(1) Waldmeier, *Astr. Mitt. Zurich*, 133 (1935); Gleissberg, *The Observatory*, **63**, 215 (1940); *Ap. J.*, **96**, 234 (1942).
(2) Ludendorff, *Zs. f. Ap.*, **2**, 370 (1931).
(3) See article on Solar Physics by Abetti, *Handbuch der Astrophysik*, **4**, 100 (1929).
(4) Ball, 'The Story of the Sun', p. 152 (1893).
(5) Wolf, *Memoirs R.A.S.*, **43**, 199 (1877).
(6) Hale, Ellerman, Nicholson, and Joy, *Ap. J.*, **49**, 153 (1919).

On the Mean Solar to Mean Sidereal Gear Ratio.

Gentlemen,—

The problem of devising a gear-train that will enable a clock keeping mean solar time to drive a dial showing sidereal time is by no means a new one. It attracted the attention of horologists and mathematicians in the early days of clock-making, but the current use of phonic motors controlled by quartz crystal oscillators has given it an added interest.

The Time Department of the Royal Observatory, Greenwich, has hitherto maintained separate pendulum clocks to provide sidereal time for astronomical purposes. Pendulums have now been gradually superseded by quartz clocks, the crystals of which are cut to provide a nominal frequency of 100 kilocycles per mean solar second. In order that sidereal time shall be available, several of the phonic motors employed have been fitted with the two-meshing gear system proposed by Esclangon*, and are thus able to provide two sets of pulses at intervals of (*a*) mean solar seconds and (*b*) mean sidereal seconds.

For the purpose of inter-comparison of the clocks it has been found most convenient to consider the numerical difference between the error of the mean solar wheel, expressed in solar seconds, and the error of the sidereal wheel, expressed in sidereal seconds. This quantity, in the sense sidereal wheel more slow than solar wheel, is denoted by k. If the solar wheel had zero rate, k would undergo a daily drift of about one-third of a millisecond owing to the small error in the gear-ratio. In practice, the driving clock will not have zero rate, and it is of interest to consider the effect that rate has on the value of k. The daily change in k can be written $\rho N - \rho'(N - r) - r$, where ρ is the number of mean sidereal seconds in one mean solar second, N is the number of seconds in a day (= 86,400), ρ' is the gear-ratio, and r is the losing rate of the driving clock in mean solar seconds per day. If r has a certain value $r' = N(\rho' - \rho)/(\rho' - 1)$ the expression for the daily change in k vanishes, and k becomes constant. For any value of r, the daily change in k may be written $(\rho' - 1)(r - r')$. In the case of the Esclangon gearing in use at Greenwich, r' is $-$ 123·8 millisec./day, which means that if a clock with this rate drives the gearing, the mean solar wheel gains 123·8 solar milliseconds per day, while the sidereal wheel gains 123·8 sidereal milliseconds per day, and k, the difference between the errors of the wheels, remains constant.

It so happens that the first clock at Greenwich to drive one of these phonic motors has a rate very near to this value, and the resulting steadiness in k simplifies the comparison of this clock with others. This would

* *Comptes Rendus*, **169**, 231, 1919.

indicate that the search for the perfect sidereal gearing is mainly one of academic interest, since a gearing with an error that compensates exactly for the rate of the driving clock is preferable in practice. It may be well to mention that since the value of ρ is increasing by 6 parts in 10^{13} each year, a "perfect" gearing will attain perfection at one particular epoch only, and will then produce the desired result only when driven by a perfect clock running with zero rate!

I am, Gentlemen,
Yours faithfully,
R. H. Tucker.

Royal Observatory,
Greenwich.
1947 July 30.