric and the period variation is caused by a revolution of the apsidal line, and MR Cyg together with AG Per of which the same seems to be true), and only one (i.e., U Oph) has so far been reported to exhibit appreciable period changes (though no displacement of the secondary minimum) which cannot be safely explained (2).

Summarizing, we may conclude that whereas a proximity of the secondary component's surface to its Roche limit seems indeed conducive to the appearance of period variation, it does *not*, by itself, represent a *sufficient* condition for such variation to occur. In point of fact, while some semi-detaches systems (U Cep, or U CrB) have for long been known to exhibit conspicuous period changes, in others (U Sge or TX UMa) the known variations of period are altogether small ; and the majority of semi-detached systems do not seem to exhibit any period fluctuations at all. This might be possibly explained by an assumption that the secular expansion of secondary component does not proceed in different eclipsing systems at a uniform rate, but spasmodically — so that epochs of slow expansion may alternative with others during which the expansion (and the consequent loss of mass) may be relatively rapid. Even if such an assumption could be supported

(1) Wood [21] included this star among the systems with variable periods without quoting any evidence for it, though Dugan and Wright (*Princ. Contr.*, 1939, No. 19), in their extensive work on period variation of eclipsing binary systems, listed AR Lac among " stars having periods probably constant ".

(2) Parenago (*Per. Zvjozdy*, **7**, 1949, 102) conjectured that the observed period changes of U Oph are periodic and may be due to the light equation in a triple orbit. No independent confirmation of the existence of any third body in this system is, however, available so far.

on independent physical grounds, however, it would still fail altogether to account for the period fluctuations known to occur — though less frequently — also in systems, of known mass-ratios (RS CVn, AR Lac (?), U Oph) whose *both* components are considerably smaller than their Roche limits and should, therefore, be completely stable from the dynamical point of view.

Moreover, a secular loss of mass can clearly bring about only a *lengthening* of the orbital period (and only if the mass lost by the secondary component is not intercepted by its mate and manages to escape from the systems), whereas the observed period fluctuations in such systems as U Cep or Algol are manifestly very much more complicated. Besides, it is also easy to show that (at least assuming a uniform rate of expansion) the consequences of a secular loss of mass under consideration would scarcely be observable. For consider, for instance, the well-known system of U Sge ($m_1 = 6.7 \odot$, $m_2 = 2.0 \odot$) and assume, for the sake of argument that the present mas defficiency of the secondary component developed as a result of (uniform) secular loss of mass which also escaped from the system in the course of 10^9 years. As the present period of U Sge is 3.38 days, a time interval of 10^9 years corresponds roughly to 10^{11} orbital cycles. Therefore, a total loss of mass of $4.7 \odot = 9 \times 10^{33}$ g would correspond to a loss of 9×10^{22} g per cycle. The corresponding fractional period variation should be given by

$$\frac{1}{\mathrm{P}}\frac{d\mathrm{P}}{dt} = -\frac{1}{2(m_1 + m_2)}\frac{dm_2}{dt} = \frac{9 \times 10^{22}}{2(8.7 \times 10^{33})} = 5 \times 10^{-12}, \tag{21}$$

in accordance with KEPLER's third law. The orbital period of U Sge would thus be increasing by about 3×10^{-6} seconds per cycle, and invoke a quadratic term of the order of 10^{-11} E^2 in the prediction of the minima. Needless to say, amounts of these orders of manitude are as yet wholly outside of any possibility of observational verification ; and would remain so even if we were to multiply them a hundred times (corresponding to a period of expansion lasting only 10^7 years). It his, therefore, rather dubious whether or not a secular loss of mass due to expansion of subgiant components in close binary systems is connected in any way with their observed period changes ; and, at any rate, the question raised by such an inquiry admits of no simple answer [1].

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[1] WOOD [21], in order to explain both the occasional increases and decreases of periods in close binary systems, postulated spasmodic ejection of matter from the surface of its components in the direction of its orbital motion (or opposite to it), on grounds of an assertion that ... " Computation of the JACOBIAN surface shows that the star is nearest instability at those portions of its surface near the ends of its shortest equatorial axis " (*op. cit.*, p. 204). This assertion is, however, incorrect. In actual fact, the star is nearest instability (i.e., the work required to remove a unit mass from its surface will be minimum) on the line joining the centres of the two stars (which is perpendicular to the shortest equatorial semi-axis). Besides, the loss of mass necessary to account for the observed period fluctuations in this way is of the order of 10^{-8} to $10^{-6} \odot$, which is inordinately large. ,, If we remember " commented MARTYNOV at the Dublin Symposium on non-stable stars in September, 1955, " that masses ejected by Nova outbursts exceed these values by one order only, it seems doubtful whether an escape of masses of the order of 10^{-7} to $10^{-6} \odot$ from eclipsing binary systems could pass otherwise unnoticed for frequently observed stars ".

What is the evolutionary significance of the secular expansion of subgiants ? Is, in particular, this expansion limited to subgiant components of close binary systems, or is it characteristic of all stars of this class irrespective of whether they occur single or in pairs ? Is it, in fact, the expansion whose consequences endow the subgiants with the particular characteristics ? In considering these questions, we should keep in mind that the disturbing effects due to the proximity of a companion scarcely exert any appreciable influence in the deep interior of a centrally-condensed star : therefore, if the cause of the expansion is to be sought in deep interiors, the presence or absence of a companion should be of little or no import (1). SANDAGE and SCHWARZSCHILD [22] pointed out some time ago that the secular expansion of a star is likely to set up in response to a diminishing hydrogen content. The subgiant components in close binary systems accompany, however, as a rule late B - or early A - type Main Sequence stars ; and as both components are no doubt of equal age (and, presumably, of initially the same chemical composition), the subgiant secondaries cannot manifestly be " old " on the same time scale.

CRAWFORD [17] proposed recently a hypothesis according to which the present subgiant secondaries are erstwhile primary components which have, in the course of time, lost enough mass to reverse the balance of the mass-ratio. The (initially) more massive primary component is, according to CRAWFORD, likely to run through its evolutionary course more rapidly than its mate, and to deplete more rapidly its hydrogen supply whose diminution may cause expansion. In the course of a sufficiently long time the star will attain its Roche limit, start discharging mass, and eventually lose so much of it that the original role of the two stars will be reversed : the former primary now becoming the secondary component, and *vice versa.* CRAWFORD's suggestion is ingenious, but not convincing. Two objections arise immediately — both of them fundamental. The first concerns a complete lack of subgiants, at their Roche limits, whose masses are equal (or nearly so) to those of their mates — and, on CRAWFORD's hypothesis, such systems should be at least as frequent as those characterised by very unequal mass-ratios. The second objection is the existence of stars of definite subgiant characteristics (RS CVn B, Z Her B, or AR Lac B, for example) whose dimensions (see Table IV) are still well inferior to their Roche limits.

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(1) The gravitational attraction of the primary (more massive) component distorts, of course, rather profoundly the outer parts of a contact component. Is it possible that the distortion alone, if sufficiently large, may bring about a loss of equilibrium which compels the star to expand ? A gravitational trigger mechanism of this kind would be likely to operate selectively on stars above certain fractional size inducing them to expand and become subgiants, while stars immune to it would probably remain on the Main Sequence. This might possibly explain a co-existence of the binaries of our Groups I and II. On the other hand, expanding subgiants would thus be limited to close binary systems, and the occurrence of subgiants elsewhere (in visual binaries, for instance) would confront us with a separate problem.

The actual reason of secular expansion of subgiant components in close binary systems may well be an incipient hydrogen defficiency ; and if so, it is tempting to surmise that the same phenomenon characterizes *all* subgiants — irrespective of whether they are single or occur in binaries — be they wide (ζ Her) or close. In the former case, expansion could presumably go on unchecked for an astronomically long time. The presence of the primary components in close binary systems provides merely an intangible upper limit (i.e., the Roche limit) which the secondary cannot exceed on dynamical grounds ; and if many such secondaries are found actually to attaint this limit, their expansive tendency is thereby revealed. It is, however, only the subgiants in close binary systems which are compelled (by the existence of the Roche limit) to shed off mass in the course of continuing expansion ; single subgiants retain all their mass, but may attain much larger dimensions.

If we consider close binary systems, there seems but little room for doubt that their components originated simultaneously and from the same primordial matter. What is the reason which impels, at some stage, both components of comparable masses to embark on different evolutionary tracks and renders one of them a subgiant ? Needless to say, the answer is not yet known ; and close pairs containing subgiant components are by no means the most drastic examples of such a state of affairs. Indeed, catalogues of binary systems — close or wide — are replete with examples defying the applicability of the RUSSELL-VOGT theorem to such stars in most flagrant manner : the symbiosis of a Main Sequence star with a white dwarf (the systems of Sirius, Procyon, etc.), or of a late-type supergiant with a dense and sub-luminous B star (VV Cep, ζ Aur, Mira Ceti, Antares, etc.) represent much greater problems to the theory of stellar evolution than the coexistence of an expanding subgiant with an ordinary Main Sequence star. The answer to this latter problem will probably emerge as a part of an understanding of broader issues, of which subgiants and their significance in the framework of stellar evolution forms only a relatively small part.

In concluding the present discussion of the physical properties of subgiants, one last observation should be added in this place. In 1950, PARENAGO and MASSEVICH [23] claimed to have shown on the basis of an extensive statistical study of single-spectrum binary systems, that the quantities $\xi = \log m/R$ and $\eta = \log L/m^5$ are linearly correlated for subgiants components — an assertion which was subsequently given additional publicity by STRUVE [24, 25]. When, in order to test this assertion, we evaluated the ξ and η parameters for two-spectra subgiant systems on the basis of the data compiled in Table I, a plot of ξ against η failed to confirm the conclusions of the Russian investigators. The distribution of the individual subgiants, in the $(\xi - \eta)$ - coordinates, revealed a scatter diagram in place of any well-defined locus. Those data, compiled in our Table I, which are relevant to subgiants are less numerous, but far more precise than the statistical material at the basis of the Russian work. As long as these accurate individual data fail to bear out the particular features announced by MASSEVICH and PARENAGO, it would seem rather premature to speculate too much on their physical significance.

VI. DETERMINATION OF MASS-RATIOS IN SUBGIANT ECLIPSING SYSTEMS.

In the preceding section observational evidence was adduced in support of the view that, in close binary systems possessing subgiant secondary components, a large majority of such subgiants happen to fill exactly their respective Roche limits. The aim of the present section will be to point out one obvious use for which this fact readily lends itself : namely, a determination of the mass-ratios of single-spectrum systems of this type.

A geometry of the Roche model discloses [13] that the fractional radii r (or, in distorded eclipsing systems, the diametrial semi-axes $a\sqrt{1-z}$) of secondary (less massive) components are related with the mass-ratios of the respective systems in accordance with the following tabulation :

TABLE VI

m_2/m_1	r_2
1.00	0.374
0.8	0.354
0.6	0.329
0.4	0.295
0.3	0.272
0.15	0.223
0.10	0.197

Whenever the secondary component actually attains its Roche limit, the foregoing relationship between r_2 and m_2/m_1 evidently opens a way for a determination of the mass-ratios of the respective system. Moreover, unlike the method of section IV applicable to Main-Sequence systems, the procedure suggested herewith should not be statistical in nature, but lead to exact results.

What is the confidence with which we can assert so in any practical case ? The answer depends primarily on the percentage of subgiants which have attained their Roche limits. If we were to apply this method to the individual systems listed in Table IV, all but three of them would have yielded mass-ratios agreeing with direct spectroscopic observations within the limits of observational errors — the probable exceptions being RS CVn, Z Her, and AR Lac, whose secondary components being significantly smaller than their Roche limits. A relative scarcity of systems of this latter type in the sample under consideration leads us to conjecture that a determination of the mass-ratios by the foregoing method is likely to lead to correct results in about eight or nine out of ten cases — a relatively high degree of reliability for a method of this nature.

From the theoretical point of view, a paucity of subgiant components smaller than their Roche limits indicates that their secular expansion is likely to proceed at a relatively rapid rate ; and the time which elapses in the course of an expansion of such a body from its equilibrium form to the Roche limit is probably but a small fraction of the time which the star will spend at this limit. From the practical point of view, a noteworthy feature emerging from Table IV is the fact that subgiant components of close binary systems do not appear to be confined to any narrow group of spectral types. If we regard the filling of the Roche limit as a sufficient (though not necessary) condition for a star to be classified in this group, such stars as μ^1 Sco B or u Her B of spectra B6-B7 can be assigned to this group as legitimately as U Sge B (spectrum G6) or U Cep B (G8). Subgiants in this sense are, therefore, apparently not limited to stars of advanced spectral classes ; though their deviation from the Main Sequence may not be as conspicuous for late B-type subgiants as among those of later spectral types.

With these features (and reservations) in mind, we may now proceed to illustrate an application of our " geometrical " method for determining the mass-ratios of binaries with subgiant components on a few selected examples. This " geometrical " method should, of course, be of particular use for single-spectrum systems where no other way may be open for such a determination. We might easily apply it to CRAWFORD's list of 18 single-spectrum systems [17], but no independent method would be available for checking the correctness of the results. In what follows we shall, instead, apply the method to other single-spectrum eclipsing systems, suspected of containing subgiants, whose velocity-curves exhibit conspicuous rotational effect during their minima.

As is well known, an analysis of the rotational effect can lead to a determination of the equatorial velocity of rotation of the component undergoing eclipse ; and if the period of axial rotation is known (being, for instance, equal to the period of orbital revolution, or bearing other known ratio with the latter), the absolute dimensions of the rotating star, and of the whole system, may thus be obtained. On the other hand, the same dimensions may alternatively be obtained from a single-spectrum absolute orbit of the primary component if the mass-ratio were known. Let us, in what follows, employ the first method on the assumption of a synchronism between rotation and revolution ; the second based on the mass-ratio deduced from the fractional dimensions of the secondary component with the aid of Table VI ; and compare the outcome to see the extent to which the two results are consistent with each other.

In more specific terms, if $R(\theta)$ denotes the magnitude of the rotational effect observed at a phase θ within eclipse, the velocity V_{obs} of axial rotation at the equation of the primary component will be given by

$$V_{obs} = \{ R(\theta)/F(\theta) \} \csc i, \tag{22}$$

where $F(\theta)$ denotes the well-known " rotation factor " which can be computed from the geometry of the eclipse [26, 27] and i, the inclination of the orbital plane to the celestial sphere. On the other hand, the same value of V may also be computed (on the assumption of synchronism between rotation and revolution) with the aid of the formula

$$V_c = \frac{2\pi}{P} (a_1 \sin i) \left(1 + \frac{m_1}{m_2}\right) \frac{r_1}{\sin i}, \tag{23}$$

where $a_1 \sin i$ denotes the projected semi-major axis of the absolute orbit of the primary component and r_1, its fractional radius ; the value of m_1/m_2 being deduced from Table VI in accordance with the secondary's radius r_2.

The following Table VII contains nine well-known single spectrum systems for which such a comparison is possible. The individual columns indicate, successively, (1) the star ; (2) the orbital period P (in days) (3) and (4), the fractional radii of the two components ; (5) the value of the mass-ratio deduced from Table VI from the observed values of r_2 on the assumption that the secondary is a contact component ; (6) the values of $a_1 \sin i$ of the single-spectrum orbit ; (7) the orbital inclination i deduced from the light curve ; (8) the equatorial velocity V_{obs} of the primary component, evaluated from equation (23) on the basis of its absolute dimensions obtained by combining the elements of the single-spectrum orbit with the mass-ratios of column (5), and column (9) lists the values V_c of the equatorial velocity deduced from the observed rotational effect with the aid of equation (22). The last column contains the references to the spectroscopic data.

A glance at columns (8) and (9) of the foregoing table reveals that agreement between the observed equatorial velocities and those computed on the assumption of contact nature of the secondary components is, on the whole, satisfactory for all systems available for comparison — with probable exception of X Tri which is, however, known to be anomalous in several respects. For all other systems the discrepances between theory and observation are likely to be within the limits of observational errors. Since the latter are relatively large, the outcome cannot be regarded as very conclusive test of our hypothesis. On the other hand, an assumption that the secondary components of the eight eclipsing systems listed in Table VII have attained their Roche limits is indeed found to be consistent with the observed rotational effects for stars rotating with Keplerian angular velocities ; and this is perhaps all that we could have expected.

In conclusion of the present section dealing with the determination of mass-

TABLE VII

STAR	PERIOD	r_1	r_2	m_1/m_2	$a_1 \sin i$ (in 10^6 km)	i	V_c (in km/sec)	V_{obs} (km/sec)	REFERENCE
—	—	—	—	—	—	—	—	—	—
RZ Cas	1ᵈ195	0.241 ± .002	0.284 ± .002	2.86	1.14	80°9	65	78	HORAK, *Ap. J.*, **115**, 1952, 61.
W Del	4.806	0.153 ± .003	0.245 ± .002	4.56	1.94	85°1	25	36	STRUVE, *Ap. J.*, **104**, 1946, 253.
Y Leo	1.686	0.21 ±	0.25 ±	4.5	1.4	88°	70	57	STRUVE, *Ap. J.*, **102**, 1945, 74.
δ Lib	2.327	0.301 ± .008	0.302 ± .006	2.26	2.42	79°5	69	64	MCLAUGHLIN, *Michigan Publ.*, **6**, 1934, 3.
RY Per	6.864	0.119	0.238	5.3	2.6	81°	29	(60) (1)	HILTNER, *Ap. J.*, **104**, 1946, 396.
ST Per	2.648	0.190	0.234	5.6	0.94	90°	32	38	STRUVE, *Ap. J.*, **104**, 1946, 253.
β Per	2.867	0.227 ± .002	0.239 ± .002	5.26	1.74	82°0	63	71	STRUVE and ELVEY, *M. N.*, **91**, 1931, 663.
λ Tau	3.953	0.22 ± .04	0.30 ± .02	2.23	3.06	73°	43	44	MCLAUGHLIN, *Michigan Publ.*, **6**, 1934, 3.
X Tri	0.972	0.283 ± .005	0.330 ± .002	1.60	1.47	88°9	81	50 (2)	STRUVE, *Ap. J.*, **104**, 1946, 253.

(1) This value is based on measurements of the hydrogen lines. The HeI lines lead to $V_{obs} \sim 100$ km/sec.

(2) STRUVE writes (*op. cit.*) that " the range of the rotational disturbance and the appearance of the lines suggest a value of $V_{rot} = 50$ km/sec ". But the form of the disturbance as observed by STRUVE suggests partial eclipse, whereas the light curve discloses that the primary eclipse is total. Hence, it is at least possible that, owing to the rapidity of light and velocity changes before and after the inner contacts of the eclipse, STRUVE did not observe the full range of the rotational disturbance.

ratios of semi-detached binary systems, we should comment briefly on the " problem of R CMa-type stars ", to which attention in the past has been called, in particular, by STRUVE [19, 20]. As is well known, this problem concerns a small group of single-spectrum eclipsing systems whose mass-functions are so minute that a great disparity between masses of the two components is required to render the latter of the right order of magnitude. Whatever may have been the uncertainty arising in the past in this connection, there appears now but little room for doubt that stars like R CMa and XZ Sgr (or, to a lesser extent, T LMi or S Vel) represent extreme cases of semi-detached systems whose subgiant components have filled their Roche limits.

Consider, for example, the case of R CMa whose elements are relatively well known. According to the photometric solution [28], the fractional radii of the two components are 0.26 ± 0.01 and 0.25 ± 0.01 respectively ; and a secondary of the latter fractional size will be at its limit of stability for the mass-ratio $m_1/m_2 = 4.6$. Inserting this in STRUVE and SMITH'S mass-function of $0.0037 \odot$ [29], we obtain $m_1 = 0.56 \odot$ and $m_2 = 0.12 \odot$ for the masses of the components. Now, according to STRUVE and SMITH (*op. cit.*), $a_1 \sin i =$ 491 000 km, leading (with DUGAN'S value of $i = 77^{\circ}$) to $a_1 + a_2 = 4.03 \odot$. Therefore, the corresponding absolute radii of the two components should be $R_1 = 1.05 \odot$ and $R_2 = 1.01 \odot$, respectively. The observed spectrum of the primary component of R CMa is F0, and the secondary's spectrum computed from its observed surface brightness should be dK1. If we adopt, in accordance with the current temperature scale, $\log T_1 = 3.88$ and $\log T_2 = 3.69$, the absolute bolometric magnitudes of both components follows as $M_1 = + 3^{m}5$ and $M_2 = + 5^{m}4$ respectively. According to these characteristics, both components of R CMa would lie closely on the Main Sequence, but deviate conspicuously (especially the secondary !) from the mass-radius relation (13), and even more so from the mass-luminosity relation ; though not again from the radius-spectrum relation obeyed statistically by Main Sequence stars.

It is of interest to note that the foregoing conclusions are consistent with the absolute magnitude of the system as deduced from its known trigonometric parallax According to SCHLESINGER'S *General Catalogue of Stellar Parallaxes* (Edition of 1935), the parallax of R CMa (No. 2317) is $0''.039 \pm 0''.008$ which, combined with the apparent photoelectric magnitude of $+ 6.2$, leads to an absolute magnitude of $+ 4.1 \pm 0.5$ for the system. The bolometric correction for the F0 primary component (which dominates the light of the system between minima) is effectively zero, and the photoelectric (λ 4 500) colour index is $+ 0.3$. Hence, the combined absolute bolometric magnitude of the system of R CMa should be $+ 3.8 \pm 0.5$, in satisfactory agreement with the value of $+ 3.4$ computed above.

The system of XZ Sgr appears to exhibit an even more extreme case of mass-ratio, though the observational data are as yet less well-founded : its only existing light curve has been published by SHAPLEY [30] twenty-four years ago, and in the hands of Sergei GAPOSCHKIN [31] yielded $r_2 = 0.20$ for the fractional radius of the secondary component. This latter value is probably quite uncertain, as secondary minimum has not been observed (1) ; but if it is taken on its face value, instability should set in, according to Table VI, for a mass-ratio $m_1/m_2 = 10$. That a value as large may indeed be required is indicated by the smallness of the mass-function of this eclipsing system which is, according to SAHADE [32], between $0.0005 \odot$ and $0.004 \odot$. The actual value of this function being as yet quite uncertain, it is premature to compute specific masses and absolute dimensions of this eclipsing system ; but they are likely to deviate from the mass-luminosity or mass-radius relations in the same sense, but even more, than the components of R CMa.

The significant feature which emerges from our discussion is the fact that the smallness of the mass-functions of R CMa, YZ Sgr, and similar systems (T LMi, S Vel) by itself demands mass-ratios which are sufficiently small to bring the secondary components to (or near) their limit of stability ; and our belief that this limit has actually been attained is based on abnormal smallness of the masses of the respective stars, suggestive of long secular loss of mass in the past. The mass-ratios deduced from the fractional dimensions of the secondaries, combined with the respective mass-functions, lead — it is true — to absolute properties which render *both* components quite abnormal with respect to mass and luminosity. This, however, is probably the consequence of old age ; for the secular expansion must have been operating a very long time to render a disparity in masses of the two components as large as indicated by a ratio of 5 : 1 (R CMa) or even 10 : 1 (YZ Sgr). Both components of such systems as R CMa, YZ Sgr (or, to a lesser extent, T LMi or S Vel) are, therefore, probably very old stars ; and their present characteristics may be indicative of the direction of evolutionary trends of Main Sequence stars of really great age. It may be added, in this connection, that a relatively high space velocity of R CMa — namely, 67 km/sec — makes it just possible that this system represents an ageing Population II object which made its way into our spiral arm from the inner parts of our Galaxy. The space velocities of other objects of this group (as far as known) do not, however, appear to be unduly large.

VII. CONTACT BINARY SYSTEMS.

In the preceding sections of this paper we have described, in turn, two distinct groups of close binary systems, differing in the fractional sizes of their components

(1) The light curve of XZ Sgr would well repay accurate photoelectric re-observation (amplitude $9^{m}2$-$11^{m}5$; period $P = 3.276$ days).

expressed in terms of those of their limiting Roche surfaces : the *detached systems*, both components of which are smaller than their Roche limits ; and *semi-detached systems*, whose less massive (subgiant) components fill exactly their respective Roche limits — and are apparently in the process of secular expansion — while their primaries are well detached from the Roche limits and apparently stable. In order to complete our survey, it remains to consider a third distinct group of close binaries whose *both* components appear to occupy their Roche limits and which are, therefore, in contact at the inner Lagrangian point L_1. Such systems will hereafter be denoted as *contact binaries*, and seem to consist of two physically rather different groups of stars : one consisting of Main Sequence dwarf pairs, of periods generally less than one day, whose prototype is the system of W UMa ; and the other consisting of early-type O and B-stars of high luminosity and abnormal masses, of which β Lyr can be considered an example. Stars of W UMa group represent, probably, the most common type of close binaries per unit volume of space in the neighbourhood of our Sun [33], while stars of the β Lyr type are probably abnormal and excessively rare.

In view of these facts, it is proper to discuss stars of the former group first. The material at our disposal is, however, relatively meagre : for all stars of this type are relatively faint, so that it was not until during the recent decade that both photometric and spectroscopic (two-spectra) orbits have been obtained for a sufficient number of W UMa-type systems to make their study as a group worth while. The observational data which will serve as a basis for the present investigation are summarized in the following Table VIII, whose individual columns indicate, successively ; (1) the star ; (2) and (3) the fractional radii (diametral semi-axis) of the primary and secondary components ; (4) the corresponding ratios of the radii r_1/r_2 ; (5) the ratios of surface brightnesses J_1/J_2 ; (6) and (7) their corresponding Roche constants C_1 and C_2 ; (8) and (9) the ratios C_0/C_1 and C_0/C_2 respectively (the values of C_0 being taken from column (5) of Table II) ; and (10) the potentials W of a particle of unit mass placed on the Roche limit of the respective system (in solar units).

In trying to assess the accuracy of the respective data it is well to keep in mind the fact that the photometric elements of all systems of this group are still quite uncertain : we possess as yet no reliable methods for a proper analysis of the complex photometric evidence exhibited by W UMa-type systems. In their absence, the " elements " as deduced by an extrapolation of conventional analysis to highly distorded systems must be regarded as very preliminary, and for individual systems may be considerably in error. The dispersion of the individual values of C_0/C_j as listed in columns (6) and (7) of Table VIII — some large, some smaller than unity — is probably due to this cause. Their *averages* — physically more meaningful quantities — are, however, found to be

$$\left\{ \begin{aligned} \overline{C_0/C_1} &= 0.97 \pm 0.02 \qquad \text{(p. e.)} \\ \overline{C_0/C_2} &= 0.992 \pm 0.014 \quad \text{(p. e)} \end{aligned} \right. \tag{24}$$

i.e., *do not deviate significantly from unity* — which encourages us to conclude that, as a class, stars of the W UMa-type do constitute contact binary systems.

TABLE VIII

CONTACT BINARY SYSTEMS

STAR	r_1	r_2	r_1/r_2	J_1/J_2	C_1	C_2	C_0/C_1	C_0/C_2	W
AB And	0.31	0.31	1.0	0.9	5.0	4.1	0.96	1.20	1.9
S Ant	0.38 ± 0.02	0.17 ± 0.02	2.2 ± 0.3	0.58	4.02	3.97	0.98	1.00	0.70
i Boo	0.36	0.34	1.1	0.71	4.6	3.8	0.86	1.04	1.55
TX Cnc	0.26	0.28	0.9	< 1	5.7	4.3	0.7	0.9	1.6
VW Cep	0.49 ± 0.02	0.29 ± 0.01	1.7 ± 0.1	0.48	3.86	3.78	1.00	1.02	1.39
RZ Com	0.34	0.34	1.0	< 1	4.8	3.8	0.8	1.07	1.7
GO Cyg	0.39	0.36	1.1	—	4.0	4.0	1.0	1.0	1.0
YY Eri	0.38 ± 0.03	0.35 ± 0.03	1.1 ± 0.1	1.0	4.3	3.8	0.91	1.04	1.35
SW Lac	0.36	0.33	1.1	0.73	4.2	4.2	0.95	0.95	1.61
V 502 Oph	0.51	0.35	1.46	0.71	3.7	3.6	1.07	1.08	1.22
ER Ori	0.30 ± 0.03	0.30 ± 0.03	1.0 ± 0.1	1.1	5.1	4.2	0.8	0.95	0.71
U Peg	0.40	0.40	1.0	0.9	3.96	3.7	1.01	1.07	1.54
RZ Tau	0.42 ± 0.03	0.40 ± 0.03	1.05 ± 0.1	1.1	4.04	3.6	0.98	1.1	3.0
W UMa	0.42	0.24	1.75	0.9	4.04	4.6	0.98	0.87	1.52
AH Vir	0.44	0.26	1.7	0.67	4.0	4.2	0.97	0.93	1.33
V 599 Aql	0.46 ± 0.03	0.26 ± 0.02	1.8 ± 0.2	1.6	3.8	4.4	1.0	0.9	2.22
EO Aur	0.32 ± 0.01	0.40 ± 0.02	0.83 ± 0.05	2.2	4.4	3.8	0.9	1.0	2.66
AO Cas	0.37 ± 0.02	0.26 ± 0.03	1.4 ± 0.2	1.0	4.0	5.0	1.0	0.8	3.06
V 450 Cyg	0.39 ± 0.02	0.33 ± 0.02	1.2 ± 0.1	1.8	4.0	4.2	1.0	0.94	2.55

This important property (which has long been conjectured, but for which the present discussion has adduced the strongest photometric evidence available so far) implies that the primary (more massive) component is the *larger* of the two, but as a rule of *later* spectral type and of *lesser* surface brightness. The primary (deeper) minima in eclipsing systems of this type are, therefore, likely to be *occultations*.

In the preceding section we have pointed out that the presence of a contact component in semi-detached binary systems opens a way for determination of the mass-ratio of the system from fractional dimensions of the contact body. In W UMa-type systems this should be doubly true : namely, their mass-ratios should specify uniquely the fractional dimensions of *both* components, as well as their forms in all details. A recent analysis of the geometry of Roche model by the present writer [13] revealed, moreover, that the fractional dimensions of contact components are likely to be such that the *sum of the fractional radii* (or, more exactly, of diametral semi-axes) *of both stars is very nearly constant* for a wide range of mass-ratios encountered in practical cases ; it is the *ratio* r_2/r_1 of such radii, rather than their sum, which is a sensitive indicator of the mass ratio. The extent to which this is true is revealed by the following tabulation, based on the geometry of the Roche model and taken from the writer's recent paper [13] :

TABLE IX

m_2/m_1	$r_1 + r_2$	r_2/r_1
—	—	—
1	0.7484	1.000
0.9	0.7486	0.950
0.8	0.7489	0.896
0.7	0.7496	0.839
0.6	0.7510	0.778
0.5	0.7529	0.711
0.4	0.7565	0.638
0.3	0.7622	0.555
0.2	0.7722	0.457
0.1	0.7935	0.331
0	1.0000	0.000

The bearing of these facts on the analysis of light curves of contact binary systems is obvious. Once we satisfy ourselves that both components have attained the sizes of their Roche limits, the sum $r_1 + r_2$ of their fractional radii can be taken as constant and equal to 0.75 to begin with, and the strategy of light curve analysis should concentrate on the determination of the ratio of the radii $r_2/r_1 = k$ and of orbital inclination i. The value of k can then be utilized to specify (within the limits of observational errors) the mass-ratio of the respective contact binary. The details of analytical processes which can be invoked to this end would exceed the scope of this paper and will be given elsewhere ; but one general observation cannot be over-emphasized : namely, that *a determination of the geometrical element of contact binary systems is inseparably connected with a determination of their mass-ratios* ; and, in particular, the fractional dimensions of both components are specified in all details by the geometry of the Roche model as soon as this mass-ratio is known. A knowledge of the mass-ratio inferred from the ratio of the radii k opens, in turn, a new way for a determination of masses and absolute dimensions of single-spectrum contact binaries, and thus is bound to augment considerably in the future the information already obtained from the study of two-spectra systems.

With regard to the physical properties of the components of two-spectra systems of W UMa-type collected in Table VIII, both primary and secondary components cluster (though not very closely) around the Main Sequence (see Fig. 5), and their spectral types exhibit a marked concentration between late A's and

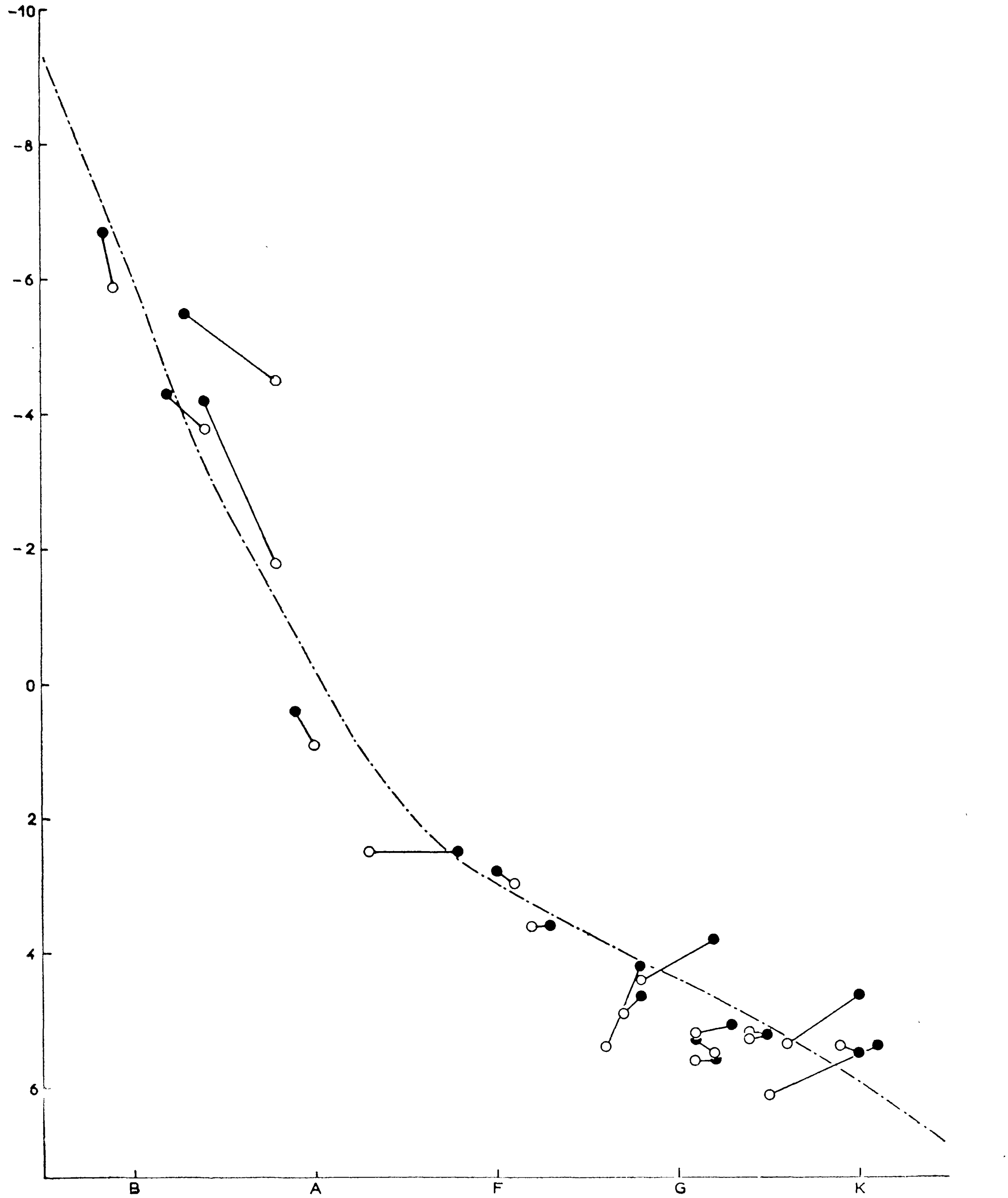

FIG. 5. — Hertzsprung-Russell Diagram of Contact Binary Systems.

early K's. They show, however, *no vestige of any statistical relation between mass and luminosity.* In this respect they differ, therefore, from the behaviour of the individual components of eclipsing systems of our Class I ; but deeper significance of this fact remains yet to be discovered.

Is the contact nature of the systems of W UMa-type likely to be due (as for the subgiants) to a secular expansion of their *both* components ? The general inequality of their masses (although not quite as pronounced as for semi-detached systems) is indeed suggestive of a secular mass transfer from the secondary to the primary component. On the other hand, the components of contact systems do not appear to deviate systematically from the Main Sequence (cf. Fig. 5) and are, in general, not too luminous for their spectra ; though they are — like the subgiants — systematically too luminous for their masses. Thus whereas the possibility that the components of contact binaries have filled their Roche limits by gradual expansion cannot as yet be dismissed, the alternative possibility that such binaries *originated in their present form* in relatively recent past, and thus represent a group of *very young* stars, should likewise be kept in mind.

In conclusion, the existence of contact binaries among early-type eclipsing systems will only briefly be mentioned in this place — not because such binaries would lack points of interest (some of the most intriguing known binaries, such as β Lyrae, are found among them), but rather because they are excessively rare and, therefore, of rather subordinate interest for a primarily statistical investigation. Apart from β Lyrae, which represents a problem for itself [34, 35, 36], V 599 Aql, EO Aur, AO Cas or V 470 Cyg possibly belong to this group (1) which — if any — deserves the name of variables of the β Lyrae-type (though β Lyrae itself, as a supergiant system, deviates significantly to the right of the Main Sequence). A closer discussion of their physical characteristics represents, however a separate problem and must be postponed for a future investigation.

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