

PHOTOMETRIC STUDIES OF ASTEROIDS. IV. THE LIGHT-CURVES OF CERES, HEBE, FLORA, AND KALLIOPE*

IMAM I. AHMAD

Yerkes Observatory

Received June 16, 1954

ABSTRACT

The variation of the magnitudes of four asteroids—Ceres, Hebe, Flora, and Kalliope—has been investigated with the photoelectric photometers on the 82-inch reflector at the McDonald Observatory and on the 12-inch refractor of the Yerkes Observatory. The magnitude of Flora was nearly constant at the epoch of observation; for the other three asteroids the light varied, and the period of variation was determined. Ceres showed a range of 0.04 mag., with a period of $9^h04^m7^s$; Hebe a range of 0.16 mag. and a period of $7^h16^m5^s$; Kalliope a range of 0.14 mag. and a period of $4^h08^m8^s$. The latter is the shortest period that has been definitely established for an asteroid. All three asteroids show two maxima and minima. Hebe is the only asteroid observed by us over a long enough interval to establish the phase factor; the value is 0.045 mag/degree.

I. INTRODUCTION

Short-period variations in the light of asteroids were first detected about the turn of the century, although the rather large variation in the light with phase angle had been established by Müller and Parkhurst. A compilation of the early results is given by Miss Harwood (1924). The early observations were made photographically or visually, and the accuracy was rarely sufficient to establish the period. This is particularly true if the asteroid has two nearly equal maxima and minima, as in the case of 15 Eunomia, where the correct period is exactly twice that announced earlier (Kuiper, Harris, and Ahmad 1953). Photoelectric observations of asteroids were first made by W. Calder in 1935. His observations of Ceres are discussed later.

In order to obtain information regarding the light-variations of asteroids, a program was initiated by Dr. G. P. Kuiper to observe a number of objects with high precision, using the photoelectric technique (Groeneveld and Kuiper 1954). The light-curve of Vesta has been published by B. Stephenson (1951), and a preliminary report on the results for several other asteroids was given at the Amherst meeting of the American Astronomical Society (Kuiper, Harris, and Ahmad 1953). The present paper gives the results of four asteroids investigated by the writer in 1953 with the 82-inch reflector of the McDonald Observatory and the 12-inch refractor of the Yerkes Observatory.

II. PHOTOMETERS AND OBSERVING PROCEDURE

A description of the McDonald photometer and the modifications added in 1952 has been published by Johnson and Morgan (1951, 1953). It employs a 1P21 photomultiplier with a cast-iron shield to reduce interference from any existing magnetic field. The cell is refrigerated with dry ice to minimize the effect of dark currents. The focal-plane diaphragms are $\frac{1}{2}$, 1, 2, 3, 4, and 8 mm in diameter. A Fabry field lens is used to minimize guiding errors. The effective wave length with the yellow filter (Corning 3384 filter) is about 5550 Å. The amplifier has very high stability, with a linear relationship between input and output and a high gain, which depends only on the ratios of the resistances of wire-wound resistors. An offset mechanism allows quick changes from star to sky and back. A short description of the Yerkes photometer has been published by S. Provin (1953). The same types of cell and filters as used at McDonald are employed, though the multiplier is not refrigerated.

* *Contributions from the McDonald Observatory, University of Texas, No. 244.*

An observation consists of two deflections on a comparison star (generally very close to the asteroid, to minimize the effect of extinction); one deflection on the adjacent sky; two or three deflections on the asteroid and sky; and then two deflections on the comparison star. The mean of the asteroid deflections is compared with the mean of the comparison deflections. Differential extinction was applied when necessary, though it rarely exceeded 0.002 mag. All the observations were made through the yellow filters only. Experience shows that the yellow measures give less scatter than the blue, because of the smaller extinction coefficient which is, furthermore, practically independent of color. Since asteroids do not show appreciable variations in colors (Kuiper, Harris, and Ahmad 1953; Groeneveld and Kuiper 1954*b*), little is gained by observing in more than one

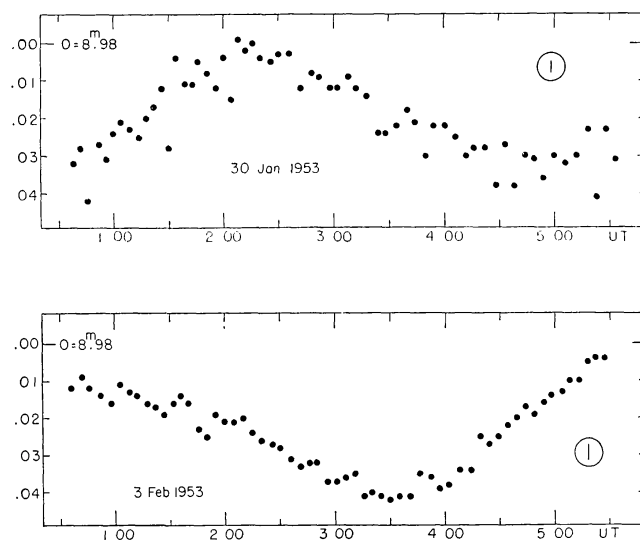


FIG. 1.—Single night's runs on 1 Ceres

color. This procedure also has the advantage of providing a large number of points as closely spaced as possible on the light-curves.

III. THE LIGHT-CURVES

CERES (NO. 1)

Ceres was announced to be variable, with a range of 0.2 mag., by Bailey and Campbell (1904), but they did not establish a period. In 1935 Calder observed Ceres photoelectrically and obtained a variation of 0.03 mag. He interpreted his observations as giving a period of $3^{\text{h}}36^{\text{m}}$. His observations extended over a sufficiently long interval of time to establish the phase coefficient of 0.050 mag/degree, a value which we have adopted in our reductions.

Ceres was observed on six nights at the McDonald Observatory, beginning on January 26, 1953. The main comparison star used was BD+20°703. The Yale zone catalogue gives its spectral type as F8; and its magnitude and colors as observed on Johnson's system (1953) are $V = 9.18$ mag.; $B - V = +0.63$ mag., and $U - B = +0.13$ mag. Other comparison stars used were reduced to this one differentially.

The individual observations amounted to 272 in all. The longest runs, made on January 30 and February 1, 3, and 8, consist of about 60 observations each, over $4\frac{1}{2}$ –5-hour intervals, and are plotted in Figures 1 and 2. The individual observations were all re-

duced to mean opposition by applying corrections for the distances from the earth and sun,

$$\Delta m = 5 \log \frac{a(a-1)}{r\Delta},$$

where r and Δ are the distances of the asteroid from the sun and earth and a is the semi-major axis of the asteroid, 2.7674 A.U. (1947). The observations were also corrected for phase angle to January 30, 1953, with Calder's coefficient of 0.050 mag/degree.

Comparison of the observations made on different nights showed that the asteroid had a period of $9^{\text{h}}04^{\text{m}}7$. The observed times were then reduced to January 30, and the mean magnitudes were determined for every five observations consecutive in phase. These normal points are plotted in Figure 3. The range is found to be only 0.037 mag. There are two maxima, differing by 0.014 mag., and two minima, whose difference is negligible. From the mean light-curve we find that Ceres — comparison star = -1.377 mag., corrected for distance changes and phase changes with respect to January 30, 1953. With

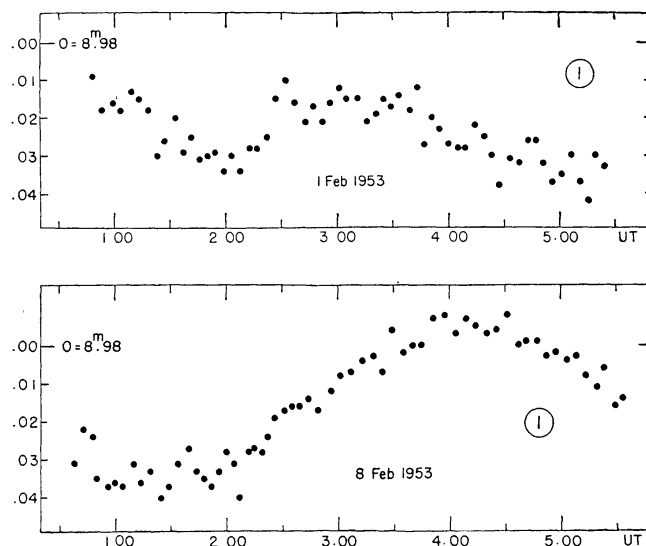


FIG. 2.—Single night's runs on 1 Ceres

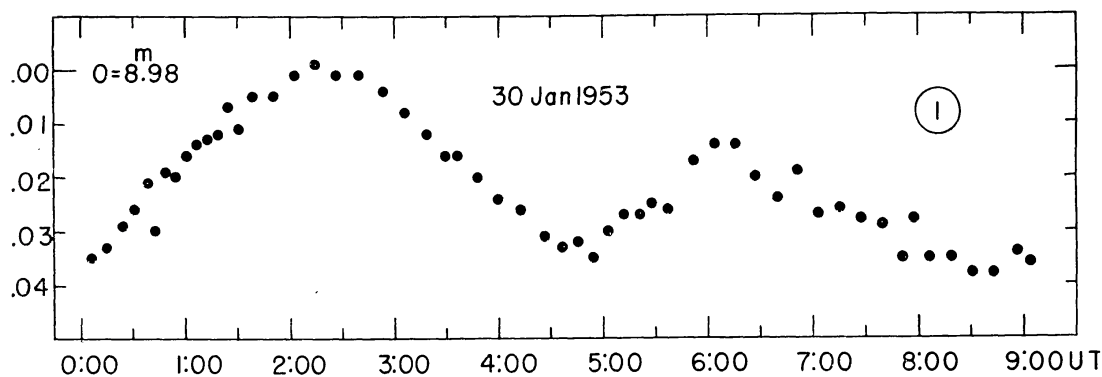


FIG. 3.—Light-curve of 1 Ceres

$V = 9.18$ mag. for the comparison star, we find Ceres = 7.803 mag. on January 30, 1953. As the phase angle was $19^{\circ}.7$, the correction required to reduce to zero phase angle is about -0.985 mag. Therefore, the magnitude at mean opposition is about 6.82 on Johnson's system. We can express the magnitude of Ceres as follows:

$$\text{Ceres} = 6^m82 + 0.05i + 5 \log r\Delta - 3.447 + \Delta m,$$

where Δm is dependent on the rotation phase. Calder's photoelectric observations were reduced to January 13, 1935, with our period of $9^h04^m.7$ and the Calder's phase coefficient of 0.050 mag/degree. These observations are plotted in Figure 4.

As the amplitude of the rotational variations is only about 0.03 mag., it may be neg-

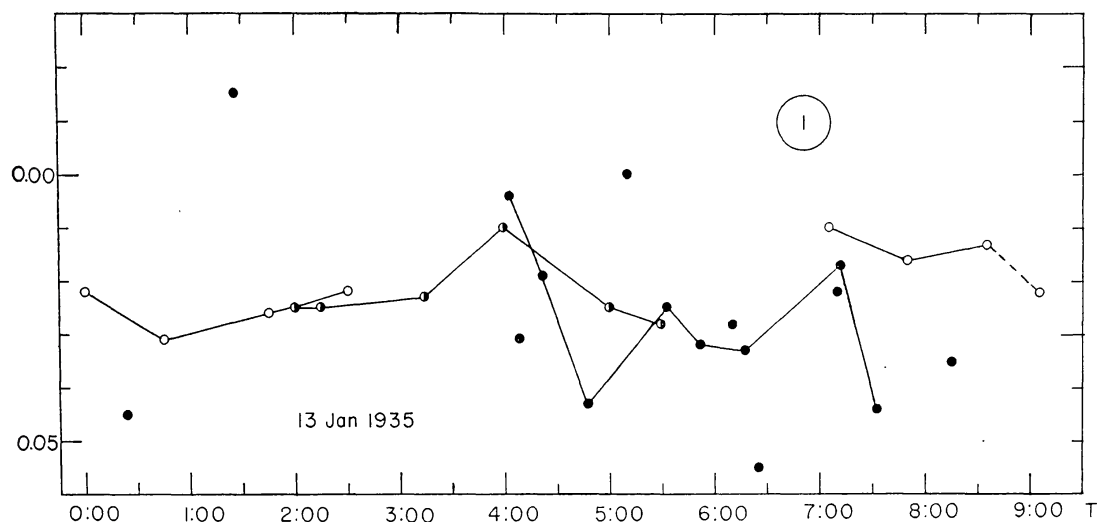


FIG. 4.—Calder's observations on 1 Ceres with the present period of $9^h04^m.7$. The connected filled circles are observations on December 15, 1934, U.T.; the half-filled circles on January 12, 1935; the open circles on January 13, 1935; and the individual filled circles are observations on other dates. The zero corresponds to Ceres *minus* the mean of the two comparison stars, HD 69786 and HD 69866 = -1.28 mag. T = E.S.T., January 13, 1935.

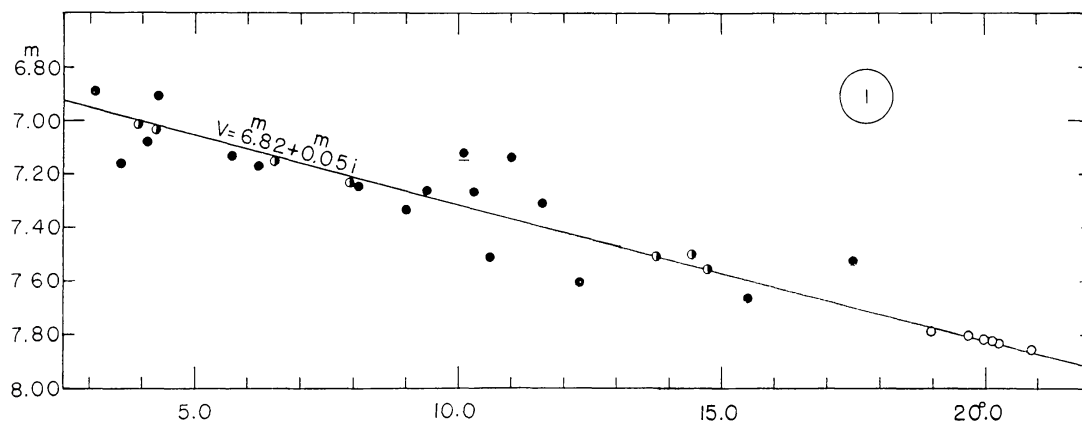


FIG. 5.—The magnitude-phase relation of 1 Ceres. The open circles are the present observations; the half-filled circles represent Calder's observations; and the filled circles are Müller's observations.

lected in the determination of the phase coefficient. Applying a zero-point correction to Calder's and Müller's observations (1893), all the observations may be plotted as in Figure 5. It is seen that the present observations, as well as Calder's, fit the magnitude-phase relation very well. Müller's observations show considerable scatter but are not in disagreement with the adopted phase coefficient of 0.050 mag/degree. Müller's value of 0.042 mag/degree is obviously of considerably lower weight. Furthermore, phase coefficients determined from different oppositions may actually differ somewhat (Groeneveld and Kuiper 1954*b*).

HEBE (NO. 6)

The variability of Hebe was reported by E. Bianchi and E. Padova in 1921. From visual observations on three nights they announced that the asteroid varied by 0.15 mag. in a period of 3^h14^m.

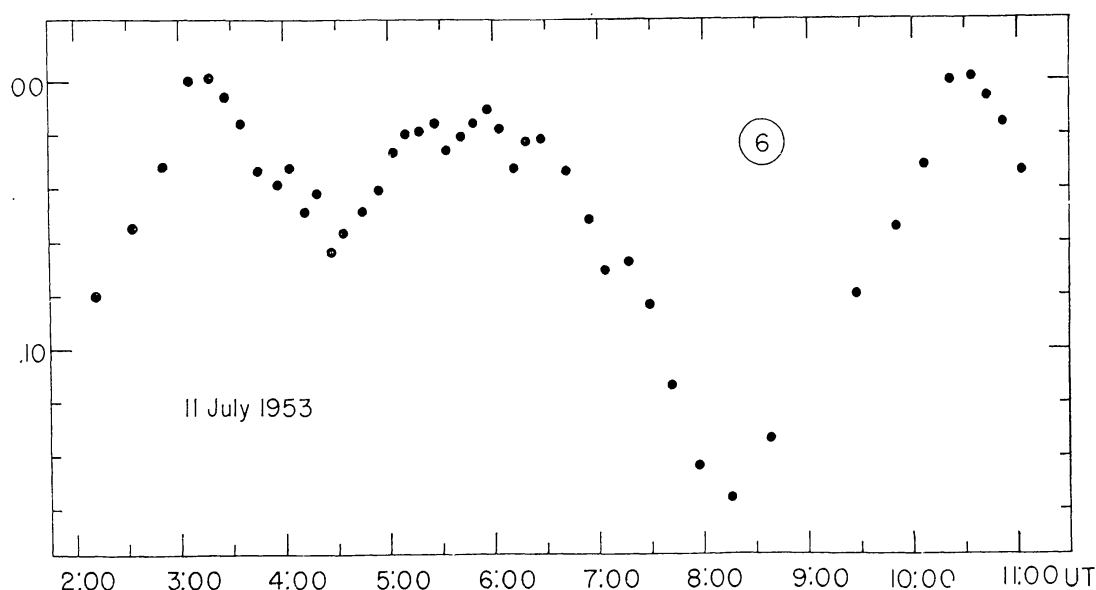


FIG. 6.—Light-curve of 6 Hebe. The zero corresponds to $\Delta\text{Hebe} - (\text{BD} + 9^\circ 52' 03'') = +0.103$ mag.

The writer observed Hebe on eight nights, beginning June 14, 1953, with the 12-inch refractor of the Yerkes Observatory. The main comparison star to which the others were reduced is BD $-9^\circ 52' 03''$. The Yale zone catalogue gives its spectral type as F2 and its magnitude as 8.3. The individual observations totaled 186 in all. By superposing plots of the observations made on different nights, we found a period of 7^h16^m5. With this period the individual observations were reduced to July 11, 1953, and normal points for each five consecutive observations in phase were formed. The normal points are plotted in Figure 6. It is seen that the asteroid varies through a range of 0.156 mag. The curve has two maxima, differing by only 0.011 mag., while the two minima differ by 0.100 mag.

The observations extend over a range in phase angle large enough for a determination of the phase coefficient. On July 11, 1953, the mean difference Hebe—comparison star was 0.163 mag. The corresponding observed magnitude differences on the other nights were corrected for distance to mean opposition, with $a = 2.4254$ (1947). A plot of the magnitude differences against phase angle gives 0.045 mag/degree for the phase coefficient of Hebe.

To obtain the magnitude of Hebe at mean opposition, we have used Müller's (1893)

observations. These have been corrected by $+0.04$ mag., based on the magnitude of Müller's comparison star, BD-4°4376 as determined recently by Harris on the Johnson-Morgan V system. Combining these observations with the phase coefficient determined above, we obtain $V_0 = 8.45$ for the magnitude of Hebe at mean opposition.

Assuming $V_0 = 8.45$, the zero point of our observed magnitude differences is reduced to the V system. Figure 7 shows a plot of our reduced magnitudes, as well as the observations of Müller versus phase angle. The line $V = 8.45 + 0.045i$ appears to represent the observations fairly well, except for the night of June 23, which deviates by 0.08 mag.

FLORA (NO. 8)

In 1921 E. Padova obtained a variation of 0.12 mag. for 8 Flora. In 1950 R. Rigollet observed no variation. No period has been announced for the suspected variability.

Flora was observed at the McDonald Observatory for 2 hours on February 11, 1953,

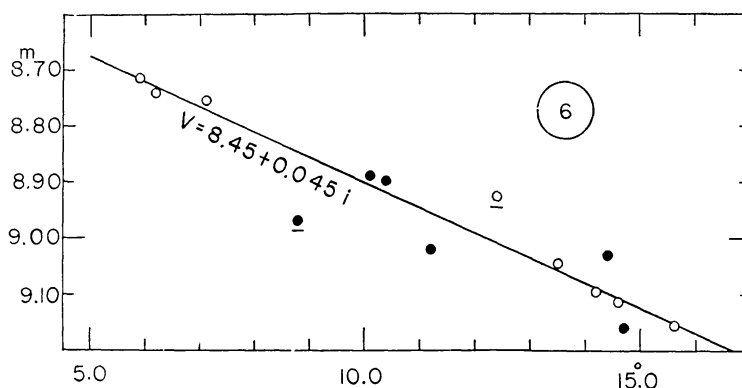


FIG. 7.—The magnitude-phase relation of 6 Hebe. Open circles are the present observations and filled circles those of Müller.

after which the observations were interrupted by clouds. On the next night a 6-hour run was obtained, with BD+15°2249 used as comparison star. The 6-hour run is plotted in Figure 8 and shows that there was no appreciable variation during the interval. If the asteroid is assumed to be constant in light over this interval, the probable error of a single observation is found to be ± 0.0028 mag.

The constancy of the light of Flora can, of course, be interpreted as due to a spherical object or to the circumstance that the axis of rotation was directed toward the earth at the epoch of observations. The observations made by Kuiper and Groeneveld on March 27, also plotted in Figure 8, still fail to show decided variation, although the aspect of the asteroid had changed. It is desirable that this asteroid be investigated further, as it is the only one so far observed photoelectrically which was found constant in light within the accuracy of measurement.

KALLIOPE (NO. 22)

A light-variation of (22) Kalliope was announced by S. Bailey in 1904, who reported a range of 0.3 mag. from seven observations on three nights.

Kalliope was observed for 2 hours on February 13, 1953, when the observations were interrupted by clouds. On February 15 a run of over 6 hours showed that this asteroid has the very short period of variation of $4^{\text{h}}08^{\text{m}}.8$. This is the shortest period so far determined in the current investigation of asteroids. The observations of February 15 are plotted in Figure 9 as filled circles; the observations of February 13 are plotted as open

circles after a zero-point adjustment of $+0.356$ mag. was made. The comparison star on February 15 was BD+30°2078.

The amplitude is 0.141 mag.; the two maxima differ by 0.016 mag. and the two minima by 0.028 mag.

IV. DISCUSSION

The periods of the light-variation of several asteroids have been published. Except for Eros, the period has been determined from observations made during a relatively short interval of time, so that the changing distance and orientation of the asteroid with respect to the sun and the earth may be neglected. These changes must be taken into consideration when the observations cover a long interval or when we discuss light-curves at different oppositions (Huruhata 1940).

By combining light-curves made at different epochs, H. N. Russell (1906) has shown

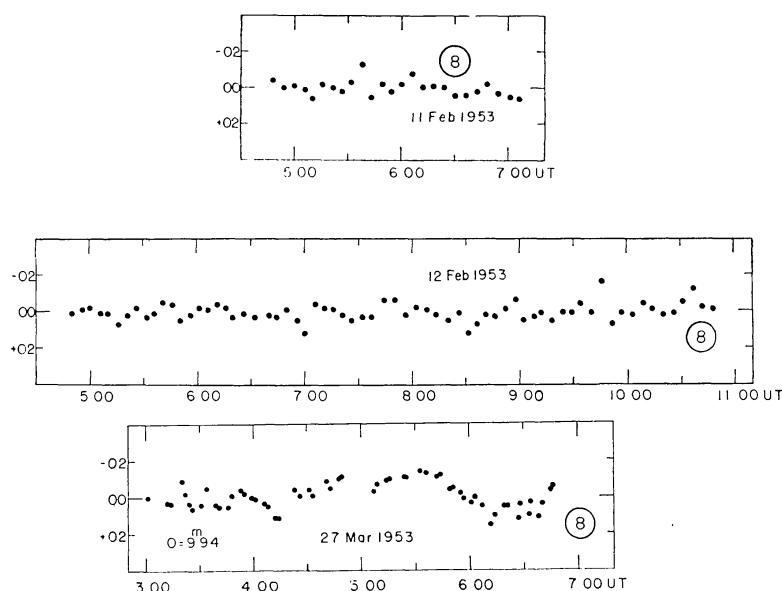


FIG. 8.—Single night's runs on 8 Flora. The zero represents $\text{Flora} - (\text{BD} + 15^\circ 2249) = -0.061$ and -0.262 mag. on February 11 and 12, respectively.

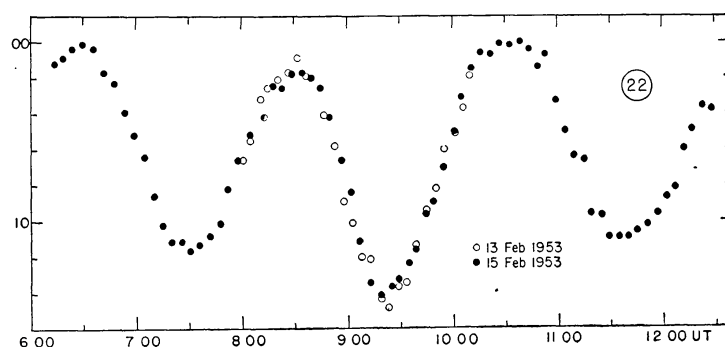


FIG. 9.—Light-curve of 22 Kalliope. Filled circles represent observations on February 15, 1953, U.T., and open circles observations on February 13, 1953, U.T. The zero corresponds to $\text{Kalliope} - (\text{BD} + 30^\circ 2078) = +0.394$ mag.

that it is possible to determine the position of the asteroid's equator. However, the sign of the inclination remains unknown. It is also impossible to determine the shape of the asteroid unambiguously or the distribution of brightness over the surface. If any continuous convex form is possible, then many other such forms are possible too; also, a great variety of brightness distributions on the surface may give rise to the same light-curve.

A list of asteroids with known periods and the observed ranges is given in Table 1,

TABLE 1

PERIODS AND AMPLITUDES OF ASTEROIDS

Asteroid	Period	Amplitude	Reference*	Asteroid	Period	Amplitude	Reference*
22 Kalliope	4 ^h 09 ^m	0 ^m 14	a	3 Juno	7 ^h 13 ^m	0 ^m 15	b
354 Elenora	4 16	.15	b	6 Hebe	7 17	.16	a
9 Metis	5 05	.11	b	1 Ceres	9 05	.04	a
511 Davida	5 10	.06	c, b	25 Phocaea	9 57	.18	b
39 Laetitia	5 11	.18	c, b	14 Irene	11?	.04	b
433 Eros	5 16	.0-1.5	d	17 Thetis	12 17	.32-0.36	b
4 Vesta	5 20†	.10-0.12	e, c, b	532 Herculina . . .	16-17?	.08	b
15 Eunomia	6 05	.49	c, b	10 Hygiea	18?	.10	b
44 Nysa	6 25	.41	b	8 Flora		≤0.01	a
7 Iris	7 07	0.22	c, b				

* References are as follows:

a: Present paper.

b: Groeneveld I., and Kuiper, G. P. 1954, *Ap. J.*, **120**, 200 and 529.

c: Kuiper, G. P.; Harris, D. L.; and Ahmad, I. I. 1953, *Sky and Telescope*, **12**, 129.

d: Watson, F. 1937, *Harvard Circ.*, No. 419; Huruata, M. 1940, *Harvard Circ.*, No. 442.

e: Stephenson, B. 1951, *Ap. J.*, **114**, 500.

† Period may be 10^h41^m.

arranged by the length of the period. All the asteroids are intrinsically bright, except for Eros. The larger range observed for Eros is undoubtedly due to its being an elongated irregular object.

The minimum rotation period may be estimated from the results given by Jeans (1919) for an incompressible and homogeneous mass of fluid,

$$P^2 = \frac{2\pi}{0.187G\rho},$$

which gives $P = 12,000$ seconds or 3.3 hours for a density of 3.5. The smallest rotation period so far observed of 4^h09^m is 5/4 times this predicted minimum.

I wish to thank Dr. D. L. Harris for suggesting this problem and for his general guidance; Dr. G. P. Kuiper for valuable discussions; and Drs. Kuiper and Groeneveld for making available for Section IV their unpublished data. I am indebted to Miss Delphine Jehoulet, Miss Lucille Schott, Mrs. Beverly Negaard, and Mrs. Phyllis Erikson for technical help.

REFERENCES

- Bailey, S., and Campbell, L. 1904, *Harvard Ann.*, **72**, 165.
 Bianchi, E., and Padova, E. 1921, *Soc. astr. ital.*, ser. 2, p. 47.
 Calder, W. A. 1935, *Harvard Bull.*, No. 904.
 Cincinnati Observatory. 1947, *Elements and Opposition Ephemerides of Minor Planets*, p. 1.
 Groeneveld, I., and Kuiper, G. P. 1954a, *Ap. J.*, **120**, 200.
 ———. 1954b, *ibid.*, **120**, 529.
 Harwood, M. 1924, *Harvard Circ.*, No. 269.
 Huruata, M. 1940, *Harvard Circ.*, No. 442.
 Jeans, J. H. 1919, *Problems of Cosmogony and Stellar Dynamics* (Cambridge: At the University Press), p. 269.
 Johnson, H. L., and Morgan, W. W. 1951, *Ap. J.*, **114**, 552.
 ———. 1953, *ibid.*, **117**, 314.

- Kuiper, G. P.; Harris, D. L.; and Ahmad, I. I. 1953, *Sky and Telescope*, **12**, 129.
Müller, G. 1893, *Publ. Astr. Obs. Potsdam*, **8**, 289 and 295.
Padova, E. 1921, *Soc. astr. ital.*, Ser. 2, p. 90.
Provin, S. 1953, *Ap. J.*, **117**, 21.
Rigollet, R. 1950, *C.R. Acad. sci., Paris*, **230**, 2077.
Russell, H. N. 1906, *Ap. J.*, **24**, 1.
Stephenson, B. 1951, *Ap. J.*, **114**, 500.