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THE COLLIMATION ERROR OF THE AIRY TRANSIT CIRCLE

P. J. D. Gething

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Summary

An analysis of azimuth errors determined from stars " doubled " with the Airy Transit Circle confirms that there is a systematic error in the collimation errors adopted for use with this instrument. The existence of such an error had been suspected for many years because of the discrepancy between clock errors determined with the Transit Circle and with Small Transit " B ". Estimates of the magnitude of the error obtained by four different methods are in fair agreement.

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1. *Introduction.*—The existence of a systematic error in the collimation errors adopted for the Airy Transit Circle at the Royal Observatory, Greenwich, has been suspected for many years. In October 1926 a small reversible transit instrument, Transit " B ", was brought into use at Greenwich in connection with the international programme of longitude determination. A comparison of the clock errors obtained with the two instruments revealed a discrepancy of about one-tenth of a second, after allowance had been made for the difference of longitude of the two instruments and for the different methods used in reducing the observations. Comparison with the time signals received from other observatories suggested that it was the non-reversible Transit Circle which was at fault; the most probable source of error appeared to be in the determination of collimation error. The clock corrections derived from the Transit Circle observations were not as smooth as those derived from the small transit observations. But the R.A.s of stars determined with the Transit Circle showed better internal agreement when reduced with the determined clock corrections than when reduced with smoothed clock corrections. This is to be expected if the collimation is at fault. There was some evidence that the results with the two instruments would have been more accordant if a constant collimation error had been adopted for the Transit Circle each year, suggesting that the observed changes in the collimation error were not real.

In view of this uncertainty, the Time Service after 1927 July was based on observations made with Transit " B ". The difference between the two instruments, referred to in this paper as TC–B, has persisted at about the same value, with variations that can be attributed partly to the scatter of the observations and partly to the existence of a small annual term. The difference of clock errors, TC–B, has been predominantly positive; clock errors are considered positive when the clock is slow. If the discrepancy is assumed to be due to an error in the determination of collimation error, it can be shown that the true collimation error must be greater than the adopted collimation error by about $0''\cdot3$; collimation error is considered to be positive when a positive correction is required to the times of transit of stars at upper culmination.

The hypothesis that the Transit Circle clock errors are erroneous, and that the error is introduced in the measurement of collimation, is examined and confirmed in this paper. The analysis is based on a method which does not require any comparison between the Transit Circle and another instrument, but only between " fundamental " azimuths as determined from different stars.

The collimation error is measured with the aid of two fixed horizontal collimators placed north and south of the instrument; the process is described in detail in Section 7. The level error is determined by finding the micrometer reading for coincidence of the movable R.A. wire with its own image formed in a pool of mercury. The micrometer reading for coincidence is subtracted from the reading for line of collimation and the resulting difference is the level error expressed in revolutions of the R.A. screw. This can be converted to arc by using the known value in arc of one revolution of the screw. The azimuth error of the instrument can be determined only by observations of stars, but no azimuth marks are available to control possible diurnal changes of azimuth. Circumpolar stars are used for the determination of azimuth, since it is important to be able to observe azimuth stars at transit both above and below the pole. An azimuth star is said to have been " doubled " when it has been observed at two culminations twelve hours apart, and the azimuth error determined from such a pair of observations is here referred to as a " double azimuth ". It has the important advantage of being independent of any error in the star's adopted place.

2. *Method of analysis.*—Suppose that at the first transit the tabular right ascension of the star was T_1, and that the observed time of transit was O_1. Then we have

$$T_1 - O_1 = a_1A_1 + b_1B_1 + c_1C_1 + E_1, \tag{1}$$

where a_1, b_1 and c_1 are the true azimuth, level and collimation errors of the instrument, A_1, B_1 and C_1 are trigonometrical factors depending on the declination δ of the star and the latitude ϕ of the observatory, and E_1 is the true clock error. a_1, b_1 and c_1 are assumed to be expressed in seconds of arc and $(T_1 - O_1)$ and E_1 in seconds of time. This equation is strictly true only if the observations on which it depends are perfect. The effect of diurnal aberration is assumed to be included in c_1 in the usual way. It is assumed that pivot errors are negligible; this point is discussed in more detail in Section 3.

Similarly, at the second transit of the same star, we have

$$T_2 - O_2 = a_2A_2 + b_2B_2 + c_2C_2 + E_2. \tag{2}$$

If we now assume that the azimuth error has not changed in the twelve-hour period between the observations, i.e. that $a_1 = a_2 = a$, the azimuth error can be determined by subtracting (2) from (1). After re-arrangement we have

$$a = \frac{(T_1 - T_2) - (O_1 - O_2) - (b_1B_1 - b_2B_2) - (c_1C_1 - c_2C_2) - (E_1 - E_2)}{(A_1 - A_2)}.$$

The quantity $(T_1 - T_2)$ is known with a high degree of accuracy and is independent of any small error in the assumed R.A. of the star. $(E_1 - E_2)$ is also known with sufficient accuracy, since it depends only on the rate of the clock in a twelve-hour period. The quantities O_1 and O_2, A_1 and A_2 are also known, so that the azimuth error a can be determined.

We shall now suppose that the adopted collimation errors are both subject to the same systematic error Δc, in such a sense that

$$\Delta c = \text{True collimation error} - \text{Adopted collimation error.}$$

The adopted level errors will also be directly affected by Δc; in fact

$$\Delta c = \text{True level error} - \text{Adopted level error.}$$

Equations (1) and (2) now become

$$T_1 - O_1 = aA_1 + (b_1 + \Delta c)B_1 + (c_1 + \Delta c)C_1 + E_1, \tag{3}$$

$$T_2 - O_2 = aA_2 + (b_2 + \Delta c)B_2 + (c_2 + \Delta c)C_2 + E_2, \tag{4}$$

where a is still the *true* azimuth error, but b_1, b_2, c_1, c_2 now denote the *adopted* level and collimation errors. Combining equations (3) and (4), we obtain

$$a = \frac{(T_1 - T_2) - (O_1 - O_2) - (b_1 B_1 - b_2 B_2) - (c_1 C_1 - c_2 C_2) - (E_1 - E_2)}{(A_1 - A_2)} + \Delta c \left[\frac{(B_2 - B_1) + (C_2 - C_1)}{(A_1 - A_2)}\right]$$

or

$$\text{True azimuth} = \text{Deduced azimuth} + \Delta c \left[\frac{(B_2 - B_1) + (C_2 - C_1)}{(A_1 - A_2)}\right].$$

The formulae for A_1, B_1 and C_1 are as follows:

$$A_1 = \tfrac{1}{15} \sec \delta \sin (\phi - \delta), \quad B_1 = \tfrac{1}{15} \sec \delta \cos (\phi - \delta), \quad C_1 = \tfrac{1}{15} \sec \delta.$$

These three expressions refer to a transit at upper culmination; the corresponding formulae for lower culmination may be obtained by substituting $(180° - \delta)$ for δ.

The quantity $(B_2 - B_1)/(A_1 - A_2)$ is a constant independent of the declination of the star, and is equal to $\tan \phi$, or 1·256 at Greenwich; indeed, this result holds even if two stars at different declinations have been observed. It follows that any systematic error in the determination of level affects all determinations of azimuth equally; they have, as will be seen, no effect on the determination of Δc by the present method.

The quantity $(C_2 - C_1)/(A_1 - A_2)$ is equal to $\sec \phi \operatorname{cosec} \delta$, or $1{\cdot}606 \operatorname{cosec} \delta$ at Greenwich. If now several stars are doubled within approximately the same twelve-hour period, the true azimuth will be common to all equations, but the deduced azimuth will vary, apart from the accidental errors of observation, with the star used, through the factor $1{\cdot}606 \operatorname{cosec} \delta$. The variation of the deduced azimuths with δ can therefore be used to give an estimate of Δc.

Suppose that two stars have been doubled and that the deduced azimuths are a' and a'' respectively. Then, since the true azimuth is the same for each star, we have

$$(a' - a'') = 1{\cdot}606 \, \Delta c \, (\operatorname{cosec} \delta'' - \operatorname{cosec} \delta')$$

or

$$\Delta c = \frac{(a' - a'')}{1{\cdot}606 \, (\operatorname{cosec} \delta'' - \operatorname{cosec} \delta')}.$$

For the azimuth stars used at Greenwich the denominator cannot exceed 0·66 even in the most favourable case, and is more likely to be about 0·2. The probable errors of azimuth determinations made with the Airy Transit Circle are about $0''{\cdot}2$ or $0''{\cdot}3$, so that, if ϵ is the probable error of Δc from a single pair of stars,

$$\epsilon > [(0''{\cdot}2)^2 + (0''{\cdot}2)^2]^{1/2}/0{\cdot}66$$
$$> 0''{\cdot}43.$$

In practice, ϵ is probably at least $1''$, so that some hundreds of pairs of azimuth stars must be analysed if the existence of a systematic error Δc of the order of $0''{\cdot}3$ is to be demonstrated conclusively.

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The impersonal R.A. micrometer was not brought into use until 1915; observations prior to this date are not considered suitable for analysis. From 1915 to 1922 the determinations of azimuth error were based on a list of stars all within $3\frac{1}{2}°$ of the pole. The range of the factor 1·606 cosec δ among these stars is only 0·002, so that this period also must be discarded. A fundamental catalogue was finished in 1938 April, and after that date only a very few double azimuths were obtained. After 1940, regular observations of double azimuths were discontinued and have not been resumed, apart from two short periods in 1953, when a special series of azimuth determinations was made for another purpose. Thus we are left with only the period 1922–1938; however, in this period about a thousand double azimuths are suitable for analysis, so that the data should be adequate.

Between 1922 and 1938 azimuths were determined also by observations of two of the stars in the azimuth star list, one observed above pole and the other below pole; the observations were not usually more than a few hours apart. Azimuths so determined are relatively free from the effects of possible diurnal changes in collimation, but are affected by any errors in the azimuth star places, and they have not been used in the present discussion for this reason.

The method of least squares was used to obtain the best estimate of Δc. An example of the way in which the calculation was set out is given below. Column 1 gives the date, column 2 the name of the star doubled and column 3 the deduced azimuth. This information has been extracted from the annual volumes of *Greenwich Astronomical Results*. The fourth column, x, is the deduced azimuth *minus* the mean of the azimuths in column 3. Column 5 is the factor $f = -1{\cdot}606 \operatorname{cosec} \delta$. Column 6, y, is the factor *minus* the mean factor. Columns 7, 8 and 9 give the products x^2, xy and y^2 respectively, each multiplied by 100 for convenience. The estimate of Δc from these entries alone is

$$\Sigma xy/\Sigma y^2 = +4{\cdot}270/3{\cdot}537 = +1''{\cdot}21.$$

The column showing $100x^2$ is needed for the determination of probable errors.

1 Date	2 Name of star	3 Deduced azimuth	4 x	5 Factor	6 y	7 $100x^2$	8 $100xy$	9 $100y^2$
1922		″	″					
Feb. 13	β Cas	+5·52	+·25	−1·878	−·045	+ 6·25	−1·125	+0·202
	α Cas	+4·96	−·31	−1·933	−·100	+ 9·61	+3·100	+1·000
	γ Cas	+5·48	+·21	−1·847	−·014	+ 4·41	−0·294	+0·020
	δ Cas	+5·56	+·29	−1.856	−·023	+ 8·41	−0·667	+0·053
	ϵ Cas	+4·39	−·88	−1·797	+·036	+77·44	−3·168	+0·130
	50 Cas	+5·71	+·44	−1·687	+·146	+19·36	+6·424	+2·132
Means		+5·27		−1·833				

The columns of $100x^2$, $100xy$ and $100y^2$ have been summed for each month throughout the period under consideration. The results are summarized in Tables I, II and III respectively. Table IV shows the number of double azimuths n, on which each entry in Tables I, II and III depends. Each table consists of two sections, the first for the period 1922–1930 and the second for 1931–1938. The rows and columns in each section of every table have been summed and the results are shown.

TABLE I

Monthly values of $100x^2$

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Yearly Sums
1922	...	326	53	116	806	218	...	...	66	335	136	146	2202
1923	92	...	40	7	...	...	...	129	...	214	95	...	577
1924	...	...	98	3	137	...	...	...	442	98	21	52	851
1925	200	49	...	157	276	...	...	3	176	616	434	130	2041
1926	114	10	359	43	11	7	...	...	205	363	0	354	1466
1927	...	114	104	206	25	...	...	52	9	389	215	306	1420
1928	2	395	98	221	350	...	...	221	79	456	222	62	2106
1929	28	...	254	58	1188	...	...	64	186	12	86	190	2066
1930	11	...	26	600	...	173	...	...	...	111	575	...	1496
Sums	447	894	1032	1411	2793	398	...	469	1163	2594	1784	1240	14225
1931	...	58	32	101	75	...	...	48	...	659	63	31	1067
1932	4	19	231	74	17	29	...	...	21	132	56	170	753
1933	...	...	1016	8	60	92	...	...	81	162	87	...	1506
1934	68	88	237	15	161	...	...	...	169	115	104	...	957
1935	8	...	215	4	308	3	...	2	91	935	50	113	1729
1936	15	118	276	223	182	...	...	13	7	20	19	...	873
1937	64	...	...	...	279	...	...	...	...	...	4	...	347
1938	...	...	73	19	...	...	...	...	...	...	...	...	92
Sums	159	283	2080	444	1082	124	...	63	369	2023	383	314	7324

Grand total of monthly values = 21549.

Table II

Monthly values of 100xy

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Yearly Sums
1922	...	+2·3	+11·5	−3·5	−43·0	+15·4	...	...	−7·0	−16·3	−22·1	+3·2	−59·5
1923	−3·9	...	−5·9	−0·7	...	...	...	−11·3	...	+10·0	+5·3	...	−6·5
1924	...	...	−14·2	−3·1	−25·7	...	...	...	−29·6	−22·4	−10·5	+12·7	−92·8
1925	+4·0	+0·1	...	−8·4	+12·3	...	...	−2·2	−4·6	+57·5	+23·6	+5·2	+87·5
1926	+11·3	−0·5	−23·7	+3·8	+2·7	−1·4	...	...	−24·6	+29·9	+0·1	−4·2	−6·6
1927	...	+6·7	+16·2	−15·6	+15·1	...	...	+2·3	−2·5	−17·3	+50·3	−8·5	+46·7
1928	−0·5	+29·5	−3·6	+17·0	+38·2	...	...	+5·6	+3·4	−16·0	+22·9	+8·3	+104·8
1929	+1·0	...	+2·6	+8·4	−19·7	...	...	+4·3	+0·8	+5·3	−12·5	+2·6	−7·2
1930	+1·3	...	+1·4	−4·4	...	−22·8	...	...	...	+21·1	−19·9	...	−23·3
Sums	+13·2	+38·1	−15·7	−6·5	−20·1	−8·8	...	−1·3	−64·1	+51·8	+37·2	+19·3	+43·1
1931	...	+0·3	−1·1	−18·3	−3·3	...	...	−2·2	...	−45·5	−17·7	+2·5	−85·3
1932	−0·3	−2·4	−9·8	+11·6	−8·0	+9·1	...	...	+0·7	+5·5	+9·0	+10·1	+25·5
1933	...	...	+45·3	−1·1	−14·8	+12·7	...	...	+4·6	+10·8	+6·1	...	+63·6
1934	−7·7	+6·1	+17·4	−6·3	−36·4	...	...	...	−0·6	+5·0	+10·8	...	−11·7
1935	+5·3	...	+8·0	+2·4	+16·5	+3·2	...	−3·1	+13·8	+67·5	+4·4	+6·7	+124·7
1936	+3·3	+3·4	−2·9	−1·9	+1·1	...	...	−1·2	+1·9	+0·9	+11·7	...	+16·3
1937	−10·3	...	...	...	−25·7	...	...	...	...	...	+2·0	...	−34·0
1938	...	...	+4·9	+2·4	...	...	...	...	...	...	...	...	+7·3
Sums	−9·7	+7·4	+61·8	−11·2	−70·6	+25·0	...	−6·5	+20·4	+44·2	+26·3	+19·3	+106·4

Grand total of monthly values = +149·5.

TABLE III

Monthly values of $100y^2$

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Yearly Sums
1922	...	5·0	2·7	2·3	28·1	2·0	...	...	0·8	27·5	11·1	1·2	80·7
1923	3·9	...	0·9	0·1	...	...	...	3·5	...	24·3	16·6	...	49·3
1924	...	...	9·7	3·1	11·0	...	...	...	2·7	5·1	5·2	5·7	42·5
1925	6·0	0·9	...	6·0	12·2	...	...	1·4	2·1	17·0	44·9	8·6	99·1
1926	4·1	0·1	12·5	0·3	4·4	4·2	...	...	5·2	11·0	0·0	2·5	44·3
1927	...	0·4	3·5	15·5	9·9	...	...	0·1	0·7	21·6	22·9	3·5	78·1
1928	0·2	4·4	2·0	11·5	26·1	...	...	0·3	1·3	34·5	16·2	1·9	98·4
1929	0·0	...	1·0	14·9	57·1	...	...	0·7	3·7	3·4	6·8	5·9	93·5
1930	0·2	...	1·3	27·3	...	24·0	...	...	...	22·2	30·4	...	105·4
Sums	14·4	10·8	33·6	81·0	148·8	30·2	...	6·0	16·5	166·6	154·1	29·3	691·3
1931	...	4·0	1·4	3·3	11·2	...	...	1·0	...	43·6	8·0	0·2	72·7
1932	0·0	0·9	2·1	18·7	3·7	3·6	...	...	1·3	2·0	1·4	8·8	42·5
1933	...	...	26·2	0·2	6·2	6·0	...	...	1·4	21·8	7·8	...	69·6
1934	1·4	1·3	3·3	3·6	25·7	...	...	...	1·2	7·2	8·6	...	52·3
1935	4·8	...	3·7	1·5	47·4	3·9	...	40·8	3·9	46·2	9·3	4·7	166·2
1936	0·8	3·8	1·8	15·6	24·0	...	...	0·1	0·5	19·4	8·0	...	74·0
1937	1·7	...	...	...	23·6	...	...	...	...	...	1·4	...	26·7
1938	...	...	2·2	3·6	...	...	...	...	...	...	...	...	5·8
Sums	8·7	10·0	40·7	46·5	141·8	13·5	...	41·9	8·3	140·2	44·5	13·7	509·8

Grand total of monthly values = 1201·1.

TABLE IV

Monthly values of n

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Yearly Sums
1922	0	14	7	8	21	7	0	0	3	25	5	7	97
1923	6	0	2	2	0	0	0	4	0	10	11	0	35
1924	0	0	12	2	9	0	0	0	5	3	2	8	41
1925	10	4	0	4	9	0	0	2	7	24	29	9	98
1926	12	3	14	2	3	3	0	0	4	10	2	17	70
1927	0	2	7	12	6	0	0	2	4	17	14	11	75
1928	2	8	8	9	18	0	0	9	6	22	8	3	93
1929	2	0	19	10	35	0	0	3	11	3	6	14	103
1930	2	0	4	19	0	12	0	0	0	14	21	0	72
Sums	34	31	73	68	101	22	0	20	40	128	98	69	684
1931	0	6	5	3	9	0	0	3	0	26	3	2	57
1932	3	3	8	11	2	4	0	0	5	7	2	12	57
1933	0	0	40	2	4	6	0	0	9	19	4	0	84
1934	4	5	11	3	16	0	0	0	7	11	8	0	65
1935	3	0	11	2	30	2	0	3	10	34	7	5	107
1936	3	13	7	12	22	0	0	2	2	10	3	0	74
1937	2	0	0	0	17	0	0	0	0	0	3	0	22
1938	0	0	11	3	0	0	0	0	0	0	0	0	14
Sums	15	27	93	36	100	12	0	8	33	107	30	19	480

3. *Possible sources of error.*—In this section we examine the assumptions inherent in the double azimuth method and consider whether the approximations involved could have any significant effect on Δc. The following possible sources of error have been considered.

(a) *Pivot errors.*—The effect of errors in the shape of the Transit Circle pivots is to make the telescope trace out in the sky a curve which departs from a true great circle. The effect of a collimation error is to make the telescope describe a small circle instead of a great circle. It follows that the effect of pivot errors can be regarded as a collimation error which varies according to the direction in which the telescope is pointed. Before we can treat Δc as a constant quantity, it is therefore necessary to consider the magnitude of the pivot errors.

The last determination of pivot errors was made in 1924, after new bearings (Y's) had been installed. The Introduction to the 1924 volume of *Greenwich Astronomical Results* gave the results of the investigation in the form of x and y, the horizontal and vertical components respectively of the motion of the western end of the axis relative to the eastern. For the present investigation, smoothed values of x and y have been resolved into a displacement of the telescope in the plane containing its optical and mechanical axes, equivalent to a variable collimation error, and a component which is equivalent to a small rotation of the telescope about its optical axis, which can be ignored. The first component is plotted in Fig. 1, in which a smooth curve has been drawn through points at every 10° of N.P.D. (North Polar Distance). Positive displacements of the curve from the zero-line are equivalent to a positive collimation error.

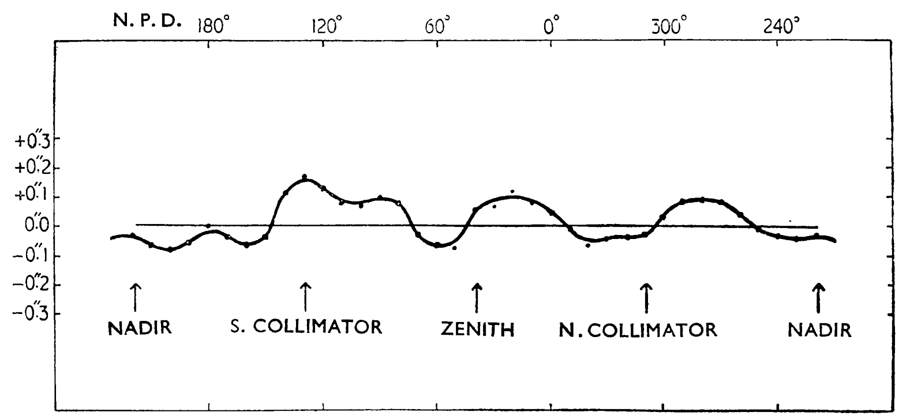

FIG. 1.—*Pivot errors of the Airy Transit Circle.*

From a study of this curve it was found that: (i) if pivot error corrections had been applied to the double azimuths used in the analysis for Δc, the value of Δc would have been increased by $+0''\cdot 11$; (ii) if pivot error corrections had been applied to the original observations for collimation error, the adopted collimations would have been decreased by $0''\cdot 06$; (iii) if pivot error corrections had been applied to all the original observations of collimation, level and times of transit of clock and azimuth stars, the clock errors derived with the Transit Circle would have been $0^s\cdot 018$ greater on average, so that the TC–B difference would have been increased by $0^s\cdot 018$.

Pivot error corrections have not been applied in the past to observations made with the Transit Circle, because of their small size and the difficulties involved in their determination. They have not been applied to the results given in this paper for the same reasons, but the above corrections can easily be applied to results relating to periods after 1924 if desired.

(b) *Variation of azimuth in a period of twelve hours.*—It was assumed in Section 2 that the azimuth error, a, of the telescope was unchanged after a period of twelve hours. Suppose, however, that the true azimuth changes by Δa, i.e. $a_2 = a_1 + \Delta a$. The deduced azimuth is increased by $A_2 \Delta a/(A_2 - A_1)$. One would expect this correction to be sometimes positive and sometimes negative, since it depends both on the sign of Δa and on the order in which the transits were observed. The effect of ignoring the correction would then be to increase the scatter of the double azimuths without affecting Δc.

The assumption that Δa can be ignored is not necessarily correct. The great majority of double azimuths are obtained by observing the above-pole transit in the morning, usually at about 10^h U.T., and the below-pole transit in the evening, at about 22^h U.T. Below-pole transits are much less commonly observed in daylight because of the difficulty of seeing stars in daylight at low altitudes. Hence, if there is an appreciable systematic difference between a_{10}, the true azimuth error at about 10^h U.T., and a_{22}, the error at about 22^h U.T., the estimates of Δc obtained from double azimuths will be in error.

An estimate of the size of $(a_{10} - a_{22})$ can be obtained by comparing an azimuth (not a " double " azimuth) deduced from two stars observed at about 10^h U.T. with an azimuth from two stars observed at about 22^h U.T. on the same day, or the preceding day. Unfortunately, the difference due to Δa cannot, in general, be *easily* separated from that due to Δc, although it is theoretically possible. In the period 1916 to 1921, however, the two effects are easy to separate, and the azimuths for this period have accordingly been analysed. Azimuths in this period were obtained by combining an azimuth star within $3\frac{1}{2}°$ of the pole with a clock star observed at about the same time. All azimuth errors deduced in this way are affected by Δc to the same extent, so that any difference between the deduced azimuths obtained at 10^h and 22^h must be due to a real change of azimuth.

The result of the analysis was that

$$(a_{10} - a_{22}) = -0''\cdot04 \pm 0''\cdot06.$$

The effect of allowing for this term would be to increase Δc by $+0''\cdot04$. For the purpose of the present investigation, it seemed reasonable to suppose that $(a_{10} - a_{22})$ does not differ significantly from zero and that the effect on estimates of Δc is not much greater than $0''\cdot04$. The data for the years 1916 to 1921 provide some evidence that the sign of $(a_{10} - a_{22})$ may reverse during the course of a year; if this is true, it may provide an explanation of the large annual term in Δc found in Section 5.

(c) *Neglect of short-period terms of nutation.*—The azimuth errors tabulated in the volumes of *Greenwich Astronomical Results* were calculated without including the short-period terms of nutation in the tabular places T_1 and T_2. Their omission introduces a small scatter into the deduced azimuths which is without any significant effect on Δc.

(d) *Personality.*—With the use of the impersonal micrometer, it is unlikely that the personality differences between different observers, and between observations made by the same observer above and below pole, are sufficiently large to have any appreciable effect on Δc. A following error, in which an observer sets the wire on a moving image either slightly in front of or slightly behind the true centre, has no effect on double azimuths obtained by that observer. A bisection error, in which the observer sets to the right or left of the true image centre, irrespective of the direction of motion, would affect the determinations of azimuth. It is known that the *differences* of personality among observers are small with the impersonal micrometer, and it must be assumed that the *absolute* personalities are also small. Nevertheless, it was considered advisable to omit from the analysis all stars which transit south of the zenith at upper culmination. This was because there may be a discontinuity in observations at the zenith introduced by changing from the " feet North " position usually adopted in observing azimuth stars to the " feet South " position. For near-zenith stars the observer's position is not always recorded. In practice, those stars with factors f numerically greater than $-2{\cdot}000$ have not been used. A star with a factor of $-2{\cdot}000$ transits nearly 2° north of the zenith, and all the stars used in the analysis would be observed " feet North " in the great majority of cases.

4. *The mean value of Δc; comparison with clock errors.*—If we divide the grand total of Table II by the grand total of Table III, we obtain the mean value of Δc for the period 1922–1938. The result is

$$\Delta c = +0''{\cdot}124 \pm 0''{\cdot}083.$$

This value is of the right sign, but is hardly significant in view of the probable error.

Values of the clock error difference, TC–B, are available from 1927 July onwards. A comparison with Δc from double azimuths is therefore possible for the period 1927 July to 1938 April. The mean value of Δc for this period is (again from Tables II and III)

$$\Delta c = +0''{\cdot}240 \pm 0''{\cdot}096.$$

If the collimation errors adopted for the Transit Circle were increased by Δc seconds of arc, the TC clock errors would be reduced by $0{\cdot}25\Delta c$ seconds of time. Assuming that the TC–B difference is due entirely to Δc, we have

$$\text{TC–B} = +0{\cdot}25\Delta c.$$

This result has been quoted in many of the annual volumes.

The mean value of $\Delta c = +0''{\cdot}24$ means that the average value of TC–B over the period considered should have been about $+0^s{\cdot}060$. A direct comparison with the mean difference of clock errors would be misleading, since the value obtained for Δc is necessarily a *weighted* mean, depending more on observations in the winter months than on the relatively few double azimuth observations in the summer. However, the monthly means of the difference TC–B, which are given in Table V, can be weighted in the same way, by multiplying each monthly mean by the appropriate monthly value of $100y^2$ given in Table III and dividing the sum by $\Sigma 100y^2$. The numbers of determinations of clock error, with both instruments, and their accuracy, are such that the weight depends only on the doubled azimuths. The result is TC–B $= +0^s{\cdot}064$, equivalent to a collimation error $\Delta c = +0''{\cdot}26$.

The agreement between the values of Δc obtained from double azimuths and from the TC–B difference is therefore very good. If the observations are divided into two groups, 1927–1932 and 1933–1938, for example, it may be shown that the close agreement for the whole period is to some extent fortuitous, but

TABLE V

Differences of clock errors, TC–B

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
	s	s	s	s	s	s	s	s	s	s	s	s
1927	...	...	...	...	...	...	+0·040	+0·031	+0·058	+0·054	+0·114	+0·134
1928	+0·142	+0·100	+0·081	+0·142	+0·052	+0·093	+0·106	+0·067	+0·076	+0·096	+0·107	+0·098
1929	+0·105	+0·101	+0·080	+0·070	+0·003	+0·050	+0·018	+0·036	−0·009	+0·058	+0·066	+0·034
1930	+0·028	+0·007	+0·053	+0·035	+0·059	+0·040	+0·017	+0·001	+0·013	+0·027	+0·037	+0·053
1931	+0·064	+0·034	+0·035	+0·086	+0·041	+0·052	+0·038	+0·053	+0·039	+0·022	+0·066	+0·074
1932	+0·083	+0·081	+0·102	+0·091	+0·067	+0·061	+0·031	−0·011	+0·007	+0·046	+0·062	+0·057
1933	+0·075	+0·084	+0·112	+0·108	+0·088	+0·064	+0·044	+0·031	+0·075	+0·067	+0·069	+0·084
1934	+0·093	+0·040	+0·074	+0·083	+0·080	+0·052	+0·029	+0·024	+0·049	+0·067	+0·042	+0·032
1935	+0·042	+0·049	+0·082	+0·064	+0·076	+0·062	+0·065	+0·098	+0·080	+0·076	+0·104	+0·100
1936	+0·103	+0·114	+0·119	+0·094	+0·106	+0·062	+0·016	−0·023	−0·014	+0·024	+0·078	+0·090
1937	+0·054	+0·088	+0·079	+0·056	+0·008	+0·038	+0·036	−0·001	+0·000	+0·016	+0·061	+0·074
1938	+0·101	+0·113	+0·092	+0·066	...	...	...	...	...	...	...	...
Means	+0·081	+0·074	+0·083	+0·081	+0·058	+0·057	+0·040	+0·028	+0·034	+0·050	+0·073	+0·075

the agreement is still sufficiently good to confirm that the TC–B discrepancy is primarily due to a systematic error in collimation.

The yearly mean values of Δc, as deduced from double azimuths, are tabulated in Table VI. The corresponding weighted estimates of Δc from the clock error difference, TC–B, are also given. The probable errors in column 2 were deduced on the assumption that the probable error of a single determination of azimuth remained at its mean value of about $\pm 0''\cdot 29$ throughout the period considered; they are inversely proportional to $\sqrt{(\Sigma y^2)}$ for each year.

TABLE VI

Yearly means of Δc

Year	Δc from double azimuths	Δc from TC–B
	″ ″	″
1927 (part)	+0·50 ± 0·41	+0·35
1928	+1·07 ± 0·29	+0·36
1929	−0·08 ± 0·30	+0·09
1930	−0·22 ± 0·28	+0·14
1931	−1·17 ± 0·34	+0·14
1932	+0·60 ± 0·44	+0·30
1933	+0·91 ± 0·35	+0·34
1934	−0·22 ± 0·40	+0·28
1935	+0·75 ± 0·22	+0·33
1936	+0·22 ± 0·34	+0·31
1937	−1·27 ± 0·56	+0·05
1938 (part)	+1·26 ± 1·20	+0·30

It will be seen that the values of Δc given in column 2 show variations from year to year which are much larger than the variations in column 3. In view of the probable errors, however, there is no significant disagreement between the results derived by the two methods. Moreover, the values of Δc in column 2 show a significant correlation coefficient of about $+0\cdot 83$ with the values in column 3.

The correlation between columns 2 and 3 would appear to imply that the variations from year to year are real; it is difficult to understand why the variations in column 2 should be so much larger than the variations in column 3.

5. *The annual term in* Δc.—In Tables II and III the columns have been summed to give totals of $100xy$ and $100y^2$ respectively for each month. These values have been analysed for an annual variation in the value of Δc. The data were divided into two groups, 1922 January–1930 December and 1931 January–1938 April. The inclusion of the data for the period 1922 January–1927 June, which cannot be compared with TC–B differences, appeared to be desirable in order to obtain as much information as possible. The two periods considered coincide with those covered by the *Second Greenwich Catalogue for 1925·0* and the *First Greenwich Catalogue for 1950·0* respectively.

The annual term in Δc was expressed as

$$\Delta c = u_0 + u_1 \cos\theta + v_1 \sin\theta + u_2 \cos 2\theta + v_2 \sin 2\theta,$$

where θ is an arbitrary phase angle equal to 30° in January, 60° in February, 90° in March and so on. A least-squares solution for the coefficients was carried out, taking account of weights. The results for the two periods were as follows:

1922 January–1930 December

$$\Delta c = -0''\cdot035 + 0''\cdot714 \cos\theta + 0''\cdot522 \sin\theta + 0''\cdot251 \cos 2\theta - 0''\cdot090 \sin 2\theta,$$

1931 January–1938 April

$$\Delta c = +0''\cdot389 + 0''\cdot532 \cos\theta + 0''\cdot150 \sin\theta - 0''\cdot214 \cos 2\theta + 0''\cdot275 \sin 2\theta.$$

A comparison of these two results suggests that there is a real variation in Δc with a period of twelve months, but the existence of a six-monthly term is doubtful. The analysis was therefore repeated, with the terms in 2θ omitted. The results were

$$1922\text{–}1930,\ \Delta c = +0''\cdot037 + 0''\cdot689 \cos\theta + 0''\cdot509 \sin\theta,$$

$$1931\text{–}1938,\ \Delta c = +0''\cdot266 + 0''\cdot579 \cos\theta + 0''\cdot133 \sin\theta.$$

A harmonic analysis of the clock error differences given in Table V yielded the series

$$1927 \text{ July–}1938 \text{ April},\ \Delta c = +0''\cdot245 + 0''\cdot049 \cos\theta + 0''\cdot089 \sin\theta.$$

This result does not confirm the existence of an annual term of the amplitude suggested by the double azimuth method.

6. *Other methods of estimating* Δc.—In view of the uncertainty in the determination of Δc it is of interest to consider other methods of obtaining an estimate of its value. Two more methods which have been investigated are discussed below.

(a) *From the R–D discordance in R.A.*—Regular observations of stars by light reflected in a mercury pool were made with the Transit Circle for many years. Observations in both coordinates, R.A. and Declination, were taken until about 1909, when the measurements in R.A. appear to have been discontinued. The results obtained by reflected light, R, showed systematic differences from those made by direct light, D. The R–D discordance in R.A. can be explained as the effect of a systematic error in the adopted level error, since the effect of a level error changes sign at the horizon, whereas the effects of collimation and azimuth errors do not.

An error Δc in the collimation will not give rise to an R–D discordance directly, but, since the line of collimation is used in deducing the level error from nadir readings, it is clear that the R–D difference can be equally well explained in terms of a collimation error or a level error. Since there is no apparent reason why the nadir readings for level error should be systematically wrong, it is reasonable to suppose that the collimation error, Δc, is responsible for the R–D discordance.

In the annual volumes of *Greenwich Astronomical Results* the value of the systematic error, Δb, in level error which could account for the R–D discordance has been tabulated for each star observed by both methods; weighted yearly means for stars transiting north and south of the zenith have been formed. Table VII, giving the values of Δc for the five years 1900–1904, is based on the assumption that $\Delta c = \Delta b$.

The north stars give a mean value of $\Delta c = +0''\cdot376$ for the five years and the south stars give $\Delta c = +0''\cdot397$. The combined mean is $\Delta c = +0''\cdot390$. An analysis of the north stars, which are all azimuth stars, for an annual term gave

$$\Delta c = +0''\cdot438 + 0''\cdot111 \cos\theta + 0''\cdot132 \sin\theta.$$

TABLE VII

Yearly means of Δc from R–D discordance

Year	North stars		South stars	
	Δc	Weight	Δc	Weight
	″		″	
1900	+0·45	103	+0·25	158
1901	+0·44	92	+0·45	224
1902	+0·39	49	+0·47	125
1903	+0·29	73	+0·50	131
1904	+0·20	46	+0·28	96

The observations could only be grouped into hours of R.A. so that it was necessary to assume, in deriving this series, that certain hours of R.A. would always be observed in certain months, e.g. $1^h\ 0^m$ to $2^h\ 59^m$ in November, $3^h\ 0^m$ to $4^h\ 59^m$ in December and so on. This assumption is not entirely satisfactory, since the observations of a particular star are likely to be spread over several months. The coefficients of $\cos\theta$ and $\sin\theta$ in the above expression are probably too small for this reason.

(b) *From an analysis of Greenwich corrections to the FK3.*—The mean effect of Δc on the determined places of clock stars is very small. Nevertheless, there is a differential effect varying with declination. This effect allows us to estimate Δc, provided we can compare the places derived with the Transit Circle with a system of star places which is free from error. Assuming the right ascensions of of the FK3 catalogue to be correct, and the Greenwich observations to be in error only as a result of collimation error, we obtain

$$\text{FK3 place} = \text{TC place} + \Delta c(\gamma - \bar{\gamma}).$$

γ is a factor defined by

$$\gamma = C + B + 3{\cdot}1A$$

and $\bar{\gamma}$ is the mean value of γ for all clock stars. The coefficient 3·1 is the average value of $(1{\cdot}256 + 1{\cdot}606 \operatorname{cosec} \delta)$ for all stars used in deriving the azimuth error.

The above equation allows us to calculate Δc from the tabulated differences of (TC − FK3). The TC places are based on observations made with the Transit Circle in the years 1922–1940. A least-squares solution for Δc yields a mean value of $+0''{\cdot}449$; when the $\cos\theta$ and $\sin\theta$ terms are included, the series for Δc becomes

$$\Delta c = +0''{\cdot}446 - 0''{\cdot}031 \cos\theta - 0''{\cdot}000 \sin\theta.$$

It was again necessary to identify hours of R.A. with the months in which they were most likely to be observed.

7. *Summary and discussion.*—Four different methods have been used to obtain estimates of Δc. The mean values obtained, and the periods to which they refer, are summarized in Table VIII.

The values of the coefficients u_0, u_1 and v_1 derived by the various methods are shown in Table IX. It must be noted that the coefficients u_1 and v_1 given in Table IX for the R–D and Greenwich corrections methods are probably smaller (numerically) than their true values.

TABLE VIII

Mean values of Δc*; summary*

Method	Period	Δc
Double azimuths	1922–1938	$+0''.124$
Double azimuths	1927–1938	$+0.240$
TC–B (weighted)	1927–1938	$+0.256$
R–D	1900–1904	$+0.390$
Greenwich corrections	1922–1940	$+0.449$

It is suggested that two main conclusions can be drawn from a study of Tables VIII and IX:

(i) there is an appreciable systematic error, Δc, in the collimation errors which have been adopted for use with the Airy Transit Circle; Δc is probably between $+0''.2$ and $+0''.5$, although there have probably been real changes in its value in the present century;

(ii) there is probably a real annual variation in Δc. The coefficients u_1 and v_1 are probably positive, but not so large as suggested by the double azimuth method alone.

TABLE IX

Values of the coefficients u_0, u_1, v_1

Method	Period	u_0	u_1	v_1
Double azimuths	1922–1930	$+0''.037$	$+0''.689$	$+0''.509$
Double azimuths	1931–1938	$+0.266$	$+0.579$	$+0.133$
TC–B	1927–1938	$+0.245$	$+0.049$	$+0.089$
R–D	1900–1904	$+0.438$	$+0.111$	$+0.132$
Greenwich corrections	1922–1940	$+0.446$	-0.031	-0.000

The advantage of the double azimuth method, considered in detail in this paper, is that it proves that it is the adopted collimation errors which are at fault. The analysis depends only on observations of times of transit of stars, of collimation and level errors and on the adoption of a clock rate. It is most unlikely that times of transit or adopted clock rates are systematically wrong. It was shown in Section 2 that an error in the adopted level error affects all determinations of azimuth equally, and has no effect on the determination of Δc. Thus, by a process of elimination, we are left with only the collimation error to account for the value of Δc.

The three other methods of estimating Δc provide valuable confirmation of its value, but are not in themselves " fundamental ". The value derived from TC–B clock differences rests on two assumptions, firstly that the tabular places of stars used with the Transit Circle are free from error and secondly that the determinations of clock error with Small Transit " B " are perfect. The R–D discordance gives a determination of $(\Delta c+\Delta b)$, where Δb is a possible systematic error in the nadir readings. We can only equate the value of $(\Delta c+\Delta b)$ with Δc on the reasonable assumption that Δb is insignificant. The analysis of Greenwich corrections rests on the assumption that the FK3 places of clock stars are entirely free from any error in R.A. which varies with declination. It is to be noted that each estimate of Δc obtained in this paper might be altered if it were possible to allow for the effects of pivot errors.

No satisfactory reason for the existence of a systematic error in the measured collimation can be suggested. The routine of observation for collimation error in the period under review was as follows.

The north collimator was first set on the south collimator, the observation being made through openings in the cube of the Transit Circle. The Transit Circle was then set in turn on the south and north collimators; the mean of the readings obtained was the deduced line of collimation. The setting of the north collimator on the south collimator was then repeated; any discrepancy between the original and final setting was attributed to " motion of the collimators " and an appropriate correction, usually small compared with the scatter of the readings, was applied. The Transit Circle, as designed by Airy and constructed, did not have a pierced cube. To set the north collimator on the south collimator the instrument had to be lifted out of its bearings. After it had been in use for some time, Airy decided to have the cube pierced, so that the setting could be made without raising the instrument, by viewing through the cube with the telescope in a vertical position. The design of the cube, with stiffening webs, was such that it was not possible to cut a circular hole in the cube. It has a number of small apertures, which produce diffraction effects, so that the definition of the south collimator wires, when viewed from the north collimator through the pierced cube, is always extremely poor.

Once a week, on Mondays, the telescope was raised out of its bearings so that an uninterrupted view of the south collimator could be obtained with the north collimator. It was found that the setting of the north collimator with the telescope raised differed systematically from the setting obtained when the south collimator was viewed through the cube. A correction to the deduced line of collimation was therefore always applied, on the assumption that the setting obtained with the telescope raised was the correct one. The values of this so-called " Up minus Down " correction applied from 1900 to 1938 are summarized in Table X. The corrections to the setting of the north collimator wire have been converted to corrections to the deduced collimation errors.

TABLE X

" Up minus Down " correction

Period	Correction to deduced collimation error
	$''$
1897–1905	+0·41
1906–1914	+0·27
1915–1921	+0·28
1922–1930	+0·31
1931–1940	+0·23

It will be seen that the " Up minus Down " correction is of approximately the same magnitude as Δc; if the correction had not been applied, the adopted collimation errors would have been smaller and the value of Δc approximately doubled.

The illumination used in the measurement of line of collimation was diffused daylight reflected into the collimators. In the absence of artificial illumination, collimation error could not be measured at night when the star observations would normally be obtained. In practice, collimation error was determined about once a day, usually in the morning. The mean of the observations for several days or weeks was taken and was used in reducing all observations in that period. The existence of a diurnal variation of collimation could introduce a systematic error into observations of stars made at night, since the adopted collimation error would differ from the true collimation error at the time of observation. However, when artificial illumination of the collimators was adopted in 1944, it was found that any diurnal term was very small and could certainly not account for a value of Δc of the order of a third of a second.

The collimation error is the difference between the observed line of collimation and the line to which times of transits of stars are referred. It seems most unlikely that the determinations of the latter could be in error by the equivalent of a third of a second of arc.

The star observations are corrected for the effect of diurnal aberration by subtracting $0''\cdot 19$ from the collimation error.

No significant correlation could be found between monthly values of Δc and monthly means of the measured line of collimation. If any annual term in Δc exists, it is of the opposite sign to the annual term in the measured collimation error itself, and the true collimation error is probably more nearly constant than the measured collimation error. This conclusion has also been suggested in *Greenwich Astronomical Results* for 1927 and 1928, in discussions of the TC–B differences.

The most probable source of error appears to lie in the determination of the setting of the north collimator. The poor definition of the image of the south wire system and the existence of an appreciable " Up minus Down " correction, referred to above, are unsatisfactory features in the determination of collimation error, although it is difficult to see how either could introduce a systematic error.

It is probable that the collimation errors of a reversible transit circle can be freed from systematic error more easily than can the collimation errors of a non-reversible instrument; it is a matter of regret that the Airy Transit Circle cannot be reversed in its bearings.

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