

THE ANNUAL FLUCTUATION IN THE RATE OF ROTATION OF THE EARTH

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Summary

The amplitude of the apparent annual fluctuation in the rate of rotation of the Earth decreased in 1950 and has since remained at the lower value. Furthermore the fluctuation observed at stations in the northern hemisphere is due in part to systematic errors in the FK₃ star places. The extreme range in the length of the day is now estimated as ± 0.5 millisecond. Comparable results are deduced from time observations made in America and Australia.

From the study of the performance over the period 1943–1949 of the quartz-crystal clocks used in the Greenwich Time Service, an apparent annual fluctuation in the length of the day of about ± 1 millisecond was deduced. The cumulative effect on time keeping was found to be approximately ± 60 milliseconds (1). This fluctuation, which was derived after corrections had been applied to the time determinations to allow for the effect of polar motion, was attributed to real variations in the rate of rotation of the Earth. It combines with the effect of polar motion to produce a non-uniformity in astronomically determined time, introducing apparent irregularities into the performance of the clocks. In the assessment of the performance of high-grade standards of frequency it is thus necessary to evaluate these effects and to establish a provisional uniform time system from which these irregularities have been removed (2).

With the continued improvement in the stability of quartz clocks over periods of two years or more, it became possible to evaluate the annual fluctuation over single periods of twelve months. Such analyses are made at regular quarterly intervals and used in the routine assessment of clock performance and in the determination of absolute frequency. In the years 1950–1951 there was no evidence of any significant change from quarter to quarter, but the amplitude determined showed a substantial reduction to about one-half of the value given by the earlier analysis (3). The quarterly analyses completed in 1952 continued to indicate an amplitude in the neighbourhood of ± 30 milliseconds. The method of analysis has been described in detail elsewhere (4), and some recent results are given in Table I. These values are based on coordinates of the pole which have recently been supplied by the Central Office of the International Latitude Service.

By international agreement, all observations for time determination are referred to the FK₃ system of star places, and the question arose whether the observed fluctuation could be due to systematic errors in the star catalogue, but it was not thought that these could account for more than a small portion of the effect.

At the U.S. Naval Observatory, Dr H. R. Morgan has been engaged on the preparation of a new star catalogue, known as N30, based on observations subsequent to those used in the construction of the FK3 catalogue and on an entirely independent system. This catalogue has not yet been published, but tables of the mean differences between N30 and FK3 in the positions and proper motions in right ascension for different hours of right ascension and for different zones of declination, for the epoch 1950, were communicated privately to the Astronomer Royal in advance of publication. It can safely be assumed that the N30 star places are systematically more nearly correct at the current epoch than those of FK3. The systematic differences between the N30 and FK3 right ascensions were accordingly adopted as corrections to the FK3 right ascension system.

TABLE I
Determinations of Annual Fluctuation

Period Covered	a_1	d_1	a_2	d_2
1950 Jan.-1950 Dec.	33.5	12	2.7	69
1950 Apr.-1951 Mar.	32.2	16	4.8	79
1950 July-1951 June	33.0	16	5.8	73
1950 Oct.-1951 Sept.	33.9	12	8.4	76
1951 Jan.-1951 Dec.	27.7	10	7.9	79
1951 Apr.-1952 Mar.	27.3	11	8.5	79
1951 July-1952 June	26.2	16	7.3	94
1951 Oct.-1952 Sept.	27.8	5	11.3	86
1951 Jan.-1952 Sept.	27.3	7	10.1	83

Cumulative Time Effect of Fluctuation, or amount by which Earth is slow, f milliseconds, is given by

$$f = a_1 \sin \frac{2\pi}{365} (d - d_1) + a_2 \sin \frac{4\pi}{365} (d - d_2),$$

where d is the day of the year.

From the known distribution of the Greenwich time observations in right ascension and declination, it was possible to calculate the contribution made by these corrections to the observed annual fluctuation. It appears that the errors in the FK3 catalogue are not negligible, and that they give rise to an apparent annual fluctuation exceeding ± 10 milliseconds in the astronomically observed Greenwich Time. The observed annual fluctuation may be regarded as the resultant of two components, one due to the star place errors, and the other attributed to a real variation in the rate of rotation of the Earth. Thus while Table I shows the amplitude and phase of the determined annual fluctuation when FK3 star places are employed, Table III shows the effect of applying corrections obtained from the N30-FK3 difference. Fig. 1 shows the cumulative time effect of the annual fluctuation referred to both the FK3 and the new N30 system of star places.

Various attempts have been made to determine the amplitude of an annual fluctuation which might arise from meteorological changes. Mintz and Munk (5) have estimated that seasonal variation in the zonal winds of the atmosphere could account for a change in the length of the day between January and July amounting to 0.5 millisecond. This they compared with a corresponding change of 1.5 milliseconds deduced from the published data relating to the annual fluctuation for 1943-1949. In fact, the Greenwich preliminary analysis indicated

that the greatest change was between March and August, and amounted to 2.0 milliseconds. An analysis by Stoyko (6) based on the results from several observatories gave a smaller six-monthly term, but the extremes ranged from +0.9 millisecond in March to -1.1 milliseconds in August. It is thus clear that the range of 2.0 milliseconds is more appropriate to the analyses of that period. The results of these analyses are shown graphically in Fig. 2.

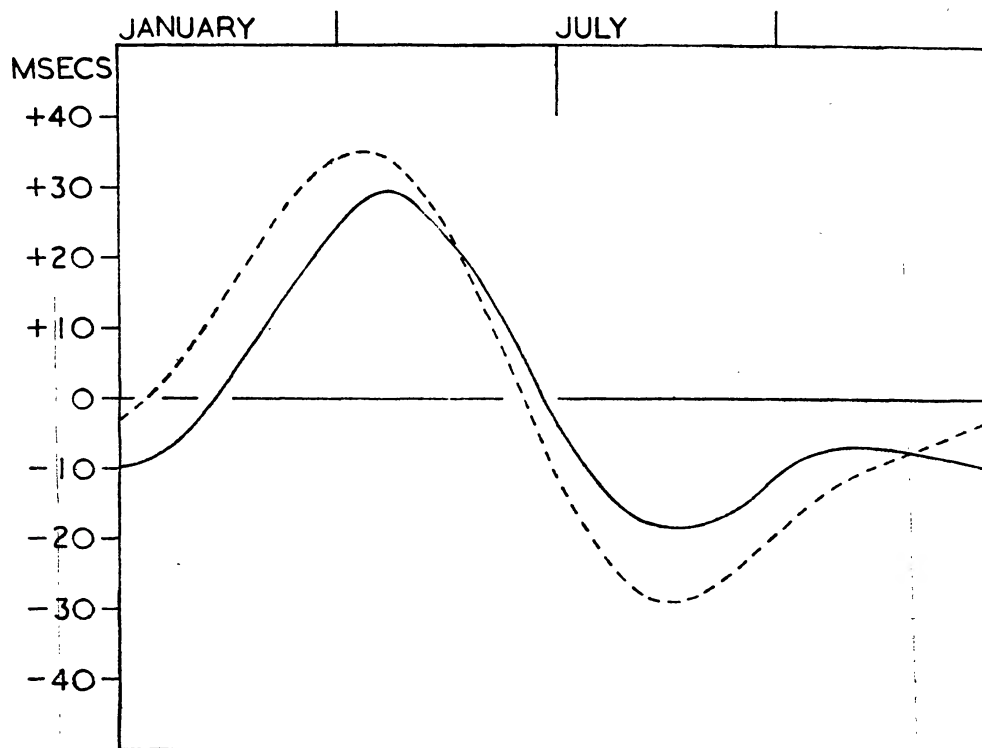


FIG. 1.—*Annual Fluctuation, 1951–1952. Cumulative Time Effect.*

Dotted line—observations referred to FK₃ star places.

Full line—observations referred to amended star places.

The more recent and more accurate determinations of the annual fluctuation based on the Greenwich Time Service data, with corrections applied to reduce the FK₃ system to the N₃₀ system, indicate an extreme range in the length of the day of about one millisecond. The larger range given by earlier investigations may be real, but may be in part attributable to the fact that the performance of the clocks on which those investigations were based was less reliable than the performance of the clocks on which the present investigation is based. If the investigation of Mintz and Munk were extended to all the months of the year, it might show that there is no longer any serious discordance between the observed annual range in the length of the day and that attributable to the seasonal variation in the zonal winds of the atmosphere.

In all analyses of the annual fluctuation it has been customary to evaluate a six-monthly term. Such a term will necessarily arise if the seasonal peaks of the fluctuation are not six months apart. This term cannot be determined with the same accuracy as the annual term, on account of its greater sensitivity to individual outstanding points. In other words, the observational material is not sufficiently accurate to give precise values of the annual range in the length

of the day, nor of the months in which the extreme values occur. For convenience of comparison with monthly meteorological data, the monthly means of the deviation of the length of the day from its mean value, based on the Greenwich observations from 1951 January to 1952 September and on the N30 star places, are given in Table II. A positive sign indicates that the length of the day exceeds its mean value, corresponding to a reduced rate of rotation.

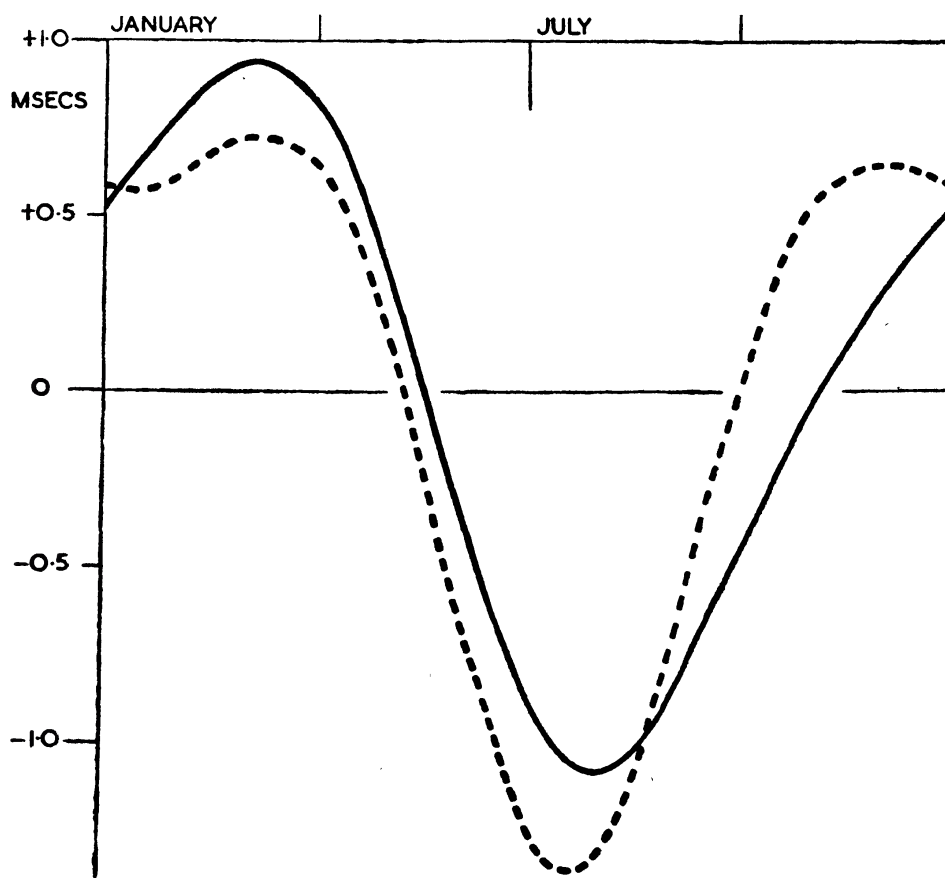


FIG. 2.—*Annual Fluctuation in the Length of the Day.*

Positive—length of day exceeds mean value.

Dotted line—Greenwich 1943-1949 (1).

Full line—mean of six observatories 1934-1939 (6).

While the table may be taken as giving values typical of the period examined, it is clear that a seasonal phenomenon arising from meteorological causes will not be strictly repetitive from year to year. The results are shown graphically in Fig. 3, together with a graph in which the rather more uncertain six-monthly term has been omitted. A comparison of these two curves may serve to indicate the extent to which the determined seasonal fluctuation differs, in a typical period, from a simple annual term. The departure from a simple sinusoidal pattern will vary from year to year, and the figures quoted in Table II must thus be regarded merely as indicative of the magnitude and general form of the fluctuation in the length of the day.

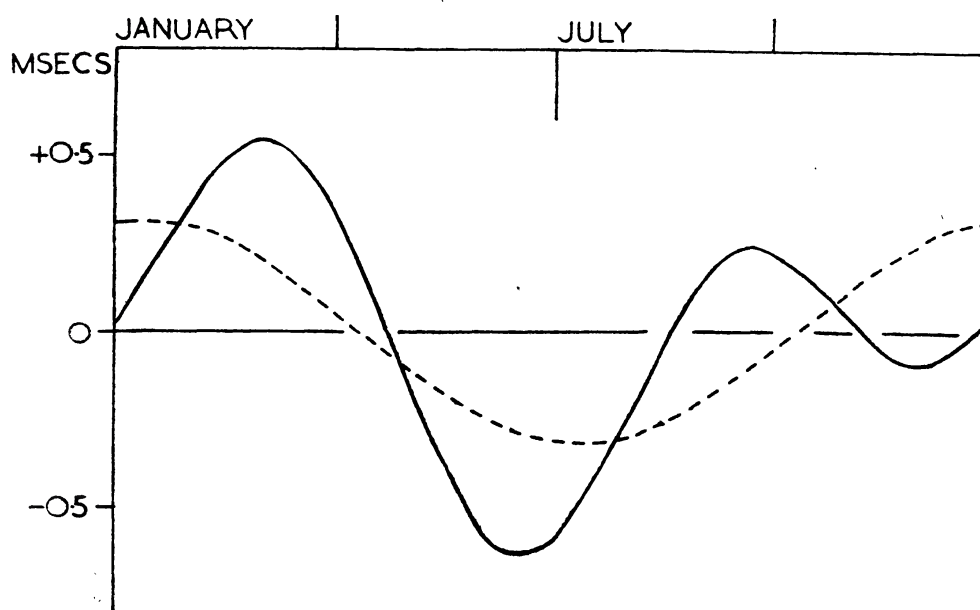
In so far as the annual fluctuation is due to a real variation in the rotation of the Earth, the observed effect should be the same at all points on the surface of the Earth. In this respect, it differs from the effect of polar variation, which

depends on the position of the observing station. It should therefore be possible to detect a comparable annual fluctuation in the time observations made at different observatories, but some investigations to this end had appeared to give conflicting results. It was accordingly decided to relate the time observations made by the U.S. Naval Observatory at Washington and Richmond to the same

TABLE II

Monthly Mean of Deviation of Length of Day from Mean Value
[milliseconds]

Jan.	Feb.	Mar.	Apr.	May	June
+0.03	+0.35	+0.54	+0.34	-0.16	-0.58
July	Aug.	Sept.	Oct.	Nov.	Dec.
-0.59	-0.24	+0.13	+0.22	+0.05	-0.10

FIG. 3.—*Annual Fluctuation in the Length of the Day.*

Positive—length of day exceeds mean value.

Full line—mean of recent Greenwich determinations.

Dotted line—12-month term only.

mean clock as had been used in the routine analyses of the Greenwich observations. A satisfactory link was provided by the measurement at Washington and Abinger of the WWV radio time signals. There is close accord between the results obtained for Washington and Richmond; it appears that the amplitude of the annual fluctuation observed in the United States is in the neighbourhood of ± 20 milliseconds.

This determination is similar to that obtained from the Greenwich observations when the latter are referred to the N30 star places. The time observations at Washington and Richmond are made with a Photographic Zenith Tube. The stars observed are too faint to be included in the FK3 catalogue; but, according to information received from the U.S. Naval Observatory, though the star positions are related to the FK3 system as a whole, they do not reflect

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the periodic errors of the FK3 system; the derived annual fluctuation is thus directly comparable with the true variation in the rate of rotation of the Earth, as given by the Greenwich observations corrected for the FK3-N30 difference. The satisfactory nature of the agreement is shown in Table III.

TABLE III

Observatory		Period	a_1	d_1	a_2	d_2
Greenwich	(FK3)	1951 Jan.-1952 Sept.	27.3	7	10.1	83
Canberra	(FK3)	1951 Jan.-1952 June	19.4	362	8.2	82
Greenwich	(N30)	1951 Jan.-1952 Sept.	18.5	26	10.5	87
Canberra	(N30)	1951 Jan.-1952 June	23.9	364	8.5	84
Washington	(PZT)	1951 Jan.-1952 June	22.3	35	11.0	119
Richmond	(PZT)	1951 Jan.-1952 June	23.0	36	10.5	111

Regular information is received from Canberra relating to the Australian Time Service, making it possible to refer a series of southern hemisphere time observations to the same Greenwich mean clock. The radio time signal link was effected in two independent, though indirect, ways which gave a good measure of agreement. The Australian observations were corrected for polar variation in the same way as the British and American observations.

The N30 catalogue indicates that the periodic errors of the FK3 system for stars in the southern hemisphere are small, but their effect was taken into account. The annual fluctuation thus derived from the Canberra observations was found to have an amplitude of approximately ± 20 milliseconds (see Table III). It would appear, therefore, that reasonable agreement exists between the annual fluctuation observed in various parts of the world, provided that the systematic errors in the FK3 star places are taken into account.

The annual fluctuation in the rate of rotation of the Earth does not affect the time intercomparisons between observatories or the determination of longitude by local astronomical observations compared with the radio time signals from a remote observatory. Both may, however, be significantly affected by the differential effect in longitude of the polar variation and by the differential errors arising from the use of different groups of stars at the two observing stations. The effect of polar variation at any station can be evaluated by means of the coordinates of the pole as published by the International Latitude Bureau, but as a rather long period must elapse before these results are available, provisional estimates must be made for current work. So that the data may be available with the minimum of delay, Professor Cecchini, the Director of the Bureau, has agreed, at the suggestion of the Astronomer Royal, to publish the coordinates of the pole at intervals in the Copenhagen circulars. As for the star places, time determinations are based by international agreement on the FK3 system, so that the results obtained at different observatories shall be directly comparable. Because of the not inconsiderable errors in the FK3 system at the present epoch, this purpose is no longer achieved. It is gratifying to know, however, that the FK3 system is to be revised at the Astronomisches Rechen-Institut, using observations obtained since its formation; and that a supplement containing the brighter stars to about the 7th magnitude is to be published.

The evidence now available suggests that the most serious systematic errors of the $\Delta\alpha_x$ type in the FK3 system occur in the star places for the northern

hemisphere: no significant periodic errors appear to be present in the equatorial zones, and in the southern half of the sky they are small. Observations made at stations in the northern hemisphere will be subject to appreciable periodic errors unless they are confined to stars in the equatorial zones or are subsequently corrected to a uniform system.

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