III, 1. THE ROYAL ROAD OF ECLIPSES

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FROM IMMEMORIAL ANTIQUITY, men have dreamed of a royal road to success — leading directly and easily to some goal that could be reached otherwise only by long approaches and with weary toil. Times beyond number, this dream has proved to be a delusion. The easy way has ended in an impasse, and the right track has led up the hill Difficulty. This has been so invariable an experience that the very suggestion that there might be a royal road is hardly to be mentioned in polite society — whether of scholars, statesmen, or saints. In the larger sense, this distrust is fully sound. We cannot expect nor hope to get anything worth while without working for it. Nevertheless, there are ways of approach to unknown territory which lead surprisingly far, and repay their followers richly.

There is probably no better example of this than eclipses of the heavenly bodies. Within the solar system, eclipses of the moon provided Aristotle with the simplest and most convincing proof of the rotundity of the earth. Eclipses of the sun revealed the existence and nature of its outer layers and envelopes. With the aid of knowledge derived from eclipses, it is now possible to observe the chromosphere, the prominences and even the corona in their absence. Many of the methods by which these are observed would probably have been invented anyhow for the study of photospheric problems, but hardly the coronagraph; and even today there are important phenomena — from the base of the flash spectrum to the outer streamers of the corona — for which we must still depend on total eclipses for all our information.

Eclipses of the satellites of the planets, when they are observable, provide the most reliable determination of the diameters of both planet and satellite, free from the grave systematic errors which affect direct measures with the micrometer.

But our greatest source of otherwise inaccessible information is found in the mutual eclipses of the components of close binary stars. For almost a century after their discovery, these were regarded as little more than curiosities; then, as their astrophysical implications were realized, they were actively studied. Now, after more than 40 years of intensive work, they still present a most promising field for new investigation, and, within the last year, it has been shown that they permit the determination of something which had previously been regarded as utterly inaccessible.

The greatest of all the discoveries which follow from the recognition of stellar eclipses was the first — namely, the existence of binary systems among the stars. Goodricke, the discoverer of the periodicity of the variation, wrote in 1783, "I should imagine that it could hardly be accounted for otherwise than . . . by the interposition of a large body revolving about Algol." [1]

This interpretation, which could hardly be better stated, was made by a young man of 18! It antedates by some 20 years Herschel's discovery of the existence of visual binaries. The latter resulted from long series of measures made with the best telescopes of the day. The former followed from a series of less than a year's visual estimates of brightness, on what was then an arbitrary scale, not reducible to numerical values.

This difference is the first illustration of the extraordinary power of eclipses in revealing the nature of the stars. Goodricke's suggestion was so far in advance of his times that it was not taken seriously for the better part of a century. He died at the age of twenty-two. It is tantalizing to think that fairly precise numerical measures of the relative brightness of stars would have been possible at that time — using simple properties of lenses, such as were employed by Sir John Herschel in 1836 in his measures of the brightness of the moon. Had Goodricke lived, he might well, despite his handicaps of deafness and ill health, have been able to present the photometric evidence of eclipse in so convincing a form that it could not have been disregarded.

It was a very long time before accurate photometric observations were made of eclipsing stars. Even in 1880, when E. C. Pickering made the first precise theoretical calculation of the variation of light arising from a stellar eclipse, he was obliged, in his discussion of Algol,[2] to use the visual estimates made by Schönfeld a decade or two earlier, and in 1896 [3] when G. W. Myers showed that the components of β Lyrae were not spherical, but considerably elongated by their mutual attraction, the best observations at his disposal were the eye-estimates made by Argelander published in 1859!

The first intensive series of observations of eclipsing variables by photometric methods of precision appear to be those by Pickering, Searle and Wendell in 1880 and 1881.[4] These gave good light curves for the principal eclipses of Algol and U Cephei (whose discovery the year before raised the number of recognized eclipsing variables to six!). The observations of Algol were all near principal minimum; for U Cephei, one sixth of them were clear of eclipse.

Pickering understood the problem thoroughly. He pointed out that the long duration of the constant phase for U Cephei was inconsistent with an annular eclipse, even by a dark body, and that neither the assumption of limb-darkening nor of orbital eccentricity could alter this conclusion. With a total eclipse by a faint component at primary, he showed that the depth of the secondary could not exceed $0^{m}.07$. He considered, however, the alternative "that the satellite consisted of a cloud of meteors so scattered that about 0.11 of the light could pass through its central portions." Twenty years ago, the "modern" reader might have smiled at this. Since the recent work on ϵ Aurigae and V444 Cygni smiles appear to be less in order.

If these photometric measures had been extended throughout the whole periods, the secondary minima of Algol and U Cephei could not have escaped detection, and the "hypothesis" of eclipse would have been raised to the status of a well-established theory before Vogel's observations of the radial velocity of Algol, in 1889, provided the crucial evidence.

The first case in which accurate photometric measures, covering the whole period of variation, were subjected to theoretical treatment appears to be Myers' discussion in 1898 of Wendell's observations of U Pegasi.[5] Unfortunately, the geometric conditions in this system are such as to make it impracticable to obtain a unique solution from photometric observations alone — which Myers, after very laborious calculations, came to realize, as did Roberts,[6] in an independent investigation later.

For U Cephei a definite solution could have been obtained from the observations of the total primary eclipse, but, unfortunately, no one attempted this, though it could have been carried out by Pickering's sound though somewhat laborious methods with much less labor than was expended upon the indeterminate cases of β Lyrae and U Pegasi. By an irony of fate, these excellent observations appear never to have been worked up until last month, when A. F. Cook, a Princeton senior, discussed them in connection with an advanced undergraduate course. They are quite sufficient to define good elements, and the resulting values are intermediate between those deduced from later observations by Shapley[7] and by Dugan.[8]

Ten years more elapsed before systems of a type which permitted a unique solution were thoroughly observed throughout the period — RT Persei by Dugan[9] and Algol by Stebbins.[10] Secondary minima and the reflection effect were independently detected in these stars by the two observers, and the modern era of discussion based on adequate observational data began. Stebbins' discovery of shallow eclipses in β Aurigae [11] at the time predicted from the spectroscopic orbit confirmed the reciprocal relation between eclipsing and spectroscopic binaries. In the following year came the development of simple methods of calculation which opened the floodgates of full discussion.

A word of reminiscence may be pardoned here. Not long before this, we received at Princeton a business-like application for our one fellowship in astronomy, accompanied with a letter of recommendation expressing a very high opinion of the applicant's abilities. It was so enthusiastic, in fact, that, though I knew that the writer (Seares) had excellent judgment, I pondered a moment, and asked myself "What sort of chap can this young man Harlow Shapley be?" I soon learned! Thousands upon thousands of published observations were crying to be worked up by the new methods. Shapley discussed all those that were worth while and added observations of his own in many cases, and his thesis on the orbits of 90 eclipsing binaries [12] added at one stroke a new branch to double-star astronomy.

There is no need to go into technical details of these calculations — especially as they have been discussed by your lecturer at a recent meeting of our society. It may be recalled, however, that the number of unknowns to be determined is large, that some of these can be found only from observations on the rising or falling parts of the curve, and that precise observations are indispensable. Estimates of brightness are very useful for determining the period and the general characteristics of a system; but reliable measures are necessary for an accurate solution. Unique results are often unobtainable unless the relative brightness of the components can be determined spectroscopically.

Let us now abandon presentation in historical order, and speak of things of various types for which our best and often our only knowledge comes from stellar eclipses.

To begin with geometrical and dynamical properties — the first direct and accurate determination of the *diameter* of a star was made by Stebbins for β Aurigae in 1911 by combining photometric and spectroscopic data — antedating Michelson's interferometric measures of α Orionis by nine years. By 1940 the radii of both components of more than 30 systems had been determined and more have since been added. This is many times the number of stars whose diameters have been directly determined in all other ways — by the interferometer or from occultations, which too are eclipses of a sort. The masses of the components must be found from spectroscopic orbits — the photometric solution removing the uncertain factor $\sin^3 i$; but the *densities* of eclipsing systems can be determined without knowing their linear dimensions or masses. Pickering in 1880 showed that the mean density of the Algol system was one ninth of the sun's; and Vogel, in 1889, assuming that Algol and its companion were of equal density, calculated the masses and diameters of both, getting much too small results because his assumption was incorrect.

It is curious that no general application to eclipsing binaries was attempted till 1899, and more so that it was made, simultaneously and independently, by a very competent amateur observer in South Africa, and by a young graduate student in this country. Their papers were sent to the same journal, reached it at almost the same time, and were published side by side.[13] Only rough upper limits to the mean density could then be made, but the fact that this was much less than the sun's was news in 1899, and influenced later theories of stellar structure.

At the present time good values of the density can be found for both components of pairs in which the mass ratio is known, and fairly good ones for the brighter components of many other pairs for which there are reliable photometric elements. The grand total comes to at least 200 stars, with densities ranging from more than 20 times the sun's for the subdwarf UX Ursae Majoris to a millionth of the sun's for the supergiant Zeta Aurigae. No eclipsing pair of white dwarfs has yet been discovered. Assuming as a fair sample a pair of stars each of half the sun's mass and 100,000 times its density, separated by four radii, the period of revolution comes out three minutes, and the duration of central eclipses thirty seconds! It is an entertaining puzzle to consider how such rapid variation could be discovered, and, if so, how a reliable light curve could be obtained.

This branch of the royal road runs farther. Eclipsing binaries, and they alone, provide information about the *distribution* of density within the inaccessible interior of a star. Each component of a pair is distorted by the tidal action of the other into a nearly ellipsoidal form and its net attraction on its companion therefore increases a little faster than the inverse square of the distance.

If the orbit is elliptic, the line of apsides will slowly advance. The rate of apsidal motion depends not only on the masses and dimensions of the components, but on the concentration of density toward their centers, and the latter may thus be found.

Variations in period are only too numerous among eclipsing binaries; but they may safely be ascribed to apsidal motion only when the intervals between the two minima are unequal, and change periodically with the time. The periods so far established range from about 25 years to several centuries — the latter rough determinations.

In every case where the observations are reliable, the concentration of density within the stars is found to be great. Assuming the standard polytropic type of distribution, the central density comes out from 50 to 450 times the mean density, in various systems. These differences are real. The well-observed systems GL Carinae and V523 Sagittarii have almost the same periods, mean densities, and ratios of dimensions of stars to orbit, but the apsidal period is 25 years for the first and certainly almost 200 for the second.

All but one of these stars belong to the main sequence, and the exception lies near it. They have widely separated components — the radii averaging 0.19 times the mean distances. With components close together, the apsidal motion would be much faster, and the eccentricity necessarily small. Whether such systems would be dynamically unstable, or merely hard to detect, is a question greatly in need of study.

No data are yet available to test the theories which predict a very much greater central condensation for giants than for the main sequence. Zeta Aurigae has an eccentric orbit; but the period of apsidal rotation, computed from Kopal's recent elements, comes out 30,000 years, assuming the smallest concentration observed in the main sequence and 300,000 years assuming the greatest, while on the aforesaid theories it should be many times longer. The supergiant W Crucis, with components of very low density and almost in contact, should show fairly rapid apsidal motion if the orbit is eccentric; but long-continued and precise photometric and spectrographic observations would be required to test this — and the star is in 58° south declination. When, at long last, astrophysical observations are begun with great southern telescopes, this star should have a high priority.

The *parallaxes* of all but a very few eclipsing systems are too small to measure directly. Their *luminosities* may be indirectly estimated, when their diameters are known, with the aid of assumptions regarding the relation between surface brightness and spectral type. The resulting values give pretty good "photometric distances" (with the usual complications due to space absorption) and show that the brighter components agree, within reasonable limits, with the normal *mass-luminosity relation.*

Many of the faint components, however, show a conspicuous discordance, which presents a very interesting example of the effects of observational selection. Eclipsing variables of large range are likely to be among the first to be discovered, and to be favored by observers above the general run of less "interesting" specimens. In such systems, the component which is in front at the deep minimum must necessarily be much fainter than the other. If it is smaller than its primary, and yet obscures almost all its light, it can be only slightly smaller, and must pass almost centrally in front of it. If it is larger, it may exceed it in size by any amount, so long as the eclipse is total or nearly so, and there is much more geometrical latitude of adjustment. The great majority of photometrically deep eclipses will, therefore, be occultations.

For normal main-sequence stars, the luminosity, the mass, the diameter, and the effective temperature diminish steadily down the sequence. Hence a fainter main-sequence star cannot totally eclipse a brighter. By selecting the most conspicuous eclipses, we are therefore automatically picking out systems in which one component, at least, is not a normal main-sequence star. If the faint component is aberrant, it must be larger and cooler than a normal main-sequence star of the same luminosity, and still larger than one of the same temperature. The point representing it in the familiar diagram will be above the main sequence, and it will be a "superdwarf" if not a subgiant. In all cases where the mass of such a component has been determined, it is found to be less than that corresponding to its luminosity according to the standard relation. These bodies are much less dense than their companions, and the greater the disparity of density, the greater is the deviation from the mass-luminosity relation.[14] No adequate theoretical explanation of this — taking internal temperature and energy generation into account — is yet available. The detailed information given by eclipsing systems should help in getting one. For example, in TX Ursae Majoris, which has been accurately observed by Dr. F. B. Wood, the small bright component has the spectral type B8, and the large faint one gF2. Apsidal motion is present, and arises almost entirely from a large distortion of the faint component by the attraction of its massive primary. By the courtesy of Dr. Wood, I am at liberty to tell that his calculations indicate that the central density of this star is about 300 times the mean density — a high value, but within the range already known for main-sequence stars.

This indicates that the peculiarities of the superdwarfs are not to be attributed to a peculiar density-model.

If the bright component is aberrant, it must be a subdwarf, below the main sequence. Its high density and spectral peculiarities show that UX Ursae Majoris belongs here; but no spectroscopic orbit is yet available, though observations are being made.

A limit to the inequality of the masses in any binary system can be set if the dimensions of the components are known, since each one must be massive enough to hold itself together against the tidal disturbing force of the other. For two point masses, revolving in a circular orbit, the limit for an outer envelope of low density is a well-known and easily calculable Jacobian surface shaped like an hourglass, with two unequal bulbs — the smaller enclosing the smaller mass. Each bulb marks the limit within which an envelope of highly rarefied fluid, rotating with the system, can remain permanently associated with the enclosed core.

The most important dimension of the bulb for our purpose is its radius in the equatorial plane perpendicular to the line joining the cores. This radius is 20 per cent of the distance of the cores if the mass of the smaller one is 10 per cent that of the larger; 25 per cent if the mass ratio is 0.22; and 30 per cent if it is 0.45. The concentration of density toward the center in actual stars is so great that the limits derived from the simplified model are practically quite satisfactory.

An interesting example is found in R Canis Majoris, for which we have two excellent light curves — visual by Dugan and photoelectric by Wood. There is a spectroscopic orbit for the bright component by Sitterly, and an older one by Jordan. These agree in leading to a very small mass function, the first to 0.0017, the second to 0.0028 (allowing for the inclination).

To make the mass of the brighter component come out equal to the sun's it is necessary to assume that the fainter one is only 0.13 or 0.15 (for the two orbits). But the crosswise radius of this component is 0.25 times the distance of center, according to Wood's elements, and 0.24 according to Dugan's. A body of this size will not be stable under the attraction of its primary unless its mass is 25 per cent as great, nearly twice the values just found.

Assuming this mass ratio, the mass of the bright component comes out from the two orbits only one sixth or one quarter of the sun's, and the other proportionally smaller. This is evidently a very remarkable system, and deserves additional intensive observation, for parallax, radial velocity, and light.[15] Some of the cases with very small mass functions, reported this afternoon by Struve, may be of the same sort.

In the study of the *radiation* from the surfaces of the stars, eclipses afford access to an even wider field. Some properties of the components, such as spectra, magnitudes, and color indices, can be found more directly for a resolved visual pair than for an eclipsing system. When the eclipse is total, the determination of these is not difficult; for partial eclipses, it is entangled with the determination of the orbital elements.

The very important determination of the *relative surface brightness* of the components is possible only for eclipsing pairs. This can be approximately found from the ratio of the depths of the two minima at corresponding phases. Correction for limb-darkening is automatically made when elements are derived for the system. It is then possible to compare the "backs" of the components—which, being undisturbed by each other's radiation, are much more fairly representative of normal stars. This determination is independent of space reddening. The *absolute magnitude* can be directly determined for very few eclipsing systems, for determinations of parallax with a small percentage probable error are prerequisite. To find the *absolute surface brightness* demands in addition a good photometric light curve and spectroscopic orbits of both components. There are only two stars at present which really pass this exacting test — μ^1 Scorpii and YY Geminorum. By extraordinary good luck, one is of class B3 and the other K6, so that we know the surface brightness of two stars near the ends of the spectral scale and of the sun in the middle.

We do not have to wait to fill in the gaps until the parallaxes of remote stars can be precisely determined. Every eclipsing system in which the components are of different temperature, if well observed in two or more wave lengths, will give a determination of the differences of color index and of surface brightness between the components, and of the spectrum of the brighter one at least. The relation between color index and spectral type is well determined. The eclipse data should enable us to find the surface brightness corresponding to a given color index.

Limb-darkening is a fundamental parameter in the theory of eclipsing variables—which afford the only chance of determining it, except for the sun. The cosine formula

$$J = J_0 \ (1 - x + x \cos \gamma),$$

where γ is the angle between the outgoing ray and the normal to the surface, provides a rather good representation of the solar observations, with x ranging from 0.23 at λ 21000 to 0.88 at λ 3230. The fall of brightness close to the limb is faster than is indicated by the cosine formula, but the difference is not serious. (Incidentally, its value close to the limb may best be determined from photometric observations of solar *eclipses* just before totality.)

The absolute value of x on a star disk can be determined in several ways. During annular eclipses of a bright star by a smaller and fainter companion, the diminution of light between internal contact and mid-eclipse arises from limb-darkening alone, and affords a good determination, in favorable cases, provided that the geometrical elements of the eclipse can be otherwise determined. For a total eclipse, the determination of x is badly correlated with that of the ratio k of the radii of the components and likewise with the difference in figure of the two. Under very favorable circumstances, and with highly precise observations, a good value of x may be obtained, as recently by Irwin for U Sagittae.[16]

With good observations, the combination of data from total and annular eclipses provides a good determination of both x and k. Through the courtesy of Dr. Kopal, I am at liberty to speak of his unpublished work on SZ Camelopardalis, for which there is an excellent light curve. Assuming $x = 0.2$, the value of k which best represents the form of the primary minimum is 0.55. The form of the secondary gives 0.38, and the depths of the two 0.45. With $x = 0.4$, the corresponding values are 0.51, 0.41, 0.44; with $x = 0.5$ they are 0.47, 0.44, 0.43. Plotting these results, it is evident that the values $x = 0.54$, $k = 0.44$ give a good representation of all the data. This new and very promising method bids fair to be of great value.

The *difference of limb-darkening* for the same star in different wave lengths is much more easily determined whenever a fairly deep total eclipse has been observed. For all normal stars it may safely be assumed that the components are sharp-edged and of definite size, so that the geometrical circumstances of eclipse, at a given instant, are the same for all wave lengths. When totality lasts long enough to give a good determination of the loss of light while it lasts—that is, of the light of the eclipsed star—the fraction α of this light which is lost at any partial phase is directly deducible from observation. Differences in α for different wave lengths at the same phase can arise only from differences in the distribution of light over the disk (provided, of course, that the measures are accurate). We then get, for the given phase, values of α_1 and α_2 corresponding to the wave lengths λ_1 and λ_2. Plotting $\alpha_2 - \alpha_1$ against α, we obtain a curve whose shape depends primarily on k, and its amplitude upon $x_2 - x_1$. With the aid of a diagram of the theoretical curves (which has been prepared by Dr. Merrill), $x_2 - x_1$ may be very easily found. An approximate value of k, resulting from a preliminary solution for the elements, is sufficient.

This method is independent of the forms of the components.

They may be spherical, ellipsoidal, or even ellipsoids of different shape, without seriously vitiating the results. The differential effects of any other cause which affects the apparent surface brightness (such as gravity-darkening or reflection) will be included.

The relations between surface brightness, color index, limb-darkening, and spectral type are of fundamental importance in the theory of the outer layers of the stars. If "gray body" assumptions could safely be made, everything could be reduced, in the first approximation, to the determination of effective temperatures, and all would be plain sailing. But there is abundant evidence that this is not the case. We have had a report on the present state of this question from Chandrasekhar this afternoon. For stars of Class B, the photospheric opacity arises mainly from free electrons; for Class A, from excited hydrogen atoms; for F and G, from the electron-affinity of unexcited atoms of hydrogen. In all three cases, the departures from gray-body theory are great.

If the outer layers of a star behaved like gray bodies, the effective temperature corresponding to the intensity of the disk would be 79 per cent as great at the limb as at the center. For monochromatic light the surface brightness would vary approximately like that of a black body at this effective temperature. This accounts in a general way for the variation of limb-darkening with wave length on the sun, but not in detail.

If, however, the opacity of the photospheric layers varies with the wave length, the situation is more complicated. Limb-darkening arises from the fact that the radiation from a given depth below the surface loses more by absorption and scattering when it escapes obliquely than when it emerges perpendicularly, so that the average effective depth from which the radiation comes is greatest at the center of the disk, and least at the limb. Since the temperature increases inward, the light from the center will come, on the average, from deeper and hotter layers, and will be brighter and bluer than that from the limb.

If, for a given wave length, the opacity is exceptionally great, the effective depth from which the light comes will be less than usual. The difference in effective depth between the center and limb will also be diminished. The first of those effects diminishes the central brightness, the second diminishes the limb-darkening. If, however, the opacity is unusually small, the light will come from greater depths, the energy curve will be raised, and the limb-darkening increased.

In the A stars the opacity diminishes steadily throughout the visible spectrum. This makes them appear too bright in the violet and too faint in the red for the actual temperature of their surfaces. *Pari passu*, the limb-darkening should be less in the red and greater in the blue than for a "gray" photosphere. This prediction is in complete agreement with the results of Kron and of Irwin, who independently find differences in x which are from 50 to 100 per cent greater than those predicted by gray-body theory.

It should be entirely possible to make photoelectric observations of suitable eclipsing binaries in the ultraviolet beyond the Balmer limit, and in the infrared outside the Paschen limit, with high accuracy, and determine the coefficient of darkening. It may be confidently predicted that this will be greater in the infrared and smaller in the ultraviolet than gray-body theory predicts, but observations are required to determine the numerical effect for a given spectral class.

In stars of Class B, where the hydrogen is almost completely ionized, its lines have lost their prominence, and the corresponding continuous absorption must be weak. The principal source of opacity in them appears to be the inescapable scattering of light by free electrons. This is independent of the wave length, so that a star, in whose outer layers the opacity was completely dominated by electron scattering, would effectively be surrounded by a dense white haze. Chandrasekhar, from a detailed analysis of this case,[17] finds that the darkening coefficient is 0.62 for all wave lengths. His prediction has been beautifully verified by Kopal's work on SZ Camelopardalis—

for which the gray-body theory prediction was that x would be about 0.25. The observed value 0.54 is just what might be expected for a blend in which electron scattering predominates.

Chandrasekhar's analysis of this problem led also to the conclusion that, with pure electron scattering, the light reaching us from any point of the limb should be polarized to the extent of 11 per cent. The plane of polarization is in the direction of the radius of the disk, so that the light of the star as a whole is unpolarized. The light from the disappearing crescent, just before total eclipse, should, however, be polarized.

Miss Edith Janssen,[18] observing with the Yerkes 40-inch, has found that the light of U Sagittae just before totality is actually partly polarized. This brilliant confirmation of theory opens up an interesting observational possibility. The position-angle of the disappearing crescent is given directly by the polariscopic observations, and can be found equally well at reappearance. From these angles, the position of the track of one star behind the other—whether central, or how much to one side, can be determined, and compared with the results of the photometric solution.

Far more exciting is the fact that these observations determine the position-angle of the line of nodes of the apparent orbit of the binary—although the maximum angular separation of the stars is of the order of 0″.0003! Within the year, I described this as one of the things which there was no hope at all that we could ever find out. It is most gratifying to be proved wrong. The royal road leads farther than the men who helped to open it up and smooth its surface dared to anticipate.

We come next to a set of related phenomena which present the most intricate part of the study of eclipsing binaries, both theoretically and in practice, namely, the *distortion of figure*, the *gravity effect*, and the *radiation effect*, which is commonly, though inaccurately, called reflection.

When the periods of rotation and revolution coincide, the forms of components are determinate if the mass ratio m_1/m_2, their mean radii, r_1 r_2, and certain parameters depending on their internal constitution are known. For the Roche model they coincide with the Jacobian surfaces, and are defined by simple and exact equations, as is also the value of gravity at any point on the surface. Otherwise, the figures may be expressed by a series of spherical harmonic terms, with coefficients proceeding by powers of r/a (where a is the radius of the orbit) beginning with r^3/a^3. These have been given in detail by Kopal as far as r^5/a^5. The general expressions for the higher terms are very complicated. For such central condensation as has been observed in the stars, the departures from the Roche model affect the leading term by about one per cent and the higher ones by smaller percentages. Since values of r/a of the order of 0.3 are common, it would appear to be a sound policy to employ the Roche model as a standard, and correct the equations for deviations from this in the rare cases where it may be desirable. Closed expressions are then available for computation which hold good up to the extreme case of the hour-glass figure.

When the distortion is moderate, the components are nearly ellipsoidal, so that this effect is commonly described as "ellipticity." The longest axes of the ellipsoids lie in the line joining their centers, and the shortest are normal to the orbit plane, but the ellipticities of the two are usually different.

The geometric theory of the eclipses of two unlike ellipsoids is complicated; but, if they are similar, it is surprisingly simple. All but highly refined discussions are therefore based upon an intermediary model composed of two similar ellipsoids. This is the simplest which is capable of representing the general properties of the light curves of actual systems, and is flexible enough to give a good representation in practically all cases. With a small modification in the assumed law of darkening, the equations connecting the light curve with the elements of the model become so similar to those for spherical stars that the methods of solution developed for these are, with slight changes, applicable in the general case.

The intermediary model furnishes a good first approximation to the actual character of the system, which may then be improved by the refined methods developed by Kopal.

The *variations of gravity* over the surface may be computed exactly for the Roche model, or, in general, by series given by Kopal. The total radiation per unit area of the surface should be proportional to the local value of gravity, provided that convection in the interior of the star is unimportant.[19] For monochromatic light this gravity effect may be represented by the formula $J = J_0\ (1 - y + yg/g_0)$, where g is the local and g_0 the mean gravity, and y a constant varying with the wave length. The values of y may readily be calculated, on gray-body assumptions, for any effective temperature and wave length, and (unlike those of x) may exceed unity.

Selective opacity in the photosphere and underlying layers should modify these values of y, but probably much less than those of x. So long as the layers concerned are thin compared with the radius of the body, the emission of radiation from any part of the star should be very similar to that from the same area of a sphere of the same composition and with the local gravity and effective temperature. Different parts of the surface of the distorted star may then differ by a few tenths of a magnitude in surface brightness, and in spectral type by a small fraction of a conventional "class." The latter effect may be neglected; the former is photometrically important.

For a central eclipse, the middle of the disk corresponds to the points farthest from the star's center, where gravity is least. This produces a negative limb-darkening, superposed upon the ordinary effect, but following a different law.

Both limb- and gravity-darkening have important effects upon the form of the light curve outside the eclipses. It has been realized for more than fifty years that a pair of ellipsoidal stars will send us more light when they are broadside-on at quadrature than when they are nearly end-on just outside eclipse. If the ellipsoids are similar, and of uniform surface brightness, this variation leads simply to a determination of the ellipticity of figure.

The presence of limb-darkening increases the variation of light for a body of given form, as does also the gravity effect — as may easily be seen geometrically in both cases. When both are present, the "photometric ellipticity," C (that is, the value derived from the light curve on the assumption of uniform surface brightness) is connected to the actual geometrical ellipticity ϵ by the relation

$$C = \epsilon \frac{(1+y)\ (15+x)}{15 - 5x}.$$

For an average star, $x \sim 0.5$ and $y \sim 1$, and the amplitude of the variation is more than doubled. The observed photometric ellipticity for the system is a mean of the values for the components, weighted according to the luminosities. If the radii and the mass ratio are known, the values of ϵ may be computed for both components (assuming the Roche model). If the values of x are known, a weighted mean of y for the two stars may be found. The scanty existing material indicates that y is not far from its theoretical value.

The hope expressed by your lecturer a year or so ago that a general relation between x and y might be obtained by combining the observed darkening on the sun with gray-body assumptions appears to have been ill-founded. There is, however, good hope that the value of x depends primarily on the spectral type. When x has been determined by observation for enough stars to give a trustworthy empirical determination of this relation, it will then be possible to find y from the photometric ellipticities for many stars even when the eclipses are partial, and theory and observation of the gravity effect can be compared.

In intermediary computation, the photometric ellipticity is found from the observations outside eclipse, and the observed light "rectified" by dividing it by the computed effect. The resulting light curve may be discussed by substantially the same methods as for spherical stars. In many systems, of the β Lyrae or W Ursae Majoris type, the ellipticity is large, and its removal greatly diminishes the rectified depths of the minima, so that that of the shallower one becomes seriously uncertain. These bid fair to be the most troublesome of all systems to the computer. All the theoretical corrections, especially those of higher order, are at their greatest; but the solution is often nearly indeterminate without additional spectroscopic data; and the computations are likely to be unusually long and laborious. Study of these stars, except the few for which good observations already exist, may well be deferred until precise observations have been made and reduced for many systems for which the geometrical and physical circumstances are more encouraging. Then, with the resulting knowledge of the relations between spectral type, darkening, etc., the harder problems may be more tractable.

In one instance, the extreme effects of distortion have been determined. Dr. Sitterly [20] has calculated the light curve which should theoretically arise from the rotation of a Jacobian hourglass figure. For this the series expansions are divergent, but numerical calculation of the form and surface brightness (for total radiation) is not difficult, and the rest can be done by a long series of quadratures. The cases which were discussed before Dr. Sitterly turned to war work give light curves strikingly similar to those arising from the eclipses of a pair of equal and similar ellipsoids separated by fully two thirds of their longest radii. The reason is that, near the conical point, gravity is very small, so that this part of the surface is so feebly luminous that it might almost as well not be there (provided that the elementary theory may be trusted, as Kopal has reminded us today).

Some known variables of this type have light curves similar enough to these to suggest that they may be systems near the point of fission. To settle this question alone should keep observers and computers busy for a long time.

The *reflection effect*, arising from the illumination of the surface of each body by the other, gives rise to the most intricate analysis in the whole field. The incident radiation penetrates the outer layers of the star on which it falls to only a moderate depth, and has no serious influence on the escape of heat from the interior. All the energy which reaches the surface is radiated away again, but its amount varies from one region to another, and its distribution changes with the angles of incidence and emission in a complicated manner. A full theory, taking account of the finite size of the stars as seen from one another, and of the penumbral zone from which only a part of one is visible from the other, has been developed by Kopal, and by Chandrasekhar, but is exceedingly complicated. At the time of eclipse, the reflection effect makes the illuminated disk appear brighter at the center than at the edge. The gravity effect acts in the opposite direction, and the two annul one another almost completely in many observable systems, and partially in all—sometimes one, sometimes the other being the stronger.

This is a very fortunate circumstance—making it permissible to ignore both effects in the intermediary discussion of eclipses, with very great simplification of the calculations.

Outside eclipses, the "reflection" enters to its full value. The principal result is to make each star appear brighter when behind its companion than when in front of it. This effect was discovered and correctly interpreted by Dugan in 1908 from his observations of RT Persei. The reflection effect upon the other component produces a term of the same period, but opposite phase. The amplitudes of the two are proportional to the surface intensities of the two stars, or, substantially, to the depths of the minima. When the minima are equally deep, the observable effect vanishes. Otherwise the brightness is greatest in the neighborhood of the shallower minima. There is also theoretically a term of half the period, for which the contributions of the components are additive. This diminishes the apparent photometric ellipticity, and must be allowed for.

Though each component reflects all the *energy* incident upon it from the other, it may re-emit more or less *light* according to its luminous efficiency. A companion of Class G exposed to the radiation from a primary of Class B will receive a great amount of short-wave radiation, and transmute it into longer waves, so that the *visible* radiation effect may be much increased. Evidence of the occurrence of this has recently been presented by Señora Pishmish-Recillas. It may not be too bold to hope that, when a number of favorable eclipsing systems have been precisely observed in several wave lengths, we may be in a position to determine, with fair precision, how great an amount of short-wave energy has thus been transformed, and so to find something about the amount of the radiation of hot stars in this otherwise inaccessible region.

The reflection effect is a maximum for the large faint components of "typical Algol" systems. Some cases are known in which the illuminated side of the companion is more than twice as bright as the other. The spectrum of this side can be observed only when blended with the much stronger one of the brighter star. Even if it could be obtained free from blending, it should be decidedly peculiar, for the ionization in the atmosphere of this side of the star may be influenced more by the diluted high-temperature radiation of the companion than by the effects of the local temperature.[21] To the extent that such a spectrum could be attributed to a type of the normal sequence, it is likely to be considerably "earlier" than that corresponding to the surface brightness or color index.

The exact calculation of the theoretical light curve of an eclipsing system must take into account not only the elementary principles so far mentioned, but a host of refinements—the difference in shape of the components, their departures from the ellipsoidal figure, the uneven distribution of brightness over their surfaces arising from the gravity and reflection effects, and the changes in these last phenomena with the wave length.

This prodigiously complex problem has been solved, in principle, by Dr. Kopal, with the aid of the tabulation of a large number of new quantities—the associated alpha-functions—and he is now at work upon their practical applications.

The inverse problem, of finding the elements from the observations, taking all these refinements into account, is far too complex to be solved directly; but it yields to perturbational methods remarkably similar to those which are used in the calculation of the definitive orbit of a comet or an asteroid.

The first orbit of a comet is calculated by methods which take account of the sun's attraction alone, and ignore those of the planets. As more observations are available, the original orbit can be improved, until it represents the observed motion over a long arc; but it will not do so exactly, because of the neglect of planetary attraction. The computer then starts with the best elements he has, and calculates the comet's positions (1) by methods ignoring the planetary attractions, and (2) by methods taking them fully into account. Subtracting the first from the second, he obtains the perturbations due to planetary influence. He then subtracts these perturbations from the observed normal right ascensions and declinations of the comet, and obtains positions "cleared of perturbations" which can be worked up as if the planets' attraction did not exist, giving the definitive orbit.

In just the same way, the computer may substitute for an actual eclipsing system a simplified intermediary model, and derive elements which would be correct if the system were built on this model. He can then calculate (1) an intermediary light curve derived from the model by the strict application of the assumptions on which it is based, and (2) a physical light curve for a system having the same fundamental physical properties as the model, but in which full account is taken of all the previously neglected refinements. The difference of these two curves represents the photometric perturbations, which are to be applied, with reversed sign, to the observed normals. The resulting values, discussed by the intermediary methods, give the refined elements. These will be definitive, if the photometric perturbations are not too large; otherwise the whole laborious process may have to be repeated.

The heart-breaking situation may be avoided by computing the physical light curve from elements which are not identical with those of the intermediary model, but derived from them by definite rules, so designed that the principal characteristics of the eclipses shall be the same for both (for example, the width at half-depth and the duration of totality, or the geometrical amount by which a partial eclipse fails of being total). The intermediary elements resulting from the discussion of the observations "cleared of perturbations" may then be transformed into physical elements by the application of the rules just mentioned. A careful study of the best methods for doing this would be highly repaying.

The final step—a least-squares solution for corrections to all the independent fundamental constants of the system—may be made from the observations cleared of perturbations. For a comet, which will not return for centuries, the computer must find how much—or how little—may be determined from the available data. But an eclipsing variable can always be reobserved, so that there is no excuse for wasting time on meticulous solutions from scanty or inaccurate data.

In some cases, solutions in which the elements vary with one degree of freedom, over a considerable range of correlated values, give almost identical light curves, and it is not possible to obtain a unique solution from photometric observations alone. This situation would be revealed by the least-squares solution, but a computer skilled in the art should always be able to recognize it in advance, and save spending his time on an indeterminate case. Merrill's nomograms will make the recognition of such cases a very simple matter.

To make a least-squares solution appear definite by arbitrarily nailing down the value of an uncertain quantity, and proceeding to calculate the rest, is not fair to the method. By this I do not mean to assert that there may not be systems for which it is legitimate to attempt a refined solution in which the value of one constant is estimated from general considerations. But to calculate and publish probable errors for the elements in such a case is likely to mislead the reader unless the situation is very clearly stated. Spectroscopic determinations of the relative brightness of the components, such as Dr. Petrie has described today, will almost always make the solution definite.

For the general run of eclipsing variables, the intermediary methods—especially the very rapid nomographic solutions—are sufficient unless the observations are of the new high standard of precision. It is to be hoped that preference will be given in such observations to systems in which results of astrophysical interest can be secured.

There are still more things which can best be studied in eclipsing systems. *Rotation* is revealed in many stars by a distinctive type of broadening of the spectral lines, and the component of the equatorial velocity in the line of sight can be estimated. During the eclipse of a rotating star the radial velocity of the obscured area differs from the mean for the disk.

This amounts to an eclipse of part of the profile of the rotationally widened spectral line—not directly observable because of blending with line-broadening due to other causes, but affecting the mean velocity. The size of this rotation effect is influenced by the distribution of brightness on the apparent disk, but may be calculated with all needful accuracy.

In most cases the observed effect is consistent with equality of the periods of rotation and orbital revolution, but not always. Double bright lines, such as would arise from a ring of luminous hydrogen revolving about the equator of the principal component, have been observed in or near the total eclipse for several stars, notably RW Tauri [22] in which the ring has an orbital velocity of 350 km/sec, and is completely hidden by the larger star at mid-totality.

This is a relatively simple case.

In RZ Scuti, for example, the absorption lines of *He* I and *Mg* II show a rotation effect with amplitude five times as great as that arising from the orbital motion. The hydrogen lines have practically the same orbital displacement, and a rotation effect similar in character, but of less than half the amplitude. Neubauer and Struve [23] conclude that "hydrogen apparently forms a slowly rotating shell around the rapidly rotating layer which gives rise to the lines of *He* I and *Mg* II." These effects appear to be strictly confined to the partial phases of eclipse, and present a puzzling kinematic and dynamical problem. Precise photometric measures of the system are fortunately under way.

Equally remarkable phenomena have been observed in U Cephei [24] and SX Cassiopeiae [25] — a very large rotation effect in the first, and abnormal displacements of the absorption lines in both. The classic example of this is, however, β Lyrae [26] in whose spectrum supernumerary lines, bright and dark, appearing regularly at certain phases, have been known for many years. Kuiper [27] has given a highly interesting explanation of these very complex phenomena, based on dynamical arguments which show that, in a "contact binary" with the stars very close together, streams of gas may escape from the component of lower density, and pursue curved paths under the attraction of the two. The interposition of these streams gives rise to the fugitive displaced lines seen only over narrow intervals of phase. A full discussion of this matter would afford ample material for a whole lecture.

The evidence is strong that some eclipsing systems may be at or beyond the limit of dynamical stability of at least one of the components. In this connection it is interesting to recall a study by P. H. Taylor [28] who found that, in the numerous eclipsing binaries which have asymmetric light curves, the radius of the larger component exceeds a certain limit (depending on the mass ratio) and suggested that the relatively low density of these bodies permitted effects of resonance or some other unusual disturbance. In many systems, an asymmetrical light curve repeats itself with great regularity. Almost nothing has been done toward theoretical interpretation of this.

A relatively simple and intelligible system is found in Zeta Aurigae, with a B-star revolving about a supergiant of Class K4, of about 70 times greater diameter. For some days before eclipse narrow sharp lines appear, caused by the absorption of the light of the small star in an outer envelope of the big one.

It remains for future study to determine (if possible) whether the former finally disappears behind a sharp edge of the latter, or gradually fades out of sight as its light traverses an ever greater thickness of atmosphere, as the setting sun's does on a hazy day.

The most perplexing photometric observations of all remain to be mentioned: RX Cassiopeiae, in which an obviously eclipsing variation of period 35 days is superposed upon an apparently intrinsic variation of period 518 days,[29] and SX Cassiopeiae, in which Dugan's visual observations and the Harvard photographs give light curves which disagree in everything except the period. Such a discordance between the results of experienced observers, covering the same interval of time, is fortunately unprecedented. Both stars show marked spectral peculiarities. They deserve intensive and prolonged observation by precise methods, and in different wave lengths.

In closing let me speak briefly of some of the things that seem most worth while doing in the immediate future.

Methods for *calculating* the elements of eclipsing systems are now well advanced. The approaching publication of Merrill's tables and nomograms will take care of the elementary and intermediary solutions, and that of Kopal's tables will do the same for advanced and refined discussions. It is still worth while to examine whether methods may not be devised for approximate calculation or tabulation so that the principal parts of the refined corrections may be made quickly.

The field of *observation* is opened wide by the new photoelectric methods, and the only question is where to begin. May I make a plea for intensive observation, preferably in several wave lengths, of a reasonable number of carefully selected systems, in which the conditions are favorable for direct determination of the figure, limb-darkening and gravity effect? Reliable knowledge of the relation between the last two coefficients and the spectral type may thus be obtained—providing a series of important tests for *astrophysical theory*, and opening the way for reliable calculations of the elements of a great number of eclipsing systems for which these quantities cannot be directly determined from the observations.

This talk has been no complete guidebook to the royal road. At best, it may have given a glimpse of some of the profitable realms to which it leads, and some good reasons for an increase of traffic on it.

In conclusion, it is a pleasure indeed to express my thanks to my friends and colleagues—Dr. Irwin, Dr. Kopal, Dr. Sitterly and Dr. Wood—who have generously permitted me to speak of their unpublished investigations; and last, but not least, to thank the American Astronomical Society, its officers, and the many friends who have contributed to the fund, for the distinguished honor of the endowment of a lectureship bearing my name, and the additional honor of inviting me to be the first lecturer.

REFERENCES

1. J. Goodricke, *Phil. Trans.*, **73,** 474, 1783; quoted by Kopal, "*Eclipsing Variables*," p. 4, *Harv. Mon.* 6, 1946.
2. E. C. Pickering, *Pr. Amer. Ac. Arts & Sc.*, **16,** 1, 1880.
3. G. W. Myers, *Dissert., Munich*, 1896; *Ap. J.*, **7,** 1, 1898.
4. E. C. Pickering, *Pr. Amer. Ac. Arts & Sc.*, **16,** 370, 1881.
5. G. W. Myers, *Ap. J.*, **8,** 163, 1898.
6. A. W. Roberts, *M. N.*, **66,** 123, 1906.
7. H. Shapley, *Ap. J.*, **36,** 269, 1912, using Wendell's later and more numerous observations.
8. R. S. Dugan, *Princ. Cont.* **5,** 1920.
9. R. S. Dugan, *Pub. A. A. S.*, **1,** 311, 1908; *Princ. Cont.* **1,** 1911.
10. J. Stebbins, *Ap. J.*, **32,** 185, 1910.
11. J. Stebbins, *Ap. J.*, **34,** 113, 1911.
12. H. Shapley, *Princ. Cont.* **3,** 1915.
13. A. W. Roberts, *Ap. J.*, **10,** 308, 1899; H. N. Russell, *Ap. J.*, **10,** 315, 1899.
14. H. N. Russell and C. E. Moore, "*The Masses of the Stars*," p. 106, *Univ. of Chicago Press*, 1940.
15. The limits imposed by stability in this case were pointed out by K. Walter in 1940 (*Zs. f. Ap.*, **19,** 157) in a paper just received in this country.
16. Unpublished, communicated by courtesy of Dr. Irwin.
17. S. Chandrasekhar, *Ap. J.*, **103,** 351, 1946.
18. E. Janssen, *Ap. J.*, **103,** 380, 1946.
19. Cf. Z. Kopal, "*Eclipsing Variables*," p. 125, *Harv. Mon.* 6, 1946.

20. B. W. Sitterly, *P. A. A. S.*, **10,** 67, 1940; **10,** 235, 1941.
21. A. S. Eddington, *M. N.*, **86,** 320, 1926.
22. A. H. Joy, *P. A. S. P.*, **54,** 35, 1942.
23. F. J. Neubauer and O. Struve, *Ap. J.*, **101,** 240, 1945.
24. O. Struve, *Ap. J.*, **99,** 222, 1944.
25. O. Struve, *Ap. J.*, **99,** 89, 1944.
26. O. Struve, *Ap. J.*, **93,** 104, 1941, and others.
27. G. P. Kuiper, *Ap. J.*, **93,** 133, 1941.
28. P. H. Taylor, *Ap. J.*, **94,** 46, 1941.
29. C. P. Gaposchkin, *Ap. J.*, **103,** 299, 1946.