ON A PERIODIC FLUCTUATION IN THE LENGTH OF THE DAY

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Summary

From a study of the performance of a number of quartz-crystal clocks employed in the Greenwich Time Service an annual periodic fluctuation in the length of the day is deduced. The variation is of the order of $\pm 0^s{\cdot}001$ and has an accumulative effect in time of approximately $\pm 0^s{\cdot}060$. This is in very close agreement with results obtained from independent data by N. Stoyko and demonstrates the persistent character of the phenomenon.

1. With the development of the quartz-crystal oscillator as a frequency standard, a demand has arisen for time signals capable of providing frequency checks to a precision consistent with the standard of performance of the crystals themselves. In undertaking to furnish these signals the Royal Greenwich Observatory has at its disposal a number of quartz clocks which serve to bridge the gap between the time determinations, the observational errors of which they also serve to smooth out. But to control the emission of a system of daily time signals that will define intervals of 24 hours to an accuracy as good as or better than one millisecond has proved a very intractable task. Of the difficulties encountered, only that of establishing the length of the current day, or alternatively, of determining the rates of the observatory clocks with the desired accuracy, is discussed in this paper.

2. Beat-frequency measurements have shown that over short periods of time frequency stabilities of the order of one part in 10^{11} can be achieved with quartz crystals, while the frequency difference of two crystals whose characteristic performances are known, can be predicted, in certain circumstances, over a period of months to within several parts in 10^8. In justice to the crystal, therefore, if the frequency of a standard oscillator is to be quoted in cycles per second, the length of the second must be known to something better than one part in 10^8. In other words, the astronomical observations must be capable of establishing the current rates of the observatory clocks to within $0^s{\cdot}001$/day.

3. At the Greenwich Observatory the available accuracy tends to be limited by two factors: (*a*) The time determinations have, up to the present, been made visually and it has not proved possible to reduce the probable error of a single determination much below $\pm 0^s{\cdot}020$. If the clock rate, assumed to be constant, is determined from two such observations, they would need to be separated by an interval of months for the rate to be derived with certainty to the desired accuracy. The interval can be reduced by increasing the number of time determinations, but the gain is by no means proportional to the increased effort. (*b*) The rates of crystal clocks are not constant, though they may be very uniform. A quartz crystal, when first set into oscillation, shows a marked frequency (rate) drift. The drift gradually diminishes until, after some months, it settles down to

a fairly constant value. The clock correction (E) can then be represented very closely by the expression $E = a + bt + ct^2$, where a, b and c are constants and t is reckoned in time from some arbitrary epoch. When t is expressed in days and E in seconds, the value of c is found in general to be between the values of $\pm 0{\cdot}00005$. The daily rate (R) is given by $R = b + 2ct$. Reliable prediction of the error of the clock requires c to be determined with great accuracy, because its effect increases with the square of the time. The existence of a frequency drift greatly increases the interval required to establish the current rate of the clock. Moreover, even the best clocks are liable to small erratic changes of b and c, and although sudden changes of rate are shown up in the daily intercomparisons, minor variations in the value of the frequency drift are not so readily detected. In order to obtain increased accuracy, and also to allow for the contingency of failures, a considerable number of clocks must be available for use in the service.

4. After quartz clocks had been introduced into the Time Department and experience with them had established their high standard of performance, the policy of predicting their errors on a long-term basis was introduced. The clock constants a, b and c were derived from a series of time determinations extending over many months. The extrapolated corrections were then heavily weighted against those indicated by current observations, which might appear discordant because of large observational errors. As observations accumulated, any persistent departure of the clock corrections from the ephemeris was allowed for graphically and the constants were subsequently adjusted to secure the best fit over the last few months. Using the adopted current corrections to the standard clocks and the daily intercomparisons it was possible to deduce the amount by which the clock controlling the time signal should be corrected prior to transmission.

5. The method appeared for a while to work fairly satisfactorily, but in the autumn of 1945 the observed clock corrections began rapidly to diverge from the predicted. The magnitude of this divergence varied somewhat from clock to clock, but all appeared to have undergone an acceleration of rate. A change in the period of rotation of the Earth was suspected. It was then noted that there had been a similar occurrence, though to a much less marked degree, during the previous autumn, which suggested the existence of a seasonal effect. There appeared to be little evidence, however, of a compensating retardation of the clocks during the spring. In subsequent years, as more clocks became available, the apparent autumnal acceleration was again observed. When, therefore, in the autumn of 1948 a discordance between the observed and predicted clock corrections again began to show, the weight given to the current observations was greatly increased. In the spring of 1949 there appeared a fairly definite indication of a retardation amongst a number of the clocks. The agreement was qualitative rather than quantitative, but it was felt that the differences between individual clocks might easily arise from erratics and the difficulty of establishing the true frequency drifts.

6. The strong suspicion of a seasonal fluctuation aroused by the spring changes led to an examination being made of the final corrections to the times of emission of the Rugby time signals, which are based on the subsequent assessment of the performance of the clocks. Since the time signals are under the control of the Greenwich Time Service, the corrections are a measure of the errors in the predicted clock corrections irrespective of which clocks are in use as

standards at the time of transmission. The four years 1944–47 agree in showing a pronounced seasonal fluctuation in the signal trend, i. e. the mean daily increase in the value of the signal corrections (signal slow). It does not appear before 1944 because the technique of long-term prediction had not then been fully developed; it disappears in 1948 because of the autumn change in technique. The mean seasonal trend of the signal for the four years 1944–47 is shown in Fig. 1. The form of this curve has necessarily been influenced to some extent by the discretion of the predictor in modifying his adopted clock corrections in the light of accumulating observations.

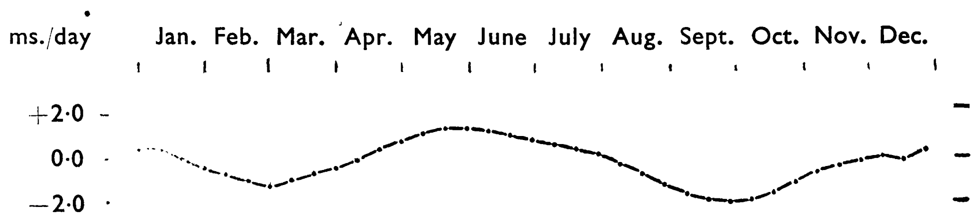

FIG. 1.—*Mean " trend " of GBR time signal at 10.00 U.T. for the years 1944–1947.*

7. The true form of the fluctuation must be derived from the observed corrections. The material suitable for this purpose is limited by various factors. It is possible to use only those clocks which have been free from disturbances occasioned by faults and have had undisturbed runs for many months. For the period 1943–47 the usable data were provided by four clocks, 4*A*, 4*B* and 4*C* in service at the Post Office, and *A*1 at the Royal Observatory, Edinburgh (where a time service was operated during the war). Considerable trouble was experienced during this period with the clocks at Abinger. From the beginning of 1948, following the replacement of the original defective beat counters of the Abinger clocks, the installation at Greenwich of six crystal clocks, and further additions at Dollis Hill, a greatly increased amount of material became available. From about the middle of 1947 the observational data were strengthened by a considerable increase in the number of time determinations and by an improvement in the accuracy consequent upon the installation at Abinger of a better transit instrument. The material falls naturally into two groups ; separate analyses have therefore been made for the two periods 1943–47 and 1948–49.

8. The analysis has been based on the assumption that the observed clock corrections can be fitted to an ephemeris of the form

$$E = a + bt + ct^2 + f,$$

where f is a periodic function of the time representing a hypothetical seasonal fluctuation, having therefore a yearly period and arising from some cause independent of the clocks. The second differences of a series of clock corrections determined at equal intervals are then of the form $2c + \Delta'' f$ and will differ from clock to clock only through the different values of the frequency drift. The third differences, $\Delta''' f$, correspond with those that would be obtained from a clock having zero values for a, b and c. Random errors will be present, but their effect is reduced by taking means of the various values obtained for $\Delta''' f$ throughout set intervals of the calendar year. Nevertheless, owing to the possible small variations in rate and frequency drift and to residual random errors, it is unlikely that the sum of the mean values of $\Delta''' f$ for the whole twelve months will be zero. The form of their distribution is not affected, however, by reducing each term by its mean value for the year. The adjusted values can be integrated to give a series of mean values for $\Delta''f$, the constant of integration being adjusted so that the mean yearly value is again zero. The integration can be continued to give first the values of $\Delta'f$ and then those of f itself.

9. The formation of the third differences was greatly eased by employing the residual clock corrections, which had been derived for the purpose of prediction, with reference to a parabolic ephemeris. These residuals had already been corrected for the effects of sudden discontinuities due to stoppages of the phonic motors and their subsequent restarting; such stoppages do not affect the rates of the clocks. The mean residual clock corrections with the corresponding mean dates of observation for each month were tabulated, after which the mean residuals were corrected for the effect of polar variation. Up to the end of 1948 these corrections were based on the values of the polar motion published by the Central Bureau of the International Latitude Service. The 1949 corrections were based upon the latitude variation data supplied by the U.S. Naval Observatory at Washington. The data were then adjusted to an adopted date for each month by correcting for the residual rate. As the correction was always small, the effect of departures of the mean from the standard date was never critical. The standard dates were selected to be separated as nearly as possible by alternate intervals of 30 and 31 days in order to make the data suitable for differencing. The weights of the final monthly values necessarily differ considerably.

10. In order to reduce the effect of observational error in relation to the fluctuation, the months were divided into two groups, odd and even, to give a uniform working interval of 61 days. The two sets of figures were separately differenced and independently reintegrated as described above. Two sets of values for f were thus obtained, which were finally dovetailed together. This treatment of the figures required a minimum undisturbed run of seven months for any clock to furnish a single third difference.

11. The values of the third differences for the two periods 1943–47 and 1948–49 are given in Table I. The clock and the year are indicated in the first two columns and at the head of each succeeding column is listed the calendar month corresponding to the centre of each seven-month period involved in the derivation of the figures below. At the foot of the table are given the mean values for each column and the weight as defined by the number of entries above. Table II shows the number of time determinations obtained in each month during the whole period 1943–49.

12. The reintegration of $\Delta'''f$ through the stages $\Delta''f$, $\Delta'f$ to f is contained in Table III; the left- and right-hand sides are devoted respectively to the odd and even months. The mean value printed at the bottom of each column is subtracted from each of the values given above it to obtain $\Delta''f$, $\Delta'f$ and f as explained above. Fig. 2 contains plots of $\Delta'''f$, $1/61 . \Delta'f$ and f for each of the two periods. It will be noted how closely the results from the odd and even months separately are in agreement. The analysis of the f curves give:

$$\text{for } 1943\text{–}47 \qquad f = +55{\cdot}7 \sin n(T+141)^\circ - 16{\cdot}8 \sin 2n(T+67)^\circ,$$

$$\text{and for } 1948\text{–}49 \qquad f = +58{\cdot}3 \sin n(T+158)^\circ - 7{\cdot}1 \sin 2n(T+48)^\circ,$$

where T is the day of the year, f is expressed in milliseconds and n is the diurnal motion of the mean Sun in degrees of arc, i.e. 0·986.

Table I

Collected Third Differences

[unit=$0^s\cdot001$]

		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
4A	1944	...	...	...	...	...	...	...	...	+ 49	+153	+175	+ 55
	1945	+ 15	−47	− 5	+ 73	+ 38	− 38	− 99	−106	− 30	+110	+115	− 37
	1946	−128	+47	+163	+ 12	+ 90	+ 30	−254	−113	+103	+ 50	− 18	+ 21
	1947	+ 13	+35	+ 97	+127	+ 3	− 53	− 2	−150	−144	+ 61	+ 51	− 38
	1948	−105	...	...	...	...	...	...	...	...	...	...	...
4B	1944	...	...	...	...	+ 11	+ 58	− 81	−214	− 30	+149	+ 85	+ 58
	1945	+ 72	−33	− 8	+ 83	+ 63	− 2	− 97	−166	−131	+ 37	+144	+ 10
	1946	−174	−43	+126	+ 83	+197	+ 60	−308	−204	+ 25	+ 14	− 53	+ 3
	1947	+ 34	−25	+145	...	...	...	...	...	...	...	...	...
4C	1943	− 17	+71	+ 63	− 27	...	...	...	...	...	...	+ 13	...
	1944	...	...	...	...	...	+ 18	−106	−166	+ 26	+157	+ 67	− 4
	1945	− 11	...	...	...	...	...	...	− 83	−134	− 85	− 20	+ 4
	1946	−121	+73	+160	...	...	...	...	...	...	...	...	...
A1	1944	...	...	...	...	...	...	...	...	...	...	+199	+ 96
	1945	+ 74	−10	− 15	+ 88	...	...	...	...	...	...	...	...
Means		− 32	0	+ 81	+ 63	+ 67	+ 10	−135	−150	− 29	+ 72	+ 69	+ 17
Wt.		11	9	9	7	6	7	7	8	9	9	11	10
C3	1948	...	...	...	+ 16	+ 58	+118	+ 14	− 58	+ 18	+ 13	+ 1	+107
	1949	+105	+70	+ 90	+ 54	+ 38	− 8	...	...	...	...	...	...
D3	1948	...	...	...	+134	+ 63	+ 7	− 85	−155	− 51	+57	+ 94	+158
	1949	+100	+22	+ 57	...	...	...	...	...	...	...	...	...
A1	1948	...	...	...	− 3	+ 28	+ 57	− 96	−189	− 54	+ 54	+ 47	+ 62
	1949	+ 14	+19	+ 94	+ 44	+ 25	...	...	...	...	...	...	...
F1	1948	...	...	...	− 54	− 72	+ 13	− 35	−142	− 83	+ 15	+ 75	+ 95
	1949	− 16	−45	+ 42	+ 48	+ 51	− 14	...	...	...	...	...	...
F3	1948	...	...	...	+ 2	− 9	+ 37	− 32	−109	...	...	...	...
	1949	...	...	...	+ 53	+ 66	− 36	...	...	...	...	...	...
5B	1948	...	...	...	+ 74	− 19	− 63	− 98	−101	+ 18	+ 74	+ 11	+ 38
	1949	+ 71	+26	− 24	...	...	...	...	...	...	...	...	...
9A	1948	...	...	...	...	+108	+ 67	− 44	− 89	+ 28	+ 91	+ 31	+ 63
	1949	+103	+88	− 24	+ 43	+ 48	− 19	...	...	...	...	...	...
Means		+ 63	+30	+ 39	+ 32	+ 32	+ 15	− 54	−120	− 21	+ 51	+ 43	+ 87
Wt.		6	6	6	11	12	11	7	7	6	6	6	6

The expressions represent that part of the clock correction caused by the seasonal variation, which is now clearly apparent. The effect upon the observed clock rates is obtained by differentiating these expressions with respect to T, when we obtain:

$$\text{for 1943–47} \qquad f' = -0{\cdot}958 \sin n(T+49)^\circ + 0{\cdot}578 \sin 2n(T+22)^\circ,$$

$$\text{and for 1948–49} \qquad f' = -1{\cdot}003 \sin n(T+67)^\circ + 0{\cdot}245 \sin 2n(T+2)^\circ,$$

the unit here being ms./day.

13. The similarity of the curves for the two periods is very marked, the deep trough occurring in August being the most striking feature of the $\Delta''' f$ curves. The second trough appearing in the $\Delta''' f$ curve for 1943–47 is the enhanced effect of the second harmonic which is very apparent in f during this period, but much less so during the period 1948–49. It is the enhancement of this harmonic which causes the f' curves to differ more markedly than the f curves. In both periods, however, the clocks appear to go fast in the spring and slow in the autumn, the minima and maxima of the f, or correction curves, occurring in May and October respectively. The date on which f passes through zero on its upward trend occurs early in August during both periods, at the time when the apparent rate, as seen in f', is a maximum. The amplitudes of the two f curves are in close accordance and are about $\pm 0^{\mathrm{s}}{\cdot}060$, while those of the f' curves indicate an annual fluctuation in the apparent rates of $\pm 0^{\mathrm{s}}{\cdot}001$/day.

TABLE II

Number of Observations Secured

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1942	...	...	...	...	...	...	...	...	...	27	33	26
1943	20	24	31	25	29	19	26	25	35	26	34	27
1944	23	17	29	24	20	15	14	23	24	20	19	24
1945	12	19	25	22	17	15	18	16	8	30	15	16
1946	16	10	10	15	16	10	14	24	16	24	20	21
1947	28	7	7	25	21	32	37	62	63	51	47	22
1948	29	48	65	50	38	22	28	25	22	47	34	25
1949	32	48	47	43	63	47	31	36	40	...	...	...

14. Having determined the seasonal fluctuation, the observed clock corrections can be corrected for its effects, and a new set of values for a, b and c for the various clocks determined. This has been done for the Post Office ring-crystal clock $5B$, which from the daily intercomparisons was known to be amongst those whose performance was the most uniform. The full line in Fig. 3 represents the performance of the clock referred to the finally adopted G.M.T., after removal of a parabolic ephemeris and the polar effect; the dotted line represents its performance further corrected by the f term. The deviations from the ephemeris over the eighteen months ending 1949 June 15, when a failure of temperature control occurred, do not exceed $\pm 0^{\mathrm{s}}{\cdot}020$. Some of the irregularities may be due to observational uncertainties, though at least one is apparent in the intercomparisons with other clocks. Nevertheless, the performance is truly remarkable and illustrates the difficulties with which the astronomer is faced when asked to check the frequencies of crystal oscillators by using the Earth as a standard.

15. The seasonal fluctuation f is one which clearly cannot be ascribed to the clocks, nor can it readily be associated with systematic errors in the observations due to faulty star places, instrumental defects or to local peculiarities.

TABLE III

Re-integration of Differences

	$\Delta'''f$	$\Delta''f$	$\Delta'f$	f	$\Delta'''f$	$\Delta''f$	$\Delta'f$	f
				Period 1943–47				
Jan.	− 32 − 35	...	...	...	...	...	...	...
Feb.	...	− 35 − 38	...	...	0 − 2	...	...	...
Mar.	+ 81 + 77	...	− 38 − 61	...	...	− 2 − 6	...	...
Apr.	...	+ 42 + 40	...	−61 −48	+ 63 + 61	...	− 6 −36	...
May	+ 67 + 64	...	+ 2 −22	...	...	+ 59 +55	...	− 36 −52
June	...	+106 +103	...	−83 −70	+ 10 + 8	...	+ 49 +19	...
July	−135 −139	...	+105 +82	...	...	+ 67 +63	...	− 17 −34
Aug.	...	− 33 − 35	...	− 1 +11	−150 −152	...	+112 +81	...
Sept.	− 29 − 32	...	+ 70 +46	...	...	− 85 −89	...	+ 64 +48
Oct.	...	− 65 − 68	...	+45 +58	+ 72 + 70	...	+ 23 − 7	...
Nov.	+ 69 + 65	...	+ 2 −21	...	...	− 15 −19	...	+ 57 +40
Dec.	...	0 − 2	...	+24 +37	+ 17 + 15	...	+ 4 −26	...
Jan.	...	...	0 −24	...	...	0 − 4	...	+ 31 +15
Feb.	...	...	...	0 +12	...	...	0 −31	...
Mar.	...	...	...	...	...	...	...	0 −17
Mean	+ 3·5	+ 2·5	+ 23·5	−12·7	+ 2·0	+ 4·0	+ 30·3	+ 16·5
				Period 1948–49				
Jan.	+ 63 + 46	...	...	...	...	...	...	...
Feb.	...	+ 46 + 16	...	...	+ 30 + 14	...	...	...
Mar.	+ 39 + 22	...	+ 16 −33	...	...	+ 14 +32	...	...
Apr.	...	+ 68 + 37	...	−33 −46	+ 32 + 16	...	+ 32 −12	...
May	+ 32 + 15	...	+ 53 + 4	...	...	+ 30 +48	...	− 12 −56
June	...	+ 83 + 53	...	−29 −42	+ 15 − 1	...	+ 80 +36	...
July	− 54 − 71	...	+106 +57	...	...	+ 29 +47	...	+ 24 −20
Aug.	...	+ 12 − 19	...	+28 +15	−120 −136	...	+127 +84	...
Sept.	− 21 − 38	...	+ 87 +38	...	...	−107 −89	...	+108 +64
Oct.	...	− 26 − 56	...	+66 +53	+ 51 + 35	...	+ 38 − 6	...
Nov.	+ 43 + 26	...	+ 31 −18	...	...	− 72 −54	...	+102 +58
Dec.	...	0 − 31	...	+48 +35	+ 87 + 71	...	− 16 −60	...
Jan.	...	...	0 −49	...	...	− 1 +17	...	+ 42 − 2
Feb.	...	...	...	− 1 −14	...	...	+ 1 −42	...
Mar.	...	...	...	...	...	...	...	0 −44
Mean	+ 17·0	+ 30·5	+ 48·8	+13·2	+ 15·8	− 17·8	+ 43·7	+ 44·0

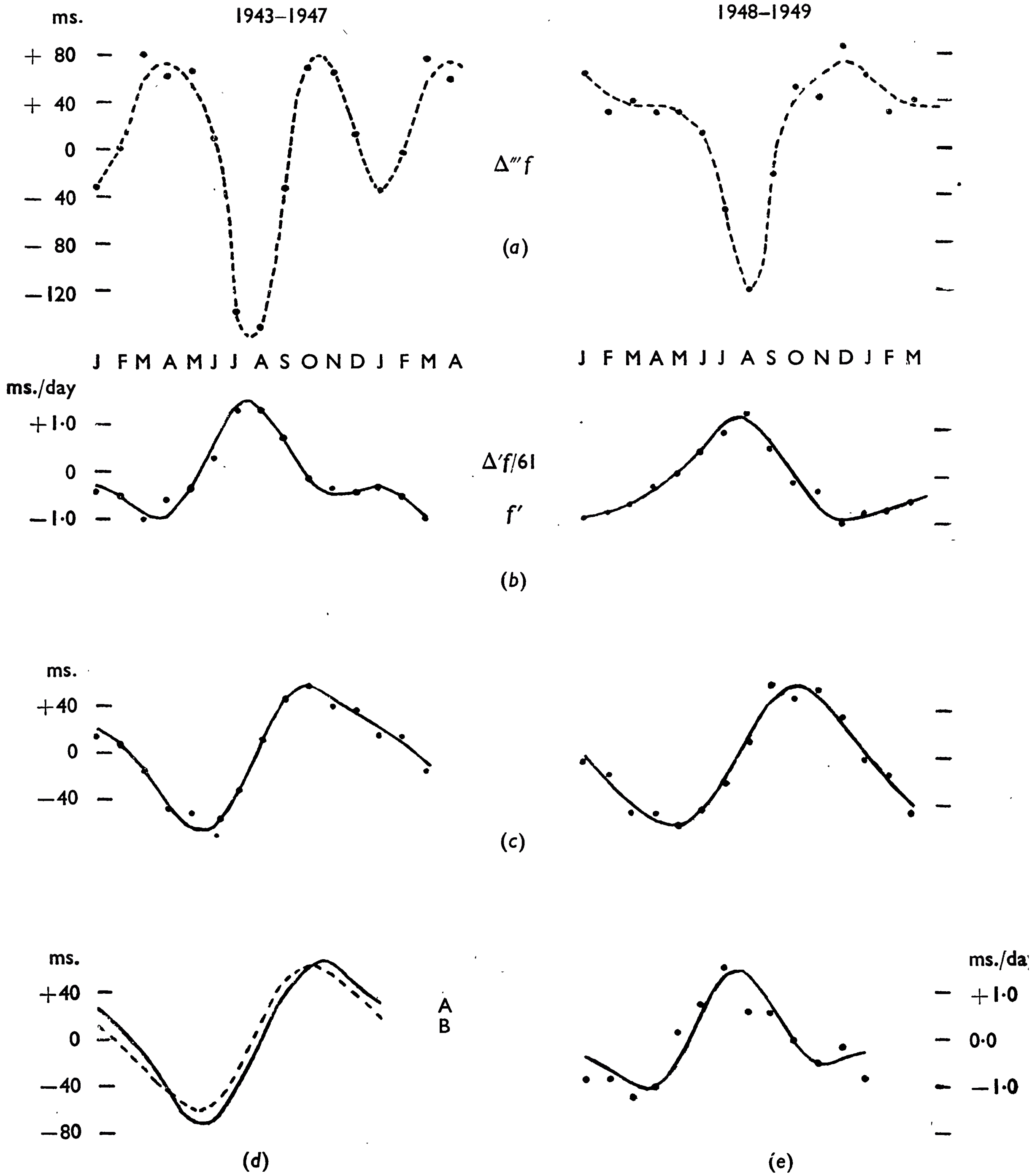

FIG. 2.—*Curves of Δ'''f, f' and f.*

It is present irrespective of which clocks are employed, and persists in the face of changes of observers, instruments and observing sites. If there were any doubt about this it would be dispelled by the fact that the fluctuations are revealed also by the Washington time observations. Reception at Abinger of the WWV radio signals renders it possible to obtain a close comparison between U.T. as determined at the Naval Observatory, Washington, and the Royal Observatory. After removal of a systematic difference of $+0^{s}{\cdot}016$ * and the

* Travel time of the signal, if allowed for, would make this about $+0^{s}{\cdot}036$.

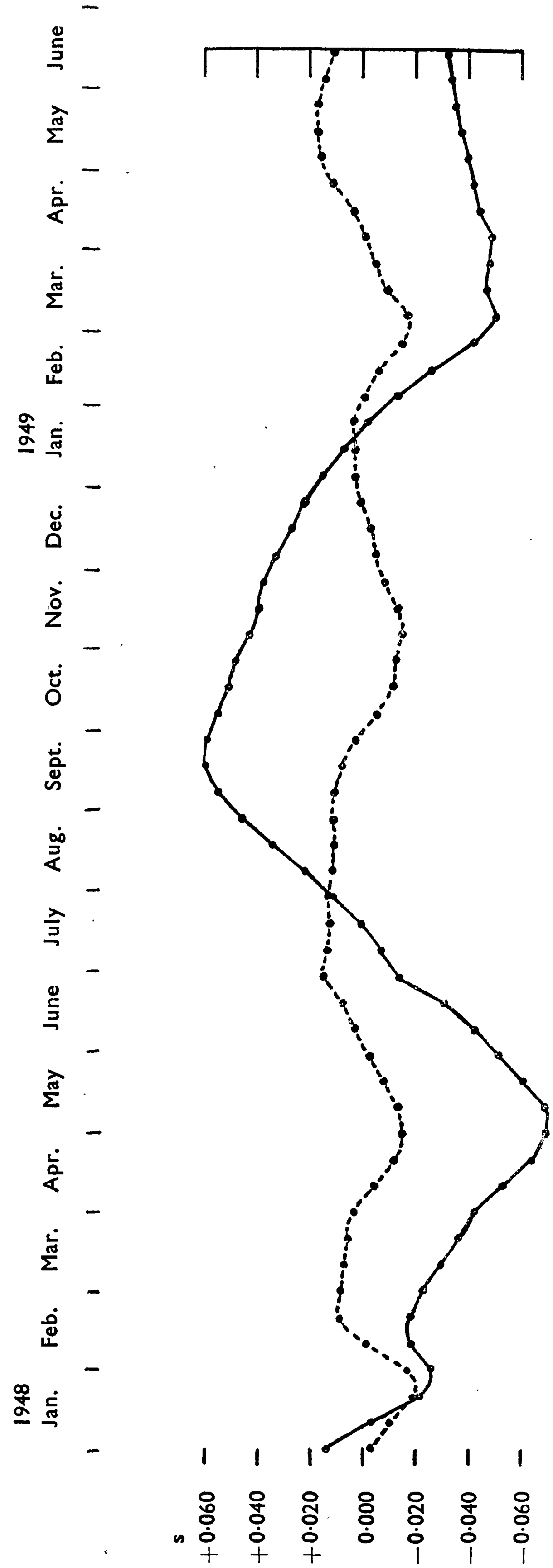

FIG. 3.—*Residual clock corrections of clock 5B, corrected for effect of polar variation.*

Full line: uncorrected for f term.
Dotted line: corrected for f term.
Ephemeris: $a = -0^s{\cdot}009$; $b = -233{\cdot}648$ *ms/d*; $c = -0{\cdot}02792$ *ms/d*^{2};
Epoch: 1948 *January* 30, 10.00 *hrs.*

effect of polar variation, the monthly mean differences over the four years are, in milliseconds, as follows:—

Jan.	+14	May	−10	Sept.	−10
Feb.	+14	June	−13	Oct.	− 1
Mar.	+11	July	−11	Nov.	+ 9
Apr.	+ 2	Aug.	−12	Dec.	+ 8

These are given as corrections to be applied to the "Greenwich" f curve to obtain the corresponding Washington curve. There appears to be a slight annual variation between the two observatories, but this is small compared with the main fluctuation. Fig. 2 (d) contains a plot of the mean f curve for the two periods referred, A to the American observations and B to the British observations. The difference shows itself as a small phase shift and a slightly increased amplitude in the American curve.

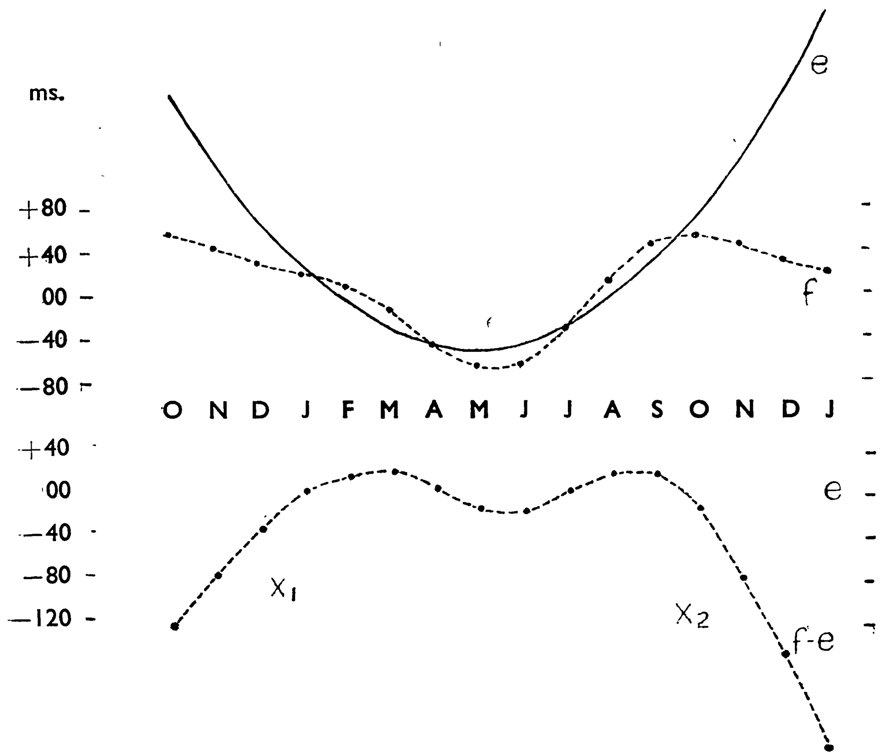

FIG 4.—*Curves showing the effect of an error in the adopted value of c.*

16. It may seem surprising that the sinusoidal character of the phenomenon had not been earlier recognized. During the period that the group 4 clocks were in operation, $4C$ was for good reasons acknowledged as being the most reliable. The value of c for this clock had been determined and the corresponding values for the others were readily derived by using their frequency drifts relative to $4C$. It would now appear that the adopted value (+0·00005) for the frequency drift of $4C$ was slightly in error, though it provided a good fit over a considerable portion of the f curve. The resulting qualitative effect can be seen on reference to Fig. 4 in which the curve f represents the seasonal fluctuation and e that part of the parabolic ephemeris resulting from the error in the adopted value of c.

Beneath is given the residual curve (f–e). Over the interval X_1–X_2 the deviations from linearity are too small to be revealed by observations subject to a comparatively large scatter in the presence of minor variations in the clock rate. At X_2 the curve rapidly dips. Since no relative change has occurred in the various values of c all the clock curves will behave in the same way and the natural interpretation would be that a *sudden* change in the rotation period of the Earth had occurred. Following the initial run-off, the original value of c will again appear to hold, but at the end of twelve months the Earth will once more appear to slow up.

17. The claim to have established for the first time an annual fluctuation in the Earth's rotation was made by Pavel and Uhink *, based on an investigation of the performance of two quartz-crystal clocks at the Geodetic Institute, Potsdam, during the period 1934 May–1935 June. Referred to the Potsdam time determinations, both clocks showed similar fluctuations in the residuals from a parabolic ephemeris. As the time determinations did not show similar fluctuations relative to the mean of the times determined at nine observatories, it was concluded that the fluctuations were not due to errors of observation but to a variable rate of rotation of the Earth. The material employed did not justify this conclusion, neither clock had an unbroken run during the period discussed, and each was used to smooth over the disturbance of the other when a stoppage occurred. The deduced variations in the rotation, with a range from the mean of from $+0{\cdot}005$ to $-0{\cdot}004$ secs./day, are too large to be real.

On the other hand, N. Stoyko, by comparing the departures of mean monthly rates from a linear rate for the clocks at Paris, Washington and Berlin during the three years 1934–37 (which cover the period of investigations by Pavel and Uhink), obtained a result † in close agreement with that found in the present paper. This paper seems to have attracted little attention, probably because further confirmation of the reality of an annual fluctuation in the Earth's rotation appeared to be required.

The present investigation had been completed when Stoyko published, without details, the results of a study of the rates of the clocks at Paris and Washington from 1946 January to 1947 April ‡, which confirmed his previous conclusions. This period is covered by the Greenwich observations; in Fig. 2 (*e*) Stoyko's results for the variations in the rate of rotation are plotted in dots upon the Greenwich f' curve for the period 1943–47. Stoyko's representation of his results can be expressed in the form:

$$f' = -1{\cdot}18 \sin n(T+59)^\circ + 0{\cdot}43 \sin 2n(T+42)^\circ,$$

which is in very close agreement with the results obtained by the Greenwich clocks. Stoyko's conclusions about the existence of an annual fluctuation in the period of rotation of the Earth are completely confirmed by the present investigation; moreover, the similarity in amplitude and phase found during the different periods show the effect to be a persistent one.

18. In a recent paper §, van den Dungen, Cox and van Miegham, using observed barometric pressure over the surface of the Earth at opposite seasons, have shown that the effect can be accounted for, at least in part, by changes in

* F. Pavel and W. Uhink, *Ast. Nach.*, **257**, 365, 1935.

† N. Stoyko, *Comptes Rendus*, **203**, 39, 1936; and **205**, 79, 1937.

‡ N. Stoyko, *Bulletin de la Classe de Science, Académie royale de Belgique*, 5e Serie, **XXXV**, 1949.

§ F. H. van den Dungen, J. F. Cox and J. van Miegham, *Bulletin de la Classe de Science, Académie royale de Belgique*, 5e Serie, **XXXV**, 1949.

the Earth's moment of inertia caused by seasonal changes in the distribution of air masses. The data are incomplete and further investigation is required, but the paper constitutes a first attempt to account quantitatively for the phenomena.

19. Since the fluctuation occurs in the unit of time it will introduce a corresponding fluctuation into the observed longitude of the Moon. The observations of occultations have shown for many years a fluctuation of approximately annual period in the Moon's mean longitude. Brouwer and Watts have analysed the results of the occultation observations and of the Washington meridian observations of the Moon *, deriving corrections to the elements of the orbits of the Moon and the Earth. They suggested that one term, not otherwise accounted for, might be attributed to an annual variation in the rate of rotation of the Earth. The variations shown by Stoyko's and by the present investigation are, however, much too small to account for this term in the Moon's longitude for which, therefore, some other explanation must be sought.

20. In conclusion, the practical importance of the fluctuation and its effect upon the estimated frequency of standard oscillators must be emphasized, together with the need for a precise definition of the Mean Solar Second which takes into account both this and the effect of the polar variation upon time.

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Abinger :
1949 *December* 15.

* D. Brouwer and C. B. Watts, *Astron. J.*, **52**, 169, 1947.

ADDENDUM

Since going to press a further paper by W. Uhink (*Ast. Nach.*, **278**, No. 3, 1950) dealing with the above subject has now become available. This paper contains an account of results obtained during recent years in Germany.