1951ApJ...113..464W

A COMET MODEL. II. PHYSICAL RELATIONS FOR COMETS AND METEORS*

FRED L. WHIPPLE
Harvard College Observatory
Received October 22, 1950

ABSTRACT

It has been shown previously that the icy conglomerate model for comets explains the anomalous accelerations of certain comets and also possible reductions in the effective attraction by the sun. These effects depend upon a moderate loss of matter, $\Delta M/M$ per period. This loss measures the loss of radius, $\Delta R/R$, while the solar radiation determines the maximum loss of radius by sublimation. By this means an upper limit of radius for seven comets has been determined. Numerical values in kilometers are: Encke, 4; Pons-Winnecke, 82; Biela, 1.7; D'Arrest, 1.4; Brooks, 1.2; Wolf I, 19; and 1905 III, 0.2. The smaller values are the most significant and are generally greater than the expected values derived from the reflected light at great solar distances.

The model predicts a large excess of unobserved hydrides, H_2O, NH_3, and CH_4 molecules, as compared to the observed CO^+, C_2, and CN. For Halley's Comet, using Wurm's calculations for the rate of loss of CO^+ and C_2 and using the total loss of ices calculated from solar radiation for a nucleus of radius 10 km, the relative abundances of CO^+ and C_2 to the combined hydrides are 10^{-5} and 10^{-3}, respectively. These abundances are roughly consistent with certain of ter Haar's calculations for molecules formed from interstellar atoms. Calculations show that the predicted excess of hydrides will produce no appreciable Rayleigh scattering in comets and also little electron scattering, should all atoms become singly ionized by photoionization. Little visible radiation from the hydrides of C, N, and O would be expected.

The comet model requires that a large cometary nucleus eject visual or photographic meteoroids with greater velocities than a small nucleus at the same perihelion distance (velocity proportional to the square root of the radius). Hence the meteor streams from the greater comets should generally be more dispersed and more uniform from year to year than streams from lesser comets with comparable orbits. Confirming examples of streams from greater comets are the Perseids and the Orionids and Eta Aquarids, if the latter streams arise from Halley's Comet; the Leonids and Bielids represent debris from dying comets. Qualitatively, the model predicts well for the meteor streams from known comets.

A. INTRODUCTION

In Paper I[1] a new comet model was described. The model consists of a conglomerate made up of *ices*, such as H_2O, NH_3, and other molecules volatile at normal temperature, mixed with meteoritic materials. The model is shown to be capable of accounting for the abnormal accelerations in mean motion and eccentricity observed for periodic Comets Encke, D'Arrest, and Wolf I, discussed previously, without requiring an excessive rate of mass loss. The acceleration is postulated to occur by a time lag in sublimation of the ices in a rotating cometary nucleus.

Some previous results of the theory applied to these three comets are summarized in Table 1 of the present paper, along with corresponding new calculations for periodic Comets Pons-Winnecke, Biela, and Brooks. The entries of the first eight lines of Table 1 are self-explanatory. The quantity $\Delta\mu/\mu$ (obs.) is the observed relative acceleration per period in the mean daily motion. Averaged values of orbital elements are used. The quantity r_0 represents the solar distance within which acceleration is assumed to act. The quantity (mean) $r^{-1/4}$ is an adopted value used to avoid the numerical integration of

* An investigation carried out under NOrd Contract No. 10449-05512 of the Naval Bureau of Ordnance.

[1] F. L. Whipple, *Ap. J.*, **111**, 375, 1950.

the integrals I_a, I_e, and I_ϖ of equations (33_1), (33_2), and (33_3) of Paper I. These integrals and their approximations are

$$I_a = \int_0^{v_0} r^{-5/4} d\, v_1 \cong \frac{v_0 + e \sin v_0}{r^{1/4} p}, \tag{1a}$$

$$I_e = \int_0^{v_0} r^{-5/4} (a p - r^2)\, d\, v_1, \tag{1b}$$

$$I_e r^{1/4} \cong a\,(v_0 + e \sin v_0) + \frac{p}{\sqrt{(1 - e^2)}} \left\{ \sin^{-1} \left(\frac{e + \cos v_0}{1 + e \cos v_0} \right) - \frac{\pi}{2} \right\} \tag{1c}$$

and

$$I_\varpi = \int_0^{v_0} r^{-1/4} \cos v_1 d\, v_1 \cong r^{-1/4} \sin v_0\,, \tag{1d}$$

where v_1 is the true anomaly and v_0 its value at r_0. The integrations have been made numerically only for Comet Encke. The quantity r_m (A.U.) in Table 1 is the solar distance

TABLE 1

SECULAR CHANGES IN COMETARY ORBITS

Periodic Comet	Encke	Encke	Pons-Winnecke	Biela	D'Arrest	Brooks	Brooks	Wolf I	1905 III
Interval	<1865	>1865	....	<1847	....	<1922	>1922	>1920	....
No. returns	13	20	5	3	6	3	3	4	0
Period (years)	3.30	3.30	6.04	6.65	6.67	7.10	6.94	8.28	297
e	0.847	0.847	0.686	0.751	0.627	0.470	0.486	0.405	0.975
q (A.U.)	0.34	0.34	1.04	0.88	1.32	1.96	1.87	2.43	1.12
$\Delta\mu/\mu$ (obs.)$\times 10^{+5}$	+9.7	+4.2	−0.16	+10	− 6.4	+ 5.6	+ 3.8	−0.30	*
Direction of rotation	Retr.	Retr.	Direct	Retr.	Direct	Retr.	Retr.	Direct	....
r_0 (A.U.)	2.0	2.0	2.0	2.0	2.0	3.0	3.0	3.0	....
(Mean) $r^{-1/4}$ (A.U.)	....	....	0.94	0.97	0.89	0.82	0.82	0.78	0.91
r_m (A.U.)	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
I_a (A.U.)$^{-5/4}$	5.8	5.8	1.30	1.64	0.86	0.61	0.65	0.46	....
I_e (A.U.)$^{3/4}$	6.2	6.2	5.4	6.8	4.6	3.3	3.4	3.0	....
I_ϖ (A.U.)	0.87	0.87	0.92	0.92	0.88	0.82	0.81	0.78	....
v_m (rad.)	2.8	2.8	1.81	1.89	2.04	1.66	1.72	1.45	1.86
$\Delta M/M$ (calc.)$\times 10^{+3}$	4.8	2.0	0.09	4.7	5.2	4.4	2.9	0.23	40
Δe (obs.)$\times 10^{+6}$	−9.4	−4	−1.5	−10.2	+17	....	....	....	....
Δe (q=const.)$\times 10^{+6}$	−9.8	−4.3	+0.3	−16.5	+16	−20	−13	+1.2	....
Δe (calc.)$\times 10^{+6}$	−8.3	−3.6	+0.3	−15	+15	−16	−11	+1.0	....
$\Delta\varpi$ (calc.)	−2″1	−0″9	−0″14	− 5″8	− 8″2	−12″0	− 7″4	−0″85	....

* Based on $\Delta k^2/k^2 = -9\ (\pm 3) \times 10^{-5}$; see Paper I, p. 393.

within which the loss of cometary material from the nucleus is assumed to be essentially complete. The true anomaly v_m corresponds to r_m.

The loss of material per period by the cometary nucleus, $\Delta M/M$, has been calculated from $\Delta\mu/\mu$ on the assumption that the dimensionless force component, γ, in the orbital plane normal to the radius vector, is 0.1 as contrasted to its maximum value $\frac{4}{9}$. This assumption is quite arbitrary. The tabulated quantity $\Delta M/M$ should be multiplied by $0.1/\gamma$ to give its value for another chosen value of γ. The sign of γ is positive for direct rotation of the cometary nucleus and negative for retrograde rotation, as assumed in the ninth line of Table 1.

The observed value of Δe per period can be compared in Table 1 with its value, Δe (q=const.), calculated on the assumption that the observed acceleration in mean daily motion arises from a tangential impulse at the instant of perihelion. It can also be compared with its value, calculated by the present theory, Δe (calc.), from the observed acceleration in mean daily motion, on the assumption that the acceleration on the nucleus occurs only at solar distances less than r_0.

The change in the direction of perihelion passage per period, $\Delta\varpi$, is calculated from the ratio of equations (33_3) and (33_1) of Paper I, viz.,

$$\Delta\varpi = \frac{1}{3ae} \frac{I_\varpi}{I_a} \frac{\zeta}{\gamma} \frac{\Delta\mu}{\mu}. \qquad (2)$$

The quantity ζ is the dimensionless component of the force acting on the cometary nucleus along the radius vector away from the sun. The ratio ζ/γ has been adopted arbitrarily as ± 4, on the assumption that γ is finite but that the radial component generally exceeds the normal component.

The motion of perihelion is essentially zero if the cometary nucleus be assumed to lose matter proportionately to the solar flux entirely around the orbit and if a negative correction be applied to the gravitational constant. In the calculation of secular accelerations in the elements of periodic comets, possible variations in ϖ tend to be absorbed in a correction to the mean anomaly and hence to the period. The order of magnitude of the calculated values of $\Delta\varpi$ are of general interest, and the writer hopes that a special search may be made for a secular effect in ϖ. An empirical determination of the ratio ζ/γ would be invaluable. The expected changes in i and Ω from similar forces perpendicular to the orbital plane are below detectability.

The source data for the observed values of $\Delta\mu/\mu$ and Δe have been given in Paper I for Comets Encke, D'Arrest, and Wolf I. The values for Comets Pons-Winnecke and Brooks were adopted from the recent investigation by A. D. Dubiago.[2] The value for Comet Biela up to the discovery of its duplicity in 1845 is based on the results of J. von Hepperger.[3] From Hepperger's and J. S. Hubbard's[4] work, it appears that no secular acceleration of the mean position of the two components is required in the period from 1846 to 1852. On the other hand, the difference in the mean motions of the two components had become considerably greater than the previous secular change per period.

The writer obtains a somewhat larger value of the accelerations in μ for Comet Biela than was obtained by Dubiago (30 per cent difference) even after correcting his values by the usual factor of 2 required to obtain agreement for Comets Encke and Wolf I. Apparently his definition of the secular change in μ differs from the present one by a factor of 2. No difference appears in Δe. The values of $\Delta\mu$ for Comets Pons-Winnecke and Brooks used here were taken from the *Bulletin of the Committee for Distribution of Astronomical Literature* rather than from the original source, making possible an uncertainty in terminology. Dubiago questions his value of Δe for Comet Pons-Winnecke, and we note that the predicted value is of opposite sign. Similarly, the predicted value of Δe for Comet Brooks is quite sizable, though Dubiago found no measurable value.

For the other comets, however, the calculated values of Δe are in good agreement with the observed values, while the calculated losses of mass in Table 1 are sufficiently small to permit considerable lifetimes for the comets. Note, however, that the calculated rate of mass loss for Comet Biela just prior to its bifurcation and disappearance is about the same as that for Comet Encke during the first half of the nineteenth century. There is no assurance of cometary longevity in a small value of $\Delta M/M$!

In evaluating the reality of the secular accelerations given in Table 1, the writer is of the opinion that the acceleration is unquestionably real for Comet Encke and is well

[2] *A.J. Soviet Union*, **25**, No. 6, 361, 1948.

[3] *Sitz-ber. Akad. Wiss. Wien*, Abt. II*a*, **109**, 299, 623, 1900; **112**, 1329, 1903.

[4] *A.J.*, **4**, 1, 1854; **6**, 137, 1860.

determined for Comets Biela and D'Arrest. The check in Δe is especially satisfactory for these three comets. For the other comets the writer has had less opportunity to study the basis of the determination. Further checks on the validity of the proposed comet model will be presented in the following sections of this paper, particularly with regard to the physical nature of comets and their relationships with meteor streams.

B. DETERMINATION OF THE RADII OF COMETS

A knowledge of the proportion of mass lost per period by a spherical cometary nucleus and of the solar heat falling upon it determines the radius of the comet in terms of certain physical characteristics of the cometary material. A comparison with other determinations of the cometary radius provides a check on the comet model or a determination of the physical characteristics, as one chooses.

We have concluded previously that the rate of absorption of solar heat by the model nucleus becomes relatively small at a solar distance of 1.5–2.0 A.U. and essentially negligible at 3 A.U. or more. Comets with perihelion distances greater than these values must be quite large or else covered with an exceedingly thin layer of meteoric material to show much activity. Hence the solar flux, F/r^2, may be ignored at true anomalies from $+v_m$ to $-v_m$. The effective time, f, of solar heating equivalent to the solar constant ($F = 0.032$ cal cm^{-2} sec^{-1} at 1 A.U.) is obtained from the averaged time integral of F_0/r^2 from $-v_m$ to $+v_m$. The result is

$$f = \frac{v_m}{\pi \sqrt{p}} \text{ years} = 1.01 \times 10^7 \frac{v_m}{\sqrt{p}} \text{ sec}, \tag{3}$$

where p (in A.U.) is the parameter of the orbit.

Suppose that the cometary ices utilize a fraction, $1/n$, of the solar radiation for sublimation; that the nucleus is spherical, of radius R_c and density ρ_c; and that the mean heat of sublimation for the ices is H calories per gram. Then the total mass of the ices, ΔM, sublimated in one orbital period, is given by:

$$\Delta M = \frac{\pi R_c^2 f F}{n H}. \tag{4}$$

The corresponding loss of radius, ΔR_c, if we neglect the meteoric material, then becomes

$$\Delta R_c = \frac{fF}{4 n H \rho_c}. \tag{5}$$

But the differential losses in mass and radius for a sphere are related by the equation

$$\frac{\Delta M}{M} = 3 \frac{\Delta R_c}{R_c}. \tag{6}$$

Hence the radius of the comet is given by

$$R_c = \frac{M}{\Delta M} \frac{3 fF}{4 n H \rho_c}. \tag{7}$$

If $\Delta M/M$ is accepted for the comets in Table 1, we have only three physical constants to evaluate in equation (7). The quantity n may be taken as unity to give a maximum value of the calculated radius. A later discussion will show that $H = 450$ cal/gm and $\rho_c = 1.0$ are reasonable values and that the neglect of the meteoric materials in computing ρ_c and the mass loss is probably not serious. With these adopted values of the physical quantities, we find the maximum radii of the six comets as given in the fourth column of Table 2. The table entries are self-explanatory after reference to Table 1 and equations (3) and (7). For Comet 1905 III (Giacobinid) the value of $\Delta M/M$ is adopted from the effective decrease in the Gaussian constant (Paper I).

In Table 2 the smaller values of the maximum radius should be considered as more significant, since they are based generally on the more reliable values of $\Delta\mu/\mu$ and hence $\Delta M/M$. Included implicitly in the calculated values of R_c (max.) is the arbitrarily adopted quantity γ, taken as 0.1. The calculated radii can be increased as much as the factor 0.444/0.1 by increasing γ, but they may be reduced indefinitely as γ is taken nearer to zero. An increase in n will reduce the calculated values proportionately.

Vorontsov-Velyaminov[5] concluded that a solid spherical nucleus for Halley's Comet cannot exceed a radius of 30 km and most probably is about 15 km in radius. Now Halley's Comet is some 5 mag. brighter intrinsically than the average comets of Table 2. Hence we see that the order of magnitude of the agreement is good if we except the large limiting radii for Comets Pons-Winnecke and Wolf I.

Comet Encke, the most reliable example in Table 2, was observed by H. M. Jeffers[6] as magnitude 18 on September 3, 1937. Reducing to the standard distance of 1 A.U. from

TABLE 2

MAXIMUM RADII OF COMETS

Comet	$\Delta M/M$ (Per Period)	f (Sec $\times 10^{-7}$)	R_c (max.) (Km)	Comet	$\Delta M/M$ (Per Period)	f (Sec $\times 10^{-7}$)	R_c (max.) (Km)
Encke (<1865)	0.0048	3.5	4.0	Brooks (<1922)	0.0044	1.0	1.2
Encke (>1865)	.0020	3.5	9.2	Brooks (>1922)	.0029	1.0	1.8
Pons-Winnecke	.0001	1.4	82	Wolf I (>1920)	.0002	0.8	19
Biela	.0047	1.5	1.7	1905 III	0.04*	1.3	0.2
D'Arrest	0.0052	1.4	1.4				

* Derived from $\Delta k^2/k^2 = -0.00009$.

both the earth and the sun by the ephemeris of the *B.A.A. Handbook*, one finds that the corresponding absolute magnitude was 16. If the albedo of the nucleus were not less than that of the moon, the nucleus of Comet Encke could not have exceeded 1 km in radius. Probably the nucleus was considerably smaller, as the comet then showed a coma-tail with a radius of some thousands of kilometers. One must conclude, therefore, by comparison with Table 2, that for Comet Encke the heating efficiency factor, $1/n$, is of the order of 0.1 or that the projection factor, γ, is of the order of 0.01 instead of 0.1.

The calculated maximum radius for Comet Biela is 1.7 km at a time just prior to its bifurcation and the rapid disappearance (if not disintegration) of its two components. It is probable, as for Comet Encke, that the actual dimensions of the nucleus were much smaller and that $(1/n) < 1$ or $\gamma < 0.1$.

The radius calculated for Comet 1905 III appears excessively small and may, of course, lack significance in case the observed value of $\Delta k^2/k^2$ arises from other causes. On the other hand, the comet at perihelion was diffuse and relatively faint, with a nucleus of 12.5 mag. on April 6, 1905, according to K. Graff.[7] We may conclude generally that the proposed comet model predicts maximum radii of cometary nuclei that are consistent with other independent data. For purposes of comparison in terms of mass, a sphere of radius 1 km and density 1 contains a mass of 4.2×10^{15} gm.

[5] *Ap. J.*, **104**, 226, 1946.

[6] *Harvard Announcement Card*, No. 433, 1937.

[7] *A.N.*, **173**, 377, 1907.

C. ORIGIN AND NATURE OF THE COMETARY CONGLOMERATE

The writer does not wish to speculate at this time on the detailed theories for the origin of comets. Nevertheless, the subject can hardly be completely avoided in making estimates of the relative abundances of the various materials. Furthermore, the basic assumptions for the comet model demand certain limitations in the conditions under which comets can come into existence. The most critical condition is that the temperature be very low. Presumably, in addition, the atoms (or molecules) must combine from the gaseous state, and the solid particles coalesce by encounter at relatively small velocities. One seems justified in assuming that the relative abundance of the elements in comets should be typical of the universe at large, with the limitation that elements not freezing or forming compounds should be rare or absent.

Reference to estimates of the cosmic abundances of elements—for example, those essentially by J. L. Greenstein, as tabulated by G. P. Kuiper[8]—shows that, on the basis of unity for O, the major contributions are 100 by H, 0.3 by C, 0.35 by N, and 0.8 by the sum of the major heavier contributors through Ni. Helium is, of course, omitted above. Since solid H_2 is unlikely to form in the interstellar space of the galaxy, H can occur only in compounds in small bodies like comets. Hence the contribution of H to the cometary masses will be small. The great abundance of H, however, leads one to expect that most of the C, N, and O will combine with H rather than with one another or with other elements.

On the basis of the above discussion, then, the meteoric materials should constitute about one-third or less the mass of cometary nuclei, the other two-thirds being made up largely of hydrides of C, N, and O. Estimates of the distribution of the chemical and physical structures of the meteoric materials must be made from studies of comets, meteors, and micrometeorites.[9] Determinations of these quantities can then lead to quantitative understanding of the physical conditions under which comets developed. This general problem constitutes one of the challenges in solar-system astronomy.

An example of this type of reasoning concerns the appearance of sodium (and other metals) in cometary spectra. Bobrovnikoff[10] points out that for certain comets the Na lines are extremely strong, even though the equilibrium temperature with sunlight could scarcely exceed the melting point of water. Neither vapor pressure from the solid state nor desorption would be likely to produce so much Na vapor from solid particles. On the other hand, if Na atoms or molecules of sodium compounds were imbedded in the ices of the present model, sublimation of the ices or sublimation plus dissociation of the molecules could release the Na atoms observed. The existence of such phenomena helps describe the conditions under which comets were formed.

Lacking, at present, sufficient information to estimate the frequency distribution of the various molecules of H, C, N, and O, the writer has arbitrarily assumed that the major mass is in the form of CH_4, NH_3, and H_2O.[11] The heat required to sublimate the above-assumed proportions of these compounds to the average surface temperatures at cometary nuclei is of the order of 450 cal/gm, as assumed in previous calculations.

The assumed high abundance of H_2O ice is the basis for expecting it to play a major part in reducing the efficiency of sublimation of cometary ices at fairly large solar distances.

If, indeed, a third of the mass of the comet is in the form of meteoric material (21 per cent, according to Harrison Brown), then the calculations of $\Delta M/M$ made previously

[8] *The Atmospheres of the Earth and Planets* (Chicago: University of Chicago Press, 1949), p. 309.

[9] Small meteoric particles that can pass through the atmosphere undamaged (see *Proc. Nat. Acad. Sci.*, **36**, 687, 1951.

[10] *Rev. Mod. Phys.*, **14**, 164, 1942.

[11] This assumption is in agreement with that made by Harrison Brown in his postulated composition of the preplanetary medium. I am indebted to Dr. Brown for a discussion of this point.

must be increased appropriately to allow for the macroscopic material carried along at low velocities by the hydrides of C, N, and O. This correction will not affect the calculated values of Δe and $\Delta\varpi$ in Table 1 but will reduce proportionately the calculated maximum radii of comets in Table 2. A low density of $\rho_c = 1.0$ has been chosen to allow for spaces between finite particles.

Of vital interest in interpreting the spectra of comets are the expected or observed percentages of the C-N-O material in the forms of compounds such as C_2, CN, C_2N_2, CO, CO_2, N_2, and NO. K. Wurm,[12] for example, has corrected the earlier estimates by Schwarzschild and Kron[13] for the mass of CO^+ lost by Halley's Comet near perihelion. He obtains a rate of loss of 1500 gm/sec, approximately ten times the previous value and about this same factor greater than was obtained by Vorontsov-Velyaminov.[14] Wurm also estimates that some 1.5×10^{32} C_2 molecules are required to produce the C_2 radiation observed for Halley's Comet near perihelion. If we accept Wurm's value of a lifetime of 10 hours for the C_2 molecule between $r = 0.5$ and $r = 1$ A.U., the rate of loss of C_2 becomes 2×10^5 gm/sec. Again his numerical results exceed the corresponding value obtained by Vorontsov-Velyaminov by about one order of magnitude.

We can now estimate the relative abundances of CO^+ and C_2 to the hydrides if we adopt a radius and heating efficiency factor for Halley's Comet. Suppose the radius to be 10 km (probably a generous estimate) and half the solar heat to be effective in sublimating the ices. Then the loss of matter from Halley's Comet at $r = 0.6$ A.U. becomes 3×10^8 gm/sec, yielding a relative abundance by mass of about 5×10^{-6} for CO^+ and 7×10^{-4} for C_2.

As an illustration of the type of argument that is applicable, without defending any implied conclusions, we note that D. ter Haar,[15] in an extension of the C. F. von Weizsäcker[16] theory of planetary formation, has calculated the relative abundances of various molecules from a medium at a temperature of a few hundred degrees K and a density of 10^{16} atom/cm^3. His calculated ratio of CO to H_2O is numerically 10^{-2} and of C_2 to CH and CH_4, 10^{-4}. Other such ratios are given by ter Haar. The agreement of the values for C_2 is excellent, and we note that C_2 might remain mixed with the ices rather than be derived from C_2N_2. Indeed, ter Haar would predict a relatively low abundance of C_2N_2. On the other hand, CO^+ must be derived from some parent-compound, such as CO or CO_2, by photoionization. Losses would be expected in the secondary processes. The abundance ratio of CO_2 to H_2O is 10^{-5}, as predicted by ter Haar, more in keeping with the observed-theoretical ratio for comets if CO^+ is derived from CO_2.

Before continuing with the abundance problem, we note that certain general restrictions on the origin of the solar system may be set by the present comet model. Three possibilities may be mentioned briefly.

a) If the comets were acquired by the system since its origin, possibly in the manner suggested by R. A. Lyttleton,[17] no implications concerning the origin of the system would be involved. In this case the material of the comets should typify interstellar solids, possibly somewhat modified. J. Oort's suggestion[18] that the comets were derived from asteroidal material is, of course, incompatible with the proposed comet model.

b) If the comets were formed with the planets *and* with the sun, limits on the physical surroundings of the protosun become established. Unless the radiation from the con-

[12] *Mitt. Hamburger Sternw., Bergedorf*, No. 51, p. 57, 1943.

[13] *Ap. J.*, **34**, 342, 1911.

[14] *Op. cit.*, p. 231.

[15] *Det. Kgl. dansk. Vid. Selsk., Mat.-fys. Medd.*, Vol. **25**, No. 3, 1948.

[16] *Zs. f. Ap.*, **22**, 319, 1944.

[17] *M.N.*, **108**, 465, 1948. It can easily be shown, however, that planetary perturbations destroy the precise convergence of particles required by Lyttleton's theory and render the process inoperative.

[18] *B.A.N.*, **11**, 91, 1950.

densing (or disrupting) system was surprisingly small, the comets must have formed at very large solar distances, as would follow, for example, from the writer's dust-cloud hypothesis of the origin of the system.[19]

c) If the comets and planets formed in a ring of matter revolving about the sun after the sun was formed, as is most commonly assumed, much depends upon the solar radiation at the time, the density of the ring, and whether solid CH_4 is actually present in cometary nuclei. At low gas pressures solid CH_4 can scarcely condense at gas temperatures above 50° K and possibly not above 30° K (ter Haar). These stringent temperature limitations almost demand that solid CH_4 be condensed in H I regions of space within large quiescent dust clouds. Whether CH_4 might condense at higher temperatures on finite particles of H_2O or meteoritic material requires further investigation, as do the general problems of physical chemistry in interstellar space.

The absence of H_2O, NH_3, and CH_4 in observed cometary spectra sets no particular limit on the relative abundances of these substances in cometary nuclei. These gases are poor radiators in the photographic, the visual, and even the far red regions of the spectra. Little is known about the detailed processes of their photodissociation. A simple calculation shows that Rayleigh scattering by these molecules is negligible compared to the direct reflection of sunlight from the cometary nucleus at the rates of sublimation assumed in the present paper. A similar calculation of electron scattering, on the assumption that each atom produces one electron immediately on its escape from the cometary nucleus, shows that the scattering by electrons near perihelion could be comparable only to the direct reflection of sunlight from the nucleus. Hence electron scattering, even with such a generous assumption, appears ineligible to account for much cometary radiation. The haze of fine particles surrounding and escaping from the nucleus would be expected to outshine the nucleus itself.

D. THE EJECTION OF METEORITIC MATERIAL

Meteoritic material near the model cometary nucleus will be forced away from the surface against gravity by momentum transfer from the outgoing gases. Very near the surface the law of force will be complicated and the actual force increased by the "multiple reflection" of outgoing gases between the surface of the nucleus and the particle. Furthermore, for a rotating nucleus, the rate of gas output will vary with time and in all cases will depend upon the position on the nucleus.

Let us consider only the meteoritic material that is already carried an appreciable distance from the nucleus and let us assume that escape is sufficiently rapid with respect to the rotation of the nucleus so that we need consider only the gases escaping from the sunlit hemisphere (perhaps somewhat rotated from the sun's direction by the heat-transfer lag). Although various solutions are possible, the following appears sufficiently representative for our present needs: similarly to equation (4), the mass of gas lost per second by the nucleus at a distance r (A.U.) from the sun is $(\pi F R_c^2)/(n r^2 H)$. At a distance R ($\gg R_c$) from the nucleus we assume that this gas escapes over the hemisphere $2\pi R^2$ with an outward velocity $\bar{v}$ (eq. [28], Paper I). Hence the momentum transferred per square centimeter per second at distance R is $(\bar{v} F R_c^2)/(2 n r^2 H R^2)$.

The meteoritic particles must be exceedingly irregular and rough, thus acting with a large accommodation coefficient for gaseous encounters. For simplicity let us assume that the accommodation coefficient is unity and that the particles are spherical, of radius s and density ρ_s. If the cometary gas of very low density is momentarily stopped by the particles and re-emitted immediately with thermal velocities, the drag coefficient is approximately $2\,(1+\frac{4}{9})$, the case of free molecular flow. Hence the outward force on a

[19] *Centennial Symposia* (Cambridge: Harvard College Observatory, 1948), p. 126.

slowly moving particle is given by $(13\pi\bar{v}s^2FR_c^2)/(18nr^2HR^2)$. The total force will be reduced by gravity so that the net outward acceleration becomes

$$\frac{d^2R}{dt^2}=\frac{(C_1-C\,R_c)\,R_c^2}{R^2}, \tag{8a}$$

where

$$C_1=\frac{39\,\bar{v}F}{72n\,r^2\,s\,\rho_s H} \tag{8b}$$

and

$$C_2=4\pi\,\rho_c\,\frac{G}{3}. \tag{8c}$$

The relative velocity, V_∞, of the particle at infinity with respect to the cometary nucleus, then, is

$$V_\infty^2=2C_1R_c-2C_2R_c^2. \tag{9a}$$

We may evaluate $\bar{v}$ numerically from equation (28), Paper I, and express equation (9*a*) numerically from solar-system data as previously for a cometary density of $\rho_c = 1$ gm cm^{-3} and a particle density of $\rho_s = 4$ gm cm^{-3}. If the particle radius s is expressed in centimeters, the cometary radius R_c in kilometers, and the solar distance r in A.U., then we find the velocity of ejection for meteoritic particles to be

$$V_\infty=\left(\frac{1}{n\,s\,r^{9/4}}-0.052R_c\right)^{1/2}R_c^{1/2}\times 328\text{ cm sec}^{-1}. \tag{9b}$$

It must be noted that equations (9*a*) and (9*b*) apply to the case when the velocity of ejection of the particle is *small* compared to the velocity of the outgoing gas. The upper limit to the particle velocity for very small particles is obviously near the gas velocity given by equation (28) of Paper I.

Since $1/n$ represents the efficiency of solar radiation in sublimating cometary gases, we see that a new comet (with $n = 1$) of radius 1 km at 1 A.U. from the sun would eject meteoritic particles of radius 1 cm and density 4 gm cm^{-3} with a velocity of about 3 meters per second. A bright photographic meteor of geocentric velocity 30 km sec^{-1} arises from a meteoroid of about this dimension.[20] For particles of this size or smaller, the velocity of ejection theoretically varies directly as the square root of the cometary radius and the efficiency factor, inversely as the square root of the particle radius, and approximately inversely as the solar distance.

Hence the ejection of the meteoritic particles from comets should be more violent as well as more frequent near perihelion. Also, larger comets should eject particles with greater velocities than smaller comets of comparable $1/n$. Observational evidence to test these conclusions must come from detailed studies of meteor showers. Certain data concerning the major meteor showers are given in Table 3. The column headings are self-explanatory. Data have been taken largely from F. G. Watson[21] and C. P. Olivier.[22] The durations of the showers are only approximate. The strength of the photographic showers are based on Harvard photographic meteors of this century and are of average values, neglecting the unusual displays from Comet Giacobini-Zinner in 1933 and 1946, which could not be photographed in Massachusetts. The showers that are most widely

[20] See, e.g., F. L. Whipple, *Proc. Amer. Phil. Soc.*, **79**, 499, 1938.

[21] *Between the Planets* (Philadelphia: Blakiston Co., 1941), chap. vii.

[22] *Meteors* (Baltimore: Williams & Wilkins Co., 1925), chaps. iv–viii.

dispersed and associated with known comets are the Perseids, the Taurids, and the *two* showers from Halley's Comet, if these last two associations are admitted. All arise from very bright comets, since Encke's Comet must originally have been massive to have persisted so long in a short-period orbit.

S. Hamid[23] finds for the Perseid meteor shower that its long persistence and the random deviations in radiants for the photographic meteors[24] can be explained adequately by planetary perturbations, particularly Jupiter's, if the shower has endured for about two hundred revolutions of the parent-comet. Initial ejection velocities as given by equation (9) are sufficient. Greater initial velocities for smaller particles probably will account for the larger spread in radiants of the faint visual Perseids, but precise data from the Super-Schmidt meteor cameras are required to prove this possibility.

If the case of Comet Encke we have an old, but not too faint, comet providing a widely spread shower. The Harvard photographic data[24] coupled with secular-perturbation theory by S. Hamid and the writer[25] indicate clearly that the dispersion among the

TABLE 3

METEOR STREAM CHARACTERISTICS

Meteor Shower	Geocentric Velocity (Km/Sec)	Associated Comet	Comet Brightness	q (A.U.)	Shower Length (Days)	Photographic Shower
Perseids*	60	1862 III	Bright	0.97	30	Strong
Leonids*	72	1866 I	Lost	0.98	5	Medium
Lyrids*	51?	1861 I	Bright	0.92	4	Weak
Androminids*	16?	Biela	Lost	0.86	5	Weak
Geminids	35	....	....	0.14	4	Strong
Eta Aquarids	66?	Halley?	Bright	0.59	6	Weak
Orionids	66	Halley?	Bright	0.59	10	Medium
Giacobinids*	23	1933 III	Faint	1.00	1	Weak?
Taurids	27	Encke	Medium	0.39	40	Strong
Delta Aquarids	50?	....	....	0.04	10	Strong
Quadrantids	46?	....	....	0.99	2	Weak

* Have presented outstanding displays.

Taurid orbits must arise from ejection in the asteroid belt rather than at perihelion. Presumably, collisions with asteroidal material are responsible for the dispersions in these orbits of low inclination and short period.

In contrast to the widely spread Perseid and Halley-Comet showers from bright comets, we find that the faint comets—Giacobini-Zinner, Biela, and the lost Leonid parent-comet—have produced showers of marked concentration. However, in the past, both the Perseid and the Lyrid showers from bright comets have also provided remarkable displays. Although the evidence from meteor showers is by no means conclusive, it is generally consistent with the conclusions from equations (9). The most extensive and widely spread showers tend to arise from large comets; although some large comets have produced concentrated showers, *all* the faint comets responsible for showers in Table 3 have done so.

A solution of equations (8) and (9) for the maximum radius, $s_{\max}$, of ejected meteoritic

[23] "The Formation and Evolution of the Perseid Meteor Stream" (doctoral thesis; Harvard, 1950).

[24] *Technical Report No. 6* ("Harvard Rept. Ser.," No. II-35 [1950]).

[25] *A.J.*, **55**, 185, 1950 (abstr.).

particles yields the following numerical result on the basis of the assumptions made above:

$$s_{\max} = \frac{19 \text{ cm}}{n\, r^{9/4} R_c}, \tag{10}$$

where R_c is expressed in kilometers and r in A.U.

One notices that the maximum radius is quite sensitive to the perihelion distance of the comet and, for small perihelion distances, corresponds to extremely bright fireballs. Table 3 contains three perihelion distances less than 0.5 A.U. All the associated showers are strong photographically, indicating the presence of relatively large meteoric particles. Although the Perseid shower is also strong photographically, the ratio of photographic to visual strength would be smaller than for most of the other showers in Table 3. A perusal of the fireball catalogue by G. von Niessl and C. Hoffmeister[26] leads to similar conclusions. A rough identification ($\pm 40°$ in radiant within the shower interval) of possible fireballs associated with the various showers indicates perhaps 50 Taurids, 12 Geminids, 12 Delta Aquarids, 7 Perseids, and 3 or less representatives of other showers in Table 3. Hence meteor showers with small perihelion distances tend definitely to show more unusually bright members.

Although the evidence of the above several paragraphs is preliminary in character and subject to alternative explanations, the writer feels that, in totality, the comet-meteor data tend to support the comet model in associating comets and meteor streams by a physical process. Much more precise testing of the theory is possible and will be carried out as the photographic and radar observations of meteors become more extensive.

A forthcoming Paper III will present a quantitative relationship associating the loss of particles by comets with the maintenance of the Fraunhofer corona and the zodiacal light via the Poynting-Robertson effect.

[26] *Denkschriften Akad. Wiss. Wien,* **100**, 1, 1926.