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THE STRUCTURE OF THE CLOUD OF COMETS SURROUNDING THE SOLAR SYSTEM, AND A HYPOTHESIS CONCERNING ITS ORIGIN,

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The combined effects of the stars and of Jupiter appear to determine the main statistical features of the orbits of comets.
From a score of well-observed original orbits it is shown that the "new" long-period comets generally come from regions between about 50000 and 150000 A.U. distance. The sun must be surrounded by a general cloud of comets with a radius of this order, containing about 10^{11} comets of observable size; the total mass of the cloud is estimated to be of the order of 1/10 to 1/100 of that of the earth. Through the action of the stars fresh comets are continually being carried from this cloud into the vicinity of the sun.
The article indicates how three facts concerning the long-period comets, which hitherto were not well understood, namely the random distribution of orbital planes and of perihelia, and the preponderance of nearly-parabolic orbits, may be considered as necessary consequences of the perturbations acting on the comets.
The theoretical distribution curve of $1/a$ following from the conception of the large cloud of comets (Table 8) is shown to agree with the observed distribution (Table 6), except for an excess of observed "new" comets. The latter is taken to indicate that comets coming for the first time near the sun develop more extensive luminous envelopes than older comets. The average probability of disintegration during a perihelion passage must be about 0·014. The preponderance of direct over retrograde orbits in the range from a 25 to 250 A.U. can be well accounted for.
The existence of the huge cloud of comets finds a natural explanation if comets (and meteorites) are considered as minor planets escaped, at an early stage of the planetary system, from the ring of asteroids, and brought into large, stable orbits through the perturbing actions of Jupiter and the stars.
The investigation was instigated by a recent study by VAN WOERKOM on the statistical effect of Jupiter's perturbations on comet orbits. Action of stars on a cloud of meteors has been considered by ÖPIK in 1932.

1. *Sketch of the Problem.*

Among the so-called long-period comets there are 22 for which, largely by the work of ELIS STRÖMGREN, accurate calculations have been made of the orbits followed when they were still far outside the orbits of the major planets [1]). Approximate calculations of the original orbits by FAYET [2]) are available for 8 other comets with well-determined osculating orbits. For the present limiting ourselves to the comets for which the perturbations were rigorously determined, and excluding 3 for which the mean error of the reciprocal major axis, $1/a$, is larger than 0·000100, the values of $1/a$ for the remaining 19 comets are distributed as shown in Table 1.

The mean errors of $1/a$ are all smaller than 0·000061; their average is ± ·000027. The steepness of the maximum for small values of $1/a$ indicates that the real mean errors of the original $1/a$ cannot greatly exceed these published mean errors. The 22 comets do not form a representative sample of the long-period comets; there has been a selection for small values of $1/a$, so that the real proportion of comets with $1/a$ > ·000 50 is much larger than indicated in the table. It can be shown, however, that the selection has not appreciably influenced the relative numbers in the rest of the table. Among the comets in the first division there are two with negative values of $1/a$, viz. — ·000 007 and — ·000 016, probably due to observational errors.

TABLE 1

Distribution of original semi-major axes (a in Astronomical Units)

$1/a$	n
< ·000 05	10
·000 05 — 10	4
10 15	1
15 20	1
20 25	1
25 50	1
·000 50 75	1
> ·000 75	0

It is evident from Table 1 that the frequency curve of $1/a$ shows a steep maximum for very small values. The average for the 10 orbits in the first interval is ·000 018, thus corresponding to a major axis of 110 000 A.U. We may conclude that a sensible fraction of the long-period comets must have come from a region of

[1]) A list of these is given by SINDING, *Danske Vidensk. Selsk., Mat.-Fys. Medd.* **24**, Nr 16, 1948, or *Publ. o. Mindre Medd. Köbenhavns Obs.* Nr 146. VAN BIESBROECK's orbit for comet 1908 III has been added to this list.

[2]) *Thèse*, Paris, 1906; also in *Ann. Paris, Mém.* **26A**, 1910.

space extending from a distance $2a = 20000$ to distances of at least 150000 A.U. from the sun; that is, almost to the nearest star. This does not mean that they are interstellar. They belong very definitely to the solar system, because they share accurately the sun's motion. Yet, the prevalence of these very large major axes has led several astronomers to investigate the question whether the comets could not be of interstellar origin. It is evident that they cannot *directly* come from interstellar space, for in that case there would have to be many more outspoken hyperbolic orbits than nearly parabolic ones. So far, no comet has been found for which the eccentricity exceeds 1 by an amount large enough to be considered as real. It is conceivable, however, that comets would be caught from an interstellar field by the action of the major planets, and would then move for a long time in orbits of large dimensions, so that the number of comets caught would gradually become far larger than the number of hyperbolic comets passing through the solar system. This suggestion has recently been studied by Dr VAN WOERKOM[1]). He concludes that this possibility must be ruled out, because the action of the major planets which causes the comets to be captured would at the same time result in a distribution of the values $1/a$ which is constant over a considerable range of negative as well as positive values. There would again be a large preponderance of hyperbolic comets, which is contradicted by observations. For a more exhaustive discussion of this problem I may refer to section 4 of VAN WOERKOM's article.

There is no reasonable escape, I believe, from the conclusion that the comets have always belonged to the solar system. They must then form a huge cloud, extending, according to the numbers cited above, to distances of at least 150000 A.U., and possibly still further. It is not necessary at this point to enter upon the question how this cloud has originated. It might conceivably be considered as part of the remnants of a disrupted planet (see section 6). An alternative hypothesis, repeatedly put forward, according to which comets would be formed by eruptions from Jupiter and the other planets, does not appear to be likely (cf. VAN WOERKOM, *l.c.* p. 464 a.f.).

Accepting this existence of a huge cloud of comets we are still faced with a difficulty that has been put into full light by VAN WOERKOM's study. Jupiter, and to a lesser extent the other planets, exert a diffusing action on the long-period comets. According to VAN WOERKOM's calculations the small perturbations by Jupiter suffered by an observable comet during its passage through the "inner" part of the planetary system will on the average change the reciprocal major axis by about 0·0005; positive and negative changes are equally probable. By these perturbations the long-period comets will gradually disappear, partly into interstellar space, partly into the families of short-period comets. In addition, the comets may gradually diminish in brightness through the sun's action, or be dissolved. It is evident from VAN WOERKOM's study that within one or two million years after their first perihelion passage practically all long-period comets will have disappeared. As it is highly improbable that the comets we observe have only originated within the last two million years we are led to conclude that comets already existing outside the region where they are subject to the perturbing action of sun and planets are continually being brought into this region.

A direct indication of the probable escape of a considerable fraction of the comets of very long period has been given by FAYET[1]). Among 36 comets for which he has made approximate calculations of the orbits which they must have described after they passed out of the action of Jupiter he found 7 for which this orbit was hyperbolic. A more complete calculation for a similar case (comet 1898 VII) where the final orbit is definitely hyperbolic, has recently been made by SINDING[2]).

If we assume that at the start the velocity distribution of the comets in the huge cloud surrounding the planetary system was a random distribution, there must have been comets, even in the outer parts of the cloud, whose velocities were so nearly directed towards the sun that these comets would eventually pass through the "observable region" (i.e. the region within about 2 A.U. from the sun). Even if the radius of the cloud was 150000 A.U. all the comets which could come into the vicinity of the earth would have done so within roughly 20 million years. All these comets will diffuse into space or be disintegrated. No new comets would come in after this period unless they were made to do so by some perturbation. VAN WOERKOM's discussions make it clear that perturbations by *planets* cannot be effective in bringing comets into the observable region: their influence on the major axis and the period is always much more important than on the perihelion distance. Their perturbations will diffuse the comets out of the long-period range long before they have caused a change of any importance in the perihelion distance.

Two alternative types of perturbations offer themselves, namely resistance by an interplanetary medium, and influence of passing stars. It seems extremely unlikely that the former mechanism could have an observable influence on the perihelia of comets. For a general influence of this kind to be effect-

1) *B.A.N.* No. 399, 1948.

1) *Ann. Bur. Longitudes* **10B**; *Comptes Rendus* **189**, 1122, 1929.

2) This was communicated to me before publication by the kindness of Mr SINDING and Prof. STRÖMGREN.

ive a density of interplanetary gas would be required that is quite inadmissible on dynamical grounds. Moreover, a resisting medium would in the first place tend to decrease the major axes, while for the nearly parabolic comets the perihelion distances would appear to be practically unaffected, so that it could never solve our problem.

The purpose of the present paper is to investigate the second possibility, the action of passing stars.

For reports on work done by other investigators of the origin of long-period comets I wish to refer to the extensive discussions given in VAN WOERKOM's paper.

2. *General Influence of Stars on the Cloud of Comets.*

I am indebted to Dr WHIPPLE for drawing my attention to an interesting article by ÖPIK in which also the action of stars on a cloud of meteors or comets is discussed. This article [1]), which I have only been able to read after the first three sections of the present paper had been written, deals with the influence of passing stars on greatly elongated orbits. The author computes in the first place the number of direct ejections from the solar system in the course of 3.10^9 years, and concludes that orbits extending to 10^6 A.U. would probably be resistant against such ejections. The discussions in the present section confirm this general conclusion; because I have also considered smaller and more gradual increases of energy, I found the limiting radius of a possible cloud surrounding the solar system somewhat smaller, viz. at most 2.10^5 A.U. In the second place ÖPIK investigates the general influence of stellar encounters on the distribution of the perihelium distances, q, of meteors. He mentions the possibility of an equilibrium distribution of q, but is more inclined to believe that all orbits large enough to be subject to stellar perturbations have been diffused away from the observable region, the only objects remaining being those whose orbits do not extend beyond a few thousand A. U. He concludes that all observed meteors belonging to the solar system must have relatively small orbits. ÖPIK's conclusion is just opposite to that reached in the present article. The difference is due to two facts. In the first place ÖPIK does not consider the influence of Jupiter's perturbations, which are, however, an equally essential factor in the household of meteors and comets as the perturbations of the stars. In the second place the idea of a very vast cloud of meteors containing also great numbers of less elongated orbits did not seem very probable to ÖPIK. His reluctance to enter upon this possibility is all the more understandable as his primary interest was in meteors, and not in the investigation of known orbits of nearly-parabolic comets.

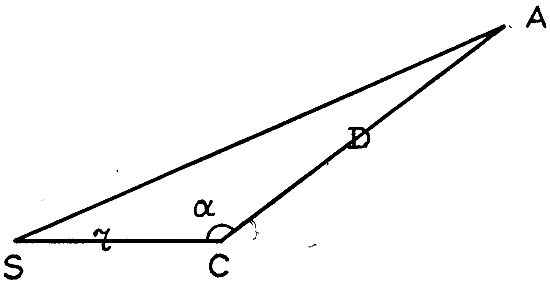
RUSSELL, however, following ÖPIK's suggestion, mentions the possible existence of a vast assembly of comets with great perihelion distances and brought occasionally nearer to the sun by planetary or stellar perturbations [1]).

We shall first briefly discuss the question whether a cloud with a radius of the order of 200000 A.U. can keep in existence for a period of the order of the age of the solar system in face of the disrupting forces of stellar encounters. In the second place we shall investigate the effect of these encounters on the shape of the cloud, and on the velocity distribution in it.

Let us consider the effect of a star with a mass equal to that of the sun moving through this cloud with a velocity of 30 km/sec. The velocity of a comet will be changed by ΔV. ΔV is directed along the perpendicular drawn from the comet to the star's path, and its absolute value is with sufficient approximation given by

$$\Delta V = \frac{2\gamma m}{D V_*}, \qquad (1)$$

where m is the star's mass, γ the constant of gravitation, V_* the velocity of the star relative to the sun, and D the shortest distance at which the star would have passed the comet if the latter had not been deflected by the star's attraction. We are interested in the differential effect on the comet relative to the sun.



Let S represent the position of the sun, C that of the comet and A that of the star at the moment of closest approach. If the distance CA is again denoted by D, the distance of the comet from the sun, CS, by r, and $\angle ACS$ by α, and if further ΔV is now taken to be the absolute value of the change in the velocity of the comet relative to the sun, we can easily compute that [2])

$$\Delta V^2 = \left(\frac{2\gamma m}{D V_*}\right)^2 \frac{r^2}{r^2 + D^2 - 2rD\cos\alpha}. \qquad (2)$$

If we consider one comet, this expression must be averaged over all values of α, the points A being, for a given value of D, evenly distributed over a sphere. We must therefore multiply (2) by $\frac{1}{2}\sin\alpha\, d\alpha$ and integrate from $\alpha = 0$ to $\alpha = \pi$. Carrying out the integration we obtain

$$\overline{\Delta V^2} = \left(\frac{2\gamma m}{D V_*}\right)^2 \frac{r}{2D} \ln \frac{1 + r/D}{\pm(1 - r/D)}, \qquad (3)$$

the sign of the denominator to be chosen so that it

[1]) *Proc. Am. Ac. Arts and Sc.* **67**, 169, 1932; *Harv. Repr.* No. 79.

[1]) *The Solar System and its Origin*, New York, 1935, p. 43.
[2]) For an improved computation see Note (1), p. 109.

becomes positive. If D is large compared to r, we get

$$\overline{\Delta V^2} = \left(\frac{2\gamma m}{DV_*}\right)^2 \frac{r^2}{D^2}, \qquad (4)$$

while for values of D that are small compared to r, evidently

$$\overline{\Delta V^2} = \left(\frac{2\gamma m}{DV_*}\right)^2. \qquad (5)$$

The number of stars with mass between m and $m + dm$ and with velocities relative to the sun between V_* and $V_* + dV_*$ passing per second between D and $D + dD$ is

$$\nu'(m)\, dm \,.\, 2\pi D dD \,.\, V_* \frac{4l^3}{\sqrt{\pi}} V_*^2 e^{-l^2 V_*^2} dV_*, \qquad (6)$$

where the distribution of the space velocities V_* relative to the sun is approximated by a Maxwellian distribution with modulus l; $\nu'(m)$ is the number of stars with mass m per cm³. For simplification I shall suppose the velocity distribution to be independent of m.

In order to find the total average of ΔV^2 per unit of time we must multiply (3) by (6) and integrate over all values of m, V_* and D. The integrations can be carried out separately.

For the integration with respect to m I have used the rough data given in Table 34, *B.A.N.* **6**, 285, 1932, from which we get $\int m^2 \nu(m)\, dm = 0{\cdot}040$ (solar masses)² per cubic parsec, or $5{\cdot}4 . 10^9$ g²/cm³. The absolutely faint stars are relatively unimportant. For the more schematical calculations which we shall use in subsequent estimates we shall assume that there are $0{\cdot}020$ stars per cubic parsec with average mass $1{\cdot}4$ times that of the sun.

The integration with respect to V_* is equally simple. We have to integrate the expression $l^3 V_* e^{-l^2 V_*^2} dV_*$ from 0 to ∞. This gives $l/2$. Estimating that the average radial velocity relative to the sun is approximately 20 km/sec, or $2{\cdot}0 . 10^6$ cm/sec, we find $l/2 = 1{\cdot}41 . 10^{-7}$ cm/sec. For schematical calculations with formula (1) I shall insert for V_* $\pi/4$ times the average space velocity relative to the sun, or 31 km/sec.

Inserting the results of the integrations with respect to m and V_*, we obtain

$$\overline{\Delta V^2} = 16\gamma^2 \sqrt{\pi} \,.\, 5{\cdot}4 . 10^9 \,.\, 1{\cdot}41 . 10^{-7} \int_{D_1}^{D_2} \frac{r\, dD}{D^2} \ln \frac{1 + r/D}{\pm(1 - r/D)} = 9{\cdot}6 . 10^{-11} \int_{D_1}^{D_2} \frac{r\, dD}{D^2} \ln \frac{1 + r/D}{\pm(1 - r/D)}, \qquad (7)$$

D_1 being the distance for which ΔV becomes equal to the velocity of escape from the cloud of comets (D_1 is computed from formula (11); for $r = 100\,000$ A.U., $D_1 = 597$ A.U.), and D_2 the distance where multiple encounters become frequent. D_2 is of the order of the average distance between the stars. I have assumed $D_2 = 10^6$ A.U. As we are dealing with differential effects, the value of D_2 has hardly any influence. If we had taken D_2 infinite, the result for $\overline{\Delta V^2}$ would have changed by only about $0{\cdot}1\%$. With regard to the limit D_1 the encounters with $D < D_1$, causing a direct escape from the cloud, will be considered separately.

Putting $r/D = x$ the integral becomes

$$\int_{r/D_2}^{r/D_1} \ln \frac{1 + x}{\pm(1 - x)}\, dx. \qquad (8)$$

For $x < \frac{1}{2}$, or $D > 2r$, we may, according to (4), replace the integral by

$$\left(\frac{1}{2}\right)^2 - \left(\frac{r}{D_2}\right)^2. \qquad (9)$$

For values of D that are less than $r/2$ we may with amply sufficient approximation insert (5) instead of (3). The integral in (7) can then be replaced by

$$2\left(\ln \frac{r}{D_1} - \ln 2\right). \qquad (10)$$

We still have to evaluate the integral (8) from $x = \frac{1}{2}$ to $x = 2$. Let us leave out of consideration passages for which the change in the sun's velocity relative to the cloud of comets exceeds the velocity of escape for comets at the boundary of the cloud ($r = 200\,000$ A.U.). During the lifetime of the solar system the probability for such a passage is only about $0{\cdot}05$, and may be neglected. The integral from $x = \frac{1}{2}$ to $x = 2$ then becomes equal to $3{\cdot}08$. Adding (9) and (10) to this, and multiplying by $9{\cdot}6 . 10^{-11}$, we get $\overline{\Delta V^2}$ per second. Multiplying by a further factor of $9{\cdot}48 . 10^{16}$ we obtain the total energy transfer per unit mass in $3 . 10^9$ years. The numerical results are shown in Table 2, in which the second column shows the velocity of escape in cm/sec from a point at a distance r from the sun, the fourth column the velocity of circular motions at the same distance; $\overline{\Delta V^2}$ in column 6 is in (cm/sec)² per $3 . 10^9$ years.

TABLE 2

r	V_e	V_e^2	V_c	D_1	$\overline{\Delta V^2}$ 1)	$\overline{V^2}$
25 000	$2{\cdot}68 . 10^4$	$7{\cdot}18 . 10^8$	$1{\cdot}89 . 10^4$	299	$0{\cdot}99 . 10^8$	$3{\cdot}74 . 10^8$
50 000	$1{\cdot}89$,,	$3{\cdot}59$,,	$1{\cdot}34$,,	423	$1{\cdot}04$,,	$1{\cdot}60$,,
100 000	$1{\cdot}34$,,	$1{\cdot}80$,,	$0{\cdot}95$,,	597	$1{\cdot}11$,,	$0{\cdot}53$,,
200 000	$0{\cdot}94$,,	$0{\cdot}89$,,	$0{\cdot}67$,,	842	$1{\cdot}17$,,	0

We see that for $r = 100\,000$ A.U., $\overline{\Delta V^2}$ is still considerably less than V_e^2, so that only a moderate fraction of the comets in this region will have escaped during the lifetime of the solar system. At $r = 200\,000$, $\overline{\Delta V^2}$ exceeds V_e^2; practically all comets originally present in this part of the cloud will have escaped; but of those which have dispersed into this region from

1) For improved values see Note (1), p. 109.

smaller values of r during the second half of the lifetime of the solar system a considerable fraction will remain. Beyond $r = 200000$ very few comets will be retained.

We see thus that the perturbative action of other stars puts a limit to the cloud at about 200000 A.U. This radius agrees remarkably with the extent indicated by the direct observations of $1/a$ as discussed on page 92.

At $r = 50000$ A.U. one half of the total amount of $\overline{\Delta V^2}$ is due to passages within $D = 8500$ A.U.; two thirds is due to passages within $D = 22900$ A.U.

For the further calculations, considered in section 4, I shall as a rule assume that all passing stars have the same mass and the same velocity relative to the cloud, and shall simply use formula (1), inserting for m 1·4 times the mass of the sun, and for V_* 31 km/sec, while the number of stars per cubic parsec will be assumed to be 0·020. These numbers are in accordance with the more rigorous results worked out above. If D is expressed in A.U. and ΔV in cm/sec, (1) then becomes

$$\Delta V = 8{\cdot}0 \,.\, 10^6 D^{-1}. \qquad (11)$$

We still have to consider the passages with $D < D_1$, which we have so far omitted. As a rough approximation we may assume that all of these will lead to permanent expulsion from the cloud. With the data just given I find that by these close passages 5% of the comets at $r = 100000$ will have been expelled during 3.10^9 years, and 9% of the comets at $r = 200000$. The effect of these direct expulsions is therefore unimportant.

It is important to see what effects the passing stars will have had on the *shape* of the cloud, and on the distribution of orbital eccentricities The fact that neither the orbital planes nor the aphelia of the long-period comets show a distinct preference for the ecliptic, has often been taken as an indication that they are of interstellar origin [1]). However, if we take account of the influence of the other stars on the cloud of comets, we see that, even if this cloud had originally been strongly concentrated towards the plane of the ecliptic, not much trace of this could have remained.

As regards the eccentricities, although *observations* cannot tell us whether the distant regions, into which we have seen that many of the elongated orbits extend, contain also comets with less eccentric orbits, we may infer from consideration of the action of stellar perturbations that this is probable. We may even conclude that, unless comets are only a recent phenomenon, their velocity distribution in these regions must be nearly isotropic.

In the next section I have attempted to give a working model for a cloud extending to 200000 A.U. In this model the average square velocity at $r = 50000$ is $1{\cdot}60.10^8$ (cf. Table 2 and Table 3, last columns) or in one co-ordinate $0{\cdot}53.10^8$. Now, according to Table 2, $\frac{1}{3}\overline{\Delta V^2} = 0{\cdot}35.10^8$. So, even if originally the velocities had all been directed along the radius vector to the sun, or had all been parallel to the ecliptic, the comets would by now have acquired velocities of practically the same amount in the other co-ordinates. The argument applies still more strongly to the comets at larger distances from the sun. It does *not* apply to smaller distances. If originally there had been a flattening of the part of the cloud within, say, 40000 A.U., the flattening should still be visible. But it is probable that most comets come originally from distances larger than 40000 A.U.; for these we cannot expect to find a sensible deviation from random distribution.

3. *Working Model for the Cloud of Comets.*

From what has been said in the first paragraph of this article it is clear that the density in the cloud of comets near 100000 A.U. cannot be of a lower order than that near 50000 A.U. If the velocity distribution had an exponential form this would imply that the average peculiar velocity must be comparable with the velocity of escape from this region. For, with a Maxwellian velocity distribution the density is proportional to $e^{3\Phi/W^2}$, where W is the mean square space velocity, and $\Phi = \gamma m/r = \frac{1}{2}V_e^2$. Now, if we should take W equal to half the velocity of escape at 50000 A.U. for instance, the density at 100000 A.U. would be only 1/20 of that at 50000 A.U., while for still smaller distances the density would increase steeply. This does not appear to be in agreement with the observed long-period comets (cf. page 100). Moreover it is clear that with a Maxwellian distribution the densities for small distances would become impossibly high. In order to get a slow decrease of density the mean velocity corresponding to a Maxwellian distribution would have to be about equal to the velocity of escape in the regions considered. As the part of the velocity distribution above the velocity of escape will have to be cut off, the exponential factor in the velocity distribution will then nowhere differ much from unity. As a simple approximation I have therefore adopted the following form for the frequency function of the space velocities V

$$3L^{-3}V^2 \quad (V < L). \qquad (12)$$

The velocities are distributed homogeneously over a

1) Statistics of the distribution of perihelia have been given, among others, by Oppenheim, *Festschrift für H. von Seeliger*, p. 131, 1924, and by Bourgeois and Cox, *B.A.* **8**, 271, and **9**, 349, 1934.

sphere with radius L[1]). There are no velocities larger than L; L is smaller than the velocity of escape, and will for the present be chosen in such a way that the outer surface of the cloud has a radius of 200000 A.U. L is a function of r. The factor $3L^{-3}$ is a normalizing factor. If we denote the velocity component along the radius vector to the sun by u, and the transverse velocity by v, the frequency function of u and v will be

$$\varphi(u,v) = \tfrac{3}{2} L^{-3} v \quad (\sqrt{u^2+v^2} < L) \ . \qquad (13)$$

It can easily be shown that if the velocity distribution has the form (12) or (13) at a certain distance r from the sun, it will be of the same form at any other distance r'. Consider a comet whose orbit passes r as well as r', and denote its velocity components at r by u, v, at r' by u', v'. We have

$$r'v' = rv$$

$$\tfrac{1}{2}(u'^2+v'^2) - \frac{\gamma m}{r'} = \tfrac{1}{2}(u^2+v^2) - \frac{\gamma m}{r}. \qquad (14)$$

It is evident that the time passed by a comet in a shell of unit thickness is inversely proportional to the radial velocity with which it crosses the shell. The numbers of corresponding comets in different shells will therefore also be inversely proportional to these radial velocities; hence

$$\frac{4\pi r'^2 \nu(r')\, \varphi(u',v')\, du'\, dv'}{4\pi r^2\, \nu(r)\, \varphi(u,v)\, du\, dv} = \frac{u}{u'}, \qquad (15)$$

$\nu(r)$ denoting the total number of comets per cubic A.U. From (14) we derive $\frac{du'\, dv'}{du\, dv} = \frac{r\, u}{r' u'}$. Inserting this into (15) we obtain

$$\nu(r')\, \varphi(u',v') = \frac{r}{r'}\, \nu(r)\, \varphi(u,v). \qquad (16)$$

If $\varphi(u, v)$ has the form (13) we get, using (14),

$$\nu(r')\, \varphi(u',v') = \tfrac{3}{2} L^{-3} \nu(r)\, v' \qquad (17)$$

$$\left[u'^2 + v'^2 < L'^2, \text{ where } L'^2 = L^2 + 2\gamma m\left(\frac{1}{r'} - \frac{1}{r}\right)\right].$$

The velocity distribution at r' is thus of the same form as that at r.

It is easily computed that with such a velocity distribution the mean of the squares of the space velocities, $\overline{V^2}$, is equal to $\tfrac{3}{5} L(r)^2$. Values of $\overline{V^2}$ are indicated in Table 3.

The space density of the comets at r' is obtained by integrating (17) over all possible values of u' and v'. We get

$$\nu(r')/\nu(r) = (L'/L)^3. \qquad (18)$$

1) The supposition of spherical symmetry is based on the arguments discussed in the last paragraphs of the preceding section.

If R is the radius of the cloud in cm, the limiting velocity at a distance r is given by

$$L = \sqrt{\frac{2\gamma m}{R}} \cdot \sqrt{\frac{R}{r} - 1}. \qquad (19)$$

For R = 200000 A.U. this gives

$$L = 9430 \sqrt{\frac{200\,000}{r} - 1} \qquad (r \text{ in A.U.}).$$

We thus find the following values of L, $\nu(r)/\nu(100\,000)$ and $N(r)/N(100\,000)$, $N(r)$ denoting the number of comets in a shell of unit thickness.

TABLE 3

r (A.U.)	L (cm/sec)	$\frac{\nu(r)}{\nu(100\,000)}$	$\frac{N(r)}{N(100\,000)}$	$\overline{V^2}$ (cm/sec)^2
25 000	25 000	18·8	1·2	3·74.10^8
50 000	16 400	5·2	1·3	1·60 ,,
100 000	9 430	1	1	0·53 ,,
150 000	5 440	0·19	0·4	0·18 ,,

I want to emphasize that this model has no pretensions beyond a simple working model for the outer parts of the cloud of comets. The density distribution in particular may be rather different from that adopted. The model can only be improved when more and better material concerning the original major axes becomes available. The present scheme seems sufficient for the rough estimates we want to make. It should be stressed, however, that it cannot be extrapolated to distances less than about 40000 A.U. The only way in which we can say anything about these latter densities is from speculations concerning the origin of the cloud of comets (cf. section 6).

4. *The Manner in which Passing Stars bring Comets into the Region of the Planets. Total Number of Comets in the Cloud.*

VAN WOERKOM's work has made it clear that, if there were no outside disturbances, all orbits passing through the inner part of the solar system would have been eliminated within about twenty million years (cf. page 92 of the present article). The removal is due to the action of Jupiter, and to a lesser extent also to that of Saturn.

We want to investigate how large the number of comets in the cloud will have to be in order to explain the observed number of "new" comets passing through perihelion.

Let us consider the comets in an element of space at a distance of, for instance, 50000 A.U. from the sun. A certain fraction of these comets will have velocities so nearly directed to the sun that their perihelion distance, q, is of the order of one A.U. These comets will eventually pass through the "observable region", which I shall assume to be a sphere of radius 1·5 A.U. around the sun (cf. p. 105). In order to have a perihelion distance less than q a comet at a distance r must have a transverse velocity v smaller than $\frac{1}{r}\sqrt{2\gamma m q}$ if r is large compared to q [1]). For $r = 50000$ A.U. the velocities of the comets with perihelia within 1·5 A.U. of the sun are therefore contained within a cylinder of the velocity space with axis pointing to the sun, and with a radius of 104 cm/sec.

All comets which can come in the vicinity of the orbits of the major planets will be affected by the perturbations mentioned in the first section. As a rough approximation I have assumed that all comets with q less than 15 A.U. will be so affected that they do not return to the distant parts of the cloud from which they came. For comets with $q = 1$ A.U. VAN WOERKOM computed that the average change in $1/a$ due to Jupiter's attraction is $0{\cdot}53.10^{-3}$; changes of similar amounts will presumably be brought about by Jupiter in all cometary orbits with q up to about 10 A.U. The perturbations by Saturn, though probably about 10 times smaller, will still be sufficient in general to prevent comets from returning to distances beyond 50000 A.U. Saturn's action may be estimated to be effective in this way to about $q = 15$ A.U.

At $r = 50000$ A.U. the velocities of the comets with $q < 15$ A.U. are contained in a cylinder of radius 328 cm/sec. But for the action of passing stars this cylinder in the velocity space would have become empty after a relatively short time. What happens in reality is that after their first perihelion passage these comets are indeed expelled from their original very large orbits, so that, if we would consider the velocity distribution of comets not too far from the sun, say at $r = 10000$ A.U., that half of the cylinder corresponding to $q = 15$ and to positive values of u (that is, to comets moving away from the sun) would be empty. If we go to greater distances we shall find that the cylinder is gradually filled up through the disturbing action of the stars. At $r = 50000$ A.U. this filling-up process will be practically completed, at least for the returning comets. For, a comet which reaches aphelion at 50000 A.U. has a period of revolution of 4·0 million years. During their outward journey the comets with velocities outside the cylinder corresponding to $q = 15$ will gradually diffuse into this cylinder. Now, according to Table 2, in two million years $\overline{\Delta V^2} = 6{\cdot}9.10^4$. For the transverse velocities $\overline{\Delta v^2} = \frac{2}{3}\,\overline{\Delta V^2} = 4{\cdot}6.10^4$, or $\sqrt{\overline{\Delta v^2}} = 217$ cm/sec. The comets with velocities just outside the cylinder will therefore have had time to fill it up nearly to its axis. The remaining deficiency near the axis will be of slight importance, as it occurs only for comets with aphelion distance Q very near 50000 A.U., and therefore with small radial velocities u. The comets, for instance, that reach aphelion near 60000 A.U. would have had time to fill up the cylinder completely before they return to 50000 A.U., for their periods are $5{\cdot}2.10^6$ years. The greater part of this will be spent beyond $r = 40000$. At the time these comets return to $r = 50000$, $\sqrt{\overline{\Delta v^2}}$ will be about 320 cm/sec, so that the cylinder will be practically filled up to the general average density in the velocity space. In the schematical calculations which follow I have assumed that the whole negative part of the cylinder at $r = 50000$ is completely filled.

By the gradual disappearance of comets from the cylinder corresponding to $q = 15$ a density gradient will be set up in the velocity space outside the cylinder, the density decreasing towards the boundary of the cylinder. Provisional estimates show that this gradient remains quite small, and may certainly be neglected in estimates like the present.

Using the velocity distribution proposed in the preceding section we can now easily express the number of "new" comets passing per century through a perihelion within 1·5 A.U. from the sun in terms of the density of the cloud at $r = 50000$. The results are presented in Table 4, for various intervals of a.

The comets considered having negligible transverse velocities the relation between the radial velocity u at a distance r and the semi-major axis of the orbit can be written

$$u = \sqrt{\frac{1{\cdot}78.10^{13}}{r}}\,.\,\sqrt{1 - \frac{r}{2a}}, \qquad (20)$$

r and a being expressed in A. U. The third column of Table 4 shows the radial velocities at $r = 50000$, in cm/sec, corresponding to the aphelion distances Q and the semi-major axes a indicated in the first two columns. Let these radial velocities corresponding to a certain interval of a be u_1 and u_2, respectively. As, in the model considered, the velocity space is homogeneously filled to a radius L cm/sec, the fraction of the comets near $r = 50000$ A.U. which have radial velocities between these limits, and transverse velocities less than 104 cm/sec (corresponding to $q = 1{\cdot}5$ A.U.) is

[1]) If the velocity in perihelium is denoted by v_p the momentum and energy integrals give the relations

$$rv = qv_p$$

$$\tfrac{1}{2}(u^2 + v^2) - \frac{\gamma m}{r} = \tfrac{1}{2}v_p^2 - \frac{\gamma m}{q},$$

γm being the sun's mass in gravitational units. Eliminating v_p we obtain

$$\tfrac{1}{2}(u^2 + v^2) - \frac{\gamma m}{r} = \frac{r^2 v^2}{2q^2} - \frac{\gamma m}{q}.$$

If r is large compared to q the left-hand member is small compared to the right-hand member, and we get

$$v = \frac{1}{r}\sqrt{2\gamma m q}.$$

$$\frac{104^2\pi\,(u_2 - u_1)}{\frac{4}{3}\pi\,L^3} = 1{\cdot}84\,.\,10^{-9}\,(u_2 - u_1). \qquad (21)$$

The number of comets between the same velocity limits moving inward per second through the sphere of radius r is obtained by multiplying this expression by the average velocity $\frac{1}{2}(u_1 + u_2)$, by the surface of the sphere, and by the density. Multiplying also by the number of seconds in a century we find for the number of new comets per century

$$6{\cdot}08\,.\,10^{-3}\,(u_2^2 - u_1^2)\,\nu(50000), \qquad (22)$$

ν being the number of comets per cubic A.U. The values (22) are shown in the first four numbers of the last column of Table 4.

TABLE 4

Number of comets passing per century through a perihelion within 1·5 A.U. from the sun.

Q	a	$u(50000)$	n
200000	100000	$1{\cdot}637.10^4$	
			$2{\cdot}31.10^5\,\nu(50000)$
141000	70700	1·518 ,,	
			3·10 ,, ,,
100000	50000	1·336 ,,	
			4·56 ,, ,,
70700	35400	1·022 ,,	
			6·32 ,, ,,
50000	25000	0	
			0·5 ,, ,,
35400	17700		
			0·14 ,, ,,
25000	12500		
			0·07 ,, ,,
17700	8840		
			0·03 ,, ,,
12500	6250		

We must now consider the comets with aphelion distances less than 50000. In order to do this I shall first consider an element of space at $r = 25000$, and in this element the comets with aphelion distance $Q = 40000$, corresponding with a period of $2{\cdot}8.10^6$ years. The comets passing through $r = 25000$ and moving towards the sun have been exposed during about $2\frac{1}{2}$ million years to the perturbing action of the stars. During this time $\sqrt{\overline{\Delta v^2}} = 236$ cm/sec. Because the radius of the cylinder corresponding to $q = 15$ is 657 cm/sec at $r = 25000$, only the outer region of the cylinder will be filled through the general action of the stars. The density in the cylinder of 208 cm/sec radius, corresponding to $q = 1{\cdot}5$, will be very much smaller than that in the velocity space outside the outer cylinder. The number of observable comets coming from this region of space will be accordingly smaller.

In order to obtain an estimate of the number of comets that yet penetrate into the inner cylinder we must investigate the effects of relatively close, single encounters. For the velocity of a comet to be changed by an amount equal to the radius of the outer cylinder, or 657 cm/sec, an "average" star should pass it, according to (11), within 12000 A.U. As we want to consider comets with $Q < 40000$, the only stars that are effective will be those that pass within, say, 45000 A.U. of the sun. From the data given in section 2 we find that on the average 0·09 stars of average mass 1·4 ☉ pass within this distance per million years, or one star in 11 million years. Now, the period of a comet with $Q = 40000$ is $2{\cdot}8.10^6$ years; for a comet with $Q = 30000$ it is $1{\cdot}8.10^6$ years. This means that about 2·8 million years after an appropriate stellar passage all comets with $Q = 40000$ will have passed perihelion and will as a consequence have been eliminated from the group of large-distance comets. The comets with $Q = 30000$ will similarly have disappeared from this group after 1·8 million years. It follows that only during about 1/4 of the time we shall observe many comets with $a = 20000$. During the remaining time the number will generally not be zero, because stars with smaller masses will be passing, but it will be very much reduced. The above calculations were schematized by assuming that there were 0·021 stars of average mass 1·4 ☉ per cubic parsec. The actual number may be as much as 3 times higher, but the average mass would then be only about 0·2 ☉.

I shall now make a rough calculation of the number of comets brought into the "inner" cylinder of 208 cm/sec radius by the passage of a single star of mass 1·4 ☉ and velocity 31 km/sec. I consider a star passing the sun at a shortest distance of 25000 A.U. During the part of its orbit lying within $r = 40000$ A.U., i.e. over a length of 62000 A.U., the star passes $\nu.\,62000.\,2\pi\,D\,dD$ comets at distances between D and $D + dD$. Introducing, by means of (11), the velocity transfer ΔV as a variable instead of D we find for the number of comets suffering a velocity change between ΔV and $\Delta V + d(\Delta V)$

$$2{\cdot}49.10^{19}\,\nu\,(\Delta V)^{-3}\,d(\Delta V).$$

The number of such comets per (cm/sec)³ of the velocity space is found by dividing by $\frac{4}{3}\pi L^3$; taking $L = 25000$ cm/sec, corresponding to $r = 25000$ A.U., we get $3{\cdot}81.10^5\,\nu\,(\Delta V)^{-3}\,d(\Delta V)$. By an elementary consideration we then find that the number of comets thrown into an element of thickness du of the inner cylinder is approximately given by

$$208^2\pi\,du\,.\,3{\cdot}81\,.\,10^5\,\nu\int_{657}^{L}(\Delta V)^{-3}\,\frac{\text{Arc cos}\,(657/\Delta V)}{\pi/2}\,d(\Delta V) = 7{\cdot}62\,.\,10^4\,\nu\,du\int_{1}^{38}x^{-3}\,\text{Arc cos}\,\frac{1}{x}\,dx. \qquad (23)$$

The integral is found to be 0·384; the exact value of the upper limit is irrelevant. If we had considered a star of mass 0·2 ⊙ the coefficient would have been 1/49th of that given.

Per million years there are, as we have seen, 0·09 passages of massive stars. Multiplying (23) by 0·09 we find that, on the average over a long time, 2700 $\nu\, du$ comets are brought into the inner cylinder per million years. This will be equal to the number of perihelion passages per million years. In order to find the number of comets with a between 12 500 and 17 700 (Q between 25000 and 35400) passing through perihelion per century we must multiply by the corresponding interval of $u(25000)$, and by 10^{-4}. The radial velocity at $r = 25000$ of comets with $Q = 35\,400$ is $1{\cdot}44.10^4$, so that the total factor becomes 1·44. We thus obtain $3900\,\nu(25000)$ comets per century, or, as $\nu(25000) = 3{\cdot}6\,\nu(50000)$ according to Table 3, $14000\,\nu(50000)$. This number has been entered in Table 4. It can easily be proved that the number of comets brought into the inner cylinder by the combined effect of two, or several, more distant passages is negligible compared to that brought in by one single passage. As has been stated, the number given represents an average over a long period. During about 1/4th of the time the frequency will be 4 times higher, during 3/4th of the time it will be 12 times lower. It may be noted that if any comets should be found coming from these distances less than 40000 A.U. they are most likely all due to the passage of one star. If this star had a mass 1·4 ⊙ the aphelia of these comets should all be confined between two great circles making an angle of about 58°; in case the star had a mass 0·2 ⊙ this angle would be 8°. But as cometary orbits of these dimensions are likely to be rare, there is little prospect of finding any such concentration. For the comets with $Q > 50000$ the number brought into the inner cylinder by one close passage is negligible compared to that brought in by general diffusion, so that there is no likelihood that any trace of the passages of specific stars would be visible in the distribution of perihelia. A plot of the directions of the perihelia of long-period comets having appeared between 1850 and 1936 was made. No trace of concentrations of the kind just mentioned could be seen.

For the comets with Q between 35400 and 50000 we shall have a mixture between the process of general diffusion and the effect of single passages. The number entered in the table for this interval is only a guess.

The numbers in the two last lines of the table have been computed in the same way as that for the interval 25000 to 35400. The numbers represent again averages over very long times. The real numbers will fluctuate greatly. For the last interval they will be zero during 19 out of 20 millions of years. Comets coming from these distances ($Q < 18000$ A.U.) will come in only during about one million years after the passage of a star within 20000 A.U. of the sun. Even if we consider stars with masses down to 0·1 ⊙ such passages will only occur once in about 20 million years. It is very unlikely therefore that we shall observe any "new" comets coming from distances less than 20000 A.U. For this reason the observations of comets cannot give any indication of the densities in the parts of the cloud within $r = 20000$.

We see from Table 4 that practically all "new" comets must have had orbits with semi-major axes larger than 25000 A.U. This agrees very well with observation (see Table 1 and the next section) and gives us a clear insight into the reason why all new comets appear to come exclusively from such very large distances.

The numbers in Table 4 enable us also to compute the density $\nu(50000)$. As we shall see in section 5 (p. 105) we may estimate that on the average 97 observable new comets pass per century through perihelion with $q < 1{\cdot}5$. According to Table 4 this number should be equal to $1{\cdot}7.10^6\,\nu(50000)$. We find therefore that

$$\nu(50000) = 5{\cdot}7.10^{-5} \text{ per (A.U.)}^3. \qquad (24)$$

The number of comets between $r = 25000$ and $r = 200000$ is then equal to

$$4\pi.\, 50000^2.\, 5{\cdot}7.10^{-5} \int_{25000}^{200000} \frac{N(r)}{N(50000)}\, dr.$$

The values of $N(r)/N(50000)$ can be inferred from Table 3. The total number of comets in the cloud is thus found to be $1{\cdot}9.10^{11}$.

There are no good estimates of the average mass of a comet, except that it must probably be larger than about 10^{14}, and smaller than 10^{20} grams. A plausible estimate is perhaps about 10^{16} g (cf. also VAN WOERKOM, *l.c.* p. 462, footnote). With such an average mass the total mass of the cloud of comets would be 10^{27}, or about 1/10th of the earth's mass. This estimate is uncertain by one or two factors of 10.

It is of some interest to see how the distribution of the semi-major axes of new comets depends on the model selected for the cloud. The model we have used so far is one with very little increase in density towards the inner parts. I shall now briefly consider the alternative of a model with strong concentration towards the centre, such as would follow if the velocity distribution were of the Maxwellian type with an average space velocity smaller than the velocity of escape from the outer parts. As an example of such a distribution I take the case already mentioned on p. 95, in which the average space velocity corresponding to the Maxwellian distribution equals half the velocity of escape at $r = 50000$. The latter is $1{\cdot}89.10^4$ cm/sec. The modulus of the Maxwellian distribution, h, is then $1{\cdot}30.10^{-4}$. If Φ is the potential the density is given by

$$\frac{\nu(r)}{\nu(50000)} = e^{2h^2\{\Phi(r) - \Phi(50000)\}}$$

$$= e^{3{\cdot}01.10^5\left(\frac{1}{r} - \frac{1}{50000}\right)}, \qquad (25)$$

r being again expressed in A.U.

This gives the following relative densities:

r	$\frac{\nu(r)}{\nu(50000)}$
100000	$0{\cdot}05$
50000	1
25000	$4{\cdot}1.10^2$
20000	$8{\cdot}3.10^3$
15000	$1{\cdot}3.10^6$
10000	$2{\cdot}9.10^{10}$
5000	$3{\cdot}4.10^{23}$

In order to explain the frequency of comets with very large major axes $\nu(50000)$ must be of the same order as in the previous model. It is evident from the above numbers that the assumption of a Maxwellian velocity distribution would thus lead to quite impossible densities in the inner parts of the cloud. Already in the shell between $r = 7500$ and $r = 12500$ the number of comets and their total mass would become entirely prohibitive. We can thus conclude that the velocity distribution in the cloud of comets can certainly not approximate a Maxwellian form.

But even if we confine attention to the outer parts, of which we have direct information through the observed "new" comets, it will be seen that a velocity distribution such as I have used gives a more satisfactory representation than a Maxwell distribution. The distribution of u,v corresponding to the latter takes the form

$$\frac{2h^3}{\sqrt{\pi}} v\, e^{-h^2u^2 - h^2v^2}.$$

The number of comets per A.U.[3] having transverse velocities less than $v_{1{\cdot}5}$ and radial velocities between u and $u + du$ is

$$\frac{h\nu}{\sqrt{\pi}} e^{-h^2u^2}\left(1 - e^{-h^2 v_{1\cdot5}^2}\right) du\,.$$

For the number of such comets moving per century through a sphere of radius r, with radial velocities between u_1 and u_2 we then find

$$\frac{7{\cdot}47.10^{-4}\,\nu r^2}{h}\left(1 - e^{-h^2 v_{1\cdot5}^2}\right)\left(e^{-h^2u_1^2} - e^{-h^2u_2^2}\right).$$

This gives also the number of comets moving per century through perihelion ($q < 1{\cdot}5$ A.U.). Taking $r = 50000$ A.U. we obtain the following numbers of comets per century between the same limits of a and u as used in Table 4.

a	n
100000 – 70700	$0{\cdot}2.10^5\,\nu(50000)$
70700 – 50000	$0{\cdot}7$,,
50000 – 35400	$3{\cdot}2$,,
35400 – 25000	$22{\cdot}$,,
25000 – 17700	(19) ,,

The number between $a = 17700$ and $a = 25000$ is a rough estimate similar to that made before.

We observe that, with this model, on the average $41.10^5\nu$ out of the $45.10^5\nu$ new comets passing through perihelion per century would have semi-major axes between 18000 and 35000 A.U., while only $1.10^5\,\nu$ would have a semi-major axis larger than 50000. In reality, 7 out of the 19 accurately known original orbits had semi-major axes in excess of 50000 A.U. It may be concluded that, also in the outer parts of the cloud, the velocity distribution certainly does not resemble a Maxwellian distribution. What the actual velocity distribution is we do not know, except that in the outer parts it must be more or less like that used in our tentative model. But we cannot use this to extrapolate inwards, nor to estimate the densities in the inner parts of the cloud. The apparent deviation from an exponential distribution of velocity and density will be related to the way in which the cloud was formed. A formation as suggested in section 6 would necessarily result in just such a density distribution as observed.

Two remarks may be added to the above discussion.

As a consequence of the fact that the cylinder in the velocity distribution corresponding to $q = 1{\cdot}5$ is smaller than the distance over which the velocities must be diffused in order to get into this cylinder, the distribution of velocity points over a section of the cylinder perpendicular to its axis must be practically uniform, regardless of how the velocities were originally distributed. This means that the orbits of the newly incoming comets can show no preference in orientation. Even if the cloud of comets were somewhat flattened towards the plane of the ecliptic no trace of this could remain in the orbital inclinations of the new comets. Such flattening should, however, still reveal itself in the distribution of their perihelia. But we have seen on p. 95 that already at $r = 50000$ very little of any flattening the cloud might originally have had, could have maintained itself up to the present time. If, as is indicated by the observations, practically all new comets come from distances *larger* than $2a = 50000$ there is little chance that any flattening might be observable. As BOURGEOIS and COX (*l.c.*) have indicated, it is very difficult to disentangle effects of observational selection completely, but so far as the available data go they confirm entirely the expectations in showing no significant deviations from randomness in the observed distribution of either the aphelia or the inclinations of nearly-parabolic comets.

In the second place we may briefly consider in how far the large cloud of comets has been depleted through the combined action of the stars and Jupiter. As a representative case we may consider comets with $a = 35400$ A.U. Their period of revolution is $6{\cdot}7.10^6$ years. Therefore, at an average distance $r = 50000$, a fraction equal to the ratio of the volume of the cylinder with radius v_{15} to that of the sphere with radius L will be lost per $6{\cdot}7.10^6$ years. This ratio is $0{\cdot}0003$. The fraction lost in 3.10^9 years will thus be $0{\cdot}13$. We see that the cloud will have suffered only a minor depletion in this manner.

5. *The Distribution of the Major Axes, and the Proportion of Retrograde and Direct Orbits.*

It is of interest to compare the observed distribution of $1/a$ with the stationary distribution that would be produced by the action of Jupiter. VAN WOERKOM [1]) has investigated this for the case that comets would continuously come in from interstellar space. Neglecting distintegration effects he remarked that the number of comets in equal intervals of $1/a$ would then be a constant for positive as well as for negative values of $1/a$. The observed distribution being entirely different he concludes that there can be no steady state of the kind studied by him.

I want to rediscuss this problem on the basis of the picture of the origin of long-period comets given in the present article.

It is clear, in the first place, that with this picture there can be no negative values of $1/a$. It is also clear that practically half of the "new" comets coming through perihelion will escape from the solar system, so that the numbers returning in subsequent intervals of $1/a$ will be reduced. For large values of $1/a$ the numbers will be further reduced by the effect of disintegration.

Let us denote by N the annual number of comets which, coming from the large cloud, pass for the first time through a perihelion in the observable region; for brevity these comets will be called *new* comets. Let, further, $n(x)$ represent the annual number of "old" comets with a reciprocal semi-major axis $x = 1/a$, and let k be the probability that a comet is disrupted during a passage through perihelion. If we make the plausible supposition that the distribution function does not change with the time, the following equation must be satisfied

$$n(x) = (1-k)\, N \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} + (1-k)\frac{h}{\sqrt{\pi}} \int_{-x}^{+\infty} n(x-\Delta)\, e^{-h^2\Delta^2}\, d\Delta\,. \qquad (26)$$

Here Δ represents the change in $1/a$ due to the planetary perturbations, for a complete passage through perihelion. The distribution of Δ is approximated by a Gaussian function with modulus h. The left-hand member of the equation represents the number disappearing annually from the interval x to $x + dx$, while the right-hand member shows the number of comets entering into this interval; the factor dx is omitted on both sides.

In order to get rid of dimensions I shall suppose that x and Δ are measured in units equal to the average value of $|\Delta|$.

It is easy to find a solution for the integral equation for large values of x. As, for Δ larger than a few units, the frequencies are negligible, and as, for large values of x, $n(x)$ will vary only little when x is varied by a few units, we can there develop $n(x-\Delta)$ in a Taylor series. As, furthermore, the first term of the right-hand member of (26) is negligible for $x > 3$, we obtain

1) *L.c.* p. 459 a.f.

$$n(x) = (1-k)\left\{n(x)\frac{h}{\sqrt{\pi}}\int_{-x}^{+\infty} e^{-h^2\Delta^2}\,d\Delta - \frac{dn(x)}{dx}\frac{h}{\sqrt{\pi}}\int_{-x}^{+\infty}\Delta\, e^{-h^2\Delta^2}\,d\Delta + \tfrac{1}{2}\frac{d^2n(x)}{dx^2}\frac{h}{\sqrt{\pi}}\int_{-x}^{+\infty}\Delta^2 e^{-h^2\Delta^2}\,d\Delta \ldots\ldots\right\}.$$

Again, for $x > 3$, the parts of the integrals below $\Delta = -x$ are negligible, and we can replace $\int_{-x}^{+\infty}$ by $\int_{-\infty}^{+\infty}$. Hence

$$n(x) = (1-k)\,n(x) + \tfrac{1}{2}(1-k)\frac{d^2n(x)}{dx^2}\bar{\Delta}^2 + \ldots\ldots \quad (27)$$

The higher-order terms can safely be neglected, so that we get

$$\frac{d^2n(x)}{dx^2} = \frac{k}{1-k}\cdot\frac{2}{\bar{\Delta}^2}\,n(x).$$

Thus,

$$n(x) = Ce^{-x/p}, \quad (28)$$

$$p = \sqrt{\frac{(1-k)\bar{\Delta}^2}{2k}} = \frac{\sqrt{\pi}}{2}\sqrt{\frac{1-k}{k}}. \quad (29)$$

The lower limit of x down to which (28) will be a good approximation is about $x = 3$. We can easily find a numerical solution for the lower values of x. For this purpose I shall consider intervals of unit length (the unit being again $\overline{|\Delta|}$). A suffix 1 will indicate the interval from $x = 0$ to $x = 1$, a suffix 2 that from $x = 1$ to $x = 2$, etc. The new comets will be considered to come at $x = 0$, while the other comets in an interval will all be supposed to be at the centre of the interval. Under these conditions the probability for a *new* comet to be moved by the perturbations to the first interval will be $0{\cdot}287(1-k)$, to the second interval $0{\cdot}157(1-k)$, and to still further intervals $0{\cdot}055(1-k)$; of the latter practically all will come in the third interval, and for simplicity I assume that they *all* come in this interval. For an *old* comet the probability to remain in its original interval will be $0{\cdot}310(1-k)$; the total fraction leaving the interval is therefore $1-0{\cdot}310(1-k) = 0{\cdot}690 + 0{\cdot}310k$. The chance for a shift into one of the adjacent intervals is $0{\cdot}229(1-k)$ for each of these intervals. For a shift into one of the next intervals it is $0{\cdot}093(1-k)$, and into one of the intervals following upon these $0{\cdot}023(1-k)$.

Equating again the numbers of comets that after a complete perihelion passage leave a certain interval (left-hand side) to those coming into it from various other intervals (right-hand side), we obtain the following equations.

$$\begin{aligned}
(\cdot690+\cdot310\,k)\,n_1 &= (1-k)\,(\cdot287\,N + \cdot229\,n_2 + \cdot093\,n_3 + \cdot023\,n_4)\\
(\cdot690+\cdot310\,k)\,n_2 &= (1-k)\,(\cdot157\,N + \cdot229\,n_1 + \cdot229\,n_3 + \cdot093\,n_4 + \cdot023\,n_5)\\
(\cdot690+\cdot310\,k)\,n_3 &= (1-k)\,(\cdot055\,N + \cdot093\,n_1 + \cdot229\,n_2 + \cdot229\,n_4 + \cdot093\,n_5 + \cdot023\,n_6)\\
(\cdot690+\cdot310\,k)\,n_4 &= (1-k)\,(\cdot023\,n_1 + \cdot093\,n_2 + \cdot229\,n_3 + \cdot229\,n_5 + \cdot093\,n_6 + \cdot023\,n_7)
\end{aligned} \quad (30)$$

. . . .

Inserting for n_4, n_5 .... the values given by (28) and (29) the first three equations can rigorously be solved. For a given value of k they yield expressions for n_1, n_2 and n_3 in the two parameters N and C. But the equations for the 4th, 5th and 6th intervals must also be satisfied (those for the 7th and higher intervals, in which N, n_1, n_2 and n_3 do not occur, are automatically satisfied by (28)). This condition relates C to N. It appears that the three equations can all be satisfied with a sufficient degree of accuracy by one and the same relation between C and N.

We have thus expressed the complete distribution of $1/a$ in the one parameter N, the annual number of new comets. Numerical results for two different values of k are given in Table 8. For the derivation of these results we must know the relation between the variable x in which the calculations just sketched were carried out, and the values of $1/a$ in A.U.$^{-1}$; that is, we must know $\overline{|\Delta|}$, the unit of x.

VAN WOERKOM has made extensive calculations of this quantity, averaging over all possible orbits with a perihelion distance of 1 A.U. He finds [1] $0{\cdot}53.10^{-3}$. This may be compared with the average of the perturbations as calculated for a number of individual comets. Approximate computations of this kind have been made by FAYET for 146 comets [2]). These calculations refer to the first half of the perihelion passage only. He gives the difference in eccentricity between the osculating orbit near perihelion and the original orbit at large distances from the planets. Before leaving the region of the major planets the comet will undergo another similar perturbation. The first half of the perturbation in $1/a$ will on the average be negative, the second half positive. Let the residual perturbations, remaining after removing the systematic part, be denoted by $\delta(1/a)$. Table 5 shows the distribution of these residuals as calculated from FAYET's results. It may be noted, in passing, that the algebraic average is found to be $-\cdot000\,500$.

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1) *L.c.* p. 445, in *Summary*.
2) *Ann. Obs. Paris, Mém.* **26A**, 1910.

TABLE 5
Distribution of $\delta(1/a)$ due to Jupiter.

$\delta(1/a)$ (A.U.$^{-1}$)	FAYET	Gauss distr.
·0000 — ·0002	50	52
2 4	45	42
4 6	33	28
6 8	9	15
·0008 ·0010	3	6
> ·0010	6	3

The distribution compares reasonably with a Gaussian distribution corresponding to $\overline{|\delta|}$ = ·000344, which is shown in the last column. Remembering that this only represents the perturbation for half of the passage through the planetary system, and assuming that the residuals for the two halves may be taken as independent, we obtain for the total average change in $1/a$: $\overline{|\Delta|} = \sqrt{2} . \overline{|\delta|}$ = ·000487. This is practically the same as the value ·000530 found by VAN WOERKOM.

Another value for $\overline{|\delta|}$ has been derived by SINDING [1]) from the 21 comets for which *rigorous* calculations of the perturbations by the major planets had been made. From his numbers I find $\overline{|\delta|}$ = ·000200, or $\overline{|\Delta|}$ = ·000283. The fact that this value is so much smaller than either VAN WOERKOM's or FAYET's values is probably due to the accidental absence of any very large perturbation among these 21 comets. I have assumed $\overline{|\Delta|}$ = ·00048.

The perturbations are generally larger for direct than for retrograde orbits. According to VAN WOERKOM [2]) the ratio between the average value of $\overline{|\Delta|}$ for orbits with inclinations distributed at random over the interval 0° to 90° and the average for orbits with inclinations between 90° and 180° is $\sqrt{22{\cdot}0/9{\cdot}5}$ = 1·52. The two groups will be briefly referred to as direct and retrograde orbits. In accordance with this value of the ratio I have assumed $\overline{|\Delta|}$ to be ·00058 and ·00038 for the direct and retrograde orbits respectively. Separate calculations for the two groups are shown in Table 8.

Only uncertain estimates can be made of the probability of disintegration at a perihelion passage. I have found mention of observations of the splitting up of 11 comets which have appeared since 1600. In his catalogue of comets A.S. YAMAMOTO [3]) gives 576 apparitions of comets after 1600. If all the splittings observed resulted in disintegration (or at least loss of visibility) the average probability of disintegration would be $k = \frac{11}{576}$ = 0·019. If we omit comets with periods less than 50 years we find $k = \frac{7}{409}$ = 0·017. The former value has been used in Table 8. It stands to reason that k will differ greatly for individual comets. There will be some comets that have a much greater resistance than others. A case of small k seems, for instance, to be presented by Encke's comet [1]). It is not possible from the scarce data available to get any idea about the dispersion in k. The phenomena may be somewhat further complicated by the fact that the disruption of a comet might give rise to two or more independent comets that remain observable. On the whole, however, there does not seem to be much evidence for this.

In order to see what the effect is of a smaller probability of disintegration, computations of the distribution function of $1/a$ have also been made with k = 0·003.

The derivation of the *observed* distribution of $1/a$ is complicated by the fact that the orbits are of very widely different quality. For the first part of the distribution, up to $1/a$ = ·001 00, we can only rely on orbits of the highest standard, for which I have selected the orbits where $1/a$ has a mean error smaller than ± ·000 10, or, if the mean error has not been given, the period during which the comet has been observed is at least 6 months. The distribution of the original $1/a$ for this category is shown under (A) in Table 6. It contains 41 comets. For 19 of these, accurate calculations of the original orbits are available, for 8 others FAYET has computed approximate values for the reduction to the original orbits. The distribution of the original $1/a$ for these 27 objects is shown in column (2). Two slightly negative values of $1/a$ were included in the first interval. For the 14 remaining objects of group (A) a correction of + ·000 65 was applied to eliminate the systematic part of the planetary perturbations, while on top of this an uncertain statistical correction was made to allow for the dispersion in these perturbations. The resulting distribution is shown in column (3). Column (4) gives the final distribution for group (A).

The next category contains the orbits for which the mean error of $1/a$ is between ± ·000 10 and ± ·000 50, or, if the m.e. is unknown, the period from which the orbit was determined is between 3 and 6 months. Among these comets there are 17 for which only parabolic elements have been computed. These have been included in the interval ·000 00 — ·001 00. It was clearly impossible to attempt a subdivision of this interval. The distribution for category (B) is shown in column (5).

There remain 138 comets in the interval 1850 to 1936 and with $1/a$ < ·040 for which the periods of observation used for the orbit determination have

1) *L.c.*, cf. footnote p. 91 of the present article.
2) *L.c.* p. 458.
3) *Kwasan Publ.* 1, No. 4, 1936.

1) Cf. VAN WOERKOM, *l.c.* p. 449.

TABLE 6

Observed distribution of original values of 1/*a*.

(1)	(2)	(3)	(4)		(5)	(6)	(7)	(8)
1/*a*	(A)				(B)	(C)	Adopted	$\bar{n}$
·00000 — ·00005	10	2	12				12	12
5 — 10	4	0	4				4	4
10 — 15	1	1	2				2	2
15 — 20	2	0	2				2	2
20 — 25	1	0	1	} 27	25	15	1	1
25 — 50	2	1	3				3	·6
50 — 75	1	0	1				1	·2
75 — 100	1	1	2				2	·4
100 — 200	3	1	4		3	4	3·2 (7)	·16
200 — 400	1	3	4		4	5	3·6 (8)	·09
400 — 600	1	0	1		2	3	1·4 (3)	·04
600 — 800	0	4	4		4	2	3·6 (8)	·09
800 — 1000	0	0	0		1	1	0·5 (1)	·01
1000 — 2000	0	1	1		6	9	2·9 (16)	·014
2000 — 3000	0	0	0		3	4	1·3 (7)	·006
·03000 — ·04000	0	0	0		1	2	0·5 (3)	·003
Total	27	14	41		49	138		

been less than 3 months. The determinations of 1/*a* must be very poor for most of these, and I have used them only to strengthen the part of the distribution function between 1/*a* ·010 and ·040. For 93 of these only parabolic elements have been published. But this does not necessarily mean that 1/*a* is small. In several cases the observations are so few that the true value of 1/*a* may well exceed 0·01. The distribution is shown in column (6) under (C). The parabolic orbits have been omitted except in the total number given at the bottom.

The distribution finally adopted is shown in column (7). For 1/*a* < ·001 00 it is based exclusively on (A); for the interval ·001 00 to ·010 00 the sum of (A) and (B) has been used, multiplied by 41/90 in order to reduce to the total number in category (A); for still larger values I have used (A) + (B) + (C), again reduced to a total of 41. The total number of orbits used in each interval has been added between parentheses.

I am indebted to Mr PELS for providing me with the data required for arranging the orbits into the different classes of quality.

The last column of the table shows the average number of comets per interval of ·000 05 in 1/*a*. It indicates the way in which the frequencies gradually diminish with increasing 1/*a*.

A comparison of columns (4), (5), (6) of Table 6 shows that group (A) contains a smaller proportion of large values of 1/*a* than group (B). This is further illustrated by the numbers in Table 7. The classification into the various columns is essentially one according to the period over which the observations extend.

TABLE 7

Distribution of 1/*a* classified according to the length of the period of observation.

1/*a*	$(A)_1$	$(A)_2$	(B)	(C)
< ·001	18	9	25	—
·001 — ·010	4	9	14	15
·010 — ·040	1	0	10	15

Group (A) has been subdivided into two parts: $(A)_1$ for which the period of observation was 8 months or longer, $(A)_2$ for which it was shorter than 8 months.

So far as it is possible to draw a conclusion from the small numbers in the table they seem to indicate that comets with small values of 1/*a* are generally visible over longer periods than those with large values of 1/*a*. It would not be surprising if this were so. The comets with large 1/*a* being older and having passed many more times through perihelion, it is quite plausible that they would have diminished in brilliance. I have had no opportunity to study this phenomenon more in detail; it might warrant a more complete investigation [1]).

A comparison of the observed distribution of 1/*a* with the steady-state distributions calculated as outlined above is shown in Table 8. The column *Obs.* gives the total numbers in the intervals indicated in the first column, taken from column (7) of Table 6. Columns *D* indicate computed distributions for orbits

[1]) An indication in the same direction was found by VSEKHSVIATSKY, *A.J.S.U.* **25**, 337, 1948 (cf. *Astr. News Letter* No. 44).

with $i < 90°$, while R refers to retrograde comets, with $i > 90°$. The actual numbers of comets to be expected in the various intervals may be obtained by multiplying the numbers given by N.

TABLE 8

Comparison of observed and computed distributions of $1/a$.

$1/a$		Obs.	k = 0·019		k = 0·003		$\bar{n}$		
			D	R	D	R	Obs.	k=·019	k=·003
·00000 —	·00050	24	1·67	2·03	1·77	2·22	2·4	0·48	0·34
50	100	3	0·68	1·00	0·84	1·28	0·3	·22	·18
100	200	3·2	1·22	1·49	1·64	2·30	·16	·18	·17
200	400	3·6	1·75	1·79	3·09	3·76	·09	·12	·15
400	1000	5·5	1·98	1·30	5·72	6·06	·046	·035	·084
1000	2000	2·9	0·48	0·13	4·26	3·03	·014	·004	·031
·02000	·04000	1·8	0·04	0·00	2·00	0·73	·004	·0001	·006

In the last section of the table a comparison is given between the average numbers of comets per unit interval of $1/a$ (·00005). The observed values are from the 8th column of Table 6, the calculated numbers, under k = ·019 and k = ·003, respectively, correspond to the average of D and R, multiplied by a factor that makes the weighted average $\bar{n}$ for the intervals from $1/a$ = ·00050 to $1/a$ = ·00400 equal to the weighted average observed $\bar{n}$.

It will be seen that, apart from the first line, the agreement is fairly satisfactory. The calculated numbers for $\bar{k}$ = ·019 become too small for $1/a >$ ·010. It may well be that the true average value of k is somewhat smaller than ·019. Moreover, the dispersion in k will play a role. A quite satisfactory representation may for instance be obtained by taking the average of the columns for k = ·019 and k = ·003, with weights 2/3 and 1/3 respectively. This would correspond to $\bar{k}$ = ·014, which agrees still well enough with the observed disruptions. Part of the deviations between observed and calculated values for large $1/a$ may, however, also be explained by the deviation that is known to exist between the actual distribution of Δ and the Gaussian law. Summing up we may conclude that the present theory seems entirely capable of explaining the observed distribution of $1/a$, except for the interval from ·00000 to ·00050, where the observed number is some 5 times higher than that which would be expected from the numbers observed in the subsequent intervals. The observed number rests on 24 comets, and is supported by the independent, though less certain evidence of category (B), so it is very probable that the difference is real. The most direct interpretation of the discrepancy would be that the new or almost new comets which come for the first few times near the sun have a greater capacity for developing gaseous envelopes, and that a large fraction of these would not be rediscovered at subsequent passages when they would be much less brilliant. This phenomenon would be similar to the gradual decrease in brightness indicated by the data of Table 7. A *quantitative* confirmation of this conclusion does not, however, seem possible at present.

From Table 6 we may estimate that on a total of 41 comets with $1/a <$ 0·04 there are about 15 new ones. This would give 83 new comets on the total of 228 with $1/a <$ 0·04 that have passed through perihelion between 1850 and 1936, or 97 per century. We have needed in the previous section the total number with $q <$ 1·5 A.U. Now, if we tabulate the numbers of long-period comets in different intervals of q it appears that the distribution is practically constant up to q = 1·2 A.U., indicating that up to this value the completeness does not depend much on q. For $q >$ 1·2 the incompleteness increases rapidly; the number between q = 1·2 and 2·4 is only 30% of that between 0·0 and 1·2, while the actual numbers in the two intervals should be equal. For still larger values of q the observed number becomes very small; only for 5% of all long-period comets is q larger than 2·4. It appears that the "complete" number of observable comets with $q <$ 1·5 must be practically equal to the observed total for all perihelion distances, so that we may estimate that per century 97 observable new comets pass through a perihelion less than 1·5 A.U. from the sun.

Comparing the columns D and R we see that for $1/a <$ ·00200 the retrograde orbits should preponderate; if we take into account, however, that the proportion of new comets, which are supposedly distributed evenly over all inclinations, is in reality much larger, the remaining preponderance of R over D will be negligible. From $1/a$ = ·00400 the circumstances are reversed, and the direct orbits should be the more frequent ones. For higher values of $1/a$ this preference becomes more and more pronounced; above a = ·02 there should be practically no retrogade orbits in the case k = ·019. This absence is likely to persist if the deviation from the Gaussian law referred to above is taken into account.

The actual distribution of the inclinations i is shown in Table 9. For $1/a <$ ·010 only orbits of categories (A) and (B) have been used. The comets for which no eccentricity has been computed are shown in a separate column under the heading $e = 1$. In case of a random distribution the numbers in each of the four intervals of i should evidently be equal. It will be seen that the expectations expressed above are well confirmed. In particular the preference for small inclinations in the column $1/a$ ·020 to ·040 is well pronounced. It is shown still better if the first interval is subdivided into two parts, from i 0° to 41°·5 and 41°·5 to 60°, respectively. With a random distribution these two parts should contain equal numbers of orbits. Actually, 6 out of the 8 orbits with i smaller than 60° come in the first part.

TABLE 9
Distribution of inclinations of cometary orbits.

i \ $1/a$	< ·000 10	·000 10 to ·001 00	·001 to ·004	·004 to ·010	·010 to ·020	·020 to ·040	$e = 1$
0° – 60°	3	3	4	6	2	8	20
60 90	6	6	5	2	8	1	30
90 120	2	3	3	3	1	1	24
120 180	6	6	4	0	5	0	36

Because the average perturbation $\overline{|\Delta|}$ depends also —though less strongly than upon the inclination— upon the shortest distance, d, at which the comet's orbit passes the orbit of Jupiter, the orbits with large $1/a$ should show some preference for small values of d. The effect should be most pronounced in the orbits with high inclination. From approximate values of d which Mr PELS has computed we found the following averages (in A.U.):

$e = 1$, and $1/a <$ ·001	1·40 (45)	2·33 (44)
$1/a$ between ·001 and ·010	1·16 (14)	1·88 (13)
$1/a$ between ·010 and ·040	1·56 (15)	1·42 (11)

The two sets of values refer, respectively, to orbital planes making angles of less than 60°, and between 60° and 90° with the ecliptic. The numbers of orbits have been added in parentheses. Though showing a trend in the expected direction the data are evidently still too few to allow a conclusion.

The present theory seems capable of accounting satisfactorily for all statistical data concerning the long-period comets down to periods of about a century, viz. the remarkable form of the distribution curve of $1/a$, the random distribution of inclinations and of the directions of perihelion, and the decreasing inclinations for orbits with a between 25 and 250 A.U. The relation of the short-period comets, in particular those of the Jupiter "family", to the long-period ones has been extensively investigated in the past, especially by H. A. NEWTON[1]), H. N. RUSSELL[2]), and most recently by VAN WOERKOM[3]). The work on this intricate problem is still far from complete, but the evidence available appears to be in at least approximate agreement with what one would expect if the population of the family is kept up by the captures by Jupiter from the field of long-period comets[4]). The Jupiter family would thus be in equilibrium with the long-period comets.

It may be noted that, except for the incompleteness in the discussion of the Jupiter and Saturn families, we have now a fairly comprehensive theoretical picture of the distribution of cometary orbits. The picture is not confined to the comets that come within the observable region, but may be extended to any perihelion distance, because for the long-period comets, as H. A. NEWTON has already remarked, the number passing through a perihelion between $q - \frac{1}{2}$ and $q + \frac{1}{2}$ may be expected to be independent of q. For the short-period comets the conditions are vastly more complicated, but nevertheless it would seem possible to work out the theory of these orbits in a statistical way.

Meteors and Zodiacal Light.

It is not unlikely that meteors and comets are the same type of objects. It has long been known that some meteor showers are directly connected with comets. We might accordingly consider part of the meteors as débris of comets. In addition, the large cloud of comets may contain independent small bodies as well as the large comets. The interesting evidence recently produced by WHIPPLE[5]) indicates that the distribution of major axes of orbits of meteors shows great analogy to that of the comets. Similarly to the short-period comets, the meteors with small orbital major axes ($a <$ 11 A.U.) are strongly concentrated towards the ecliptic, while the larger orbits show a random distribution in inclination.

If the meteors are débris of comets the number of meteors would increase relative to that of the comets as we proceed from smaller to larger values of $1/a$. As

1) "On the Capture of Comets by Planets, Especially their Capture by Jupiter." *Mem. Nat. Ac. Washington* **6**, 7, 1893.
2) "On the Origin of Periodic Comets", *A.J.* **33**, 49, 1920.
3) *L.c.* section 5.
4) The essential difference between these captures and the small perturbations considered in the present paper is that the captures are due to one, or a very few, quite large perturbations. The comets in the Jupiter group may therefore well be "younger" on the average than the comets which have come down to orbits with major axes of the order of 100 A.U. by successive, small perturbations. The family is partly made up of comets whose original orbits happened to come exceptionally close to the orbit of Jupiter.
5) *A.J.* **54**, 53, 1948 (*Abstract*).

the frequency of comets diminishes by disintegration the number of meteors increases. According to Table 8 about half of the comets would have disintegrated at $1/a = \cdot 003$; in the interval from $1/a$ ·01 to ·04, however, already about 95% have disintegrated. It is clear that this factor would cause considerable differences between the distributions of $1/a$ for comets and meteors. Another circumstance causing a difference is that the probability of disintegration may be different for the two kinds of objects.

It would seem worthwhile to investigate the resultant space distribution of the meteoric particles. It is a not unlikely supposition that the zodiacal light would be due to similar particles, so that it would be of interest to confront the theoretical density distribution with the intensity distribution in the zodiacal light. Such an investigation would involve a somewhat more extended study of the Jupiter family of comets than has hitherto been made.

6. *Hypothesis of a Common Origin of Comets and Minor Planets.*

The enormous size of the cloud of comets presents an interesting problem in itself. It seems most unlikely that in the regions between 50000 and 200000 A.U. from the sun, where probably the general gas density will never have been much higher than the average density in interstellar space, bodies as large as the comets could have been built up by condensation or accretion. If, as there is some reason to believe, meteorites belong to the comet species, the argument becomes still more stringent. It seems impossible that the peculiar inner structure of a meteorite could have been the result of gradual condensation and accretion at low temperatures. It would be very interesting in this connection, if it could be decided whether the orbital characteristics of at least part of the known meteorites resemble those of the comets.

It appears far more probable that instead of having originated in these far away regions, comets were born among the planets. It is natural to think in the first place of a relation with the minor planets. There are indications that these two classes of objects belong to the same "species". Like the minor planets the comets' nuclei seem to consist of solid blocks of considerable dimension. I am indebted to MINNAERT for pointing out that there is at least one case in which the evidence on this point is quite unambiguous, namely that of comet 1843 I, which moved through the corona; from which one can compute that the solid block, or blocks, of which the nucleus consists must have a diameter of at least 1/2 km. The known asteroids are all rather larger than this, but there is good evidence that their number increases considerably when we extend the search to fainter limits. The observed difference in size may therefore be an effect of observational selection, only the larger objects being observed as asteroids.

It seems a reasonable hypothesis to assume that the comets originated together with the minor planets, and that those fragments whose orbits deviated so much from circles between the orbits of Mars and Jupiter that they became subject to large perturbations by the planets, were diffused away by these perturbations, and that, as a consequence of the added effect of the perturbations by stars, part of these fragments gave rise to the formation of the large cloud of comets which we observe today.

It can hardly be doubted that at least a certain number of the minor planets must have disappeared in the course of time through the action of Jupiter in exactly the same way as comets of the Jupiter family disappear through this action. Even at the present time there are some minor planets whose orbits cross the sphere with radius equal to Jupiter's mean distance from the sun, or the spheres with radii equal to Mars' or the earth's mean distance. Such asteroids are likely to suffer at some time in the future so large a perturbation by one of the planets that they will be brought into long-period orbits. After which they will be gradually diffused out of the solar system by small perturbations that bring them into more and more elongated orbits. Most of the minor planets that have in the past been thus diffused outwards through Jupiter's perturbations must have escaped into interstellar space, but not all. During the diffusing process a certain fraction will get orbits with semi-major axes between 25000 and 100000 A.U. Now, it is clear from the data given on p. 97 that practically all the asteroids that happened to get into this range of orbits would, during the 4 to 30 million years they needed to complete such an orbit, have been diverted into orbits that did no longer come near the large planets. In as much as they would generally have been brought into these orbits soon after their origin, the continued action of the stars would afterwards have dispersed their velocities such as to give them a random distribution.

For an asteroid there are thus two types of more or less stable orbits in which it can get: nearly circular orbits between Mars and Jupiter like the known asteroids, or orbits with mean distances of the order of 100000 A.U. into which it can be brought through the combined action of Jupiter and the stars.

At the present time the number of minor planets being transferred into long-period cometary orbits is certainly very small. As we have seen in the preceding sections the main process is now the inverse one, that of a slow transfer of comets from the large cloud into short-period orbits. But at the epoch at which the minor planets were formed, when presumably there were a large number of fragments with considerable orbital eccentricities and inclinations, the trend must have been opposite, many more objects being transferred from the asteroid region to the comet cloud than vice versa.

The present data about minor-planet orbits will probably contain no clue by which we could estimate how large a fraction of the fragments once present have escaped by planetary perturbations. This would certainly depend mostly upon the unknown original distribution of the orbital elements. It may well have been a very large fraction. For, the aggregate mass of the present minor planets is very much smaller than would have been expected if there had once been a mass of the order of that of an ordinary planet in the open space between Mars and Jupiter. According to RUSSELL, DUGAN and STEWART[1]) a reasonable estimate for the mass of all the asteroids, known and unknown, would be 1/1000 of the earth's mass. It is tempting to suppose that the mass has indeed at the outset been of a dimension comparable to that of an ordinary planet, but that the large majority of the asteroids that were formed from this mass has escaped, the only ones that remained being those whose orbits happened to have small eccentricities and small inclinations.

The hypothesis offers a direct and simple explanation of the huge cloud of comets and its characteristics. The asteroids would have to be started on their way out by one, or a few, large perturbations. If the average change in $1/a$ that they would undergo *after* this start is estimated to be about ·001 A.U.$^{-1}$ per perihelion passage, the probability of their coming at some time during the diffusing process into the interval between $a = 25000$ and 200000, or between $1/a$ ·000005 and ·000040, is evidently about 1/30. Therefore, about 1/30 of all the mass escaped from the asteroid region would have become part of the large cloud of comets. If the mass in the asteroid region had once been of the order of the earth's mass a cloud of comets should have been formed with a total mass of the order of 1/10 of the earth's mass, which is of the same order as the actual mass of the cloud of comets as estimated on p. 99. The mechanism proposed would give rise to a density distribution showing exactly those characteristics that are exhibited by the cloud of comets. A cloud so formed would necessarily extend to the limit set by the dissolving action of the stars, that is, to about 200000 A.U. The inner limit should lie near 25000 A.U., where the perturbing action of the stars is no longer strong enough to have shifted bodies from the elongated orbits into orbits that remain outside the region of the large planets; the cloud would therefore contain practically no members with mean distances less than 25000 A.U. The escaping fragments being evenly distributed over $1/a$ the comets in the cloud should have a similar distribution. This fits in remarkably with the slowness of the outward density decrease indicated by the well-determined orbits of new comets. Other modes of formation might have been expected to lead in general to distributions approximating an exponential form with a steeply decreasing density (cf. pp. 99 and 100). The observational data on the density distribution being still very scarce it would seem of much interest to make further calculations of definitive (and original) orbits of nearly-parabolic comets that have been well observed, in order to verify whether the density distribution really accords with that predicted by the mechanism considered.

Although the present hypothesis offers a natural explanation of the existence of the cloud of comets, it should be emphasized that the hypothesis should be thoroughly studied from the point of view of celestial mechanics before it can put a claim to be accepted as a good working hypothesis. In particular, it would be essential to inquire in how far it is compatible with the observed distribution of orbital elements of the minor planets.

The difference in general appearance between minor planets and comets can certainly not be taken as an indication of a difference in origin. The fragments that we now call comets having disappeared from the regions near the sun soon after their birth, and having passed their entire further existence at distances where the sun's radiation can have had little or no effect, must have kept the larger part of the gases that were included within them at their origin. The fact that comets can apparently keep up their gas emission until they have got into short-period orbits indicates that it is legitimate to assume that, during the comparable period that would have elapsed between their birth in the asteroid region and their transfer into the cloud of comets, they would not have lost all their volatile constituents. The objects, however, that have remained in the region between Mars and Jupiter will since long have lost their capacity for emitting gases.

If, as seems to be indicated by the fact that when heated they develop the same gases as comets [1]), and by the apparently continuous transition between meteors, fireballs and meteorites, the meteorites — or at least part of them — belong to the same class as the comets, this would form an additional argument in favour of an identical origin of comets and minor planets. From their peculiar inner struc-

[1]) "Astronomy", I, Revised Ed. 1945, p. 353.

[1]) The latter phenomena have recently been extensively rediscussed by LEVIN in *Russ. A. J.* 1943. My attention was drawn to this article by Professor MINNAERT.

ture most authors conclude that meteorites appear to be fragments of a broken-up planet. The apparently irregular shapes of many asteroids have led to a similar inference with regard to these bodies.

In the past many theories have been proposed to explain the origin of the comets. A review of these theories, which may be found in various general text-books, would be outside the scope of this article.

Quite recently an entirely different hypothesis has been given by LYTTLETON[1]), who supposes that the comets have been formed from small solid particles captured by the sun during past slow passages through interstellar clouds.

I cannot enter here upon a discussion of LYTTLETON's interesting suggestion. From the little which is known about frequency and motions of interstellar clouds there seems, however, to be only a slight chance that during its existence the sun would have passed through such a cloud at a sufficiently low velocity. There is also the general difficulty mentioned earlier in this section, that it seems impossible to understand the structure of meteorites on the basis of a gradual growth from small particles. LYTTLETON's theory would therefore involve that we must accept an entirely different origin for meteorites and comets.

1) *M.N.* **108**, 465, 1948.

NOTES ADDED TO PROOF

(1) Mr VAN WOERKOM, who has read the manuscript of the foregoing article, has drawn my attention to the effects of an incompleteness in the formulae used for computing the perturbations of stars on the motions of the comets. Formula (2) rests on the tacit, assumption that the points of closest approach of the star to the sun and the comet coincide (in A). As a consequence of Mr VAN WOERKOM's remark I have realized that this restriction does not give a sufficient approximation. This is mainly due to the fact that it does not take adequate account of the cases where the stars pass the *sun* at relatively small distance. A much better approximation may be obtained by considering separately (a) the encounters for which the shortest distance to the *comet* is less than half the distance r between the comet and the sun, and (b) the encounters for which the shortest distance of the star to the *sun* is less than $r/2$. We shall get a fair average for (a) by neglecting the effects upon the sun, while similarly for (b) we may safely neglect the effects upon the comet. The contribution of each of these groups of encounters to the integral in (7) will therefore be given by (10); for $r = 100000$ A.U. this amounts to 8·87, so that the total contribution from the two groups is 17·74. By a somewhat more complicated calculation it may be shown that more distant encounters will contribute 2·52. The total integral thus becomes 20·26, and the total average value of ΔV^2 in 3.10^9 years $1{\cdot}84.10^8$ (cm/sec)^2, which is 1·66 times higher than the value given in Table 2, and is about equal to the square of the velocity of escape.

This correction only strengthens the conclusion reached in section 2: Once we accept that comets have existed in the solar system since its origin it follows as a necessity that, at the great distances from which they come, their velocity distribution is determined entirely by the effect of stellar encounters, and that it must therefore be isotropic and nearly homogeneous over a sphere with radius of the same order as the velocity of escape. It follows with the same necessity that the orbital planes and directions of perihelia of long-period comets must be distributed at random, such as is shown by observation.

(2) Prompted by Mr VAN WOERKOM's further remarks I have made estimates of the distribution of orbits which may ensue as a consequence of a rupture of a planet. As they may possibly also interest other readers I am appending a summary of these crude calculations.

Suppose a planet similar in size to the earth, and moving in a circular orbit with a radius corresponding to the mean orbital radius of the known minor planets, or 2·8 A.U., had been disrupted by some internal cause. It appears reasonable to suppose that the velocities with which the fragments escaped from the "sphere of attraction" of the parent planet will not have been of a smaller order than the velocity of escape from this planet, or, say, 10 km/sec. Let us denote these velocities relative to the parent planet by v, and assume that they are evenly distributed over all directions. We shall assume that outside the "sphere of attraction", which may be supposed to be of negligible size, the fragments' velocities v are no longer influenced by the original planet. Orbital elements were computed for three different values of v and for various directions. By rough integration of these data over the various directions I then derived the data given in Table 10.

TABLE 10.

v (km/sec)	(a) hyp. f	(b) $Q > 5{\cdot}20$ f	$\bar{i}$	(c) "minor planets" f	$\bar{i}$	$\bar{e}$	(d) f	(e) f
5	·00	·31	6°	·50	9°	·24	·19	·00
10	·20	·34	16	·17	22	·36	·10	·19
20	·56	·19	42	·03	66	·3	·07	·15

In the respective divisions the columns f show the fractions of the fragments which are thrown into: (a) hyperbolic orbits, (b) orbits extending beyond the sphere with radius equal to Jupiter's mean distance, (c) orbits contained entirely between the spheres with radii equal to Mars' and Jupiter's mean distances (these are shown under the heading "minor planets"), (d) orbits for which the perihelia lie between the spheres with radii equal to the earth's and Mars' mean distances, and (e) orbits for which the perihelia lie inside the sphere with a radius of one A.U.; in the latter two groups only those fragments are included that had not already been included in (a) or (b).

For $v = 5$ km/sec almost half of the fragments will describe orbits between the orbits of Mars and Jupiter. The average major axis of these orbits is very near to that of the hypothetical parent planet, the average inclination and eccentricity are about 9° and 0·24, respectively. These objects might be identified with the observed minor planets, for which the inclinations average 9°·5 and the eccentricities 0·15.

The orbital velocity of the original planet being 17·8 km/sec there can evidently be no hyperbolic orbits for $v = 5$ km/sec. For a fraction of about 0·31, however, the aphelion distance is larger than the radius of Jupiter's orbit.

For $v = 10$ km/sec the proportion of category (c) is much smaller, and the mean eccentricity and inclination are higher, as shown under $\bar{i}$ and $\bar{e}$ in the table. Is is quite beyond the scope of the present article to investigate whether a considerable fraction of these bodies may not in the long run be expelled from the asteroid region by planetary perturbations. For $v = 20$ km/sec hardly any fragments will remain in group (c). For still higher velocities practically all parts will move away in hyperbolic orbits, so that these velocities need not be considered.

The fragments in column (b) will probably all be removed either into the large cloud of comets or to interstellar space by Jupiter's perturbations. For the orbits concerned, which, for instance for $v = 10$ km/sec have an average inclination of about 16° and a reciprocal semi-major axis of about 0·15, an approach to Jupiter within 0·05 A.U. would cause a change in $1/a$ amounting to roughly 0·1 A.U.$^{-1}$. We may thus estimate that approaches to within 0·1 A.U. would, on the average, suffice to bring the fragments into long-period orbits. Such approaches will be brought about as a consequence of the rotations of the apsidal lines. As a rough guess we may estimate that in about 10 000 years, or some 700 revolutions, all bodies of group (b) will have been removed into the long-period range. WHIPPLE's study of comet Encke, which showed that it has been moving in the inner regions of the planetary system for at least 10 000 years, indicates that even a comet moving much nearer to the sun than those considered above, can keep up its capacity for developing gases during a period of this length.

We have seen in section 6 that of fragments removed by Jupiter's perturbations perhaps 1/30 enter into the cloud of comets. It is evident, therefore, that for $v = 5$ km/sec the number of comets in the cloud would be rather smaller than the number of minor planets. It is only for velocities v in the vicinity of 20 km/sec that the number of comets could much surpass that of minor planets. Now we have seen that the number of comets in the cloud must be estimated to be about a million times higher than that of observable minor planets. The hypothesis of a common origin of minor planets and comets can therefore only be upheld if the number of minor planets of sizes comparable to the comets very greatly surpasses that of the observed minor planets, which does not seem impossible, or else the number of fragments expelled with values of v between 20 and 40 km/sec were of a much larger order than that expelled with v from 5 to 10 km/sec. It is also possible that a large fraction of the objects that remained in the asteroid region have subsequently disintegrated.

In conclusion I wish to point out that the results developed in the first five sections of the present article depend in no way upon the speculative discussions in this note and in section 6. In whatever manner the comets have originated we may, if their age is comparable to that of the solar system, apply the considerations set forth in the former sections.

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