

PHOTOMETRIC STUDIES OF ASTEROIDS. I*

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ABSTRACT

This paper is the first of a series describing a systematic photometric study of the asteroids. Fourteen light-curves based on 27 nights, are communicated here, and four color-curves. The principal data are summarized in Table 1 and the diagrams. The rotation of Eunomia is discussed in Section IV and found to be retrograde.

I. PLAN OF INVESTIGATION

A systematic photometric study of the asteroids was organized and initiated at the McDonald Observatory by Mr. Kuiper in 1949. It consisted of two parts: (A) A systematic survey with a 10-inch, $f/7$ telescope, which was to cover all asteroids down to about 16.5 mag. photographic. The purpose of this photometry was to obtain approximate magnitudes on the international photographic scale and to reach a known degree of completeness, so that the statistics of the asteroids down to $m = 16$ could be placed on a reliable basis. (B) Detailed photometry of all asteroids brighter than about 10 and of selected fainter ones; the purpose of this work was to obtain periods of rotation; the characteristics of the light-curves; the colors and possible color variations; and, by continued observation over a number of years, some information on the shapes of these bodies and on the orientations of their rotational axes in space. The present report, the first of a series, is concerned largely with results obtained under program B. It seems useful, however, to make at this time a brief statement about the status of program A.

Program A was carried out with the 10-inch Ross-Fecker telescope belonging to the Cook Observatory of the University of Pennsylvania, which, through the gracious cooperation of Dr. Olivier and the officers of the University of Pennsylvania, was acquired on a loan basis for a period of 5 years. Later the period of the loan was extended. It was shipped to the McDonald Observatory during the summer of 1949 and, after the necessary overhaul and repairs, was put into operation in a new housing in June, 1950. The observations for the regular asteroid program started in August, 1950, and were completed May 26, 1952. The program of observations consisted of the photography during each dark-of-the-moon period of an area of $40^\circ \times 40^\circ$, centered on the opposition point of the sun at the time of new moon. This program was continued without interruption for 23 lunations, so that a belt 40° wide, centered on the ecliptic, was covered in nearly two complete annual cycles. Since the synodic period of an asteroid situated at 2.8 astronomical units is 15 months, ample overlap was provided in this manner. Essentially all asteroids brighter than 16 and with a ≥ 1.7 will have been included; only occasional objects with high inclinations will have been missed. The plates are all 8×10 inches in size, exposed for 10 minutes each, taken in pairs separated by about 1 hour. This caused the asteroid images to appear essentially stellar and to move from 2 to 6 image diameters between the two plates. Usually 48 plate pairs were needed to cover the monthly area of $40^\circ \times 40^\circ$. Consecutive monthly centers were about 30° apart, so that a safe margin of overlap was provided each month. In all, over 2400 program plates were taken, including Selected Areas, one for each night. The observations were made almost entirely by two night assistants, first, Mr. W. Braun and, later, Mr. H. Rubingh, who carried out their assignments with great devotion and care.

The reduction of the plates consists of four stages: (1) the blinking of all the plate

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pairs, in order that an entirely independent survey of the asteroid belt becomes available down to $m = 16$ or 16.5; (2) the measurement of the positions and daily motions of the asteroids found; (3) the identification of the known objects and the re-examination of the plates for known objects missed; and (4) the measurement of the magnitudes of known and new asteroids.

At this time of writing, over 500 pairs have been blinked, about 45 per cent of the total, while the measurements have nearly kept pace with the blinking. The reduction work was started by Messrs. A. Fujita and J. Kent and is currently continued by Miss I. Groeneveld, and Messrs. G. Van Biesbroeck, A. Gehrels, and C. J. Van Houten, ably assisted by Mrs. H. E. Thorson. The results are being prepared for publication in consultation with the Minor Planet Center at Cincinnati.

Program B became practicable after the introduction of the new photoelectric techniques with the 1P21 photomultiplier tubes. Nearly all observations were made with the 82-inch telescope. They were begun in 1949 by Messrs. Kuiper and Shatzel and have been continued at regular intervals since. It was clear that the most efficient observing procedure would be to make long continuous runs on first-class nights, of preferably 8 hours or more, so as to simplify the problem of finding the period and the shape of the light-curve. Often two or three consecutive nights were used to cover one object.

Mr. Shatzel is publishing the result of 44 Nysa in a separate paper, while some other 1949 observations are given below. The work was continued in 1950 and 1952 by Messrs. Harris and Kuiper; most of these results are incorporated in this paper. Early in 1953 Messrs. Ahmad and Harris obtained extensive observations on asteroids 1, 8, and 22, which were reduced by Mr. Ahmad and will be published by him. Later in 1953 the present authors observed Nos. 8, 15, 17, 39, and 511; most of that material is included here. The reductions of the observations here discussed were made by Miss Groeneveld and Messrs. Ahmad and Harris. Mr. Harris' contributions to this program have been so extensive that he may in several ways be regarded as a co-author of this paper. Some of the comparison stars used in this paper were kindly measured by Mr. Hiltner, who has also advised us on procedures of measurement and reduction. Additional measures, made in December, 1953—January, 1954, are now being prepared for publication. A preliminary report on some of the material included in this paper was presented by Kuiper and Ahmad (1953) at the December, 1952, meeting of the A.A.S.

II. THE PHOTOELECTRIC OBSERVATIONS

All observations were made with the 82-inch telescope. The photometers and amplifiers used were constructed by Messrs. Hiltner and Johnson, to whom the authors are greatly indebted. The blue filter used in 1949 and 1950 had a somewhat wider transmission band than that used thereafter, which is the standard combination introduced by Johnson and Morgan (1951); furthermore, since 1953, ultraviolet observations have been added. The 1949–1950 colors are therefore not on the “standard” system, defined later, but can be used to check for the presence of color variations.

The magnitude system adopted is that defined by Johnson and Morgan (1953). The standard stars were all taken from that paper. The normal procedure was to observe, first, a pair of standard stars of very different color, then the asteroid and its comparison star, in turn, throughout the entire run; and, finally, another pair of standard stars. The standards were observed in three colors, B , V , and UV , but the comparisons between the asteroid and the comparison star were mostly made in yellow light, with only occasional measures made at the two other wave lengths. In several of the 1950 runs all comparisons were made in both yellow and blue. Since, however, it was found that the asteroids do not change their colors appreciably in the course of their light-variation, this procedure was abandoned, and blue comparisons were made thereafter only for color measurements two or three times per night. All photometer readings were made in duplicate, separated by a sky reading in the same color.

The data were recorded on a Brown potentiometer, with individual readings lasting about 40 seconds. The comparison stars were selected for their proximity to the asteroid, usually within $\frac{1}{4}^\circ$, and for their similarity in color and magnitude. Comparison stars used for the same asteroid on different nights were intercompared afterward.

The coefficient of extinction was occasionally determined from the observations of the standards and the comparison star; usually, a mean extinction coefficient was used to correct for the differential effects between the standards and the comparison star. Differential extinction between the comparison star and the asteroid was allowed for when necessary, but was always very small. The differential extinction effects caused by the small differences in color were negligible for the yellow measures used in the light-curves. The only remaining correction to the standard system is that introduced by the use of different multiplier tubes at different times, which may cause slight shifts in effective wave length even if used in combination with the standard filters. This correction is in the nature of a color term and has been evaluated and applied.

III. THE LIGHT-CURVES

The light-curves derived as described earlier are reproduced in the figures. These are arranged according to a uniform procedure: the number of the asteroid, the date of observation, and the U.T. are given, as well as the magnitude of the asteroid at maximum light expressed in the standard system. In a few cases the magnitude of the maximum is missing because no suitable standards had been measured. Individual pairs of measures are plotted as points; open circles represent uncertain values, unless a different explanation is given in the legend. Supplementary data, among which are the geocentric coordinates of the asteroid at the time of observation, are given in Table 1. These coordinates determine the *aspect* of the object and are needed when different light-curves of the same asteroid are compared. The U.T. values given are as observed, uncorrected for light-time.

The principal derived quantities are collected in Table 1. The columns contain, respectively, the number of the asteroid, the date of observation, the observers, the visual magnitude of the maximum, $B-V$ and $U-B$, all three in the standard system; the amplitude in visual light, the period and its estimated uncertainty, the mean magnitude derived from the entire light-curve; the resulting quantities m_0 and g used in ephemerides; the phase coefficient; the interval covered by the observations, in hours; the R.A. and Dec. of the asteroid at the time of observation (aspect); and the figure number showing the observations. For two cases the phase coefficients were computed from our own data, as mentioned in the discussion of the light-curves. For five others Müller's (1897) phase coefficients were used; they are designated by M . For 97 and 511 the adopted values are somewhat arbitrary. Improved phase coefficients will lead to more accurate values of m_0 and g . Dr. W. Strobel, of the Rechen Institut at Heidelberg, kindly supplied us with the heliocentric and geocentric distances of the majority of the asteroids observed here, while Dr. D. L. Harris kindly computed the remainder.

Vesta (No. 4).—Reference is made to a previous publication by Stephenson (1951), based on a one-night run with the 82-inch telescope by Johnson and plotted in our Figure 1 and six nights with the Yerkes Observatory 12-inch telescope. The period derived was 5^h20^m , but the possibility of the double period (10^h41^m) was left open. Stephenson also listed previously announced periods and color changes. Mr. Harris secured a 10-hour run with the 82-inch telescope, which is shown in Figure 2. This long run shows two very similar cycles and would suggest that the shorter period is correct. However, additional observations from different aspects are needed before this can be considered established. For comparison, the earlier 82-inch observations in Figure 1 are shown with the short period; the original observations are plotted as dots and have been replotted one period earlier or later as triangles, so as to bring out the shape of the light-curve. It appeared necessary to add a vertical displacement of 0.013 mag. per cycle, which is in approxi-

TABLE 1
SUMMARY OF PHOTOMETRIC RESULTS

No.	Day	Month & Year	Obs.	V of Zero	V - B	U - B	Amp.	Period	Mean V of One Period	m ₀	g	Phase Coeff.	Interval Observed (Hours)	R.A.	Dec.	Fig.			
4.....	{ 22 7 }	Dec., 1950	J	7.30			0.13	5 ^b 20 ^m .5	7.36	6.12	3.58	0.022M	6.0	2 ^h 26 ^m	+ 6°5	1			
		Mar., 1952	H	6.10	0.78	0.45	.12	(see text)	6.15	6.04	3.50	.022M	10.1	10 44	+19°0	2			
7.....	{ 12 13 14 16 28 }	Aug., 1950	K, H	*			.29	7 07.5 ± 2	*	*	*	.019M	{ 2.2 3.7 2.4 1.7 }	18 06	-19°5	3			
		Aug., 1950	K, H																
		Aug., 1950	K, H																
		Aug., 1950	K, H																
9.....	{ 16 28 }	Aug., 1950	K, H	8.11	.84	.41	.22	7 03 ± 2	8.22	8.67	6.08	.019M	7.9	7 54	+13°9	4			
		Jan., 1952	K, H	8.27			.11	5 03 ± 2	8.34	6.31	8.91	.041M	6.6	2 41	+10°5	5			
15.....	{ 11 15 25 7 }	Nov., 1949	K, S	*			.43	6 05	*	*	*	.042	{ 4.6 3.6 6.0 4.2 }	19 18 19 16 5 58 12 03	-20°3 -20°2 +26°0 -18°6	6 (8) 7 (8) 9 10			
		Aug., 1950	K, H																
		Aug., 1950	K, H					6 05					8.86				8.63	5.44	(.042)
		Jan., 1952	K, H		8.62	.82		.44					.48					9.83†	8.66†
17.....	{ 28 31 2 19 }	Apr., 1953	G, K	9.66	.812	.41	.42						2.3	11 46 11 45 11 42 11 33	+ 9°7 +10°1 +10°3 +11°3	14 11 (13) 12 (13) 15			
		Mar., 1953	G, K	10.82	.865														
		Mar., 1953	G, K	10.871	.843		.32	12 16.5	11.02†	10.87†	8.07†	.040							
		Apr., 1953	G, K	10.912	.822	.414	.32												
25.....	{ 5 6 7 }	Apr., 1953	G, K	11.188	.859	.460	.36						{ 3.2 5.1 0.9 4.7 }	19 11	+22°3	16			
		Aug., 1950	K, H	9.93†			.18	9 56.7	10.02:	10.90:	8.27:	.025M							
39.....	{ 29 4 10 }	Aug., 1950	K, H				.18	5 11	10.28	9.83	6.38	.022M	6.5	6 26	+11°6	17			
		Jan., 1952	K, H	10.17	.89	.52							(5.7)	12 15	+ 6°2				
		Apr., 1953	G, K	10.53	.86		.22	5 09.5 ± 1	10.65	9.87	6.42	.022M	5.7	12 11	+ 6°8	18			
		Apr., 1953	G, K		.88	.49													
97.....	{ 4 5 }	Nov., 1949	K, S	*								(.040)	4.0	1 09	- 8°8	19			
		Nov., 1949	K, S	*									3.8	1 08	- 8°8				
511.....	{ 26 8 }	Jan., 1952	K, H	9.95	.71	0.34	.06	5 07 ± 5	9.98	10.62	6.40	(.030)	7.3	7 36	+25°0	20			
		Apr., 1953	G, K	11.309	0.724		.25	5 13 ± 4	11.44	10.87	6.65	(0.030)	5.9	13 47	+12°7	21			

* Data incomplete; to be published later. † Half-period used only. ‡ Without color reduction.

mate agreement with the shift expected from the motion of the asteroid. The change in the light-curve between Figures 1 and 2 is apparent. If the short period is confirmed in future observations, Vesta will be the only object so far observed showing one maximum and one minimum during a cycle; this circumstance would lead one to look on the 5-hour period with suspicion.

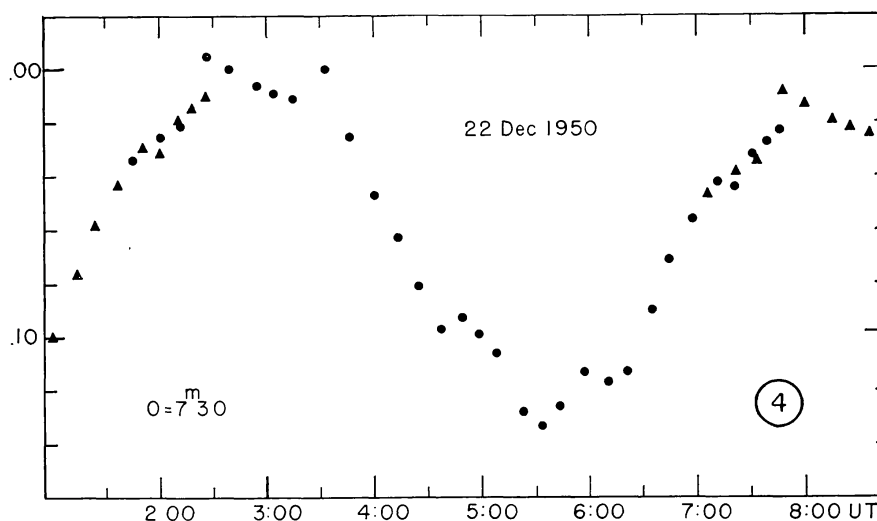


FIG. 1.—Light-curve of 4 Vesta. Triangles are observations replotted one cycle earlier or later and moved vertically 0.013 mag.

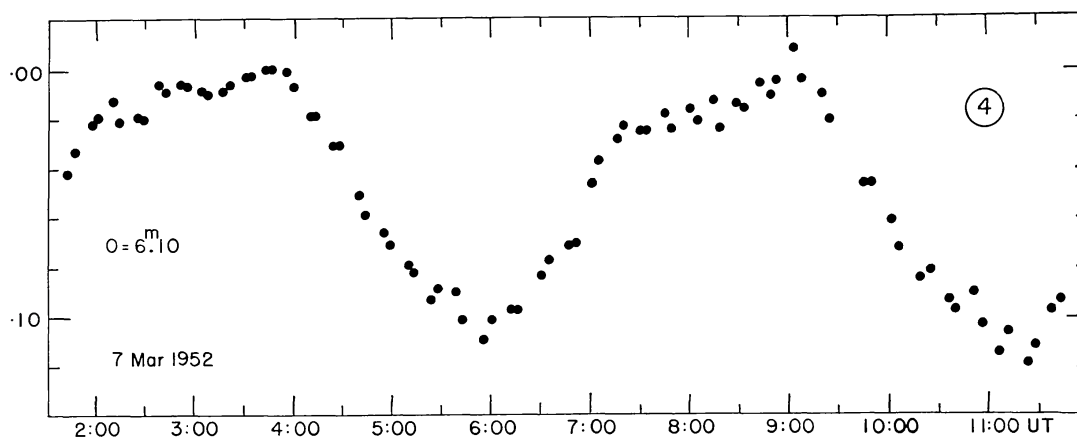


FIG. 2.—Light-curve of 4 Vesta. Single night's run

The phase angles of the 1950 and 1952 observations are $18^{\circ}0$ and $4^{\circ}3$, respectively. The g values may be brought into coincidence, at 3.48, by using the coefficient 0.028 mag/degree.

Reference is made to an important series of 138 photoelectric measures of Vesta published by Giclas (Slipher *et al.* 1951), covering the period of October 8, 1950, to March 6, 1951. The measures were made in both blue and yellow light and lead to phase coefficients of 0.035 and 0.033 mag/degree, respectively. Actually, some small deviation from a linear relationship between magnitude and phase angle was found. Giclas derived the period

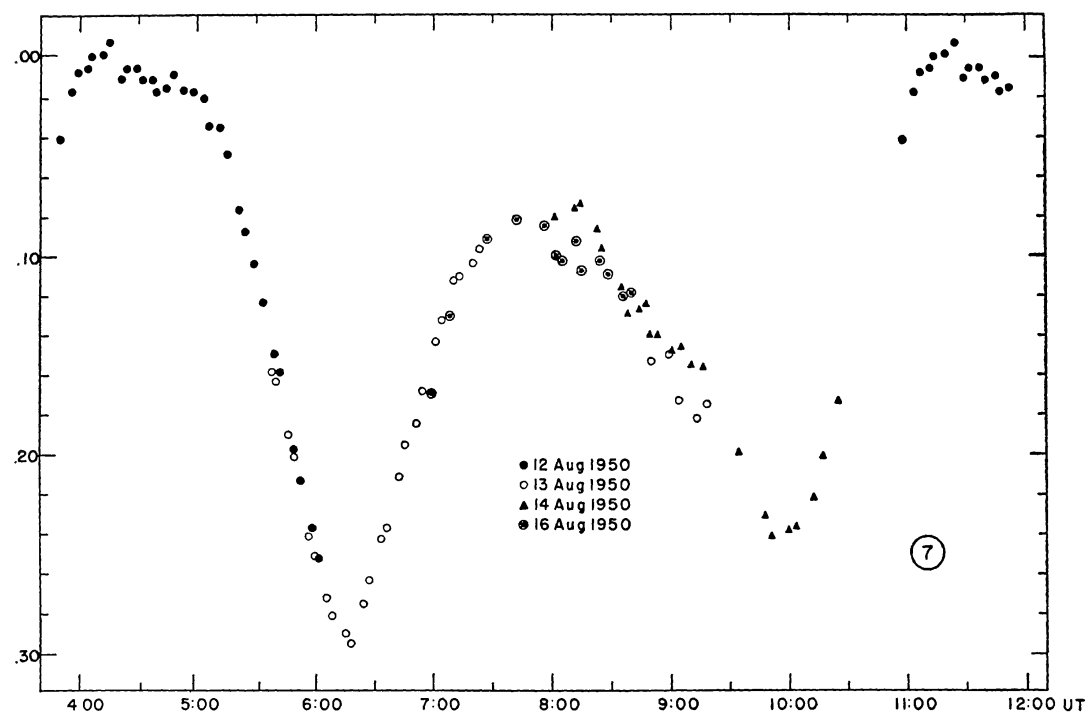


FIG. 3.—Light-curve of 7 Iris. Times refer to August 12 observations; magnitudes all reduced to August 12 (see text). Observations 11^h–12^h are replotted from earlier cycle to complete light-curve.

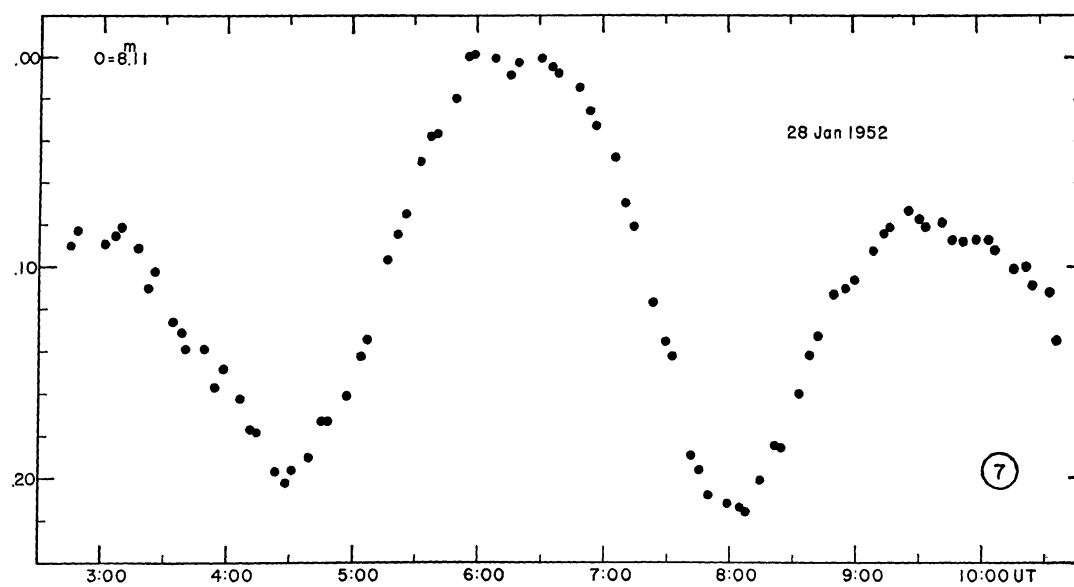


FIG. 4.—Light-curve of 7 Iris

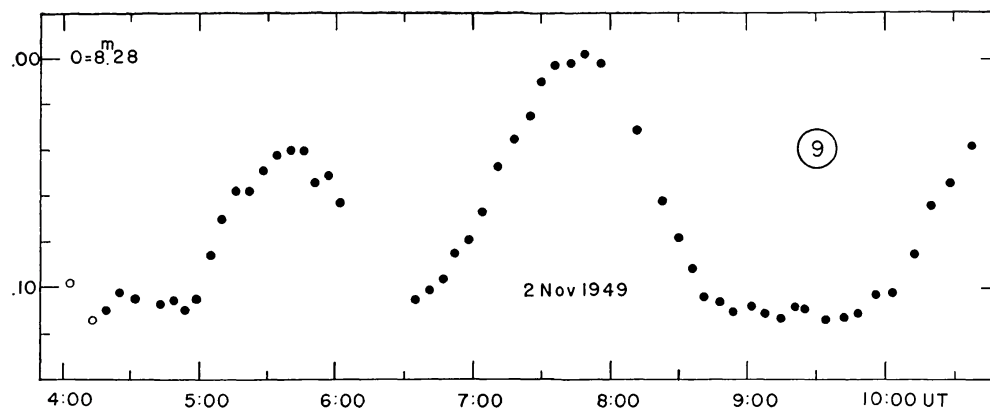


FIG. 5.—Light-curve of 9 Metis. The zero point is 8.27 (corrected)

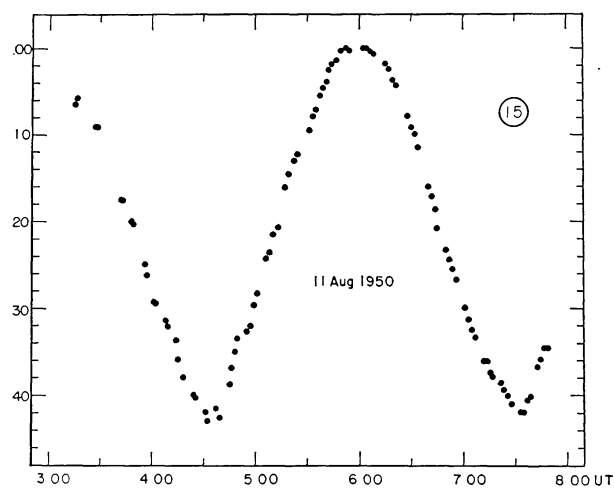


FIG. 6.—Observations of 15 Eunomia

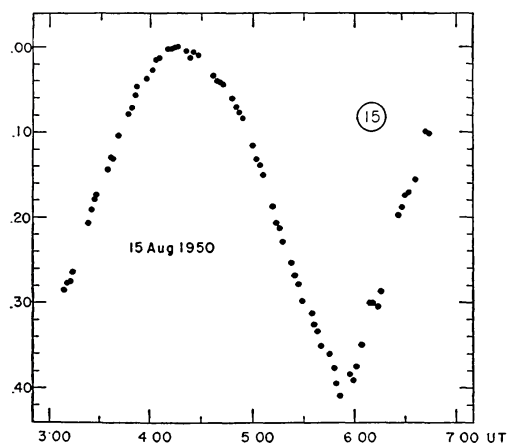


FIG. 7.—Observations of 15 Eunomia

$5^h20^m56.53^s = 0.222615$ days, in full accord with that found by Stephenson from observations during the same opposition. The light-curves found were also quite similar.

Iris (7).—Observations were made at two epochs; they are shown in Figures 3 and 4. Figure 3 is based on measures taken on four nights, all reduced to August 12, 1950. Allowance was made for the changes in distance of the asteroid to the sun and to the earth (corrections -0.006 , -0.013 , and -0.027 mag.) and for changes in phase angle by using Müller's phase coefficient of 0.019 mag/degree (corrections -0.006 , -0.011 , and -0.022 mag.).

Comparison of the two light-curves shows a considerable change in amplitude but a general resemblance in shape. The high minimum occurs in both cases before the high maximum. The periods derived from the two curves are mutually just consistent, as is

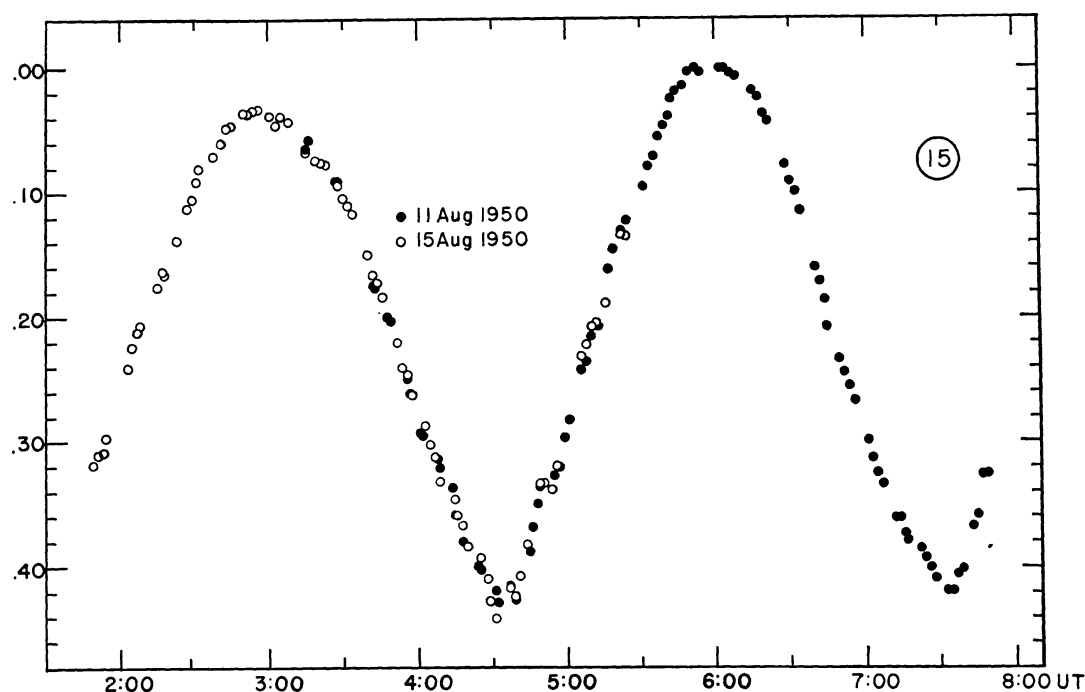


FIG. 8.—Curves 6 and 7 combined, reduced to August 11 (see text)

seen from the data in Table 1. The number of cycles between the two sets of data is still uncertain.

Metis (9).—The observations (Fig. 5) are exceptionally accordant. A second series was obtained in January, 1954, which is published in Paper II.

Eunomia (15).—Figures 6 and 7 show the observations of the separate nights, and Figure 8 the combined result of the 1950 observations. The August 15 measures were reduced to August 11 by allowing for the change in the distances (0.007 mag.) and in phase. The phase angle changed $1^\circ.6$, and the two light-curves indicate a shift of 0.068 mag. due to phase, corresponding to a phase coefficient of 0.042 mag/degree. This is slightly higher than Müller's value of 0.028 mag.

The 1952 observations (Fig. 9) show a somewhat increased amplitude, but an otherwise very similar light-curve, with two unequal maxima and a small difference between the minima; the deepest minimum precedes the highest maximum. The periods derived are the same. The 1953 observations (Fig. 10) show one of the maxima flanked by the two minima. The period may now be determined more accurately by comparing the three

sets of data. This is done in Section IV. It indicates that the maximum in Figure 10 is a primary maximum.

Thetis (17).—This object was observed on four nights, in March and April, 1953. The long runs of March 31 (Fig. 11) and April 2 (Fig. 12) appeared to define a very similar light-curve and could accordingly be combined (Fig. 13). The reduction of March 31 to April 2 appeared to be $+0.042$ mag. This is interpreted as due to a distance correction of $+0.002$ mag. and a phase correction of $+0.040$ mag., caused by an increase in phase of $0^\circ.9$ (from 8° to 9°). This indicates a phase coefficient of the order of 0.04 mag/degree. The amplitude of the light-curve of April 19 (Fig. 15) was distinctly different, and even the shorter run on March 28 (Fig. 14) showed a somewhat different slope. The period, $12^h16^m.5$, is the longest among the asteroids here included.

Phocaea (25).—The period of this curious light-curve (Fig. 16) is nearly 10 hours. The measures, obtained during four nights, are reduced to August 5.25 by correcting for the varying distances but not for changes in phase, which were very small (from $24^\circ.78$ to $25^\circ.15$ during the 5-day interval).

Laetitia (39).—Light-curves for two nights are reproduced here as Figures 17 and 18. The curves are similar and define precisely the same period; but the amplitudes are 0.17 and 0.18 mag. on Figure 17, and 0.21 and 0.22 mag. on Figure 18. The secondary maximum is quite low in Figure 17. Observations on a third night (April 4, 1953) were disturbed by clouds and cannot be used except for a check on the period. A secondary maximum occurred on 7^h0^m U.T., April 4, 1953.

Because the epoch of the April 4 observation may be uncertain by possibly as much as 5 minutes, the number of cycles between the 1952 and the 1953 observations cannot yet be determined with certainty. The combined 1953 observations make the period

$$P = 5^h10^m75 \pm 0^m.2 = 0.21580 \pm 0.00014 \text{ days}.$$

If Müller's phase coefficient 0.022 mag/degree is used, the g values become 6.38 and 6.42 mag. Coincidence would be obtained by using 0.013 /degree; but since the aspect has changed by nearly 90° , such precise coincidence is not required.

Klotho (97).—Two runs, on consecutive nights, of about 4 hours each, are shown in Figure 19. The second night was reduced to the first by a shift of -0.020 mag., based on the correction for the distances (0.004 mag.) and phase, with an assumed phase coefficient of 0.040 mag/degree. The total range shown is 0.07 mag., but the period is still unknown and the light-curve incomplete.

Davidia (511).—The two light-curves (Figs. 20 and 21) look very different, but define the same period of approximately 5^h10^m . The scatter near the minima on Figure 21 is somewhat surprising. The g values differ by about 0.25 mag., suggesting that the average area visible in Figure 20 was larger than that in Figure 21. The aspects differed about 90° . To attribute the difference to phase effects would lead to wholly spurious results; the phase angles are 3° and 6° . This points up a general difficulty: that determinations of the phase coefficient may contain an effect of *area change*, resulting from a change in aspect.

The reductions for light-time have not been made in the abscissae used in the light-curves. They are listed separately in Table 2, which also gives the phase angles corresponding to the light-curves.

IV. PERIOD, AXIS OF ROTATION, AND SHAPE

The observed period of an asteroid during a limited interval of time (say, a week or two) will not, in general, be the true period of rotation but the instantaneous synodic period. If it be assumed that the true period is constant and that the axis of rotation is fixed in space (a dangerous assumption that will need verification for each object), a simple relation exists between the true period and the synodic period.

Consider a nonrotating co-ordinate system, centered on and moving with the asteroid;

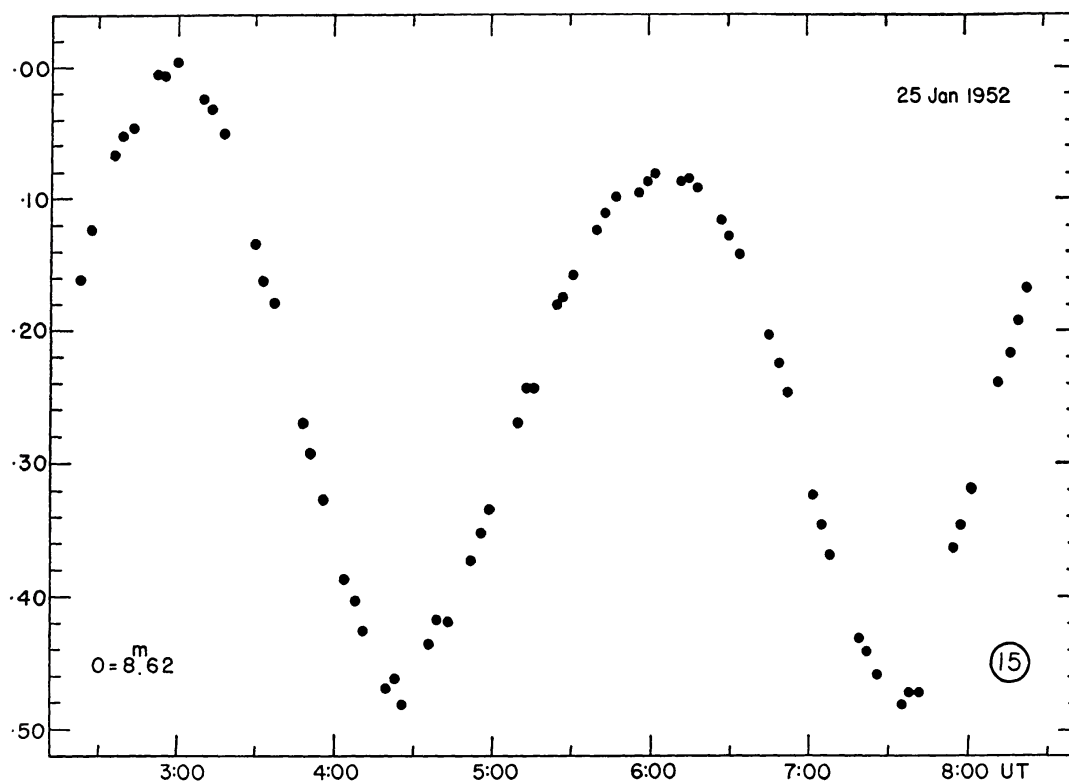


FIG. 9.—Light-curve of 15 Eunomia

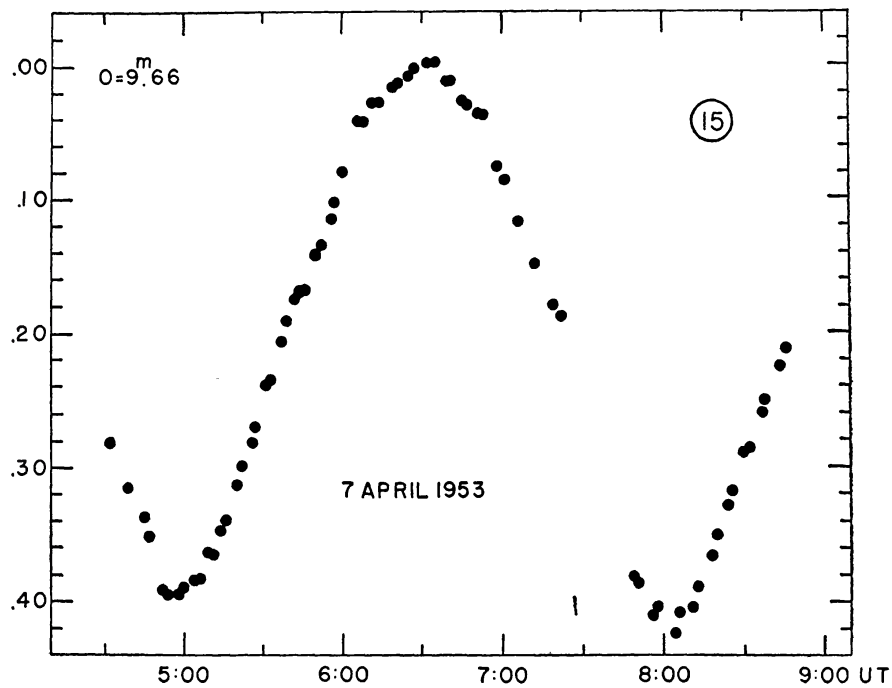


FIG. 10.—Observations of 15 Eunomia

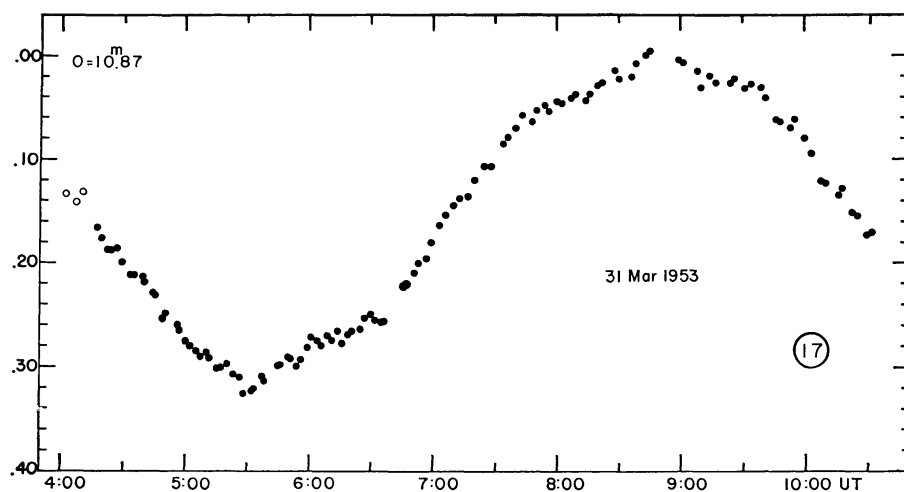


FIG. 11.—Observations of 17 Thetis

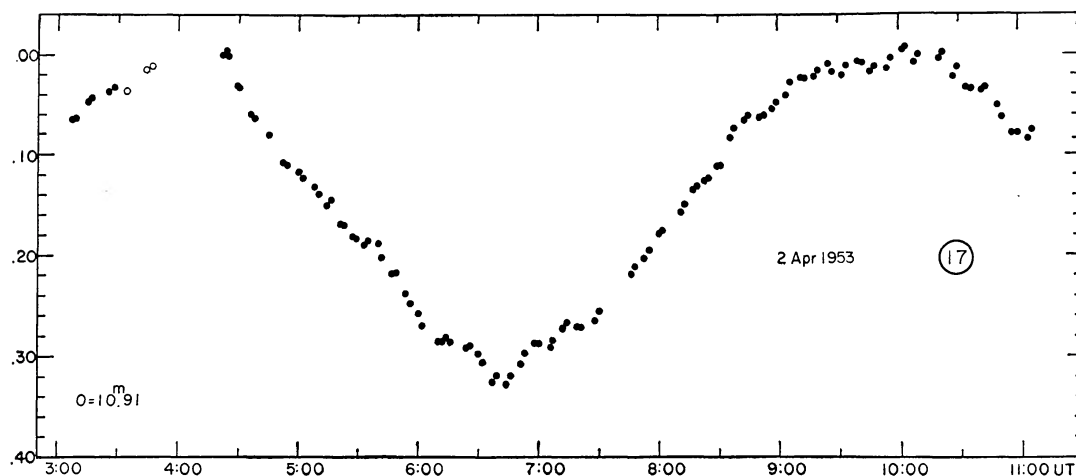


FIG. 12.—Observations of 17 Thetis

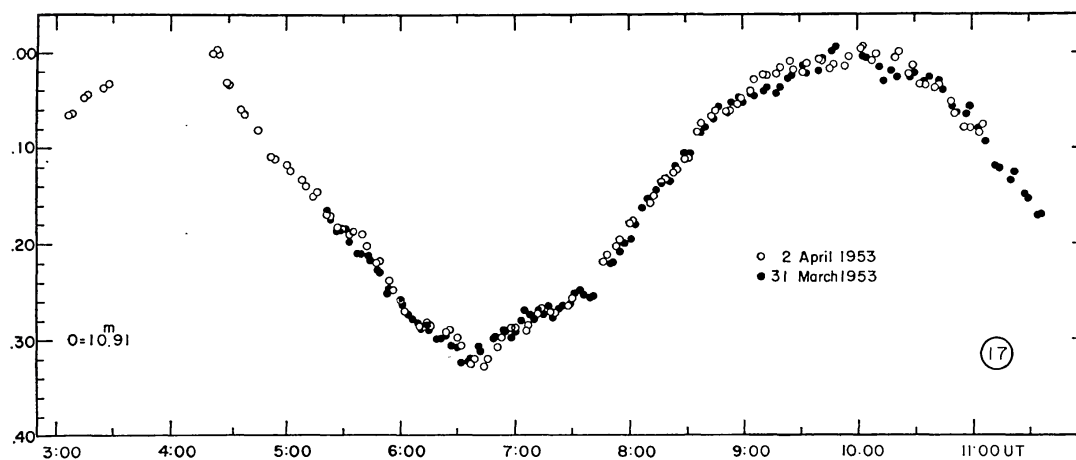


FIG. 13.—Combination of curves 12 and 13, reduced to April 2, 1953

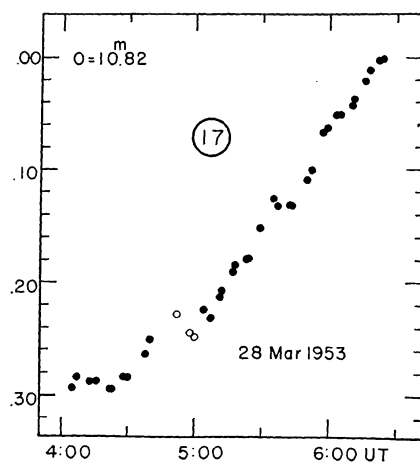


FIG. 14.—Observations of 17 Thetis

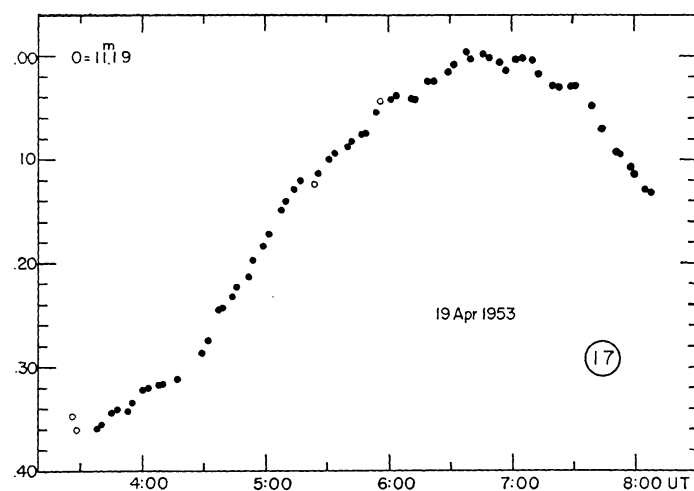
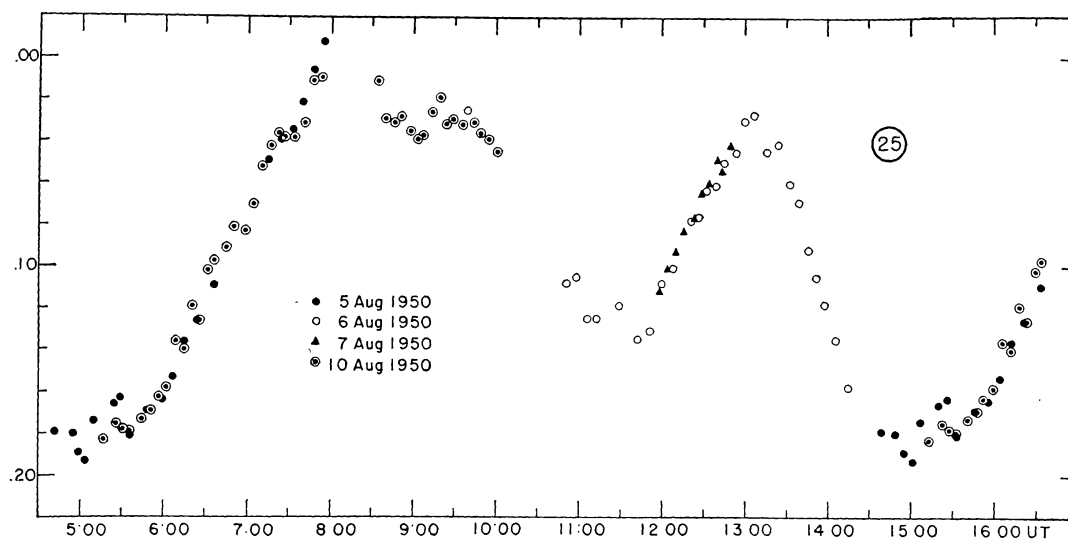


FIG. 15.—Observations of 17 Thetis

FIG. 16.—Light-curve of 25 Phocaea, reduced to August 5, 1950 (see text). Observations beyond 14^h30^m replotted to define light-curve.

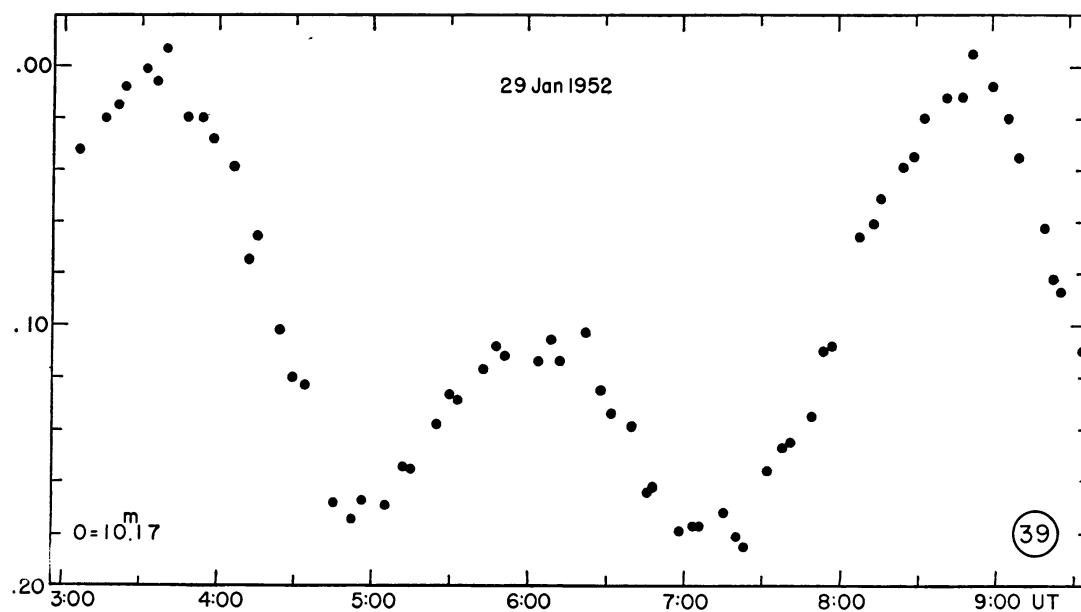


FIG. 17.—Light-curve of 39 Laetitia

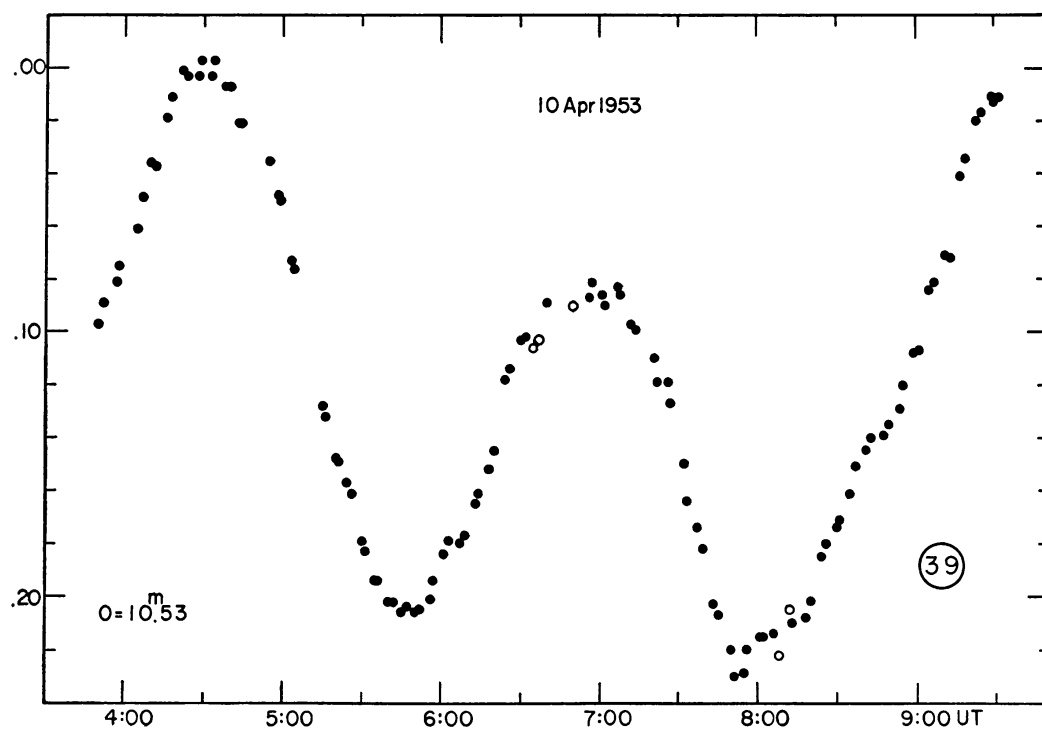


FIG. 18.—Light-curve of 39 Laetitia

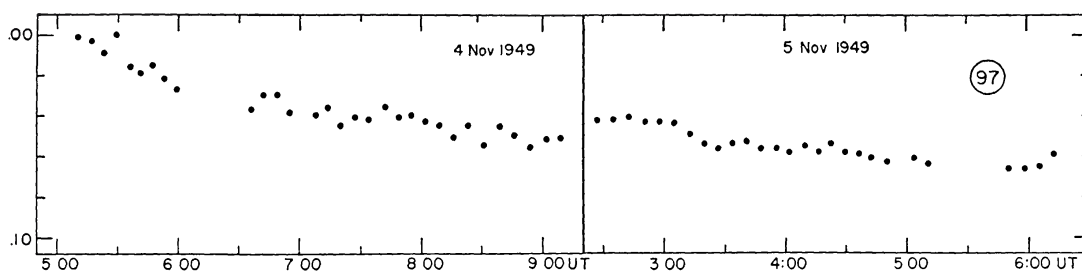


FIG. 19.—Observations of 97 Klotho

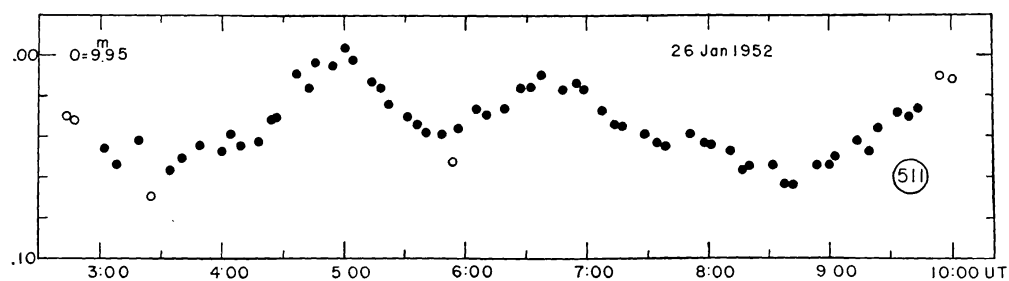


FIG. 20.—Light-curve of 511 Davida

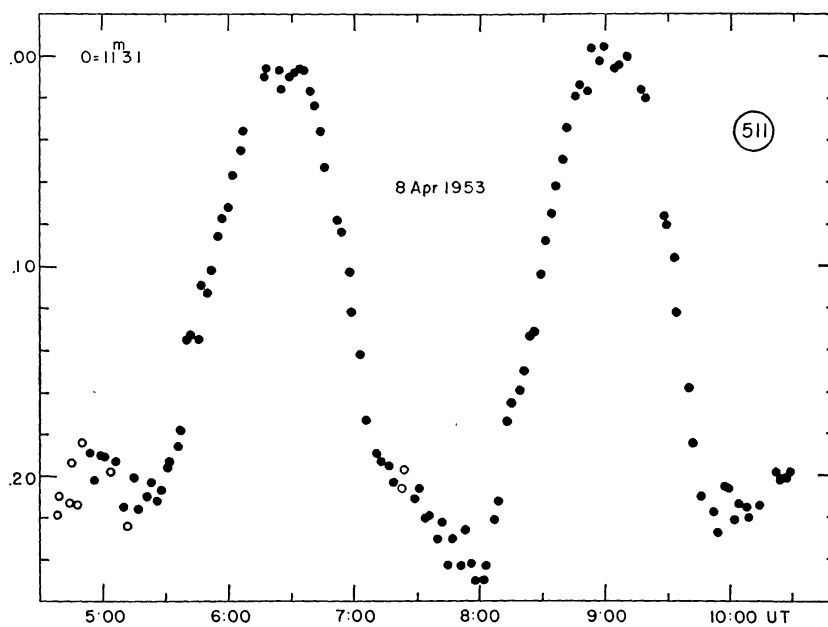


FIG. 21.—Light-curve of 511 Davida

and let the instantaneous position of the earth be on the positive y -axis (see Fig. 22). Further, let the xy -plane be so chosen that it contains the velocity vector of the earth, taken relative to the asteroid, with the negative x -direction parallel to the velocity vector. Now let the obliquity of the instantaneous rotational axis of the asteroid be θ (the angle with the z -axis), and let the azimuth of the axis from the xz -plane be $-\phi$. Assume that the asteroid, as seen from a distant stationary point in the plane of its equator, shows two light maxima and two minima, somewhat like a rotating book or box having its axis in the principal plane of symmetry. One can then derive the phase relations observed from a moving earth by watching the sweep of the normal to the broad face of the object.

TABLE 2
LIGHT-TIMES AND PHASE ANGLES

No.	Date	Light-Time (Minutes)	ϕ
4.....	{ 22 Dec. 1950	- 15.4	18°0
	{ 7 Mar. 1952	- 11.4	4.3
7.....	{ 14 Aug. 1950	- 14.2	17.8
	{ 28 Jan. 1952	- 9.8	5.1
9.....	2 Nov. 1949	- 9.4	2.7
15.....	{ 13 Aug. 1950	- 13.6	12.0
	{ 25 Jan. 1952	- 12.8	13.3
	{ 7 Apr. 1953	- 17.8	6.0
17.....	{ 28 Mar. 1953	- 11.7	6.8
	{ 31 Mar. 1953	- 11.7	8.1
	{ 2 Apr. 1953	- 11.8	8.9
	{ 19 Apr. 1953	- 12.4	16.0
25.....	7 Aug. 1950	- 7.8	24.9
39.....	{ 29 Jan. 1952	- 16.0	11.2
	{ 10 Apr. 1953	- 17.7	6.8
97.....	4 Nov. 1949	- 9.8	14.6
511.....	{ 26 Jan. 1952	- 14.6	5.0
	{ 8 Apr. 1953	- 20.3	6.6

The normal will sweep out the equator and have an inclination on the xy -plane of θ and the ascending node, N , at $90-\phi$. The earth will see maximum light when the normal reaches the point on the sphere closest to the y -axis. Let this point be designated by E . Then the great-circle arc $\overline{NE}$ (and hence the fraction of the period after the nodal passage) may be found from the spherical triangle NEY , in which Y is the point on the unit sphere defined by the positive y -axis. Now $\angle ENY = \theta$, $\angle NEY = 90^\circ$, and $\overline{NY} = \phi$. Accordingly, from spherical trigonometry,

$$\tan \overline{NE} = \cos \theta \tan \phi . \tag{1}$$

We are not interested in equation (1) itself but in how the epoch of maximum light will be delayed if the earth now moves by a known amount, reflected by an equal geocentric motion of the asteroid between the stars. Or we must find the change δE in $\overline{NE}$ as ϕ

changes by $\delta\phi$, owing to the motion of the earth. Differentiating equation (1), we find, after some reduction,

$$\delta E = \frac{\cos \theta}{\cos^2 \phi + \sin^2 \phi \cos^2 \theta} \delta \phi. \quad (2)$$

This equation gives the correction to the epoch of maximum for a given geocentric displacement $\delta\phi$ of the asteroid between the stars. More general formulae have been derived by Stobbe (1940), Huruhata (1940), and others in connection with the study of Eros; but the simple relation (2) shows the essentials of the problem.

Application of equation (2) to a determination of the orientation of the rotational axis, θ and ϕ , is straightforward, although the accuracy is quite limited, as will be seen. In the first place, θ and ϕ refer to the instantaneous co-ordinate system defined above. Since the true period will not, in general, be known, one may combine two series of observations, one before and one after the stationary point, so that the co-ordinates for the

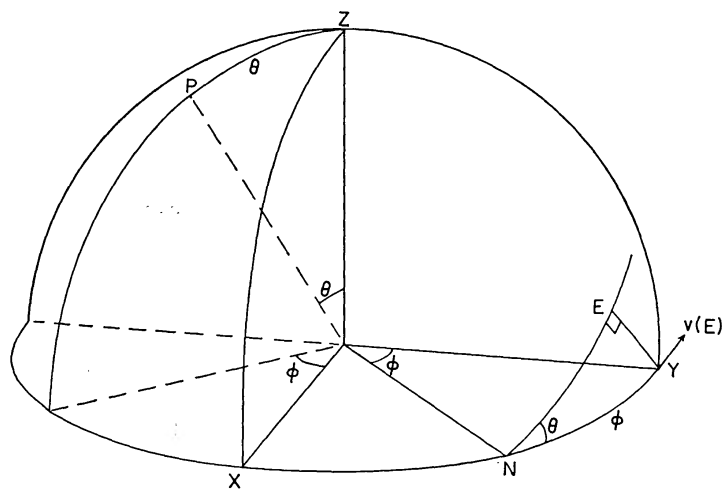


FIG. 22.—Co-ordinate system defining symbols used

two runs will be essentially the same. Since the values of $\delta\phi$ are known and since the sign reverses while θ and ϕ are constant (by assumption), the true period is found by imposing the condition that δE is proportional to $\delta\phi$; at the same time, one determination of the coefficient of $\delta\phi$ is made. A similar set of data taken at a different opposition, with the asteroid about 90° displaced, completes the determination of the orientation of the axis in space.

While the method described is correct in principle (provided that the axis keeps its orientation during the required 20 months or so), it is not very accurate. The epochs of maximum may, under favorable conditions, be determined with a precision of about 1 minute, so that an interval bounded by two such epochs is determined to ± 0.001 day. If, during each opposition, one establishes three epochs, one at the stationary point, one 2–3 months earlier, when the asteroid was, say, 10° from the stationary point, and one some 2 months *after* the stationary point, when the asteroid had returned to its first position, then $\delta\phi$ is of the order of 10° and δE also. Hence δE is about $\frac{1}{36}$ of the period, or about 0.007 days. The trigonometric coefficient in equation (2) is then known with an accuracy of some 14 per cent, which is hardly good enough.

Optimum conditions for the use of equation (2) are obtained by the use of the stationary points only. The average synodic period of rotation is then found from comparison of the epochs of one opposition with those of the next, while the two stationary points

provide the maximum possible value of $\delta\phi$, covering the entire interval of retrograde motion. For an asteroid moving in the ecliptic with a circular orbit and $a = 2.8$ a.u., the stationary points are $13^{\circ}.40$ apart. The trigonometric coefficient in equation (2) may therefore be determined with the precision of about 10 per cent. The method requires the derivation of accurate times of maximum at *four epochs* instead of six, covering a total interval of about 18–20 months.

A second method for the determination of the orientation of the axis depends on the *shape* of the light-curve and its time variations. The method has been used successfully for Eros (Watson 1937; Huruhata 1940; Stobbe 1940; Beyer 1953). In this case the amplitude varies between about 1.5 and 0.0 mag., and a good determination of the orientation may be made. The rotation of Eros is found to be direct and the position of the pole variable, though there is disagreement on the amount of variation.

In the case of 15 Eunomia, however, the amplitude of the light-curve does not vary greatly at the epochs observed. One may provisionally conclude that the axis of rotation is roughly at right angles to the ecliptic and attempt to establish the *sense of rotation* from the epochs. One thus assumes $\theta = 0^{\circ}$ in equation (2), so that $\delta E = \delta\phi$. Unless one can make use of the occasional retrograde motion of the asteroid, one would find only the synodic period and could not determine the sense of the rotation. However, the four epochs covered by our observations appear to be sufficiently irregular for this purpose. The light-curves define the epochs, uncorrected for light-time, given in the accompanying tabulation; dates are JD – 2430000. From these data the mean intervals between the four

3504.690.....	I Min.	4036.625.....	I Max.
.750.....	I Max.	.683.....	II Min.
.814.....	II Min.	.753.....	II Max.
		.815.....	I Min.
3508.678.....	II Max.		
.746.....	I Min.	4474.706.....	I Min.
		.772.....	I Max.
		.835.....	II Min.

cardinal points on the light-curve may be found, and thereafter they can be combined into four main epochs of primary maximum. These are given in Table 3. The third column gives the Julian day corrected for light-time.

TABLE 3
EPOCHS OF PRIMARY MAXIMUM, 15 EUNOMIA

No.	JD	JD(c)	R.A.	Dec.	λ	β
1.....	3504.751	3504.742	19 ^b 18 ^m	–20°3	288°	+ 2°
2.....	3508.804	3508.795	19 16	–20.2	287.5	+ 2
3.....	4036.623	4036.614	5 58	+20.5	89.5	– 3
4.....	4474.771	4474.759	12 02	–18.6	186	–17

A further correction needs to be made when the phase angle exceeds 10° or 15° , as was shown by Huruhata (1940); it arises from the illumination of the asteroid depending on the phase angle and the resulting shift in the epochs of maximum and minimum light. It is readily shown that for small values of ϕ the displacement varies as ϕ^2 ; the effects on our observations of 15 Eunomia may be neglected.

The uncertainty of the four epochs is estimated to be between ± 0.001 and 0.002 day; we adopt ± 0.0015 day. Table 1 gives $P = 6^{\text{h}}05^{\text{m}} \pm 1 = 0.2535 \pm 0.0007$ day. Hence

$16P = 4.056 \pm 0.011$ days, while the interval (2-1) in Table 3 gives 4.053 ± 0.002 days. Since $\lambda_1 \cong \lambda_2$, the true period is $(4.053 \pm 0.002)/16 = 0.25331 \pm 0.00013$ day.

The light-curves of (15) suggest that the axis of rotation is oriented roughly at right angles to the ecliptic. We must then be able to represent the epochs either by direct or retrograde rotation.

	λ
JD 3506.768.....	288°
4036.614.....	89.5
4474.759.....	186

Now $\Delta_1\lambda = 161.5^\circ = 0.114$ day; $\Delta_2\lambda = 96.5^\circ = 0.068$ day.

With direct rotation $n_1P = 529.844 - 0.114 = 529.730$ days
 and $n_2P = 438.145 - 0.068 = 438.077$ days.
 With retrograde rotation $n_1P = 529.844 + 0.114 = 529.958$ days
 and $n_2P = 438.145 + 0.068 = 438.213$ days.

With $n_1 = 2091$ and $n_2 = 1729$, P is found to be 0.2533381 and 0.2533702 day for direct rotation; and 0.2534472 and 0.2534488 day for retrograde. The discrepancy in P for direct rotation can be reduced by using $n_1 = 2092$ and $n_2 = 1730$; then P is found to be 0.2532170 and 0.2532237 day.

All these six periods are compatible with the true period derived earlier from the 4-day interval, while no others would be. The long intervals give $\Delta P = 0.0000067$ for direct and 0.0000016 for retrograde rotation. The uncertainty of each interval being ± 0.002 day, the two P values are uncertain by ± 0.0000010 and ± 0.0000012 day, respectively; their difference by ± 0.0000016 day. Therefore, *retrograde rotation* is compatible with the data, but direct rotation is not. Clearly, some extra uncertainty may be introduced in ΔP by the assumptions made above concerning the position of the rotational axis.

The *shape* of an asteroid is, strictly speaking, undeterminable from its light-curve, as Russell (1906) has shown, because the light-curve depends at the same time on the spottiness of the surface. It may turn out, however, that in most cases the average brightness over a hemisphere does not vary considerably, so that the light-variations would be due primarily to variations in the visible area—at least, when observations are made close to opposition so that shadows have no effect. There is some reason to anticipate this situation: from preliminary results on asteroid diameters, obtained with the disk meter on the 82-inch telescope, it appears that the asteroid albedos are quite similar, with, so far, only one definite exception, Vesta, which is brighter. Some of the tests proposed by Russell will disclose for which objects the hypothesis of constant surface brightness is valid. In those cases at least a simplified model may be found which, for objects brighter than the ninth magnitude, can be tested visually with the largest telescopes. The case of Eros, seen elongated by Van den Bos and Finsen in 1931, is a case in point.

The results of this section may be summarized as follows: For an asteroid with light-curve like 15 Eunomia, which shows little change in amplitude with aspect, the most efficient observational procedure is to measure accurate epochs during the two stationary points of each opposition. The combination of the results of two or more oppositions will then disclose whether the rotation of the asteroid is direct or retrograde. If the epochs have great accuracy, ± 0.001 day or better, the trigonometric factor in equation (2) may be determined for each opposition, and a rough determination of the orientation of the axis may be made. If the light-curve shows large amplitude variations with aspect, utilization of these data will greatly increase the precision of the axis position.

V. ASTEROID COLORS

Color measurements of the asteroids observed were usually made at the beginning and the end of each nightly run. In a few cases the color measurements were made con-

tinuously through the entire run. The latter results are shown in Figure 23. Comments on the three asteroids included follow.

Vesta (4).—The apparent color variations are not more than 0.015 mag. If the 5-hour period is correct, a slight systematic error (about 0.005 mag.) appears to be present, causing the second cycle to be slightly bluer than the first. On the double period this difficulty disappears. If we consider only the most accurate part of the curve (smallest

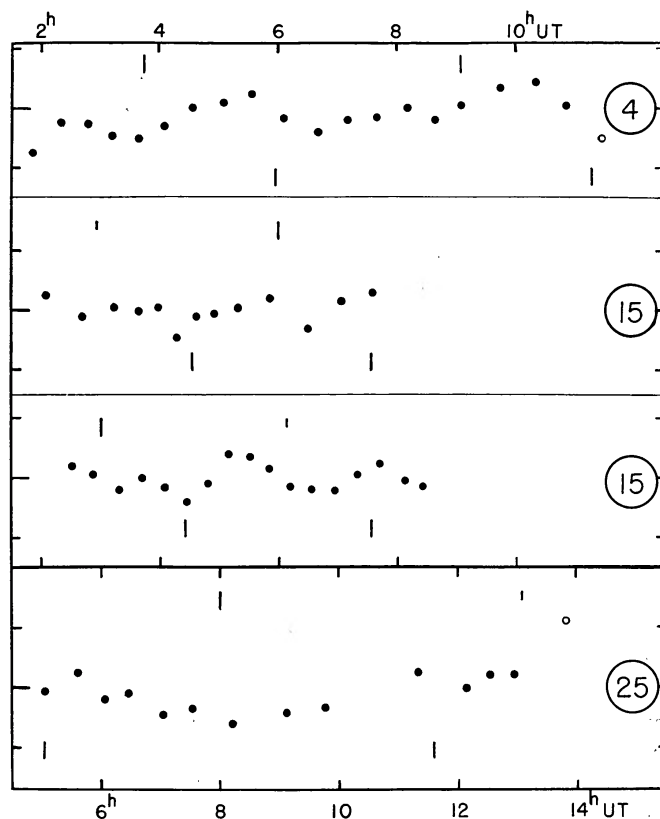


FIG. 23.—Color index ($B-V$) of three asteroids versus time (or phase). Vertical divisions are 0.02 mag. apart, with red toward bottom. Dots are averages of four measures, consecutive in phase, for 4 Vesta; six and four for 15 Eunomia; and seven for 25 Phocaea. The abscissae corresponding to the maxima are marked with short lines *above* the color curves; the minima *below*. For 15 and 25, slightly less than one cycle each is given, while two maxima and two minima occur in each cycle; the primary maximum is marked with a longer index line. The dates are, respectively, March 7, 1953; August 11, 15, 1950; January 25, 1952; August 5, 6, 7, 10, 1950. The light-curves corresponding to the color-curves are found in Figures 2, 8, 9, and 16.

zenith distances), namely, from maximum to maximum, we find 11 points which deviate, on the average, ± 0.003 mag. from the mean and which show a systematic variation of no greater than 0.010 mag.

Eunomia (15).—The first curve shows no distinct variation; the color index appears constant to within 0.01 mag.; the average residual with respect to the mean is 0.003 mag. The second curve indicates a minor variation of 0.01 mag. total amplitude; but if the color index is assumed to be constant, the mean residual is also 0.003 mag., so that we may suppose that no variation is established.

Phocaea (25).—If one ignores the uncertain point near 14 hours (obtained at $2\frac{1}{2}$ air masses), a wave of 0.01 mag. total amplitude seems indicated. However, if one assumes a constant color index, the mean residual is 0.005 mag.

The color variations of these three objects are remarkably small and suggest that color measurements cannot, in general, be used to determine the period of rotation or even remove ambiguities (single or double period).

Since the color variations of the asteroids appear to be small, representative color indices may be determined from occasional measures. The values so far derived are listed in Table 1. The $B-V$ colors of Nos. 7, 17, and 39 are very similar to those of Mercury and the moon; but Nos. 15, 4, and 511 are somewhat bluer by increasing amounts. These small differences are of distinct physical interest. The ultraviolet colors show similar differences.

VI. CONCLUDING REMARKS

Examination of the photometric results included in this paper shows that a greatly increased body of data will be needed before an adequate picture of the rotation and the shapes of asteroids can be constructed. The present program calls for the repeated observation of all asteroids reaching tenth magnitude at opposition, plus one or two dozen fainter objects. It has been shown that a single asteroid must be observed on at least four epochs during two oppositions, preferably near the stationary points, before something definite can be said about the axis of rotation and the shape of the body; but because of the uncertainty of the assumption of a constant orientation of the axis, observations at additional oppositions are necessary as checks. The period of rotation can usually be found during one long night or otherwise on two or three consecutive nights. Long, continuous runs on excellent nights are by far the most productive; because of the declination of the opposition point, winter nights are far more suitable than summer nights. From inspection of the light-curves it is clear also that, in general, visual or photographic observations would have been insufficiently accurate to allow a determination of the period. Comparison of our results with occasional earlier data obtained photographically confirms this conclusion. Even photoelectric work requires really good nights for successful observation.

All asteroids observed so far, with the possible exception of Vesta, show two maxima and two minima during one cycle. This is in accord with expectation if one is dealing with irregularly shaped bodies having a fairly uniform reflectivity over the surface, since the projected area will show two maxima and two minima during one cycle. On the other hand, a distribution of spots could give one maximum and one minimum per cycle. It is too early to make conclusions about the line of demarcation between *fragments*, resulting from violent collisions in the asteroid ring (such as leading to the Hirayama families; cf. Brouwer 1951); and *original condensations*, which would be roughly spherical, though they may have experienced minor collisions. The very frequent collisions expected in the asteroid ring (Kuiper 1953) may well have led to a roughly *random* distribution of axial orientations, i.e., to a roughly equal number of direct and retrograde rotations, with all possible obliquities. At the same time, all sorts of angles between the instantaneous axis of rotation and the axes of the inertia ellipsoid must have resulted. Unless dissipative forces are present, similar to those which damp the terrestrial latitude variation (Jeffreys 1952), the orientations of the rotational axes must be variable in periods of hours or days instead of years, while the absence of such rapid variations would prove the presence of dissipative forces in the asteroid interiors. This whole subject is full of promise for a deeper understanding of the origin and the physical properties of asteroids, which will repay the very large amount of painstaking photometric work that will be required in the solution.

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